Enhanced Resolution and Accuracy of the 2D-MUSIC Algorithm Using Virtual Antennas for DOA Estimation

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***Abstract*—To efficiently transmit desired signals to devices via beamforming, accurate estimation of the Direction of Arrival (DOA) is essential. For two-dimensional (2D) Multiple-Input Multiple-Output (MIMO) systems, a DOA estimation method based on the Multiple Signal Classification (MUSIC) algorithm using virtual antennas has been proposed. However, because the virtual antennas are positioned between the real antennas, the effective aperture cannot be extended, resulting in limited resolution. Furthermore, the conventional way of noise generation for virtual antennas often produces unstable variance, leading to larger estimation errors. This paper proposes an improved method that enhances resolution by extending virtual antenna placement beyond the real array and stabilizes the noise variance through a refined generation process. Simulation results confirm that the proposed method achieves superior resolution and lower estimation error compared to conventional techniques.**

***Index Terms*—DOA, MUSIC, 2D MIMO, virtual antenna.**

I. Introduction

In recent years, a wide variety of devices, including IoT devices, have become increasingly widespread in society. Consequently, in wireless communication environments, situations where multiple signals are simultaneously received have become more prominent [1]. In such environments, the use of beamforming technology with two-dimensional (2D) Multiple-Input Multiple-Output (MIMO) systems is crucial for base stations to efficiently transmit desired signals to each device [2]. Implementing this technology requires highly accurate Direction of Arrival (DOA) estimation of each device's signal in order to form transmit beams properly [3].

Over the years, various DOA estimation methods have been proposed, including the classical Capon method [4] and high-resolution techniques such as the Multiple Signal Classification (MUSIC) algorithm [5] and the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm [6]. In this paper, we focus on the MUSIC algorithm, which is widely recognized as a highly accurate DOA estimation method based on the eigenvalues and eigenvectors of the received signal correlation matrix.

In two-dimensional MIMO systems, a separation-based approach has been proposed, in which the one-dimensional MUSIC algorithm is independently applied to the elevation and azimuth domains, thereby extending it to two dimensions for DOA estimation [7]. Furthermore, a modified method has been proposed that enables DOA estimation even when the conditions for applying the MUSIC algorithm are not satisfied, that is, when there are more incoming signals than the number of antenna elements arranged in the rows or columns of a planar array, by virtually adding antennas [8]. However, in this approach, since the virtual antennas are placed between the existing elements, the effective aperture of the array is not expanded, and thus the resolution is not improved. Moreover, because these virtual antennas do not physically exist, they contain no noise. As a result, the assumption of spatially uniform white noise, which underlies the MUSIC algorithm, is violated, potentially leading to performance degradation [9].

Therefore, it is necessary to add noise to the introduced virtual antennas with a variance as close as possible to that of the existing antennas. However, in conventional noise generation methods, the variance tends to become unstable, often resulting in large errors. To address this issue, we propose placing the virtual antennas outside the ends of the existing array and utilizing the noise data from antennas that are not employed in the MUSIC processing. This approach aims to enhance resolution and reduce estimation errors.

The remainder of this paper is organized as follows. Section II describes the system model and conventional methods. Section III presents the proposed method. Section IVprovides simulation results to demonstrate the effectiveness of the proposed approach. Finally, Section V concludes the paper.

II. System Model and Conventional Methods

In this section, we describe the system model of a two-dimensional MIMO system and review conventional methods, including the two-dimensional MUSIC algorithm presented in [7].

Fig. 1 shows a 2D MIMO system with a uniform antenna array with the base station (BS). The BS antenna array consists of $K \times L$ elements arranged as a uniform planar array (UPA) with element spacing d, placed on the xz-plane. For simplicity,

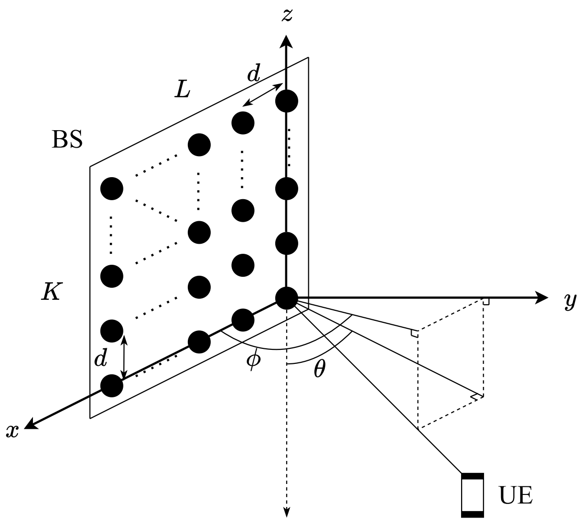

Fig. 1. 2D MIMO system moodel.

