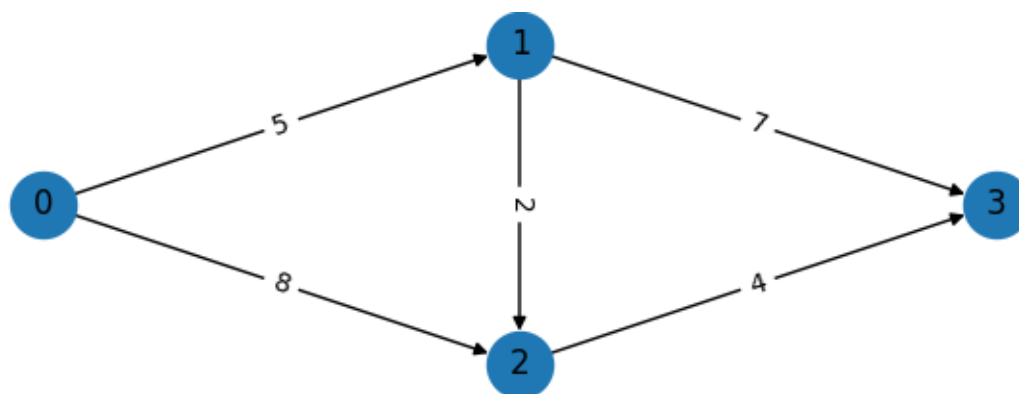


## Fórmulas



### 1 Formulación del problema

- **Objetivo:**

$$\min(5X_{01} + 8X_{02} + 2X_{12} + 7X_{13} + 4X_{23})$$

- **Restricciones:**

$$X_{01} + X_{02} = 1 \quad (1)$$

$$X_{01} = X_{12} + X_{13} \quad (2)$$

$$X_{02} + X_{12} = X_{23} \quad (3)$$

$$X_{13} + X_{23} = 1$$

### 2 Función de coste (QUBO)

$$\begin{aligned}
 C(X) = & 5X_{01} + 8X_{02} + 2X_{12} + 7X_{13} + 4X_{23} + \\
 & P(X_{01} + X_{02} - 1)^2 + \\
 & P(X_{01} - X_{12} - X_{13})^2 + \\
 & P(X_{02} + X_{12} - X_{23})^2
 \end{aligned}$$

$$P = 1 + \sum_{(i,j) \in E} w_{ij} = 27$$

### 3 Correspondencia

$$X_{01} \rightarrow z_o$$

$$X_{02} \rightarrow z_1$$

$$X_{12} \rightarrow z_2$$

$$X_{13} \rightarrow z_3$$

$$X_{23} \rightarrow z_4$$

$$z_a = \begin{cases} 1 & \text{si } X_{ij} = 0 \\ -1 & \text{si } X_{ij} = 1 \end{cases}$$

$$X_{ij} \rightarrow \frac{1 - z_a}{2}$$

### 4 Función de coste (Ising)

$$C(X) \rightarrow g(z) = 5\frac{1-z_0}{2} + 8\frac{1-z_1}{2} + 2\frac{1-z_2}{2} + 7\frac{1-z_3}{2} + 4\frac{1-z_4}{2} +$$

