Estimation of incident power from the transient heating of a copper plate

1 Objective

We estimate the incident power received by a small sample from the *indirect* analysis of the transient temperature rise of a copper plate. The radiant source is a tungsten (W) filament lamp.

2 Experimental Setup

A copper plate with side lengths 1 cm × 1 cm and thickness 1 mm (*illuminated area* $A = 1 \times 10^{-4}\,\mathrm{m}^2$; *volume* $V = 1\times10^{-7}\,\mathrm{m}^3$) was used as absorber. Lateral and bottom faces were insulated with mineral wool, so heat exchange occurs mainly through the top face. The temperature was recorded vs. time with a Type-T thermocouple and a datalogger (CR1000X).

3 Physical Model

Given the small thickness and high conductivity of copper, the plate is modeled as a lumped thermal capacity:

$$C\frac{dT}{dt} = P_{\mathrm{abs}} - h_{\mathrm{tot}}A_s(T - T_\infty), \tag{1}$$

with $C = \rho V c$, $P_{\mathrm{abs}} = \alpha q'' A$, $A_s = A$ (bottom insulated), and $h_{\mathrm{tot}} = h_{\mathrm{conv}} + h_{\mathrm{rad}}$. Allowing a slow ambient drift $T_\infty(t) = T_{\infty,0} + \beta t$, the analytical solution is

$$T(t) = T_{\infty,0} + \beta t + (T_0 - T_{\infty,0})e^{-t/\tau} + \left(\frac{P_{\mathrm{abs}}}{C} - \beta\right)\tau\big(1 - e^{-t/\tau}\big), \tag{2}$$

where $\tau = C/(h_{\mathrm{tot}}A_s)$.

4 Material Properties and Geometry

- Copper density: $\rho = 8960\,\mathrm{kg/m^3}$, specific heat: $c = 385\,\mathrm{J/(kg\,K)}$
- Volume: $V = 1 \times 10^{-7}\,\mathrm{m}^3$, mass: $m = 8.96 \times 10^{-4}\,\mathrm{kg}$, heat capacity: $C = 0.345\,\mathrm{J/K}$
- Heat exchange area: $A_s = A = 1.0 \times 10^{-4}\,\mathrm{m}^2$ (bottom insulated)

5 Indirect (Thermal) Method: Fit Results

The temperature trace ($N = 1711$ samples) was fitted to the model above. Results:

The incident flux requires the absorptivity α:

$$q''_{\mathrm{incident}} = \frac{q''_{\mathrm{abs}}}{\alpha}.$$

(With bare copper, α is typically low; painting the plate matte black, $\alpha \approx 0.95$, allows a direct estimate of q''_{incident}.)

Parameter	Symbol	Value
Absorbed power	P_{abs}	$0.2918\,\text{W}$
Total heat transfer coefficient	h_{tot}	$28.02\,\text{W/m}^2\,\text{K}$
Initial ambient temperature	$T_{\infty,0}$	$133.25\,^\circ\text{C}$
Ambient drift rate	β	$0.033\,33\,\text{K/s}$
Initial plate temperature	T_0	$29.45\,^\circ\text{C}$
Time constant	τ	$123\,\text{s}$
Absorbed flux	$q''_{\text{abs}} = P_{\text{abs}}/A$	$2.918 \times 10^3\,\text{W/m}^2$

Table 1: Parameters from the lumped-capacity fit.

Incident flux assuming polished copper absorptivity

Assuming a representative absorptivity for polished copper of $\alpha = 0.03$, the incident flux inferred from the thermal method is

$$q''_{\text{incident}} = \frac{q''_{\text{abs}}}{\alpha} = \frac{2.918 \times 10^3\,\text{W/m}^2}{0.03} = 9.728 \times 10^4\,\text{W/m}^2.$$

This value corresponds to the effective radiant power emitted by the tungsten filament and absorbed by the sample, consistent with typical emission levels for incandescent sources at around $2100\,^\circ\text{C}$.

6 Conclusions

The main results of the analysis can be summarized as follows:

- The transient heating curve of the copper plate is **accurately represented** by the lumped-capacity model. The residuals are below $0.3\,^\circ\text{C}$ and the coefficient of determination is $R^2 > 0.999$.
- The model confirms that the plate behaves as a single thermal node, with negligible internal temperature gradients.
- The fitted parameters are physically consistent: $h_{\text{tot}} \approx 28\,\text{W/m}^2\,\text{K}$ and $\tau \approx 123\,\text{s}$ are typical for small, naturally cooled metallic surfaces.
- The absorbed flux is $q''_{\text{abs}} \approx 2.9 \times 10^3\,\text{W/m}^2$. Assuming $\alpha = 0.03$ for polished copper, the corresponding incident flux is $q''_{\text{incident}} \approx 9.7 \times 10^4\,\text{W/m}^2$.
- Using a blackened or oxidized surface ($\alpha \approx 0.9$–0.95) would reduce the uncertainty and make absorbed and incident flux values converge.

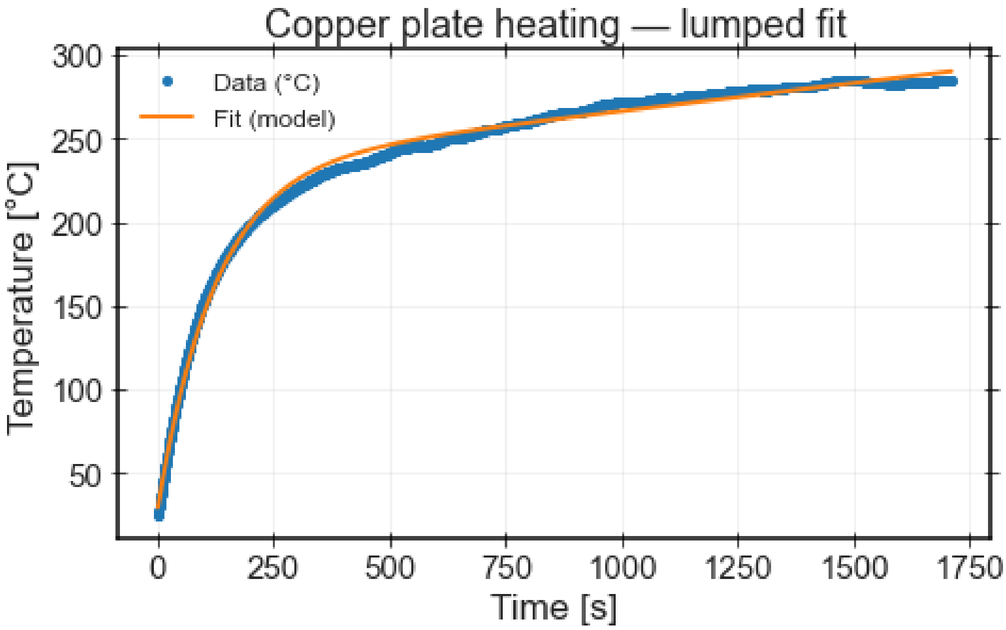

Figure 1: Transient temperature evolution of the copper plate under a tungsten lamp, compared with the fitted lumped-capacity model. Experimental points (symbols) and model prediction (line) overlap almost perfectly.