Differential Games & iLQG

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Abstract

Contents

1 Introduction

1.1 Background & Tools

1.1.1 LQR Control Problem

The Linear Quadratic Regulator problem is an optimal control problem concerning dynamic systems with linear dynamics and quadratic cost functions. The objective is to find the optimal control u that minimizes the cost J [1].

- A linear time-varying system:

$$\dot{x} = A(t)x + B(t)u,\ x(t_0) = x_0,\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m$$

- Quadratic cost function:

$$J(x,u,t) = \int_{t_0}^{t_f} (x^T(t)Qx(t) + u^T(t)Ru(t))dt + x^T(t_f)Q_f x(t_f),$$

$$Q = Q^T \geq 0,\ Q_f = Q_f^T \geq 0,\ R = R^T > 0$$

  The running cost's matrices Q and R penalize the size of the state and the control effort respectively.

We derive the necessary conditions for the minimization of J by applying Pontryagin's Maximum Principle. This yields that the optimal control is a linear state feedback law of the form:

$$u^*(t) = -R^{-1}(t)B^T(t)P(t)x^*(t)$$

where P must be a solution of the matrix differential equation (Riccati differential equation (RDE)):

$$\dot{P}(t) = -P(t)A(t) - A^T(t)P(t) - Q(t) + P(t)B(t)R^{-1}(t)B^T(t)P(t),\ P(t_f) = Q_f$$

A special case of the LQR problem described above is the infinite-horizon LQR problem. Now, we assume that A, B, Q and R are constant (transient phenomena have subsided) and as $t_f \to \infty$ the terminal cost becomes negligible, thus $Q_f = 0$. So, the dynamics of the control system become time-invariant and the cost function becomes $J(x,u,t) = \int_{t_0}^{\infty}(x^T(t)Qx(t)+u^T(t)Ru(t))dt$. The optimal solution is given by:

1. $u^*(t) = -R^{-1}B^T P x^*(t)$
2. The limit $P = \lim_{t_f \to \infty} P(t_0, t_f)$ of the solution of the RDE is a constant matrix that satisfies the Algebraic Riccati Equation (ARE):

$$PA + A^T P + Q - PBR^{-1}B^T P = 0$$

3. The optimal cost is: $J(u^*) = x_0^T P x_0$
4. The closed-loop system $\dot{x}^* = (A - BR^{-1}B^T P)x^*$ is exponentially stable, as long as the couple $(A, \sqrt{Q})$ is detectable.

The theoretical foundation of the LQR problem is necessary for the understanding of dynamic environments featuring multiple agents. In classical optimal control, our objective is to minimize a unique cost function. In contrast, in multi-player games, the optimization process is coupled. Actions of one agent influence the perfrormance and the state of the others. This transforms the Riccati equation to a system of coupled Riccati equations. Their solutions lead to a Nash equilibrium point, not a minimum. This framework is the basis of the iLQG algorithm which utilizes LQR iteratively by approximating non-linear dynamics as LQ problems.

1.1.2 Games

References

[1] Daniel Liberzon. *Calculus of Variations and Optimal Control Theory: A Concise Introduction*. Princeton University Press, Princeton, NJ, 2012.