each user equipment (UE) is assumed to be equipped with a single antenna. Furthermore, the following assumptions are made in this study:
(i) Each UE is located sufficiently far from the BS so that the impinging waves can be approximated as plane waves.
(ii) The transmitted signals from different users are mutually uncorrelated.
(iii) The signals are narrowband, and the propagation delays between array elements can be represented as phase shifts.

A. DOA Estimation by the Separation Approach in 2D MIMO Systems

In this subsection, we describe the DOA estimation method for 2D MIMO systems presented in [7], which serves as the basis of this paper.

In the uplink transmission of a 2D MIMO system, the signal received at the (k, l)-th BS antenna element ($1 \leq k \leq K$, $1 \leq l \leq L$) is expressed as

$$y_{k,l} = \sum_{p=1}^{P} x_p e^{j\{(k-1)\mu_p + (l-1)\nu_p\}} + w_{k,l}, \tag{1}$$

$$\mu_p = \frac{2\pi d}{\lambda} \cos\theta_p, \nu_p = \frac{2\pi d}{\lambda} \cos\phi_p, \tag{2}$$

where x_p denotes the transmitted signal from the p-th UE, $w_{k,l}$ represents Additive White Gaussian Noise (AWGN) with zero mean and variance σ^2, λ is the wavelength, θ_p is the elevation angle, and ϕ_p is the azimuth angle. Here, the phase shifts of the incoming waves at each antenna element can be expressed in vector form in two different ways, depending on the order of the Kronecker product. These two representations are later utilized to separately estimate the elevation and azimuth angles:

$$\mathbf{a}_p = \mathbf{a}(\nu_p) \otimes \mathbf{a}(\mu_p) \in \mathbb{C}^{KL\times 1}, \tag{3}$$

$$\mathbf{a}'_p = \mathbf{a}(\mu_p) \otimes \mathbf{a}(\nu_p) \in \mathbb{C}^{LK\times 1}, \tag{4}$$

where $\otimes$ denotes the Kronecker product, and

$$\mathbf{a}(\mu_p) = [\,1, e^{j\mu_p}, \ldots, e^{j(K-1)\mu_p}\,]^T, \tag{5}$$

$$\mathbf{a}(\nu_p) = [\,1, e^{j\nu_p}, \ldots, e^{j(L-1)\nu_p}\,]^T, \tag{6}$$

are the steering vectors corresponding to the elevation and azimuth angles, respectively. From the above, the received signals at the UPA antenna elements can be expressed in vector form for each representation in (3) and (4) as

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{Z}, \tag{7}$$

$$\mathbf{Y}' = \mathbf{A}'\mathbf{X} + \mathbf{Z}', \tag{8}$$

where

$$\mathbf{A} = [\,\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_P\,] \in \mathbb{C}^{KL\times P}, \tag{9}$$

$$\mathbf{A}' = [\,\mathbf{a}'_1, \mathbf{a}'_2, \ldots, \mathbf{a}'_P\,] \in \mathbb{C}^{LK\times P}, \tag{10}$$

$$\mathbf{X} = [\,x_1, x_2, \ldots, x_P\,]^T \in \mathbb{C}^{P\times 1}, \tag{11}$$

with $\mathbf{A}$ and $\mathbf{A}'$ denoting the steering vector matrices, and $\mathbf{X}$ representing the transmitted signals from the UEs. In addition, $\mathbf{Z} \in \mathbb{C}^{KL\times 1}$ and $\mathbf{Z}' \in \mathbb{C}^{LK\times 1}$ denote the AWGN vectors corresponding to each antenna element. From (3), (4), (9), and (10), the matrices $\mathbf{A}$ and $\mathbf{A}'$ can be decomposed as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_\mu \\ \mathbf{A}_{\mu,\nu} \end{bmatrix}, \quad \mathbf{A}' = \begin{bmatrix} \mathbf{A}'_\nu \\ \mathbf{A}'_{\nu,\mu} \end{bmatrix}, \tag{12}$$

