Problema 3.

$q_L = 1 - q_C$

1.-

	C	L
C	5,5	0,3
L	3,0	3,3

$E[u_j(C)] = 5q_C + 0(1-q_C)$

$E[u_j(L)] = 3 \cdot q_C + 3(1-q_C) = 3$

$$E[u_j(C)] = E[u_j(L)] \iff 5q_C = 3 \iff \left[q_C = \frac{3}{5}\right] \Rightarrow \left[q_L = \frac{2}{5}\right]$$

$E[u_c(C)] = 5 \cdot p_C + 0 \cdot p_L = 5 \cdot p_C$

$E[u_c(C)] = 3 \cdot p_C + 3 \cdot p_L = 3p_C + 3(1-p_C) = 3$

$\Rightarrow E[u_c(C)] = E[u_c(L)] \iff 5p_C = 3 \iff \left[p_C = \frac{3}{5}\right] \Rightarrow \left[p_L = \frac{2}{5}\right]$

Equilibrios de Nash para estrategias mixtas:

- $(p_C, p_L) = \left(\frac{3}{5}, \frac{2}{5}\right) \longrightarrow$ Agente fila
- $(q_C, q_L) = \left(\frac{3}{5}, \frac{2}{5}\right) \longrightarrow$ Agente columna

2.-

	F	D
F	-1,-1	2,1
D	1,2	0,0

$E[u_j(F)] = -1 \cdot q_F + 2 \cdot q_D = -1 \cdot q_F + 2(1-q_F) = 2 - 3q_F$

$E[u_j(D)] = 1 \cdot q_F + 0 \cdot q_D = 1 \cdot q_F$

$$E[u_j(F)] = E[u_j(D)] \iff 2 - 3q_F = q_F \iff \left[q_F = \frac{1}{2}\right] \Rightarrow \left[q_D = \frac{1}{2}\right]$$

$E[u_c(F)] = -1 \cdot p_F + 2 \cdot p_D = -1p_F + 2 \cdot (1-p_F) = 2 - 3p_F$

$E[u_c(D)] = 1 \cdot p_F + 0 \cdot p_D = 1 \cdot p_F$

$$E[u_c(F)] = E[u_c(D)] \iff 2 - 3p_F = p_F \iff \left[p_F = \frac{1}{2}\right] \Rightarrow \left[p_D = \frac{1}{2}\right]$$

Equilibrios:

- $(p_F, p_D) = \left(\frac{1}{2}, \frac{1}{2}\right) \rightarrow A_F$
- $(q_F, q_D) = \left(\frac{1}{2}, \frac{1}{2}\right) \rightarrow A_C$

3.-

	A	B
A	10, 10	0, 0
B	0, 0	5, 5

$E[u_J(A)] = 10 \cdot q_A + 0 \cdot q_B = 10 q_A$

$E[u_J(B)] = 0 \cdot q_A + 5 q_B = 5(1-q_A) = 5 - 5q_A$

$E[u_J(A)] = E[u_J(B)] \iff 10q_A = 5 - 5q_A \iff 15q_A = 5 \iff \boxed{q_A = \frac{1}{3}} \Rightarrow \boxed{q_B = \frac{2}{3}}$

$E[u_C(A)] = 10 p_A + 0 \cdot p_B = 10 p_A$

$E[u_C(B)] = 0 \cdot p_A + 5 \cdot p_B = 5(1-p_A) = 5 - 5p_A$

$E[u_C(A)] = E[u_C(B)] \iff 10p_A = 5 - 5p_A \iff \boxed{p_A = \frac{1}{3}} \Rightarrow \boxed{p_B = \frac{2}{3}}$

Equilibrios:

- $(p_A, p_B) = \left(\frac{1}{3}, \frac{2}{3}\right) \rightarrow A_F$; $\cdot (q_A, q_B) = \left(\frac{1}{3}, \frac{2}{3}\right) \longrightarrow A_C$