$$P\left(\frac{1-z_0}{2} + \frac{1-z_1}{2} - 1\right)^2$$

$$P\left(\frac{1-z_0}{2} - \frac{1-z_2}{2} - \frac{1-z_3}{2}\right)^2$$

$$P\left(\frac{1-z_1}{2} + \frac{1-z_2}{2} - \frac{1-z_4}{2}\right)^2$$

$$g(z) = 11z_0 - 17.5z_1 - 28z_2 - 17z_3 + 11.5z_4 +$$

$$13.5(-z_0z_2 + z_1z_2 + z_2z_3 - z_2z_4 + z_0z_1 - z_0z_3 - z_1z_4) +$$

$$(13.5z_0^2 + 13.5z_1^2 + 13.5z_2^2 + 6.75z_3^2 + 6.75z_4^2) +$$

$$26.5 =$$

$$11z_0 - 17.5z_1 - 28z_2 - 17z_3 + 11.5z_4 +$$

$$13.5(-z_0z_2 + z_1z_2 + z_2z_3 - z_2z_4 + z_0z_1 - z_0z_3 - z_1z_4) +$$

$$54 + 26.5$$

$$(z_i)^2 = 1, \forall i \in \{-1, 1\}$$

## 5 Hamiltonian problem y mixer

Donde  $Z$  es una puerta Pauli-Z y  $X$  una puerta Pauli-X

$$\begin{aligned} H_p = & 11Z_0 - 17.5Z_1 - 28Z_2 - 17Z_3 + 11.5Z_4 + \\ & 13.5(+Z_0 \otimes Z_1 - Z_0 \otimes Z_2 - Z_0 \otimes Z_3 + Z_1 \otimes Z_2 \\ & - Z_1 \otimes Z_4 + Z_2 \otimes Z_3 - Z_2 \otimes Z_4) + \\ & 80.5 \end{aligned}$$

Debido al postulado de medición en mecánica cuántica la fase global es despreciable, por lo que  $e^{i\gamma_i 80.5} \cdot e^{i\gamma_i H_p} = e^{i\gamma_i H_p}$ .

$$\begin{aligned} U(H_p, \gamma_i) = e^{-i\gamma_i H_p} = & Rz_0(11 * 2\gamma_i) \cdot Rz_1(-17.5 * 2\gamma_i) \cdot Rz_2(-28 * 2\gamma_i) \cdot Rz_3(-17 * 2\gamma_i) \cdot Rz_4(11.5 * 2\gamma_i) \cdot \\ & Rz_0z_1(+13.5 * 2\gamma_i) \cdot Rz_0z_2(-13.5 * 2\gamma_i) \cdot Rz_0z_3(-13.5 * 2\gamma_i) \cdot Rz_1z_2(+13.5 * 2\gamma_i) \cdot \\ & Rz_1z_4(-13.5 * 2\gamma_i) \cdot Rz_2z_3(+13.5 * 2\gamma_i) \cdot Rz_2z_4(-13.5 * 2\gamma_i) \end{aligned}$$

Pero en el paper:

$$\begin{aligned} U(H_p, \gamma_i) = e^{-i\gamma_i H_p} = & Rz_0(11) \cdot Rz_1(-17.5) \cdot Rz_2(-28) \cdot Rz_3(-17) \cdot Rz_4(11.5) \cdot \\ & Rz_0z_2(-13.5\gamma_i) \cdot Rz_1z_2(+13.5\gamma_i) \cdot Rz_2z_3(+13.5\gamma_i) \cdot Rz_2z_4(-13.5\gamma_i) \cdot \\ & Rz_0z_1(+13.5\gamma_i) \cdot Rz_0z_3(-13.5\gamma_i) \cdot Rz_1z_4(-13.5\gamma_i) \end{aligned}$$

$$H_m = \sum_{i=0}^{n-1} X_i$$

$$U(H_m, \beta_i) = e^{-i\beta_i H_m} = \prod_{i=0}^{n-1} Rx_i(2\beta_i)$$

### 5.1 Definiciones

$$Rx_i(\lambda) = \exp(-i\frac{\lambda}{2}X_i)$$

$$Rz_i(\lambda) = \exp(-i\frac{\lambda}{2}Z_i)$$

$$Rz_iz_j(\lambda) = \exp(-i\frac{\lambda}{2}Z_i \otimes Z_j)$$

## 6 Hamiltonian

- **Parámetros:**

$$\vec{\beta} = [\beta_0, \dots, \beta_{p-1}]$$

$$\vec{\gamma} = [\gamma_0, \dots, \gamma_{p-1}]$$

- **Hamiltonian**

$$H(\vec{\beta}, \vec{\gamma}) = U(H_m, \gamma_{p-1})U(H_p, \beta_{p-1}) \dots U(H_m, \gamma_0)U(H_p, \beta_0)$$