PUZZLER

This airplane is used by NASA for astronaut training. When it flies along a certain curved path, anything inside the plane that is not strapped down begins to float. What causes this strange effect? *(NASA)*

web

For more information on microgravity in general and on this airplane, visit **http://microgravity.msfc.nasa.gov/** and **http://www.jsc.nasa.gov/coop/kc135/kc135.html**

chapter

4 Motion in Two Dimensions

Chapter Outline

In this chapter we deal with the kinematics of a particle moving in two dimensions. Knowing the basics of two-dimensional motion will allow us to examine—in future chapters—a wide variety of motions, ranging from the motion of satellites in orbit to the motion of electrons in a uniform electric field. We begin by studying in greater detail the vector nature of displacement, velocity, and acceleration. As in the case of one-dimensional motion, we derive the kinematic equations for two-dimensional motion from the fundamental definitions of these three quantities. We then treat projectile motion and uniform circular motion as special cases of motion in two dimensions. We also discuss the concept of relative motion, which shows why observers in different frames of reference may measure different displacements, velocities, and accelerations for a given particle.

4.1 THE DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

In Chapter 2 we found that the motion of a particle moving along a straight line is completely known if its position is known as a function of time. Now let us extend this idea to motion in the *xy* plane. We begin by describing the position of a particle by its position vector **r**, drawn from the origin of some coordinate system to the particle located in the *xy* plane, as in Figure 4.1. At time t_i the particle is at point Ⓐ, and at some later time t_f it is at point Ⓑ. The path from Ⓐ to Ⓑ is not necessarily a straight line. As the particle moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$, its position vector changes from $\mathbf{r}_i$ to $\mathbf{r}_f$. As we learned in Chapter 2, displacement is a vector, and the displacement of the particle is the difference between its final position and its initial position. We now formally define the **displacement vector Δr** for the particle of Figure 4.1 as being the difference between its final position vector and its initial position vector:

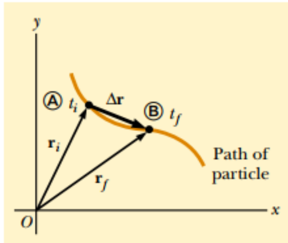


Figure 4.1 A particle moving in the *xy* plane is located with the position vector **r** drawn from the origin to the particle. The displacement of the particle as it moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$ is equal to the vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$.

$$\Delta\mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i \quad (4.1)$$

Displacement vector

The direction of $\Delta\mathbf{r}$ is indicated in Figure 4.1. As we see from the figure, the magnitude of $\Delta\mathbf{r}$ is *less* than the distance traveled along the curved path followed by the particle.

As we saw in Chapter 2, it is often useful to quantify motion by looking at the ratio of a displacement divided by the time interval during which that displacement occurred. In two-dimensional (or three-dimensional) kinematics, everything is the same as in one-dimensional kinematics except that we must now use vectors rather than plus and minus signs to indicate the direction of motion.

We define the **average velocity** of a particle during the time interval Δt as the displacement of the particle divided by that time interval:

$$\bar{\mathbf{v}} \equiv \frac{\Delta\mathbf{r}}{\Delta t} \quad (4.2)$$

Average velocity

Multiplying or dividing a vector quantity by a scalar quantity changes only the magnitude of the vector, not its direction. Because displacement is a vector quantity and the time interval is a scalar quantity, we conclude that the average velocity is a vector quantity directed along $\Delta\mathbf{r}$.

Note that the average velocity between points is *independent of the path* taken. This is because average velocity is proportional to displacement, which depends

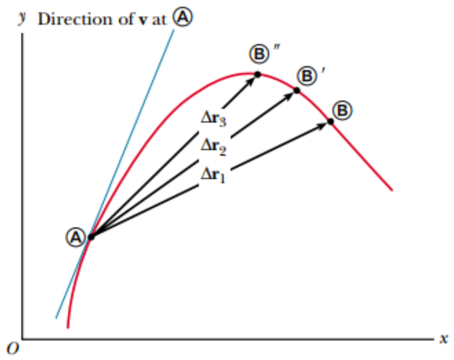


Figure 4.2 As a particle moves between two points, its average velocity is in the direction of the displacement vector $\Delta\mathbf{r}$. As the end point of the path is moved from Ⓑ to Ⓑ′ to Ⓑ″, the respective displacements and corresponding time intervals become smaller and smaller. In the limit that the end point approaches Ⓐ, Δt approaches zero, and the direction of $\Delta\mathbf{r}$ approaches that of the line tangent to the curve at Ⓐ. By definition, the instantaneous velocity at Ⓐ is in the direction of this tangent line.

