PUZZLER

Jennifer is holding on to an electrically charged sphere that reaches an electric potential of about 100 000 V. The device that generates this high electric potential is called a *Van de Graaff generator.* What causes Jennifer's hair to stand on end like the needles of a porcupine? Why is she safe in this situation in view of the fact that 110 V from a wall outlet can kill you? *(Henry Leap and Jim Lehman)*

chapter

# 25 Electric Potential

## Chapter Outline

The concept of potential energy was introduced in Chapter 8 in connection with such conservative forces as the force of gravity and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. In this chapter we see that the concept of potential energy is also of great value in the study of electricity. Because the electrostatic force given by Coulomb's law is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential*. Because the electric potential at any point in an electric field is a scalar function, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the concepts of the electric field and electric forces. In later chapters we shall see that the concept of electric potential is of great practical value.

## 25.1 POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

11.8 When a test charge $q_0$ is placed in an electric field $\mathbf{E}$ created by some other charged object, the electric force acting on the test charge is $q_0\mathbf{E}$. (If the field is produced by more than one charged object, this force acting on the test charge is the vector sum of the individual forces exerted on it by the various other charged objects.) The force $q_0\mathbf{E}$ is conservative because the individual forces described by Coulomb's law are conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. For an infinitesimal displacement $d\mathbf{s}$, the work done by the electric field on the charge is $\mathbf{F} \cdot d\mathbf{s} = q_0\mathbf{E} \cdot d\mathbf{s}$. As this amount of work is done by the field, the potential energy of the charge–field system is decreased by an amount $dU = -q_0\mathbf{E} \cdot d\mathbf{s}$. For a finite displacement of the charge from a point $A$ to a point $B$, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad (25.1)$$

Change in potential energy

The integration is performed along the path that $q_0$ follows as it moves from $A$ to $B$, and the integral is called either a *path integral* or a *line integral* (the two terms are synonymous). Because the force $q_0\mathbf{E}$ is conservative, **this line integral does not depend on the path taken from $A$ to $B$.**

### Quick Quiz 25.1

If the path between $A$ and $B$ does not make any difference in Equation 25.1, why don't we just use the expression $\Delta U = -q_0 Ed$, where $d$ is the straight-line distance between $A$ and $B$?

The potential energy per unit charge $U/q_0$ is independent of the value of $q_0$ and has a unique value at every point in an electric field. This quantity $U/q_0$ is called the **electric potential** (or simply the **potential**) $V$. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \qquad (25.2)$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

The **potential difference** $\Delta V = V_B - V_A$ between any two points $A$ and $B$ in an electric field is defined as the change in potential energy of the system divided by the test charge $q_0$:

Potential difference

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.3}$$

Potential difference should not be confused with difference in potential energy. The potential difference is proportional to the change in potential energy, and we see from Equation 25.3 that the two are related by $\Delta U = q_0 \Delta V$.

**Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field. However, when we speak of potential energy, we are referring to the charge–field system.** Because we are usually interested in knowing the electric potential at the location of a charge and the potential energy resulting from the interaction of the charge with the field, we follow the common convention of speaking of the potential energy as if it belonged to the charge.

Because the change in potential energy of a charge is the negative of the work done by the electric field on the charge (as noted in Equation 25.1), the potential difference $\Delta V$ between points $A$ and $B$ equals the work per unit charge that an external agent must perform to move a test charge from $A$ to $B$ without changing the kinetic energy of the test charge.

Just as with potential energy, only *differences* in electric potential are meaningful. To avoid having to work with potential differences, however, we often take the value of the electric potential to be zero at some convenient point in an electric field. This is what we do here: arbitrarily establish the electric potential to be zero at a point that is infinitely remote from the charges producing the field. Having made this choice, we can state that the **electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.** Thus, if we take point $A$ in Equation 25.3 to be at infinity, the electric potential at any point $P$ is

$$V_P = -\int_\infty^P \mathbf{E} \cdot d\mathbf{s} \tag{25.4}$$

In reality, $V_P$ represents the potential difference $\Delta V$ between the point $P$ and a point at infinity. (Eq. 25.4 is a special case of Eq. 25.3.)

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a **volt** (V):

Definition of volt

$$1 \text{ V} \equiv 1 \frac{\text{J}}{\text{C}}$$

That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Equation 25.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1 \frac{\text{N}}{\text{C}} = 1 \frac{\text{V}}{\text{m}}$$

A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as **the energy an electron (or proton) gains or loses by moving through a potential difference of 1 V.** Because 1 V = 1 J/C and because the fundamental charge is approximately $1.60 \times 10^{-19}$ C, the electron volt is related to the joule as follows:

The electron volt

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ C} \cdot \text{V} = 1.60 \times 10^{-19} \text{ J} \qquad \textbf{(25.5)}$$

For instance, an electron in the beam of a typical television picture tube may have a speed of $3.5 \times 10^7$ m/s. This corresponds to a kinetic energy of $5.6 \times 10^{-16}$ J, which is equivalent to $3.5 \times 10^3$ eV. Such an electron has to be accelerated from rest through a potential difference of 3.5 kV to reach this speed.

## 25.2 POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

Equations 25.1 and 25.3 hold in all electric fields, whether uniform or varying, but they can be simplified for a uniform field. First, consider a uniform electric field directed along the negative $y$ axis, as shown in Figure 25.1a. Let us calculate the potential difference between two points $A$ and $B$ separated by a distance $d$, where $d$ is measured parallel to the field lines. Equation 25.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0° \, ds = -\int_A^B E \, ds$$

Because $E$ is constant, we can remove it from the integral sign; this gives

Potential difference in a uniform electric field

$$\Delta V = -E \int_A^B ds = -Ed \qquad \textbf{(25.6)}$$

The minus sign indicates that point $B$ is at a lower electric potential than point $A$; that is, $V_B < V_A$. **Electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 25.1a.

Now suppose that a test charge $q_0$ moves from $A$ to $B$. We can calculate the change in its potential energy from Equations 25.3 and 25.6:

$$\Delta U = q_0 \, \Delta V = -q_0 Ed \qquad \textbf{(25.7)}$$

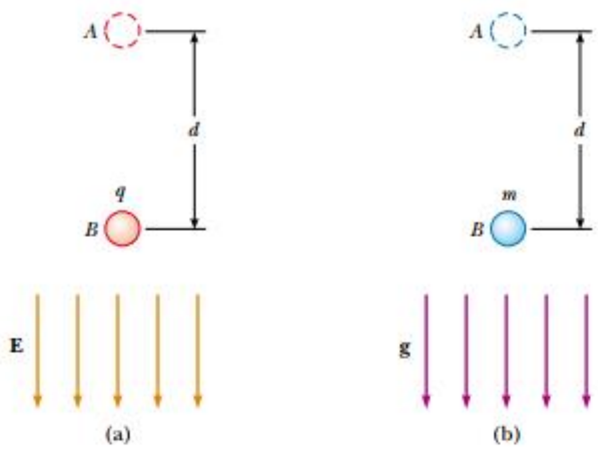



***Figure 25.1*** (a) When the electric field **E** is directed downward, point $B$ is at a lower electric potential than point $A$. A positive test charge that moves from point $A$ to point $B$ loses electric potential energy. (b) A mass $m$ moving downward in the direction of the gravitational field **g** loses gravitational potential energy.

**QuickLab**

It takes an electric field of about 30 000 V/cm to cause a spark in dry air. Shuffle across a rug and reach toward a doorknob. By estimating the length of the spark, determine the electric potential difference between your finger and the doorknob after shuffling your feet but before touching the knob. (If it is very humid on the day you attempt this, it may not work. Why?)

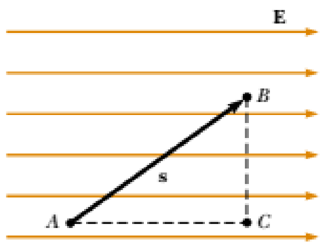



**Figure 25.2** A uniform electric field directed along the positive $x$ axis. Point $B$ is at a lower electric potential than point $A$. Points $B$ and $C$ are at the *same* electric potential.

From this result, we see that if $q_0$ is positive, then $\Delta U$ is negative. We conclude that **a positive charge loses electric potential energy when it moves in the direction of the electric field.** This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field. (This is analogous to the work done by the gravitational field on a falling mass, as shown in Figure 25.1b.) If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of $\mathbf{E}$ (downward in Fig. 25.1a). Therefore, it accelerates downward, gaining kinetic energy. **As the charged particle gains kinetic energy, it loses an equal amount of potential energy.**

If $q_0$ is negative, then $\Delta U$ is positive and the situation is reversed: **A negative charge gains electric potential energy when it moves in the direction of the electric field.** If a negative charge is released from rest in the field $\mathbf{E}$, it accelerates in a direction opposite the direction of the field.

Now consider the more general case of a charged particle that is free to move between any two points in a uniform electric field directed along the $x$ axis, as shown in Figure 25.2. (In this situation, the charge is not being moved by an external agent as before.) If $\mathbf{s}$ represents the displacement vector between points $A$ and $B$, Equation 25.3 gives

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int_A^B d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s} \tag{25.8}$$

where again we are able to remove $\mathbf{E}$ from the integral because it is constant. The change in potential energy of the charge is

$$\Delta U = q_0\, \Delta V = -q_0\, \mathbf{E} \cdot \mathbf{s} \tag{25.9}$$

11.9 Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see this in Figure 25.2, where the potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$. (Prove this to yourself by working out the dot product $\mathbf{E} \cdot \mathbf{s}$ for $\mathbf{s}_{A\to B}$, where the angle $\theta$ between $\mathbf{E}$ and $\mathbf{s}$ is arbitrary as shown in Figure 25.2, and the dot product for $\mathbf{s}_{A\to C}$, where $\theta = 0$.) Therefore, $V_B = V_C$. **The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.**

An equipotential surface

Note that because $\Delta U = q_0 \Delta V$, no work is done in moving a test charge between any two points on an equipotential surface. The equipotential surfaces of a uniform electric field consist of a family of planes that are all perpendicular to the field. Equipotential surfaces for fields with other symmetries are described in later sections.

