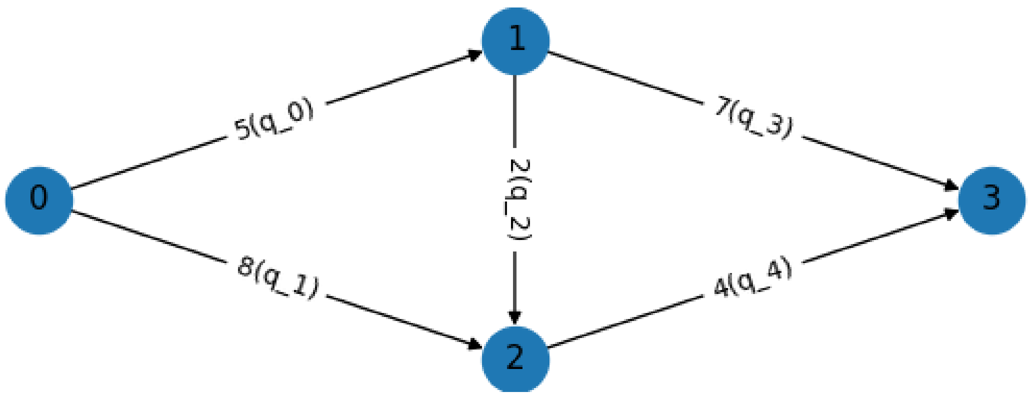
Fórmulas



1 Formulación del problema

- **Objetivo:**

$$min(5X_{01} + 8X_{02} + 2X_{12} + 7X_{13} + 4X_{23})$$

- **Restricciones:**

$$X_{01} + X_{02} = 1 \quad (1)$$
$$X_{01} = X_{12} + X_{13} \quad (2)$$
$$X_{02} + X_{12} = X_{23} \quad (3)$$
$$X_{13} + X_{23} = 1$$

2 Función de coste (QUBO)

$$\begin{aligned} C(X) = &5X_{01} + 8X_{02} + 2X_{12} + 7X_{13} + 4X_{23} + \\ &P(X_{01} + X_{02} - 1)^2 \\ &P(X_{01} - X_{12} - X_{13})^2 \\ &P(X_{02} + X_{12} - X_{23})^2 \end{aligned}$$

$$P = 1 + \sum_{(i,j)\in E} w_{ij} = 27$$

3 Correspondencia

$$X_{01} \to z_o$$
$$X_{02} \to z_1$$
$$X_{12} \to z_2$$
$$X_{13} \to z_3$$
$$X_{23} \to z_4$$

$$z_a = \begin{cases} 1 \text{ si } X_{ij} = 0 \\ -1 \text{ si } X_{ij} = 1 \end{cases}$$

$$X_{ij} \to \frac{1 - z_a}{2}$$

4 Función de coste (Ising)

$$\begin{aligned} C(X) \to g(z) =& 5\frac{1-z_0}{2} + 8\frac{1-z_1}{2} + 2\frac{1-z_2}{2} + 7\frac{1-z_3}{2} + 4\frac{1-z_4}{2} + \\ & P(\frac{1-z_0}{2} + \frac{1-z_1}{2} - 1)^2 \\ & P(\frac{1-z_0}{2} - \frac{1-z_2}{2} - \frac{1-z_3}{2})^2 \\ & P(\frac{1-z_1}{2} + \frac{1-z_2}{2} - \frac{1-z_4}{2})^2 \end{aligned}$$

$$\begin{aligned} g(z) =& 11z_0 - 17.5z_1 - 28z_2 - 17z_3 + 11.5z_4 + \\ & 13.5(-z_0z_2 + z_1z_2 + z_2z_3 - z_2z_4 + z_0z_1 - z_0z_3 - z_1z_4) + \\ & (13.5z_0^2 + 13.5z_1^2 + 13.5z_2^2 + 6.75z_3^2 + 6.75z_4^2) + \\ & 26.5 = \\ & 11z_0 - 17.5z_1 - 28z_2 - 17z_3 + 11.5z_4 + \\ & 13.5(-z_0z_2 + z_1z_2 + z_2z_3 - z_2z_4 + z_0z_1 - z_0z_3 - z_1z_4) + \\ & 54 + 26.5 \end{aligned}$$