4.-

	O	N
O	0, 0	1, -2
N	-2, 1	2, 2

$E[u_J(O)] = 0 \cdot q_O + 1 \cdot q_N = q_N$

$E[u_J(N)] = -2 \cdot q_O + 2 \cdot q_N = -2(1-q_N) + 2q_N = 4q_N - 2$

$E[u_J(O)] = E[u_J(N)] \iff q_N = 4q_N - 2 \iff \boxed{q_N = \frac{2}{3}} \Rightarrow \boxed{q_O = \frac{1}{3}}$

$E[u_C(O)] = 0 \cdot p_O + 1 p_N = p_N$

$E[u_C(N)] = -2p_O + 2p_N = -2(1-p_N) + 2p_N = 4p_N - 2$

$E[u_C(O)] = E[u_C(N)] \iff p_N = 4p_N - 2 \iff \boxed{p_N = \frac{2}{3}} \Rightarrow \boxed{p_O = \frac{1}{3}}$

Equilibrios:

- $(p_O, p_N) = \left(\frac{1}{3}, \frac{2}{3}\right) \rightarrow$ Ajuste fijo
- $(q_O, q_N) = \left(\frac{1}{3}, \frac{2}{3}\right) \rightarrow$ Ajuste columna

5.-

	L	R
U	1,-1	3,0
D	4,2	0,-1

$E[u_f(U)] = q_L \cdot 1 + q_R \cdot 3 = q_L + 3(1-q_L) = 3 - 2q_L$

$E[u_f(D)] = 4q_L + 0 \cdot q_R = 4q_L$

$E[u_f(U)] = E[u_f(D)] \iff 3 - 2q_L = 4q_L \iff \left[q_L = \frac{1}{2}\right] \Rightarrow \left[q_R = \frac{1}{2}\right]$

$E[u_c(L)] = p_U \cdot (-1) + 2 \cdot p_D = 2p_D - 1(1-p_D) = 3p_D - 1$

$E[u_c(R)] = p_U \cdot 0 - p_D = -p_D$

$E[u_f(U)] = E[u_c(R)] \iff 3p_D - 1 = -p_D \iff \left[p_D = \frac{1}{4}\right] \Rightarrow \left[p_U = \frac{3}{4}\right]$

Equilibrios:

- $(p_U, p_D) = \left(\frac{3}{4}, \frac{1}{4}\right) \longrightarrow$ Agente fila
- $(q_L, q_R) = \left(\frac{1}{2}, \frac{1}{2}\right) \longrightarrow$ Agente columna

6.-

	D	E	F
A	1,-2	-2,1	0,0
B	-2,1	1,-2	0,0
C	0,0	0,0	1,1

$E[u_f(A)] = 1 \cdot q_D - 2 \cdot q_E + 0 \cdot q_F$

$E[u_f(B)] = -2 \cdot q_D + 1 q_E + 0 \cdot q_F$

$E[u_f(C)] = 1 \cdot q_F = (1 - q_D - q_E)$

$E[u_f(A)] = E[u_f(B)] \iff 1q_D - 2q_E = -2q_D + q_E \iff 3q_D = 3q_E \iff q_D = q_E$

$E[u_f(A)] = E[u_f(C)] \iff 1q_D - 2q_E = 1 - q_D - q_E \iff -1q_E = 1 - 2q_E \iff [q_E = q_D = 1] \Rightarrow$

$\Rightarrow \underline{q_F = -1} \Rightarrow$ El método no sirve. No damos cuenta, no obstante, de que $(q_E, q_D, q_F) = (0,0,1)$ es un equilibrio de Nash, para acciones del agente columna.

Sucede lo mismo para el caso del agente fila (dada la distribución simétrica de las recompensas) y $(p_A, p_B, p_C) = (0,0,1)$ también es un equilibrio de Nash para este caso particular de estrategia mixta del agente fila.