**Quick Quiz 25.2**

The labeled points in Figure 25.3 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from $A$ to $B$; from $B$ to $C$; from $C$ to $D$; from $D$ to $E$.

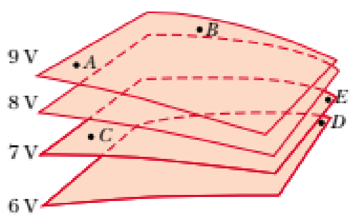



**Figure 25.3** Four equipotential surfaces.

## EXAMPLE 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.4. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider points near the plate edges.) Find the magnitude of the electric field between the plates.

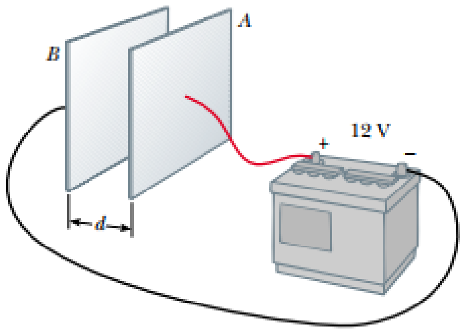



**Figure 25.4** A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference $\Delta V$ divided by the plate separation $d$.

**Solution** The electric field is directed from the positive plate ($A$) to the negative one ($B$), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3\text{ V/m}}$$

This configuration, which is shown in Figure 25.4 and called a *parallel-plate capacitor,* is examined in greater detail in Chapter 26.

## EXAMPLE 25.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of $8.0 \times 10^4$ V/m and is directed along the positive $x$ axis (Fig. 25.5). The proton undergoes a displacement of 0.50 m in the direction of $\mathbf{E}$. (a) Find the change in electric potential between points $A$ and $B$.

**Solution** Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential.

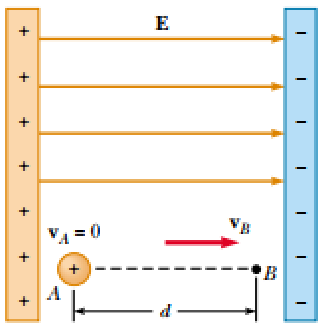



**Figure 25.5** A proton accelerates from $A$ to $B$ in the direction of the electric field.

From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m})$$
$$= \boxed{-4.0 \times 10^4\text{ V}}$$

(b) Find the change in potential energy of the proton for this displacement.

**Solution**

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})$$
$$= \boxed{-6.4 \times 10^{-15}\text{ J}}$$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time loses electric potential energy (because energy is conserved).

***Exercise*** Use the concept of conservation of energy to find the speed of the proton at point $B$.

***Answer*** $2.77 \times 10^6$ m/s.

[1] The electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integral $\int \mathbf{E} \cdot d\mathbf{s}$ between any two points in the conductor must be zero. A more complete discussion of this point is given in Section 25.6.

## 25.3 ELECTRIC POTENTIAL AND POTENTIAL ENERGY DUE TO POINT CHARGES

Consider an isolated positive point charge $q$. Recall that such a charge produces an electric field that is directed radially outward from the charge. To find the electric potential at a point located a distance $r$ from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

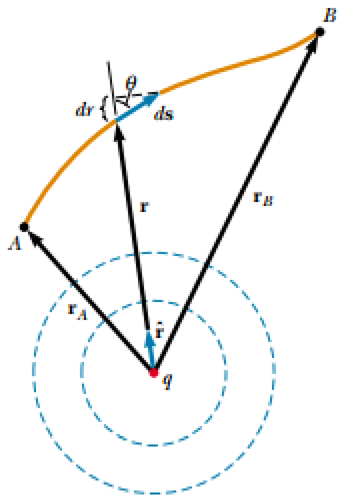



**Figure 25.6** The potential difference between points $A$ and $B$ due to a point charge $q$ depends *only* on the initial and final radial coordinates $r_A$ and $r_B$. The two dashed circles represent cross-sections of spherical equipotential surfaces.

where $A$ and $B$ are the two arbitrary points shown in Figure 25.6. At any field point, the electric field due to the point charge is $\mathbf{E} = k_e q\hat{\mathbf{r}}/r^2$ (Eq. 23.4), where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the field point. The quantity $\mathbf{E} \cdot d\mathbf{s}$ can be expressed as

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

Because the magnitude of $\hat{\mathbf{r}}$ is 1, the dot product $\hat{\mathbf{r}} \cdot d\mathbf{s} = ds \cos\theta$, where $\theta$ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$. Furthermore, $ds \cos\theta$ is the projection of $d\mathbf{s}$ onto $\mathbf{r}$; thus, $ds \cos\theta = dr$. That is, any displacement $d\mathbf{s}$ along the path from point $A$ to point $B$ produces a change $dr$ in the magnitude of $\mathbf{r}$, the radial distance to the charge creating the field. Making these substitutions, we find that $\mathbf{E} \cdot d\mathbf{s} = (k_e q/r^2)dr$; hence, the expression for the potential difference becomes

$$V_B - V_A = -\int E_r\, dr = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \textbf{(25.10)}$$

The integral of $\mathbf{E} \cdot d\mathbf{s}$ is *independent* of the path between points $A$ and $B$—as it must be because the electric field of a point charge is conservative. Furthermore, Equation 25.10 expresses the important result that the potential difference between any two points $A$ and $B$ in a field created by a point charge depends only on the radial coordinates $r_A$ and $r_B$. It is customary to choose the reference of electric potential to be zero at $r_A = \infty$. With this reference, the electric potential created by a point charge at any distance $r$ from the charge is

Electric potential created by a point charge

$$V = k_e \frac{q}{r} \quad \textbf{(25.11)}$$

Electric potential is graphed in Figure 25.7 as a function of $r$, the radial distance from a positive charge in the $xy$ plane. Consider the following analogy to gravitational potential: Imagine trying to roll a marble toward the top of a hill shaped like Figure 25.7a. The gravitational force experienced by the marble is analogous to the repulsive force experienced by a positively charged object as it approaches another positively charged object. Similarly, the electric potential graph of the region surrounding a negative charge is analogous to a "hole" with respect to any approaching positively charged objects. A charged object must be infinitely distant from another charge before the surface is "flat" and has an electric potential of zero.

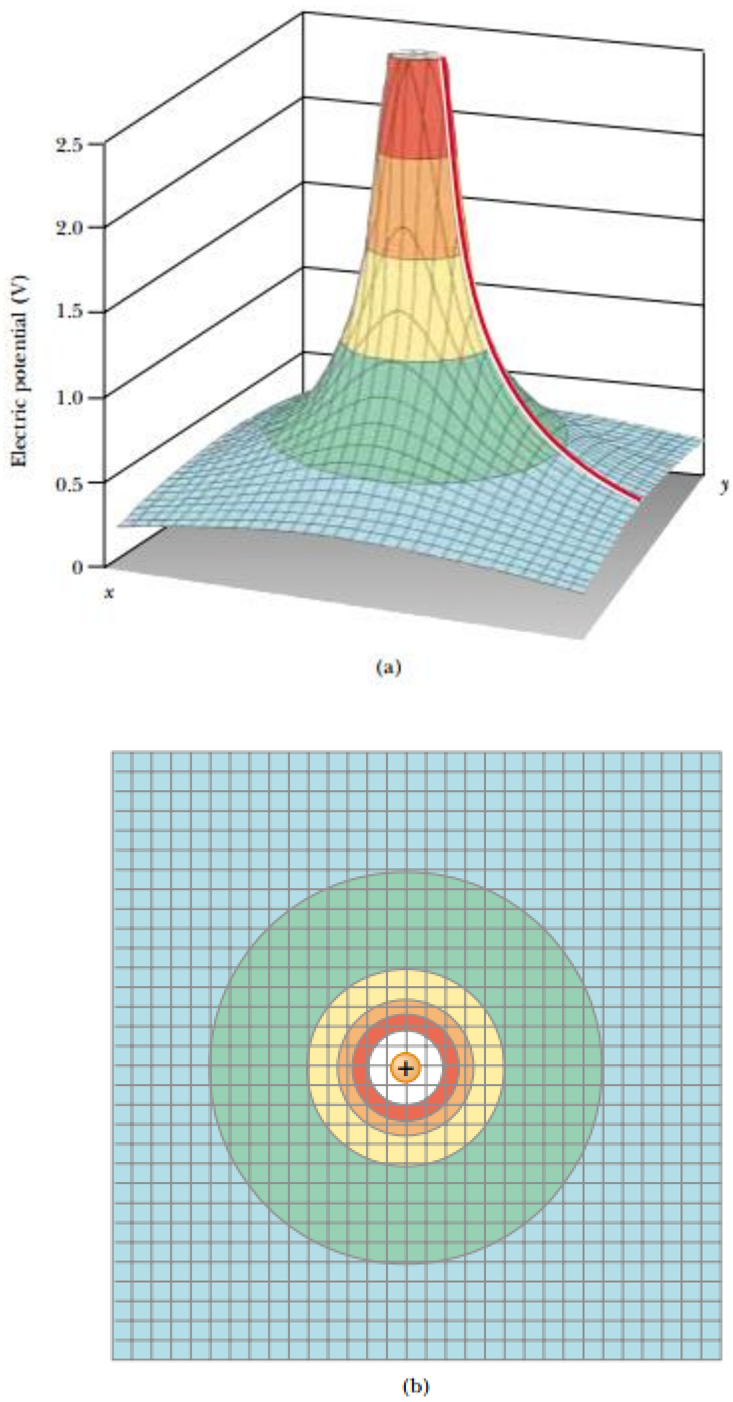



***Figure 25.7*** (a) The electric potential in the plane around a single positive charge is plotted on the vertical axis. (The electric potential function for a negative charge would look like a hole instead of a hill.) The red line shows the $1/r$ nature of the electric potential, as given by Equation 25.11. (b) View looking straight down the vertical axis of the graph in part (a), showing concentric circles where the electric potential is constant. These circles are cross sections of equipotential spheres having the charge at the center.