$(z_i)^2 = 1, \forall i \in \{-1, 1\}$

5 Hamiltonian problem y mixer

Donde Z es una puerta Pauli-Z y X una puerta Pauli-X

$$
\begin{aligned}
H_p = & 11Z_0 - 17.5Z_1 - 28Z_2 - 17Z_3 + 11.5Z_4 + \\
& 13.5(-Z_0 \otimes Z_2 + Z_1 \otimes Z_2 + Z_2 \otimes Z_3 - Z_2 \otimes Z_4 + Z_0 \otimes Z_1 - Z_0 \otimes Z_3 - Z_1 \otimes Z_4) + \\
& 80.5
\end{aligned}
$$

Debido al postulado de medición en mecánica cuántica la fase global es despreciable, por lo que $e^{i\gamma_i 80.5} \cdot e^{i\gamma_i H_p} = e^{i\gamma_i H_p}$.

$$
\begin{aligned}
U(H_p, \gamma_i) = e^{-i\gamma_i H_p} = & Rz_0(11 * 2\gamma_i) \cdot Rz_1(-17.5 * 2\gamma_i) \cdot Rz_2(-28 * 2\gamma_i) \cdot Rz_3(-17 * 2\gamma_i) \cdot Rz_4(11.5 * 2\gamma_i) \cdot \\
& Rz_0z_2(-13.5 * 2\gamma_i) \cdot Rz_1z_2(+13.5 * 2\gamma_i) \cdot Rz_2z_3(+13.5 * 2\gamma_i) \cdot Rz_2z_4(-13.5 * 2\gamma_i) \cdot \\
& Rz_0z_1(+13.5 * 2\gamma_i) \cdot Rz_0z_3(-13.5 * 2\gamma_i) \cdot Rz_1z_4(-13.5 * 2\gamma_i)
\end{aligned}
$$

Pero en el paper:

$$
\begin{aligned}
U(H_p, \gamma_i) = e^{-i\gamma_i H_p} = & Rz_0(11) \cdot Rz_1(-17.5) \cdot Rz_2(-28) \cdot Rz_3(-17) \cdot Rz_4(11.5) \cdot \\
& Rz_0z_2(-13.5\gamma_i) \cdot Rz_1z_2(+13.5\gamma_i) \cdot Rz_2z_3(+13.5\gamma_i) \cdot Rz_2z_4(-13.5\gamma_i) \cdot \\
& Rz_0z_1(+13.5\gamma_i) \cdot Rz_0z_3(-13.5\gamma_i) \cdot Rz_1z_4(-13.5\gamma_i)
\end{aligned}
$$

$$H_m = \sum_{i=0}^{n-1} X_i$$

$$U(H_m, \beta_i) = e^{i\beta_i H_m} = \prod_{i=0}^{n-1} Rx_i(2\beta_i)$$

5.1 Definiciones

$$Rx_i(\lambda) = exp(-i\frac{\lambda}{2}X_i)$$

$$Rz_i(\lambda) = exp(-i\frac{\lambda}{2}Z_i)$$

$$Rz_iz_j(\lambda) = exp(-i\frac{\lambda}{2}Z_i \otimes Z_j)$$

6 Hamiltonian

- **Parámetros:**

$$\vec{\beta} = [\beta_0, ..., \beta_{p-1}]$$
$$\vec{\gamma} = [\gamma_0, ..., \gamma_{p-1}]$$

- **Hamiltonian**

$$H(\vec{\beta}, \vec{\gamma}) = U(H_p, \gamma_0)U(H_m, \beta_0)...U(H_p, \gamma_{p-1})U(H_m, \beta_{p-1})$$