where, $\mathbf{A}_\mu$ and $\mathbf{A}'_\nu$ are submatrices that depend only on the elevation angle θ_p and the azimuth angle ϕ_p, respectively, and can be explicitly written as

$$\mathbf{A}_\mu = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\mu_1} & e^{j\mu_2} & \cdots & e^{j\mu_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(K-1)\mu_1} & e^{j(K-1)\mu_2} & \cdots & e^{j(K-1)\mu_P} \end{bmatrix}, \tag{13}$$

$$\mathbf{A}'_\nu = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\nu_1} & e^{j\nu_2} & \cdots & e^{j\nu_P} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(L-1)\nu_1} & e^{j(L-1)\nu_2} & \cdots & e^{j(L-1)\nu_P} \end{bmatrix}. \tag{14}$$

This indicates that the antenna elements corresponding to the first row and the first column of the UPA are separately utilized. Based on this separation approach, the elevation and azimuth angles are estimated using the received signals of the antenna elements in the first row and first column of the UPA, denoted by $\mathbf{Y}_\mu \in \mathbb{C}^{K\times 1}$ and $\mathbf{Y}'_\nu \in \mathbb{C}^{L\times 1}$, respectively. Accordingly, the autocorrelation matrices of the received signals $\mathbf{Y}_\mu$ and $\mathbf{Y}'_\nu$ can be expressed as

$$\mathbf{R}_{Y_\mu} \triangleq \mathbb{E}[\,\mathbf{Y}_\mu \mathbf{Y}_\mu^H\,] = \mathbf{A}_\mu R_x \mathbf{A}_\mu^H + \sigma^2 \mathbf{I}_K, \tag{15}$$

$$\mathbf{R}_{Y'_\nu} \triangleq \mathbb{E}[\,\mathbf{Y}'_\nu \mathbf{Y}'^H_\nu\,] = \mathbf{A}'_\nu R_x \mathbf{A}'^H_\nu + \sigma^2 \mathbf{I}_L, \tag{16}$$

where $R_x \triangleq \mathbb{E}[\mathbf{X}\mathbf{X}^H]$. The parameter σ^2 corresponds to the eigenvalue equal to the thermal noise power, and the eigenvectors associated with this eigenvalue are generally referred to as the eigenvectors of the noise subspace, which are orthogonal to the steering vectors of any incoming signals.

Therefore, by performing Singular Value Decomposition (SVD) of $\mathbf{R}_{Y_\mu}$ and $\mathbf{R}_{Y'_\nu}$, the orthogonal eigenvectors of the noise subspace, denoted by $\mathbf{V}_{Y_\mu}$ and $\mathbf{V}_{Y'_\nu}$, can be obtained.

As a result, the MUSIC spectra for the elevation and azimuth angles are computed as

$$Q(\theta) = \frac{1}{\mathbf{a}(\mu)^H \mathbf{V}_{Y_\mu} \mathbf{V}_{Y_\mu}^H \mathbf{a}(\mu)}, \tag{17}$$

$$Q(\phi) = \frac{1}{\mathbf{a}(\nu)^H \mathbf{V}_{Y'_\nu} \mathbf{V}_{Y'_\nu}^H \mathbf{a}(\nu)}, \tag{18}$$

and by performing a peak search on the obtained spectra, the elevation and azimuth angles can be separately estimated.

B. Conventional Methods

In this subsection, we describe the conventional methods. When estimating the elevation angle θ and the azimuth angle ϕ using the 2D MUSIC algorithm described in Section II-A, DOA estimation becomes infeasible if the number of users exceeds the number of antennas along each axis, since in such cases the noise subspace cannot be obtained in (15) and (16). To overcome this limitation, a method has been proposed in 2D MIMO systems in which the antenna array is virtually extended and the resulting virtual received data are utilized [8].

Consider the case where L antenna elements are placed along the x-axis and the azimuth angles ϕ of P users are to be estimated. Similar to a linear array, DOA estimation is feasible; however, when $L \leq P$, the noise subspace cannot be obtained, and DOA estimation becomes impossible. Therefore, at least $(P - L + 1)$ additional antennas must be introduced. To address this, as illustrated in Fig. 2, virtual antennas that do not physically exist are inserted between the real antenna elements.