**Quick Quiz 25.3**

A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon increase, decrease, or remain the same? How about the magnitude of the electric field? The electric flux?

We obtain the electric potential resulting from two or more point charges by applying the superposition principle. That is, the total electric potential at some point $P$ due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at $P$ in the form

Electric potential due to several point charges

$$V = k_e \sum_i \frac{q_i}{r_i} \tag{25.12}$$

where the potential is again taken to be zero at infinity and $r_i$ is the distance from the point $P$ to the charge $q_i$. Note that the sum in Equation 25.12 is an algebraic sum of scalars rather than a vector sum (which we use to calculate the electric field of a group of charges). Thus, it is often much easier to evaluate $V$ than to evaluate $\mathbf{E}$. The electric potential around a dipole is illustrated in Figure 25.8.

We now consider the potential energy of a system of two charged particles. If $V_1$ is the electric potential at a point $P$ due to charge $q_1$, then the work an external agent must do to bring a second charge $q_2$ from infinity to $P$ without acceleration is $q_2V_1$. By definition, this work equals the potential energy $U$ of the two-particle system when the particles are separated by a distance $r_{12}$ (Fig. 25.9). Therefore, we can express the potential energy as[2]

Electric potential energy due to two charges

$$U = k_e \frac{q_1 q_2}{r_{12}} \tag{25.13}$$

Note that if the charges are of the same sign, $U$ is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because like charges repel). If the charges are of opposite sign, $U$ is negative; this means that negative work must be done against the attractive force between the unlike charges for them to be brought near each other.

If more than two charged particles are in the system, we can obtain the total potential energy by calculating $U$ for every pair of charges and summing the terms algebraically. As an example, the total potential energy of the system of three charges shown in Figure 25.10 is

$$U = k_e \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \tag{25.14}$$

Physically, we can interpret this as follows: Imagine that $q_1$ is fixed at the position shown in Figure 25.10 but that $q_2$ and $q_3$ are at infinity. The work an external agent must do to bring $q_2$ from infinity to its position near $q_1$ is $k_e q_1 q_2/r_{12}$, which is the first term in Equation 25.14. The last two terms represent the work required to bring $q_3$ from infinity to its position near $q_1$ and $q_2$. (The result is independent of the order in which the charges are transported.)

[2] The expression for the electric potential energy of a system made up of two point charges, Equation 25.13, is of the *same* form as the equation for the gravitational potential energy of a system made up of two point masses, $Gm_1m_2/r$ (see Chapter 14). The similarity is not surprising in view of the fact that both expressions are derived from an inverse-square force law.

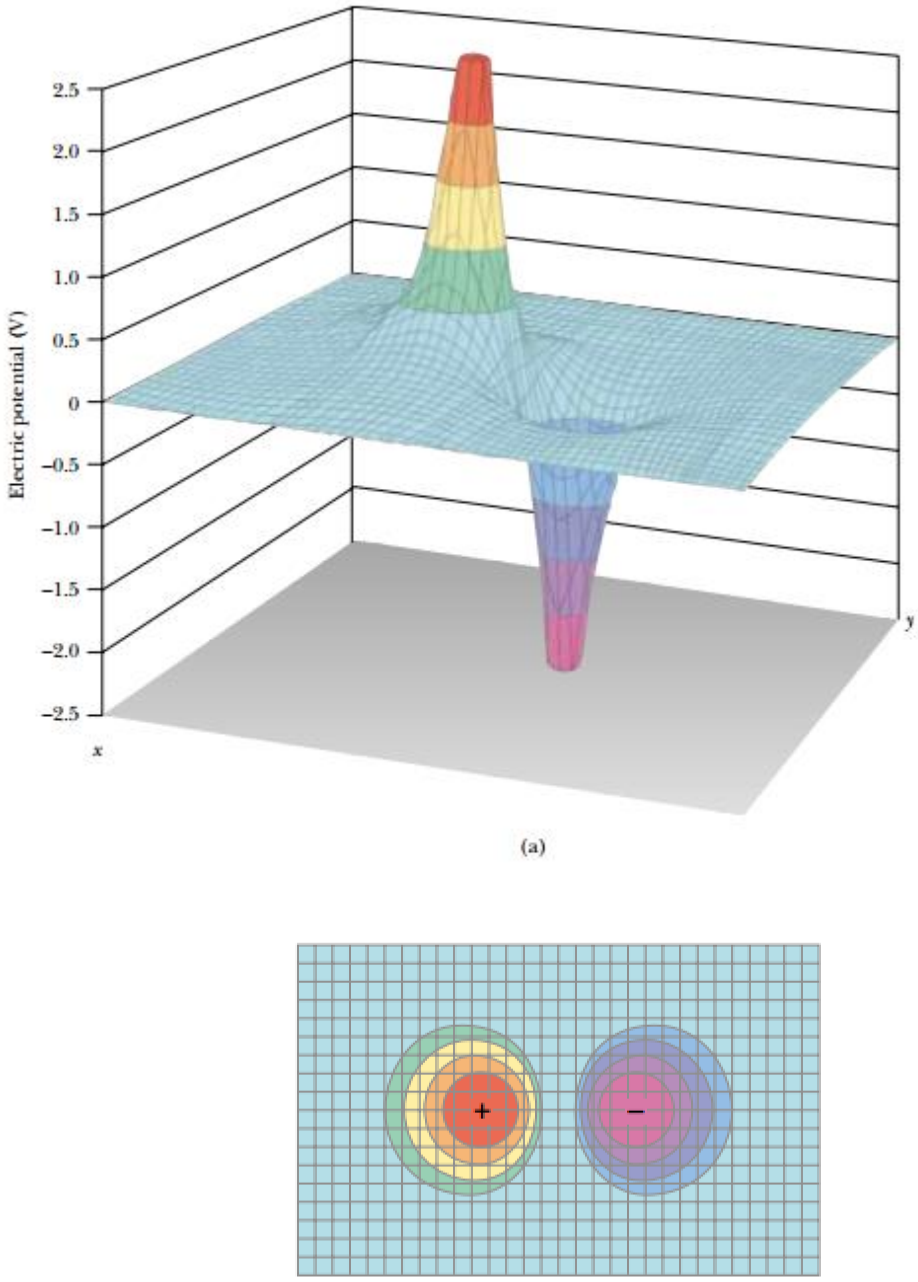



**Figure 25.8** (a) The electric potential in the plane containing a dipole. (b) Top view of the function graphed in part (a).

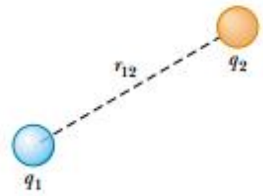



**Figure 25.9** If two point charges are separated by a distance $r_{12}$, the potential energy of the pair of charges is given by $k_e q_1 q_2 / r_{12}$.

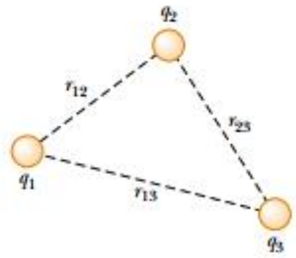



**Figure 25.10** Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 25.14.

### EXAMPLE 25.3 The Electric Potential Due to Two Point Charges

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 25.11a. (a) Find the total electric potential due to these charges at the point $P$, whose coordinates are (4.00, 0) m.

**Solution** For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9\ \frac{N\cdot m^2}{C^2}\left(\frac{2.00 \times 10^{-6}\ C}{4.00\ m} + \frac{-6.00 \times 10^{-6}\ C}{5.00\ m}\right)$$

$$= \boxed{-6.29 \times 10^3\ V}$$

(b) Find the change in potential energy of a 3.00-$\mu$C charge as it moves from infinity to point $P$ (Fig. 25.11b).

**Solution** When the charge is at infinity, $U_i = 0$, and when the charge is at $P$, $U_f = q_3V_P$; therefore,

$$\Delta U = q_3V_P - 0 = (3.00 \times 10^{-6}\ C)(-6.29 \times 10^3\ V)$$

$$= \boxed{-18.9 \times 10^{-3}\ J}$$

Therefore, because $W = -\Delta U$, positive work would have to be done by an external agent to remove the charge from point $P$ back to infinity.