only on the initial and final position vectors and not on the path taken. As we did with one-dimensional motion, we conclude that if a particle starts its motion at some point and returns to this point via any path, its average velocity is zero for this trip because its displacement is zero.

Consider again the motion of a particle between two points in the *xy* plane, as shown in Figure 4.2. As the time interval over which we observe the motion becomes smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at Ⓐ.

Instantaneous velocity

The **instantaneous velocity v** is defined as the limit of the average velocity $\Delta\mathbf{r}/\Delta t$ as Δt approaches zero:

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \qquad \textbf{(4.3)}$$

That is, the instantaneous velocity equals the derivative of the position vector with respect to time. The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion (Fig. 4.3).

The magnitude of the instantaneous velocity vector $v = |\mathbf{v}|$ is called the *speed,* which, as you should remember, is a scalar quantity.

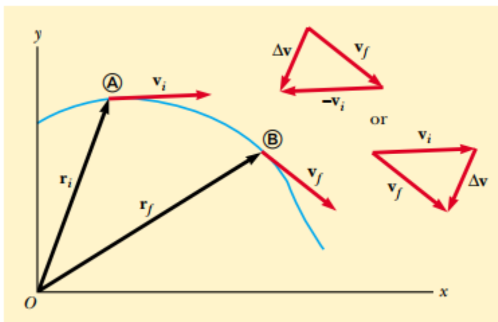


Figure 4.3 A particle moves from position Ⓐ to position Ⓑ. Its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. The vector diagrams at the upper right show two ways of determining the vector $\Delta\mathbf{v}$ from the initial and final velocities.

As a particle moves from one point to another along some path, its instantaneous velocity vector changes from $\mathbf{v}_i$ at time t_i to $\mathbf{v}_f$ at time t_f. Knowing the velocity at these points allows us to determine the average acceleration of the particle:

The **average acceleration** of a particle as it moves from one position to another is defined as the change in the instantaneous velocity vector $\Delta\mathbf{v}$ divided by the time Δt during which that change occurred:

$$\overline{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta\mathbf{v}}{\Delta t} \tag{4.4}$$

Average acceleration

Because it is the ratio of a vector quantity $\Delta\mathbf{v}$ and a scalar quantity Δt, we conclude that average acceleration $\overline{\mathbf{a}}$ is a vector quantity directed along $\Delta\mathbf{v}$. As indicated in Figure 4.3, the direction of $\Delta\mathbf{v}$ is found by adding the vector $-\mathbf{v}_i$ (the negative of $\mathbf{v}_i$) to the vector $\mathbf{v}_f$, because by definition $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$.

When the average acceleration of a particle changes during different time intervals, it is useful to define its instantaneous acceleration $\mathbf{a}$:

The **instantaneous acceleration a** is defined as the limiting value of the ratio $\Delta\mathbf{v}/\Delta t$ as Δt approaches zero:

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \tag{4.5}$$

Instantaneous acceleration

3.5 In other words, the instantaneous acceleration equals the derivative of the velocity vector with respect to time.

It is important to recognize that various changes can occur when a particle accelerates. First, the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion. Second, the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (two-dimensional) motion. Finally, both the magnitude and the direction of the velocity vector may change simultaneously.

Quick Quiz 4.1

The gas pedal in an automobile is called the *accelerator.* (a) Are there any other controls in an automobile that can be considered accelerators? (b) When is the gas pedal not an accelerator?

4.2 TWO-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Let us consider two-dimensional motion during which the acceleration remains constant in both magnitude and direction.