The received signals at the real and virtual antennas are respectively given by

$$\mathbf{Y}_{\mathrm{r}} = \mathbf{A}_{\mathrm{r}} \mathbf{X}_{\mathrm{r}} + \mathbf{Z}_{\mathrm{r}}, \tag{19}$$

$$\mathbf{Y}_{\mathrm{v}} = \mathbf{A}_{\mathrm{v}} \mathbf{X}_{\mathrm{v}} + \mathbf{Z}_{\mathrm{v}}, \tag{20}$$

where the steering vector of the virtual antennas, $\mathbf{A}_{\mathrm{v}}$, can be accurately constructed by computing the phase differences based on the propagation distance differences between the real and virtual antennas. The complex amplitudes of the signals, $\mathbf{X}_{\mathrm{v}}$, are obtained from the data of the real antennas.

On the other hand, the noise corresponding to the virtual antennas, $\mathbf{Z}_{\mathrm{v}}$, is generated using the statistical properties of the noise observed at the real antennas. Specifically, for the noise sequence obtained from the L antenna elements, $\mathbf{Z}_{\mathrm{r}}$ $(l = 1, 2, \ldots, L)$, the mean and variance are estimated for each snapshot as

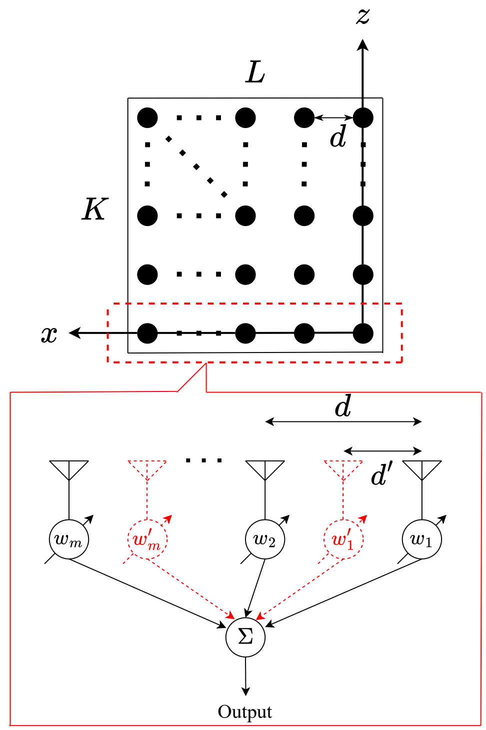

Fig. 2. Conventional virtual antenna arrangement.

$$\bar{Z}_{\mathrm{r}} = \frac{1}{L} \sum_{l=1}^{L} Z_{\mathrm{r}}[l], \tag{21}$$

$$\sigma_{\mathrm{r}}^2 = \frac{1}{L} \sum_{l=1}^{L} \left(Z_{\mathrm{r}}[l] - \bar{Z}_{\mathrm{r}}\right)^2, \tag{22}$$

and the virtual noise samples are then generated as

$$Z_{\mathrm{v}}[l_v] = \bar{Z}_{\mathrm{r}} + \sigma_{\mathrm{r}} \cdot \epsilon[l_v], \tag{23}$$

where $\epsilon[l_v]$ denotes Gaussian white noise with zero mean and a standard deviation of one $(l_v = 1, 2, \ldots, L_v)$.

Consequently, the received signals of the virtual antennas, as expressed in (20), can be generated. By generating L_v virtual antenna signals, the effective number of antennas can be extended to $L + L_v > P$, thereby enabling the extraction of the eigenvectors of the noise subspace from the correlation matrix. Similarly, along the z-axis, the effective number of antennas can be extended to $K + K_v > P$, allowing the eigenvectors to be extracted as well. Therefore, by combining the received signals of the real and virtual antennas, DOA estimation of both elevation and azimuth angles using the MUSIC algorithm becomes feasible even under the conditions $L \leq P$ and $K \leq P$.

III. Proposed Method

This paper is based on the separation-based DOA estimation approach for 2D MIMO systems described in Section II-A. In contrast to the conventional method discussed in the previous section, the proposed method aims to improve angular resolution and reduce estimation errors by devising both the placement of virtual antennas and the procedure for generating their associated noise.

A. Outer Placement of Virtual Antennas

In the conventional method, virtual antennas are placed between the elements of the real antenna array. However, in this configuration, the effective aperture length does not change even when virtual antennas are included, and thus the angular resolution cannot be improved. Moreover, when the spacing between real antennas is $\frac{\lambda}{2}$, the introduction of virtual antennas results in a spacing of $\frac{\lambda}{4}$ between real and virtual antennas. This leads to a high correlation between the real and virtual antennas, thereby degrading the estimation accuracy.