***Exercise*** Find the total potential energy of the system illustrated in Figure 25.11b.

***Answer*** $-5.48 \times 10^{-2}$ J.

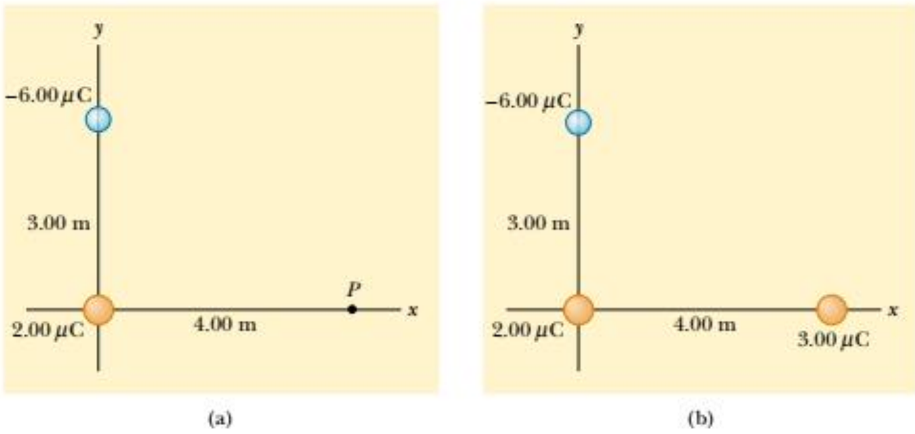



***Figure 25.11*** (a) The electric potential at $P$ due to the two charges is the algebraic sum of the potentials due to the individual charges. (b) What is the potential energy of the three-charge system?

## 25.4 OBTAINING THE VALUE OF THE ELECTRIC FIELD FROM THE ELECTRIC POTENTIAL

The electric field **E** and the electric potential $V$ are related as shown in Equation 25.3. We now show how to calculate the value of the electric field if the electric potential is known in a certain region.

From Equation 25.3 we can express the potential difference $dV$ between two points a distance $ds$ apart as

$$dV = -\mathbf{E}\cdot d\mathbf{s} \tag{25.15}$$

If the electric field has only one component $E_x$, then $\mathbf{E}\cdot d\mathbf{s} = E_x\,dx$. Therefore, Equation 25.15 becomes $dV = -E_x\,dx$, or

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

That is, the magnitude of the electric field in the direction of some coordinate is equal to the negative of the derivative of the electric potential with respect to that coordinate. Recall from the discussion following Equation 25.8 that the electric potential does not change for any displacement perpendicular to an electric field. This is consistent with the notion, developed in Section 25.2, that equipotential surfaces are perpendicular to the field, as shown in Figure 25.12. A small positive charge placed at rest on an electric field line begins to move along the direction of **E** because that is the direction of the force exerted on the charge by the charge distribution creating the electric field (and hence is the direction of **a**). Because the charge starts with zero velocity, it moves in the direction of the change in velocity—that is, in the direction of **a**. In Figures 25.12a and 25.12b, a charge placed at rest in the field will move in a straight line because its acceleration vector is always parallel to its velocity vector. The magnitude of **v** increases, but its direction does not change. The situation is different in Figure 25.12c. A positive charge placed at some point near the dipole first moves in a direction parallel to **E** at that point. Because the direction of the electric field is different at different locations, however, the force acting on the charge changes direction, and **a** is no longer parallel to **v**. This causes the moving charge to change direction and speed, but it does not necessarily follow the electric field lines. Recall that it is not the velocity vector but rather the acceleration vector that is proportional to force.

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance $r$, then the electric field is radial. In this case, $\mathbf{E} \cdot d\mathbf{s} = E_r\, dr$, and thus we can express $dV$ in the form $dV = -E_r\, dr$. Therefore,

$$E_r = -\frac{dV}{dr} \qquad \textbf{(25.17)}$$

For example, the electric potential of a point charge is $V = k_e q/r$. Because $V$ is a function of $r$ only, the potential function has spherical symmetry. Applying Equation 25.17, we find that the electric field due to the point charge is $E_r = k_e q/r^2$, a familiar result. Note that the potential changes only in the radial direction, not in

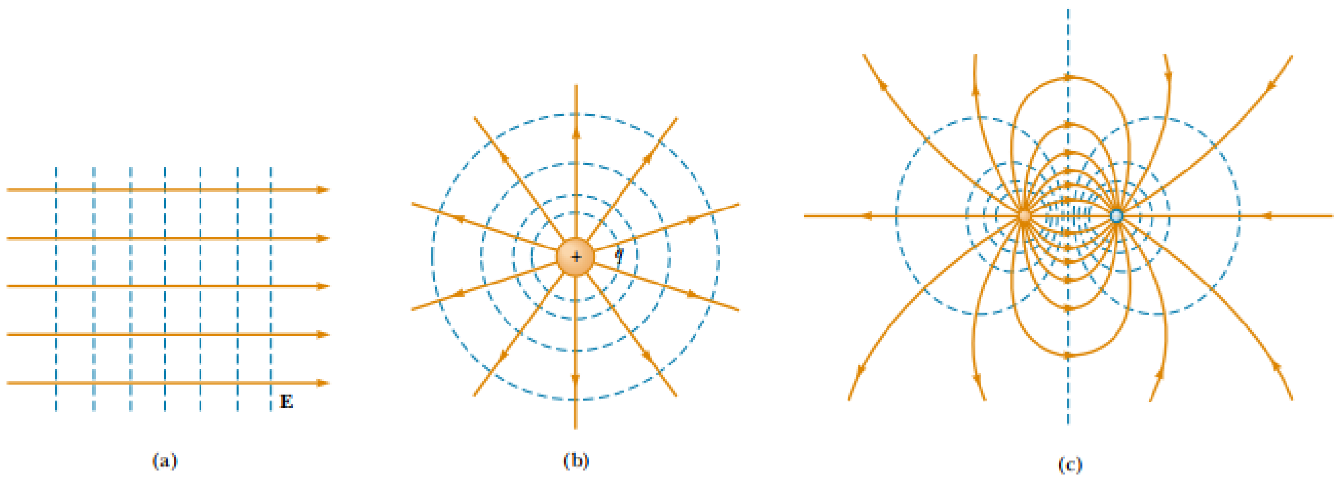



***Figure 25.12*** Equipotential surfaces (dashed blue lines) and electric field lines (red lines) for (a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are *perpendicular* to the electric field lines at every point. Compare these drawings with Figures 25.2, 25.7b, and 25.8b.

Equipotential surfaces are perpendicular to the electric field lines

any direction perpendicular to $r$. Thus, $V$ (like $E_r$) is a function only of $r$. Again, this is consistent with the idea that **equipotential surfaces are perpendicular to field lines.** In this case the equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution (Fig. 25.12b).

The equipotential surfaces for an electric dipole are sketched in Figure 25.12c. When a test charge undergoes a displacement $d\mathbf{s}$ along an equipotential surface, then $dV = 0$ because the potential is constant along an equipotential surface. From Equation 25.15, then, $dV = -\mathbf{E} \cdot d\mathbf{s} = 0$; thus, $\mathbf{E}$ must be perpendicular to the displacement along the equipotential surface. This shows that the equipotential surfaces must *always* be *perpendicular* to the electric field lines.

In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the cartesian coordinates, the electric field components $E_x$, $E_y$, and $E_z$ can readily be found from $V(x, y, z)$ as the partial derivatives[3]

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

For example, if $V = 3x^2y + y^2 + yz$, then

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(3x^2y + y^2 + yz) = \frac{\partial}{\partial x}(3x^2y) = 3y\frac{d}{dx}(x^2) = 6xy$$

## EXAMPLE 25.4 The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure 25.13. The dipole is along the $x$ axis and is centered at the origin. (a) Calculate the electric potential at point $P$.

**Solution** For point $P$ in Figure 25.13,

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \boxed{\frac{2k_e qa}{x^2 - a^2}}$$

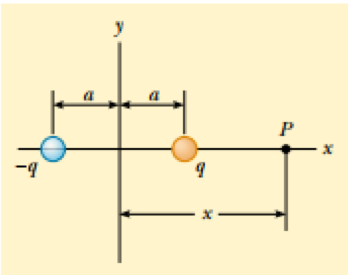



**Figure 25.13** An electric dipole located on the $x$ axis.

(How would this result change if point $P$ happened to be located to the left of the negative charge?)

(b) Calculate $V$ and $E_x$ at a point far from the dipole.

**Solution** If point $P$ is far from the dipole, such that $x \gg a$, then $a^2$ can be neglected in the term $x^2 - a^2$, and $V$ becomes

$$V \approx \boxed{\frac{2k_e qa}{x^2}} \qquad (x \gg a)$$

Using Equation 25.16 and this result, we can calculate the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \boxed{\frac{4k_e qa}{x^3}} \qquad (x \gg a)$$

(c) Calculate $V$ and $E_x$ if point $P$ is located anywhere between the two charges.

**Solution**

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{a-x} - \frac{q}{x+a}\right) = -\frac{2k_e qx}{x^2 - a^2}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qx}{x^2 - a^2}\right) = 2k_e q\left(\frac{-x^2 - a^2}{(x^2 - a^2)^2}\right)$$

[3] In vector notation, $\mathbf{E}$ is often written

$$\mathbf{E} = -\nabla V = -\left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right)V$$

where $\nabla$ is called the *gradient operator.*

We can check these results by considering the situation at the center of the dipole, where $x = 0$, $V = 0$, and $E_x = -2k_e q/a^2$.

***Exercise*** Verify the electric field result in part (c) by calculating the sum of the individual electric field vectors at the origin due to the two charges.