The position vector for a particle moving in the xy plane can be written

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \tag{4.6}$$

where x, y, and $\mathbf{r}$ change with time as the particle moves while $\mathbf{i}$ and $\mathbf{j}$ remain constant. If the position vector is known, the velocity of the particle can be obtained from Equations 4.3 and 4.6, which give

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} \tag{4.7}$$

Because **a** is assumed constant, its components a_x and a_y also are constants. Therefore, we can apply the equations of kinematics to the x and y components of the velocity vector. Substituting $v_{xf} = v_{xi} + a_x t$ and $v_{yf} = v_{yi} + a_y t$ into Equation 4.7 to determine the final velocity at any time t, we obtain

Velocity vector as a function of time

$$\begin{aligned}\mathbf{v}_f &= (v_{xi} + a_x t)\mathbf{i} + (v_{yi} + a_y t)\mathbf{j} \\ &= (v_{xi}\mathbf{i} + v_{yi}\mathbf{j}) + (a_x\mathbf{i} + a_y\mathbf{j})t \\ \mathbf{v}_f &= \mathbf{v}_i + \mathbf{a}t \qquad \textbf{(4.8)}\end{aligned}$$

This result states that the velocity of a particle at some time t equals the vector sum of its initial velocity $\mathbf{v}_i$ and the additional velocity $\mathbf{a}t$ acquired in the time t as a result of constant acceleration.

Similarly, from Equation 2.11 we know that the x and y coordinates of a particle moving with constant acceleration are

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

Substituting these expressions into Equation 4.6 (and labeling the final position vector $\mathbf{r}_f$) gives

Position vector as a function of time

$$\begin{aligned}\mathbf{r}_f &= (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\mathbf{i} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\mathbf{j} \\ &= (x_i\mathbf{i} + y_i\mathbf{j}) + (v_{xi}\mathbf{i} + v_{yi}\mathbf{j})t + \tfrac{1}{2}(a_x\mathbf{i} + a_y\mathbf{j})t^2 \\ \mathbf{r}_f &= \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \textbf{(4.9)}\end{aligned}$$

This equation tells us that the displacement vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$ is the vector sum of a displacement $\mathbf{v}_i t$ arising from the initial velocity of the particle and a displacement $\frac{1}{2}\mathbf{a}t^2$ resulting from the uniform acceleration of the particle.

Graphical representations of Equations 4.8 and 4.9 are shown in Figure 4.4. For simplicity in drawing the figure, we have taken $\mathbf{r}_i = 0$ in Figure 4.4a. That is, we assume the particle is at the origin at $t = t_i = 0$. Note from Figure 4.4a that $\mathbf{r}_f$ is generally not along the direction of either $\mathbf{v}_i$ or $\mathbf{a}$ because the relationship between these quantities is a vector expression. For the same reason, from Figure 4.4b we see that $\mathbf{v}_f$ is generally not along the direction of $\mathbf{v}_i$ or $\mathbf{a}$. Finally, note that $\mathbf{v}_f$ and $\mathbf{r}_f$ are generally not in the same direction.