To address this issue, the real antennas are arranged with a spacing of λ, and the virtual antennas are placed such that the spacing between real and virtual antennas becomes $\frac{\lambda}{2}$. This configuration enables improvement in estimation accuracy. However, when DOA estimation is performed without using virtual antennas under the conditions $K > P$ and $L > P$, the spacing between real antenna elements is $\lambda > \frac{\lambda}{2}$, which causes spatial aliasing due to the sampling theorem. Therefore, as illustrated in Fig. 3, the virtual antennas are placed outward from one end of the real antenna array. When m virtual antennas are introduced at intervals of $\frac{\lambda}{2}$ from that end, the effective aperture length is extended by $m \times \frac{\lambda}{2}$. Furthermore, because the spacing between the real antennas is kept at $\frac{\lambda}{2}$ while introducing the virtual antennas, spatial aliasing does not occur even when no virtual antennas are employed.

B. Noise Generation for Virtual Antennas

In the conventional method, Gaussian noise is regenerated by statistical processing, specifically by calculating the mean and variance of the noise data at each antenna element for every snapshot, as shown in (21), (22), and (23). However, in this approach, the number of samples available for statistical processing is limited to K in the z-axis direction and L in the x-axis direction. As a result, the estimates of the mean and variance become unstable,leading to larger errors in DOA estimation. To address this issue, we propose a method that utilizes the time-series AWGN data obtained from existing antenna elements not involved in separation-based DOA estimation with virtual antennas. These data are assigned as the received noise of the virtual antennas. The received signal of the existing antennas is expressed by (7) as $\mathbf{Y} \in \mathbb{C}^{LK\times 1}$. In practice, however, time-series data are received over s snapshots. Taking this into account, the received signal of the existing antennas, $\mathbf{Y}_{\mathrm{r_{all}}}$, can be written as

$$\mathbf{Y}_{\mathrm{r_{all}}} = \mathbf{A}\mathbf{X}_{\mathrm{r_{all}}} + \mathbf{Z}_{\mathrm{r_{all}}} \in \mathbb{C}^{LK\times s}, \tag{24}$$

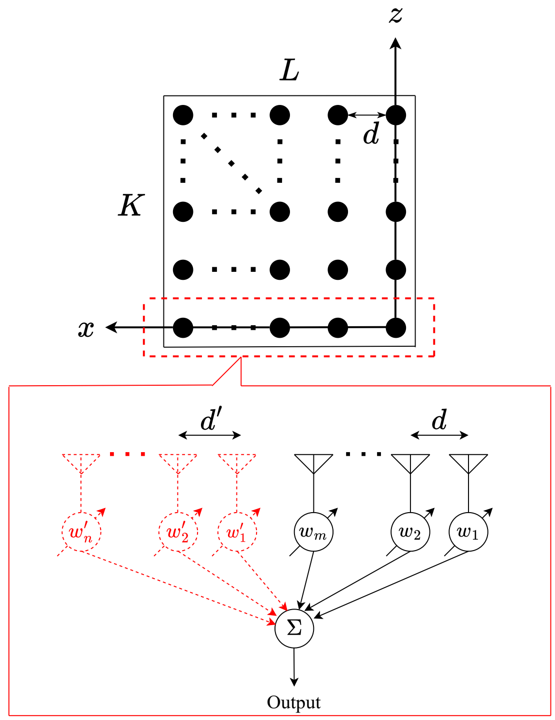

Fig. 3. Virtual antenna arrangement of the proposed method.

where s denotes the number of snapshots, and

$$\mathbf{X}_{\mathrm{r_{all}}} = [\,\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_P\,]^T \in \mathbb{C}^{P\times s}, \tag{25}$$

$$\mathbf{Z}_{\mathrm{r_{all}}} = [\,\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_{LK}\,]^T \in \mathbb{C}^{LK\times s}, \tag{26}$$