## 25.5 ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

We can calculate the electric potential due to a continuous charge distribution in two ways. If the charge distribution is known, we can start with Equation 25.11 for the electric potential of a point charge. We then consider the potential due to a small charge element $dq$, treating this element as a point charge (Fig. 25.14). The electric potential $dV$ at some point $P$ due to the charge element $dq$ is

$$dV = k_e \frac{dq}{r} \tag{25.18}$$

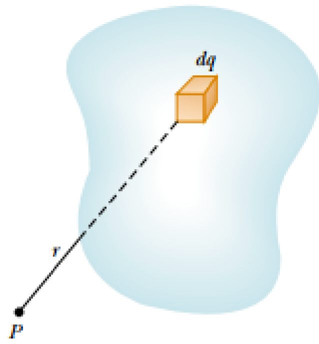



***Figure 25.14*** The electric potential at the point $P$ due to a continuous charge distribution can be calculated by dividing the charged body into segments of charge $dq$ and summing the electric potential contributions over all segments.

where $r$ is the distance from the charge element to point $P$. To obtain the total potential at point $P$, we integrate Equation 25.18 to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point $P$ and because $k_e$ is constant, we can express $V$ as

$$V = k_e \int \frac{dq}{r} \tag{25.19}$$

In effect, we have replaced the sum in Equation 25.12 with an integral. Note that this expression for $V$ uses a particular reference: The electric potential is taken to be zero when point $P$ is infinitely far from the charge distribution.

If the electric field is already known from other considerations, such as Gauss's law, we can calculate the electric potential due to a continuous charge distribution using Equation 25.3. If the charge distribution is highly symmetric, we first evaluate $\mathbf{E}$ at any point using Gauss's law and then substitute the value obtained into Equation 25.3 to determine the potential difference $\Delta V$ between any two points. We then choose the electric potential $V$ to be zero at some convenient point.

We illustrate both methods with several examples.

### EXAMPLE 25.5 Electric Potential Due to a Uniformly Charged Ring

(a) Find an expression for the electric potential at a point $P$ located on the perpendicular central axis of a uniformly charged ring of radius $a$ and total charge $Q$.

**Solution** Let us orient the ring so that its plane is perpendicular to an $x$ axis and its center is at the origin. We can then take point $P$ to be at a distance $x$ from the center of the ring, as shown in Figure 25.15. The charge element $dq$ is at a distance $\sqrt{x^2 + a^2}$ from point $P$. Hence, we can express $V$ as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element $dq$ is at the same distance from point $P$, we can remove $\sqrt{x^2 + a^2}$ from the integral, and $V$ reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{x^2 + a^2}}} \tag{25.20}$$

The only variable in this expression for $V$ is $x$. This is not surprising because our calculation is valid only for points along the $x$ axis, where $y$ and $z$ are both zero.

(b) Find an expression for the magnitude of the electric field at point $P$.

**Solution** From symmetry, we see that along the $x$ axis $\mathbf{E}$ can have only an $x$ component. Therefore, we can use Equation 25.16:

$$E_x = -\frac{dV}{dx} = -k_eQ\frac{d}{dx}(x^2 + a^2)^{-1/2}$$
$$= -k_eQ(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$
$$= \frac{k_eQx}{(x^2 + a^2)^{3/2}} \qquad \textbf{(25.21)}$$

This result agrees with that obtained by direct integration (see Example 23.8). Note that $E_x = 0$ at $x = 0$ (the center of the ring). Could you have guessed this from Coulomb's law?

**Exercise** What is the electric potential at the center of the ring? What does the value of the field at the center tell you about the value of $V$ at the center?

**Answer** $V = k_eQ/a$. Because $E_x = -dV/dx = 0$ at the center, $V$ has either a maximum or minimum value; it is, in fact, a maximum.

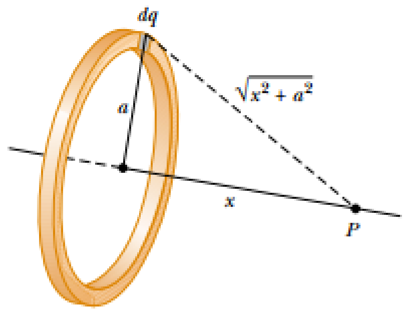



**Figure 25.15** A uniformly charged ring of radius $a$ lies in a plane perpendicular to the $x$ axis. All segments $dq$ of the ring are the same distance from any point $P$ lying on the $x$ axis.

## EXAMPLE 25.6 Electric Potential Due to a Uniformly Charged Disk

Find (a) the electric potential and (b) the magnitude of the electric field along the perpendicular central axis of a uniformly charged disk of radius $a$ and surface charge density $\sigma$.

**Solution** (a) Again, we choose the point $P$ to be at a distance $x$ from the center of the disk and take the plane of the disk to be perpendicular to the $x$ axis. We can simplify the problem by dividing the disk into a series of charged rings. The electric potential of each ring is given by Equation 25.20. Consider one such ring of radius $r$ and width $dr$, as indicated in Figure 25.16. The surface area of the ring is $dA = 2\pi r\,dr$; from the definition of surface charge density (see Section 23.5), we know that the charge on the ring is $dq = \sigma\,dA = \sigma 2\pi r\,dr$. Hence, the potential at the point $P$ due to this ring is

$$dV = \frac{k_e\,dq}{\sqrt{r^2 + x^2}} = \frac{k_e\sigma 2\pi r\,dr}{\sqrt{r^2 + x^2}}$$

To find the *total* electric potential at $P$, we sum over all rings making up the disk. That is, we integrate $dV$ from $r = 0$ to $r = a$:

$$V = \pi k_e\sigma\int_0^a \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e\sigma\int_0^a (r^2 + x^2)^{-1/2}\,2r\,dr$$

This integral is of the form $u^n\,du$ and has the value $u^{n+1}/(n+1)$, where $n = -\frac{1}{2}$ and $u = r^2 + x^2$. This gives

$$V = 2\pi k_e\sigma[(x^2 + a^2)^{1/2} - x] \qquad \textbf{(25.22)}$$

(b) As in Example 25.5, we can find the electric field at any axial point from

$$E_x = -\frac{dV}{dx} = 2\pi k_e\sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right) \qquad \textbf{(25.23)}$$

The calculation of $V$ and $\mathbf{E}$ for an arbitrary point off the axis is more difficult to perform, and we do not treat this situation in this text.

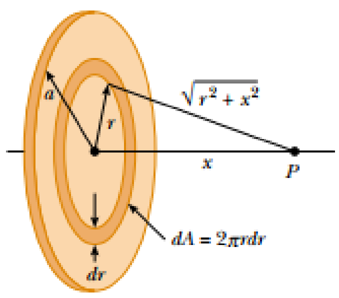



**Figure 25.16** A uniformly charged disk of radius $a$ lies in a plane perpendicular to the $x$ axis. The calculation of the electric potential at any point $P$ on the $x$ axis is simplified by dividing the disk into many rings each of area $2\pi r\,dr$.

## EXAMPLE 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length $\ell$ located along the $x$ axis has a total charge $Q$ and a uniform linear charge density $\lambda = Q/\ell$. Find the electric potential at a point $P$ located on the $y$ axis a distance $a$ from the origin (Fig. 25.17).

**Solution** The length element $dx$ has a charge $dq = \lambda\, dx$. Because this element is a distance $r = \sqrt{x^2 + a^2}$ from point $P$, we can express the potential at point $P$ due to this element as

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at $P$, we integrate this expression over the limits $x = 0$ to $x = \ell$. Noting that $k_e$ and $\lambda$ are constants, we find that

$$V = k_e \lambda \int_0^\ell \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^\ell \frac{dx}{\sqrt{x^2 + a^2}}$$

This integral has the following value (see Appendix B):

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

Evaluating $V$, we find that

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right) \qquad \textbf{(25.24)}$$

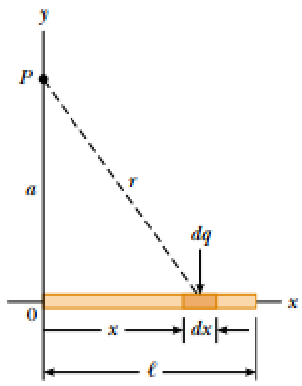



**Figure 25.17** A uniform line charge of length $\ell$ located along the $x$ axis. To calculate the electric potential at $P$, the line charge is divided into segments each of length $dx$ and each carrying a charge $dq = \lambda\, dx$.

## EXAMPLE 25.8 Electric Potential Due to a Uniformly Charged Sphere

An insulating solid sphere of radius $R$ has a uniform positive volume charge density and total charge $Q$. (a) Find the electric potential at a point outside the sphere, that is, for $r > R$. Take the potential to be zero at $r = \infty$.

**Solution** In Example 24.5, we found that the magnitude of the electric field outside a uniformly charged sphere of radius $R$ is

$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when $Q$ is positive. In this case, to obtain the electric potential at an exterior point, such as $B$ in Figure 25.18, we use Equation 25.4 and the expression for $E_r$ given above:

$$V_B = -\int_\infty^r E_r\, dr = -k_e Q \int_\infty^r \frac{dr}{r^2}$$

$$\boxed{V_B = k_e \frac{Q}{r}} \qquad (\text{for } r > R)$$

Note that the result is identical to the expression for the electric potential due to a point charge (Eq. 25.11).

Because the potential must be continuous at $r = R$, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as $C$ shown in Figure 25.18 is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

(b) Find the potential at a point inside the sphere, that is, for $r < R$.

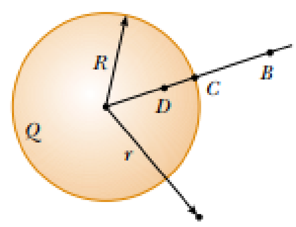



**Figure 25.18** A uniformly charged insulating sphere of radius $R$ and total charge $Q$. The electric potentials at points $B$ and $C$ are equivalent to those produced by a point charge $Q$ located at the center of the sphere, but this is not true for point $D$.