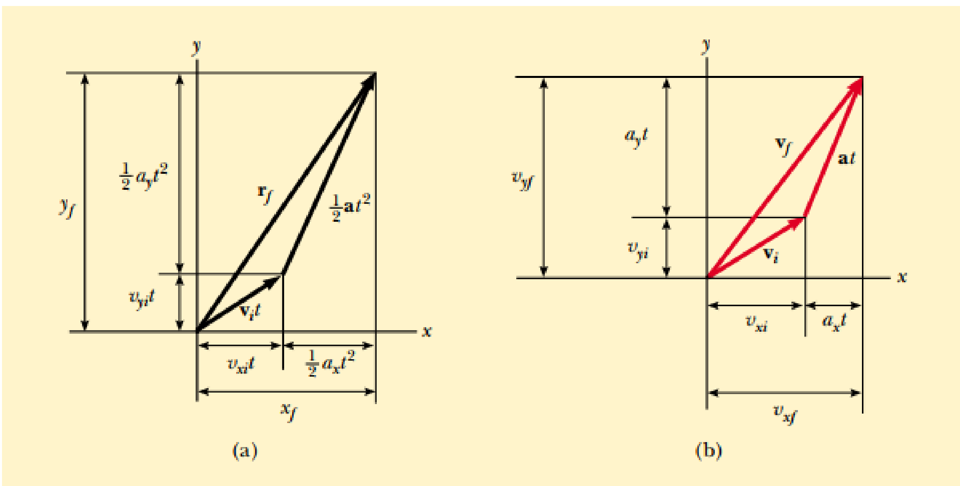


Figure 4.4 Vector representations and components of (a) the displacement and (b) the velocity of a particle moving with a uniform acceleration **a**. To simplify the drawing, we have set $\mathbf{r}_i = 0$.

Because Equations 4.8 and 4.9 are vector expressions, we may write them in component form:

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases} \tag{4.8a}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \begin{cases} x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \end{cases} \tag{4.9a}$$

These components are illustrated in Figure 4.4. The component form of the equations for $\mathbf{v}_f$ and $\mathbf{r}_f$ show us that two-dimensional motion at constant acceleration is equivalent to two *independent* motions—one in the x direction and one in the y direction—having constant accelerations a_x and a_y.

EXAMPLE 4.1 Motion in a Plane

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0\ \text{m/s}^2$. (a) Determine the components of the velocity vector at any time and the total velocity vector at any time.

Solution After carefully reading the problem, we realize we can set $v_{xi} = 20$ m/s, $v_{yi} = -15$ m/s, $a_x = 4.0\ \text{m/s}^2$, and $a_y = 0$. This allows us to sketch a rough motion diagram of the situation. The x component of velocity starts at 20 m/s and increases by 4.0 m/s every second. The y component of velocity never changes from its initial value of -15 m/s. From this information we sketch some velocity vectors as shown in Figure 4.5. Note that the spacing between successive images increases as time goes on because the velocity is increasing.

The equations of kinematics give

$$v_{xf} = v_{xi} + a_x t = (20 + 4.0t)\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_y t = -15\ \text{m/s} + 0 = -15\ \text{m/s}$$

Therefore,

$$\mathbf{v}_f = v_{xf}\mathbf{i} + v_{yf}\mathbf{j} = \boxed{[(20 + 4.0t)\mathbf{i} - 15\mathbf{j}]\ \text{m/s}}$$

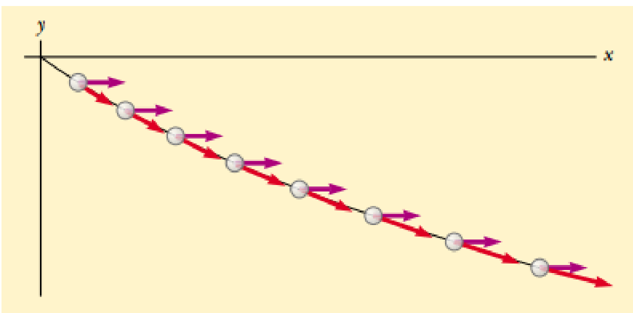


Figure 4.5 Motion diagram for the particle.

We could also obtain this result using Equation 4.8 directly, noting that $\mathbf{a} = 4.0\mathbf{i}\ \text{m/s}^2$ and $\mathbf{v}_i = (20\mathbf{i} - 15\mathbf{j})$ m/s. According to this result, the x component of velocity increases while the y component remains constant; this is consistent with what we predicted. After a long time, the x component will be so great that the y component will be negligible. If we were to extend the object's path in Figure 4.5, eventually it would become nearly parallel to the x axis. It is always helpful to make comparisons between final answers and initial stated conditions.

(b) Calculate the velocity and speed of the particle at $t = 5.0$ s.