Focusing on the elements along the x-axis, the noise data actually used are $[\,\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_L\,]$ within $\mathbf{Z}_{\mathrm{r_{all}}}$, while the remaining $[\,\mathbf{z}_{L+1}, \mathbf{z}_{L+2}, \dots, \mathbf{z}_{LK}\,]$ are not utilized. Therefore, these unused noise data are assigned as the noise of the virtual antennas. When L_v virtual antennas are introduced, the received signal of the virtual antennas placed along the x-axis, denoted by $\mathbf{Y}_{\mathrm{vx}}$, can be expressed as

$$\mathbf{Y}_{\mathrm{vx}} = \mathbf{A}_{\mathrm{v}}\mathbf{X}_{\mathrm{vx}} + [\,\mathbf{z}_{L+1}, \dots, \mathbf{z}_{L+L_v}\,]^T \in \mathbb{C}^{L_v\times s}, \tag{27}$$

In the case where K_v virtual antennas are introduced along the z-axis, the noise is assigned in the same manner.

As a result, the virtual antennas are provided with noise that has the same variance as that of the existing antennas, which is expected to reduce the DOA estimation error. Moreover, since the noise data directly received by the existing antennas are utilized, the processing required in (21), (22), and (23) for each snapshot can be eliminated, providing an additional advantage.

IV. Simulation Results

A. Simulation Parameters

The simulation parameters are summarized in Table I.

TABLE I
SIMULATION PARAMETERS

Structure of Array	UPA
Number of source signals (P)	6
Antenna elements ($K \times L$)	4×4
Virtual Antenna elements ($K_v \times L_v$)	3×3
Inter-element space	$\frac{\lambda}{2}$
Number of snapshots (s)	100
SNR[dB]	30
Angle of arrivals (θ, ϕ) [deg .]	(10, 150),(35,120), (50,100),(65,80), (75,50),(85,20)
Sampling interval of the angle domain [deg .]	0.1

For the evaluation of estimation errors, the Root Mean Square Error (RMSE) is employed as the performance metric. In this paper, RMSE is defined by (28), where γ_p denotes the angular parameter of the p-th source, and $\hat{\gamma}_p$ represents its estimated value.

$$\mathrm{RMSE}_\gamma = \sqrt{\frac{1}{P}\sum_{p=1}^{P}\left(\hat{\gamma}_p - \gamma_p\right)^2}, \quad \gamma \in \{\theta, \phi\}. \tag{28}$$

B. Resolution Comparison with Virtual Antenna Placement

In this subsection, the numbers of virtual antennas in both directions are set to $K_v = L_v = 3$. In the conventional method, the virtual antennas are placed at the center between the real antennas, resulting in a spacing of $\frac{\lambda}{4}$ between real and virtual antennas. In contrast, in the proposed method, the virtual antennas are introduced successively from one end of the real antenna array toward the outside, with a spacing of $\frac{\lambda}{2}$.

Fig. 4 and Fig. 5 show the simulation results of DOA estimation using the MUSIC algorithm. In the MUSIC spectrum of Fig. 4 for the elevation angle θ, the conventional antenna placement fails to separate the incident angles, whereas the proposed placement yields distinct peaks corresponding to the incident angles. In the MUSIC spectrum of Fig. 5 for the azimuth angle ϕ, peaks are obtained for each incident angle in both antenna placements; however, the proposed method produces sharper peaks, enabling clearer separation of the incident angles. This improvement is attributed to the fact that, by introducing virtual antennas successively from one end of the real antenna array toward the outside, the effective aperture length is extended, which in turn enhances the resolution.

C. Comparison of Noise Generation Methods for Virtual Antennas

In this subsection, the placement of virtual antennas is unified to the proposed configuration, and the DOA estimation accuracy of the MUSIC algorithm is compared as the number of virtual antennas is increased from $K_v = 7$, $L_v = 3$ (the minimum numbers required for DOA estimation) up to $K_v = L_v = 10$. For the elevation angle θ, DOA estimation is theoretically possible from $K_v = 3$. However, in Fig. 6, the starting point of the graph is set to 7. This is because the definition range of θ is narrow and the incident directions are densely distributed, resulting in insufficient resolution when the effective array aperture length is short. As shown in both Fig. 6 and Fig. 7, the RMSE decreases as the number of virtual antennas increases. This is due to the fact that a larger number of virtual antennas extends the effective aperture length, thereby improving the resolution and enabling more accurate estimation of the incident angles. When comparing cases with the same number of virtual antennas, the proposed method demonstrates superior RMSE performance compared to the conventional method. This improvement arises because, in the proposed method, the noise variance of the virtual antennas becomes closer to that of the real antennas, which stabilizes the eigenvalues of the noise subspace. Furthermore, the advantage of the proposed method is particularly pronounced at low SNRs. As the SNR decreases, the relative contribution of noise becomes larger, and the stability of the noise variance has a greater impact. Therefore, the proposed method is especially effective in improving DOA estimation accuracy under low-SNR conditions, particularly in the range of 0 dB to 5 dB.