**Solution** In Example 24.5 we found that the electric field inside an insulating uniformly charged sphere is

$$E_r = \frac{k_e Q}{R^3} r \qquad (\text{for } r < R)$$

We can use this result and Equation 25.3 to evaluate the potential difference $V_D - V_C$ at some interior point $D$:

$$V_D - V_C = -\int_R^r E_r\, dr = -\frac{k_e Q}{R^3}\int_R^r r\, dr = \frac{k_e Q}{2R^3}(R^2 - r^2)$$

Substituting $V_C = k_e Q/R$ into this expression and solving for $V_D$, we obtain

$$\boxed{V_D = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)} \qquad (\text{for } r < R) \qquad \textbf{(25.25)}$$

At $r = R$, this expression gives a result that agrees with that for the potential at the surface, that is, $V_C$. A plot of $V$ versus $r$ for this charge distribution is given in Figure 25.19.

***Exercise*** What are the magnitude of the electric field and the electric potential at the center of the sphere?

***Answer*** $E = 0$; $V_0 = 3k_e Q/2R$.

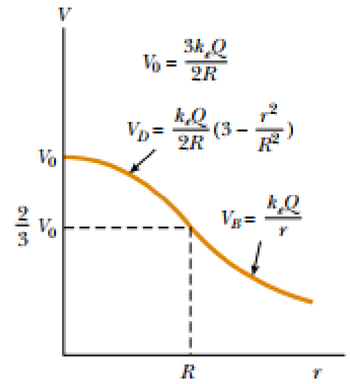



***Figure 25.19*** A plot of electric potential $V$ versus distance $r$ from the center of a uniformly charged insulating sphere of radius $R$. The curve for $V_D$ inside the sphere is parabolic and joins smoothly with the curve for $V_B$ outside the sphere, which is a hyperbola. The potential has a maximum value $V_0$ at the center of the sphere. We could make this graph three dimensional (similar to Figures 25.7a and 25.8a) by spinning it around the vertical axis.

## 25.6 ELECTRIC POTENTIAL DUE TO A CHARGED CONDUCTOR

In Section 24.4 we found that when a solid conductor in equilibrium carries a net charge, the charge resides on the outer surface of the conductor. Furthermore, we showed that the electric field just outside the conductor is perpendicular to the surface and that the field inside is zero.

We now show that **every point on the surface of a charged conductor in equilibrium is at the same electric potential.** Consider two points $A$ and $B$ on the surface of a charged conductor, as shown in Figure 25.20. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement $d\mathbf{s}$; there-

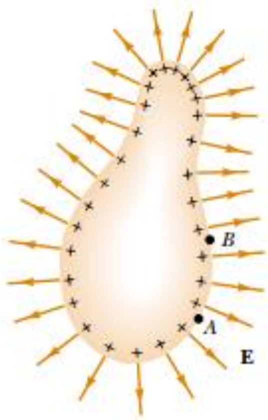



***Figure 25.20*** An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the direction of $\mathbf{E}$ just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface. Note from the spacing of the plus signs that the surface charge density is nonuniform.

fore $\mathbf{E} \cdot d\mathbf{s} = 0$. Using this result and Equation 25.3, we conclude that the potential difference between $A$ and $B$ is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, $V$ is constant everywhere on the surface of a charged conductor in equilibrium. That is,

the surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude from the relationship $E_r = -dV/dr$ that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

The surface of a charged conductor is an equipotential surface

Because this is true about the electric potential, no work is required to move a test charge from the interior of a charged conductor to its surface.

Consider a solid metal conducting sphere of radius $R$ and total positive charge $Q$, as shown in Figure 25.21a. The electric field outside the sphere is $k_eQ/r^2$ and points radially outward. From Example 25.8, we know that the electric potential at the interior and surface of the sphere must be $k_eQ/R$ relative to infinity. The potential outside the sphere is $k_eQ/r$. Figure 25.21b is a plot of the electric potential as a function of $r$, and Figure 25.21c shows how the electric field varies with $r$.

When a net charge is placed on a spherical conductor, the surface charge density is uniform, as indicated in Figure 25.21a. However, if the conductor is nonspherical, as in Figure 25.20, the surface charge density is high where the radius of curvature is small and the surface is convex (as noted in Section 24.4), and it is low where the radius of curvature is small and the surface is concave. Because the electric field just outside the conductor is proportional to the surface charge density, we see that **the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.**

Figure 25.22 shows the electric field lines around two spherical conductors: one carrying a net charge $Q$, and a larger one carrying zero net charge. In this case, the surface charge density is not uniform on either conductor. The sphere having zero net charge has negative charges induced on its side that faces the

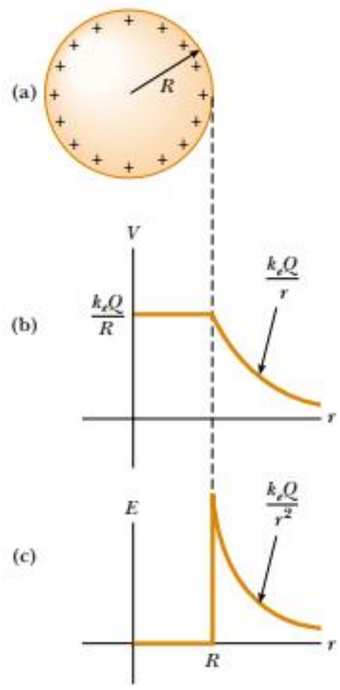
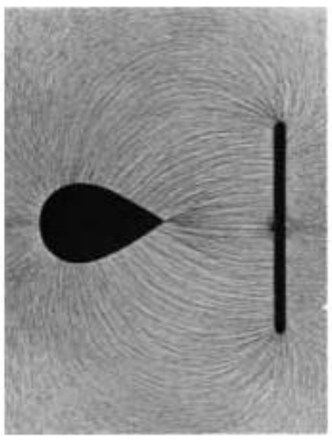



**Figure 25.21** (a) The excess charge on a conducting sphere of radius $R$ is uniformly distributed on its surface. (b) Electric potential versus distance $r$ from the center of the charged conducting sphere. (c) Electric field magnitude versus distance $r$ from the center of the charged conducting sphere.



Electric field pattern of a charged conducting plate placed near an oppositely charged pointed conductor. Small pieces of thread suspended in oil align with the electric field lines. The field surrounding the pointed conductor is most intense near the pointed end and at other places where the radius of curvature is small.

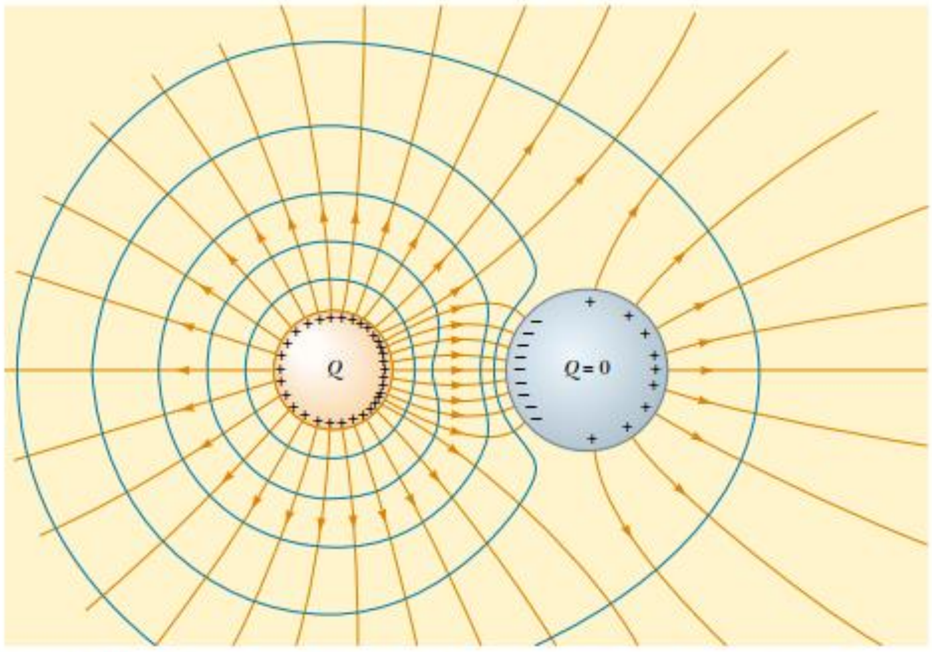



**Figure 25.22** The electric field lines (in red) around two spherical conductors. The smaller sphere has a net charge $Q$, and the larger one has zero net charge. The blue curves are cross-sections of equipotential surfaces.

charged sphere and positive charges induced on its side opposite the charged sphere. The blue curves in the figure represent the cross-sections of the equipotential surfaces for this charge configuration. As usual, the field lines are perpendicular to the conducting surfaces at all points, and the equipotential surfaces are perpendicular to the field lines everywhere. Trying to move a positive charge in the region of these conductors would be like moving a marble on a hill that is flat on top (representing the conductor on the left) and has another flat area partway down the side of the hill (representing the conductor on the right).

## EXAMPLE 25.9 Two Connected Charged Spheres

Two spherical conductors of radii $r_1$ and $r_2$ are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire, as shown in Figure 25.23. The charges on the spheres in equilibrium are $q_1$ and $q_2$, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.

**Solution** Because the spheres are connected by a conducting wire, they must both be at the same electric potential:

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

Therefore, the ratio of charges is

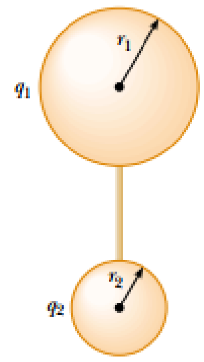



**Figure 25.23** Two charged spherical conductors connected by a conducting wire. The spheres are at the *same* electric potential $V$.

$$(1) \qquad \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

Because the spheres are very far apart and their surfaces uniformly charged, we can express the magnitude of the electric fields at their surfaces as

$$E_1 = k_e \frac{q_1}{r_1^2} \quad \text{and} \quad E_2 = k_e \frac{q_2}{r_2^2}$$

Taking the ratio of these two fields and making use of Equation (1), we find that

$$\boxed{\frac{E_1}{E_2} = \frac{r_2}{r_1}}$$

Hence, the field is more intense in the vicinity of the smaller sphere even though the electric potentials of both spheres are the same.

## A Cavity Within a Conductor

Now consider a conductor of arbitrary shape containing a cavity as shown in Figure 25.24. Let us assume that no charges are inside the cavity. **In this case, the electric field inside the cavity must be zero** regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

To prove this point, we use the fact that every point on the conductor is at the same electric potential, and therefore any two points $A$ and $B$ on the surface of the cavity must be at the same potential. Now imagine that a field $\mathbf{E}$ exists in the cavity and evaluate the potential difference $V_B - V_A$ defined by Equation 25.3:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

If $\mathbf{E}$ is nonzero, we can always find a path between $A$ and $B$ for which $\mathbf{E} \cdot d\mathbf{s}$ is a positive number; thus, the integral must be positive. However, because $V_B - V_A = 0$, the integral of $\mathbf{E} \cdot d\mathbf{s}$ must be zero for all paths between any two points on the conductor, which implies that $\mathbf{E}$ is zero everywhere. This contradiction can be reconciled only if $\mathbf{E}$ is zero inside the cavity. Thus, we conclude that a cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

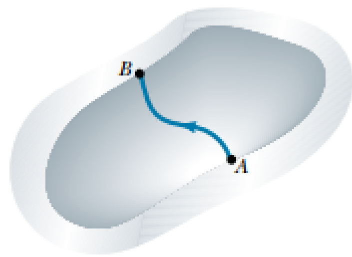



**Figure 25.24** A conductor in electrostatic equilibrium containing a cavity. The electric field in the cavity is zero, regardless of the charge on the conductor.

### Corona Discharge

A phenomenon known as **corona discharge** is often observed near a conductor such as a high-voltage power line. When the electric field in the vicinity of the conductor is sufficiently strong, electrons are stripped from air molecules. This causes the molecules to be ionized, thereby increasing the air's ability to conduct. The observed glow (or corona discharge) results from the recombination of free electrons with the ionized air molecules. If a conductor has an irregular shape, the electric field can be very high near sharp points or edges of the conductor; consequently, the ionization process and corona discharge are most likely to occur around such points.

**Quick Quiz 25.4**

(a) Is it possible for the magnitude of the electric field to be zero at a location where the electric potential is not zero? (b) Can the electric potential be zero where the electric field is nonzero?

*Optional Section*

## 25.7 THE MILLIKAN OIL-DROP EXPERIMENT

During the period from 1909 to 1913, Robert Millikan performed a brilliant set of experiments in which he measured $e$, the elementary charge on an electron, and demonstrated the quantized nature of this charge. His apparatus, diagrammed in Figure 25.25, contains two parallel metallic plates. Charged oil droplets from an atomizer are allowed to pass through a small hole in the upper plate. A horizontally directed light beam (not shown in the diagram) is used to illuminate the oil droplets, which are viewed through a telescope whose long axis is at right angles to the light beam. When the droplets are viewed in this manner, they appear as shining stars against a dark background, and the rate at which individual drops fall can be determined.[4]

Let us assume that a single drop having a mass $m$ and carrying a charge $q$ is being viewed and that its charge is negative. If no electric field is present between the plates, the two forces acting on the charge are the force of gravity $m\mathbf{g}$ acting downward and a viscous drag force $\mathbf{F}_D$ acting upward as indicated in Figure 25.26a. The drag force is proportional to the drop's speed. When the drop reaches its terminal speed $v$, the two forces balance each other ($mg = F_D$).

Now suppose that a battery connected to the plates sets up an electric field between the plates such that the upper plate is at the higher electric potential. In this case, a third force $q\mathbf{E}$ acts on the charged drop. Because $q$ is negative and $\mathbf{E}$ is directed downward, this electric force is directed upward, as shown in Figure 25.26b. If this force is sufficiently great, the drop moves upward and the drag force $\mathbf{F}'_D$ acts downward. When the upward electric force $q\mathbf{E}$ balances the sum of the gravitational force and the downward drag force $\mathbf{F}'_D$, the drop reaches a new terminal speed $v'$ in the upward direction.

With the field turned on, a drop moves slowly upward, typically at rates of hundredths of a centimeter per second. The rate of fall in the absence of a field is comparable. Hence, one can follow a single droplet for hours, alternately rising and falling, by simply turning the electric field on and off.

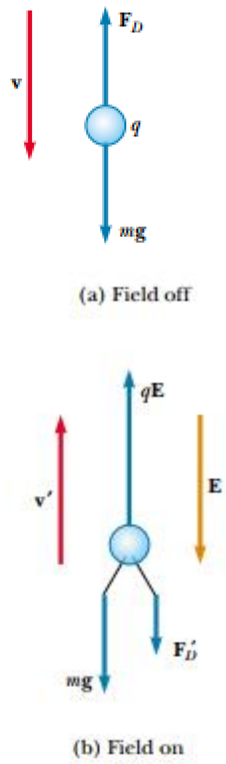



**Figure 25.26** The forces acting on a negatively charged oil droplet in the Millikan experiment.

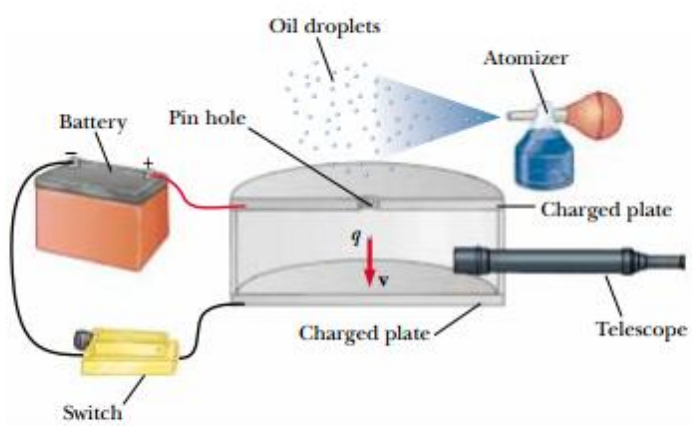



**Figure 25.25** Schematic drawing of the Millikan oil-drop apparatus.

[4] At one time, the oil droplets were termed "Millikan's Shining Stars." Perhaps this description has lost its popularity because of the generations of physics students who have experienced hallucinations, near blindness, migraine headaches, and so forth, while repeating Millikan's experiment!

After recording measurements on thousands of droplets, Millikan and his coworkers found that all droplets, to within about 1% precision, had a charge equal to some integer multiple of the elementary charge $e$:

$$q = ne \qquad n = 0, -1, -2, -3, \ldots$$

where $e = 1.60 \times 10^{-19}$ C. Millikan's experiment yields conclusive evidence that charge is quantized. For this work, he was awarded the Nobel Prize in Physics in 1923.

*Optional Section*

## 25.8 APPLICATIONS OF ELECTROSTATICS

The practical application of electrostatics is represented by such devices as lightning rods and electrostatic precipitators and by such processes as xerography and the painting of automobiles. Scientific devices based on the principles of electrostatics include electrostatic generators, the field-ion microscope, and ion-drive rocket engines.

### The Van de Graaff Generator

11.10 In Section 24.5 we described an experiment that demonstrates a method for transferring charge to a hollow conductor (the Faraday ice-pail experiment). When a charged conductor is placed in contact with the inside of a hollow conductor, all of the charge of the charged conductor is transferred to the hollow conductor. In principle, the charge on the hollow conductor and its electric potential can be increased without limit by repetition of the process.

In 1929 Robert J. Van de Graaff (1901–1967) used this principle to design and build an electrostatic generator. This type of generator is used extensively in nuclear physics research. A schematic representation of the generator is given in Figure 25.27. Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material. The high-voltage electrode is a hollow conductor mounted on an insulating column. The belt is charged at point $A$ by means of a corona discharge between comb-like metallic needles and a grounded grid. The needles are maintained at a positive electric potential of typically $10^4$ V. The positive charge on the moving belt is transferred to the hollow conductor by a second comb of needles at point $B$. Because the electric field inside the hollow conductor is negligible, the positive charge on the belt is easily transferred to the conductor regardless of its potential. In practice, it is possible to increase the electric potential of the hollow conductor until electrical discharge occurs through the air. Because the "breakdown" electric field in air is about $3 \times 10^6$ V/m, a sphere 1 m in radius can be raised to a maximum potential of $3 \times 10^6$ V. The potential can be increased further by increasing the radius of the hollow conductor and by placing the entire system in a container filled with high-pressure gas.