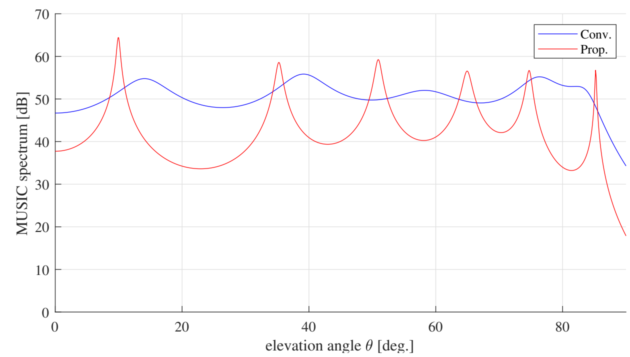

Fig. 4. MUSIC spectrum of elevation angles.

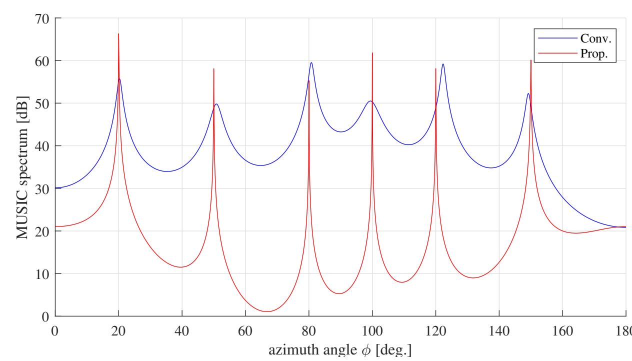

Fig. 5. MUSIC spectrum of azimuth angles.

V. CONCLUSION

In this paper, we proposed a DOA estimation method for 2D MIMO systems that achieves higher resolution and accuracy by modifying both the placement of virtual antennas and the

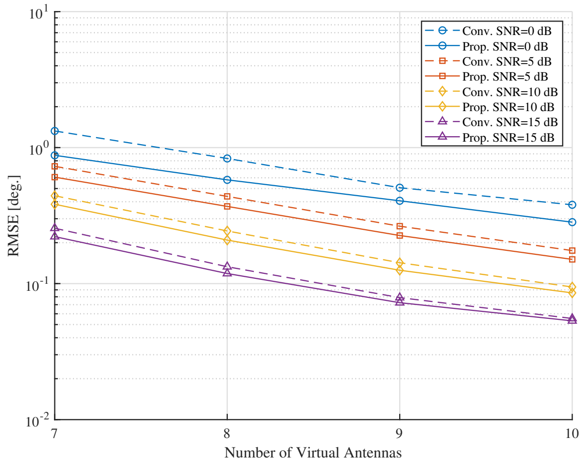

Fig. 6. RMSE performances of elevation estimated angles.

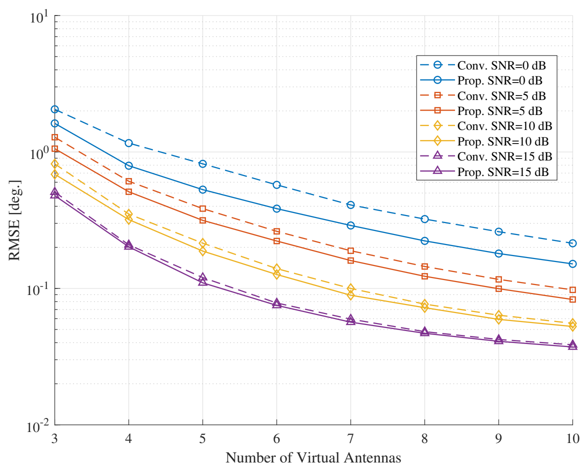

Fig. 7. RMSE performances of azimuth estimated angles.

method of noise generation. Through computer simulations, the effectiveness of the proposed method was verified by comparing it with conventional approaches to virtual antenna placement and noise generation. The results demonstrated that modifying the antenna placement improves angular resolution, and that utilizing noise from unused antennas effectively reduces estimation errors.

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