Van de Graaff generators can produce potential differences as large as 20 million volts. Protons accelerated through such large potential differences receive enough energy to initiate nuclear reactions between themselves and various target nuclei. Smaller generators are often seen in science classrooms and museums. If a person insulated from the ground touches the sphere of a Van de Graaff generator, his or her body can be brought to a high electric potential. The hair acquires a net positive charge, and each strand is repelled by all the others. The result is a

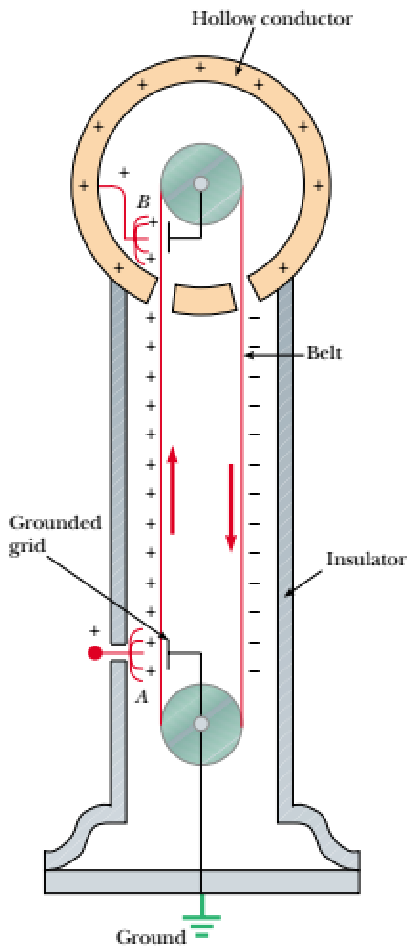



**Figure 25.27** Schematic diagram of a Van de Graaff generator. Charge is transferred to the hollow conductor at the top by means of a moving belt. The charge is deposited on the belt at point $A$ and transferred to the hollow conductor at point $B$.

scene such as that depicted in the photograph at the beginning of this chapter. In addition to being insulated from ground, the person holding the sphere is safe in this demonstration because the total charge on the sphere is very small (on the order of 1 $\mu$C). If this amount of charge accidentally passed from the sphere through the person to ground, the corresponding current would do no harm.

## The Electrostatic Precipitator

**QuickLab**

Sprinkle some salt and pepper on an open dish and mix the two together. Now pull a comb through your hair several times and bring the comb to within 1 cm of the salt and pepper. What happens? How is what happens here related to the operation of an electrostatic precipitator?

One important application of electrical discharge in gases is the *electrostatic precipitator.* This device removes particulate matter from combustion gases, thereby reducing air pollution. Precipitators are especially useful in coal-burning power plants and in industrial operations that generate large quantities of smoke. Current systems are able to eliminate more than 99% of the ash from smoke.

Figure 25.28a shows a schematic diagram of an electrostatic precipitator. A high potential difference (typically 40 to 100 kV) is maintained between a wire running down the center of a duct and the walls of the duct, which are grounded. The wire is maintained at a negative electric potential with respect to the walls, so the electric field is directed toward the wire. The values of the field near the wire become high enough to cause a corona discharge around the wire; the discharge ionizes some air molecules to form positive ions, electrons, and such negative ions as $O_2^-$. The air to be cleaned enters the duct and moves near the wire. As the electrons and negative ions created by the discharge are accelerated toward the outer wall by the electric field, the dirt particles in the air become charged by collisions and ion capture. Because most of the charged dirt particles are negative, they too are drawn to the duct walls by the electric field. When the duct is periodically shaken, the particles break loose and are collected at the bottom.


(a) (b) (c)

**Figure 25.28** (a) Schematic diagram of an electrostatic precipitator. The high negative electric potential maintained on the central coiled wire creates an electrical discharge in the vicinity of the wire. Compare the air pollution when the electrostatic precipitator is (b) operating and (c) turned off.

In addition to reducing the level of particulate matter in the atmosphere (compare Figs. 25.28b and c), the electrostatic precipitator recovers valuable materials in the form of metal oxides.

## Xerography and Laser Printers

The basic idea of xerography[5] was developed by Chester Carlson, who was granted a patent for the xerographic process in 1940. The one feature of this process that makes it unique is the use of a photoconductive material to form an image. (A *photoconductor* is a material that is a poor electrical conductor in the dark but that becomes a good electrical conductor when exposed to light.)

The xerographic process is illustrated in Figure 25.29a to d. First, the surface of a plate or drum that has been coated with a thin film of photoconductive material (usually selenium or some compound of selenium) is given a positive electrostatic charge in the dark. An image of the page to be copied is then focused by a lens onto the charged surface. The photoconducting surface becomes conducting only in areas where light strikes it. In these areas, the light produces charge carriers in the photoconductor that move the positive charge off the drum. However, positive

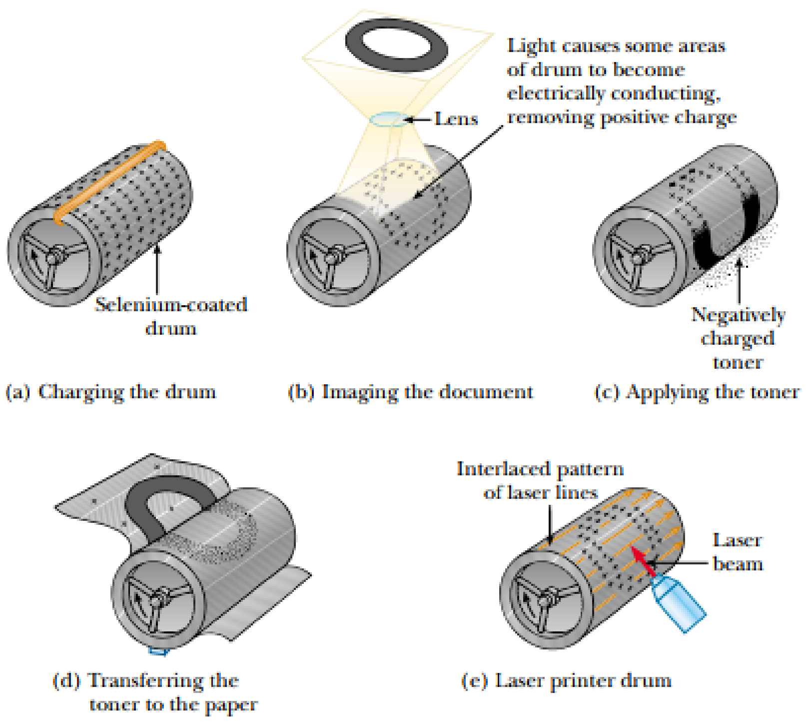



**Figure 25.29** The xerographic process: (a) The photoconductive surface of the drum is positively charged. (b) Through the use of a light source and lens, an image is formed on the surface in the form of positive charges. (c) The surface containing the image is covered with a negatively charged powder, which adheres only to the image area. (d) A piece of paper is placed over the surface and given a positive charge. This transfers the image to the paper as the negatively charged powder particles migrate to the paper. The paper is then heat-treated to "fix" the powder. (e) A laser printer operates similarly except the image is produced by turning a laser beam on and off as it sweeps across the selenium-coated drum.

[5] The prefix *xero-* is from the Greek word meaning "dry." Note that no liquid ink is used anywhere in xerography.

charges remain on those areas of the photoconductor not exposed to light, leaving a latent image of the object in the form of a positive surface charge distribution.

Next, a negatively charged powder called a *toner* is dusted onto the photoconducting surface. The charged powder adheres only to those areas of the surface that contain the positively charged image. At this point, the image becomes visible. The toner (and hence the image) are then transferred to the surface of a sheet of positively charged paper.

Finally, the toner is "fixed" to the surface of the paper as the toner melts while passing through high-temperature rollers. This results in a permanent copy of the original.

A laser printer (Fig. 25.29e) operates by the same principle, with the exception that a computer-directed laser beam is used to illuminate the photoconductor instead of a lens.

## SUMMARY

When a positive test charge $q_0$ is moved between points $A$ and $B$ in an electric field $\mathbf{E}$, the **change in the potential energy** is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.1}$$

The **electric potential** $V = U/q_0$ is a scalar quantity and has units of joules per coulomb (J/C), where 1 J/C $\equiv$ 1 V.

The **potential difference** $\Delta V$ between points $A$ and $B$ in an electric field $\mathbf{E}$ is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.3}$$

The potential difference between two points $A$ and $B$ in a uniform electric field $\mathbf{E}$ is

$$\Delta V = -Ed \tag{25.6}$$

where $d$ is the magnitude of the displacement in the direction parallel to $\mathbf{E}$.

An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define $V = 0$ at $r_A = \infty$, the electric potential due to a point charge at any distance $r$ from the charge is

$$V = k_e \frac{q}{r} \tag{25.11}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The **potential energy associated with a pair of point charges** separated by a distance $r_{12}$ is

$$U = k_e \frac{q_1 q_2}{r_{12}} \tag{25.13}$$

This energy represents the work required to bring the charges from an infinite separation to the separation $r_{12}$. We obtain the potential energy of a distribution of point charges by summing terms like Equation 25.13 over all pairs of particles.

**TABLE 25.1 Electric Potential Due to Various Charge Distributions**

| Charge Distribution | Electric Potential | Location |
| --- | --- | --- |
| Uniformly charged ring of radius $a$ | $V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$ | Along perpendicular central axis of ring, distance $x$ from ring center |
| Uniformly charged disk of radius $a$ | $V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$ | Along perpendicular central axis of disk, distance $x$ from disk center |
| Uniformly charged, *insulating* solid sphere of radius $R$ and total charge $Q$ | $V = k_e \frac{Q}{r}$ | $r \geq R$ |
| | $V = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$ | $r < R$ |
| Isolated *conducting* sphere of radius $R$ and total charge $Q$ | $V = k_e \frac{Q}{r}$ | $r > R$ |
| | $V = k_e \frac{Q}{R}$ | $r \leq R$ |

If we know the electric potential as a function of coordinates $x$, $y$, $z$, we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the $x$ component of the electric field is

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

The **electric potential due to a continuous charge distribution** is

$$V = k_e \int \frac{dq}{r} \tag{25.19}$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

Table 25.1 lists electric potentials due to several charge distributions.