Solution With $t = 5.0$ s, the result from part (a) gives

$$\mathbf{v}_f = \{[20 + 4.0(5.0)]\mathbf{i} - 15\mathbf{j}\}\ \text{m/s} = \boxed{(40\mathbf{i} - 15\mathbf{j})\ \text{m/s}}$$

This result tells us that at $t = 5.0$ s, $v_{xf} = 40$ m/s and $v_{yf} = -15$ m/s. Knowing these two components for this two-dimensional motion, we can find both the direction and the magnitude of the velocity vector. To determine the angle θ that $\mathbf{v}$ makes with the x axis at $t = 5.0$ s, we use the fact that $\tan\theta = v_{yf}/v_{xf}$:

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\ \text{m/s}}{40\ \text{m/s}}\right) = \boxed{-21°}$$

where the minus sign indicates an angle of 21° below the positive x axis. The speed is the magnitude of $\mathbf{v}_f$:

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(40)^2 + (-15)^2}\ \text{m/s} = \boxed{43\ \text{m/s}}$$

In looking over our result, we notice that if we calculate v_i from the x and y components of $\mathbf{v}_i$, we find that $v_f > v_i$. Does this make sense?

(c) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution Because $x_i = y_i = 0$ at $t = 0$, Equation 2.11 gives

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = \boxed{(20t + 2.0t^2)\text{ m}}$$

$$y_f = v_{yi}t = \boxed{(-15t)\text{ m}}$$

Therefore, the position vector at any time t is

$$\mathbf{r}_f = x_f\mathbf{i} + y_f\mathbf{j} = \boxed{[(20t + 2.0t^2)\mathbf{i} - 15t\mathbf{j}]\text{ m}}$$

(Alternatively, we could obtain $\mathbf{r}_f$ by applying Equation 4.9 directly, with $\mathbf{v}_i = (20\mathbf{i} - 15\mathbf{j})$ m/s and $\mathbf{a} = 4.0\mathbf{i}$ m/s^2. Try it!) Thus, for example, at $t = 5.0$ s, $x = 150$ m, $y = -75$ m, and $\mathbf{r}_f = (150\mathbf{i} - 75\mathbf{j})$ m. The magnitude of the displacement of the particle from the origin at $t = 5.0$ s is the magnitude of $\mathbf{r}_f$ at this time:

$$r_f = |\mathbf{r}_f| = \sqrt{(150)^2 + (-75)^2}\text{ m} = 170\text{ m}$$

Note that this is *not* the distance that the particle travels in this time! Can you determine this distance from the available data?

4.3 PROJECTILE MOTION

Assumptions of projectile motion

Anyone who has observed a baseball in motion (or, for that matter, any other object thrown into the air) has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration $\mathbf{g}$ is constant over the range of motion and is directed downward,[1] and (2) the effect of air resistance is negligible.[2] With these assumptions, we find that the path of a projectile, which we call its *trajectory,* is *always* a parabola. **We use these assumptions throughout this chapter.**

To show that the trajectory of a projectile is a parabola, let us choose our reference frame such that the y direction is vertical and positive is upward. Because air resistance is neglected, we know that $a_y = -g$ (as in one-dimensional free fall) and that $a_x = 0$. Furthermore, let us assume that at $t = 0$, the projectile leaves the origin ($x_i = y_i = 0$) with speed v_i, as shown in Figure 4.6. The vector $\mathbf{v}_i$ makes an angle θ_i with the horizontal, where θ_i is the angle at which the projectile leaves the origin. From the definitions of the cosine and sine functions we have

3.5

$$\cos\theta_i = v_{xi}/v_i \qquad \sin\theta_i = v_{yi}/v_i$$

Therefore, the initial x and y components of velocity are

$$v_{xi} = v_i\cos\theta_i \qquad v_{yi} = v_i\sin\theta_i$$

Substituting the x component into Equation 4.9a with $x_i = 0$ and $a_x = 0$, we find that

Horizontal position component

$$x_f = v_{xi}t = (v_i\cos\theta_i)t \tag{4.10}$$

Repeating with the y component and using $y_i = 0$ and $a_y = -g$, we obtain

Vertical position component

$$y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = (v_i\sin\theta_i)t - \tfrac{1}{2}gt^2 \tag{4.11}$$

Next, we solve Equation 4.10 for $t = x_f/(v_i\cos\theta_i)$ and substitute this expression for t into Equation 4.11; this gives

$$y = (\tan\theta_i)x - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x^2 \tag{4.12}$$

[1] This assumption is reasonable as long as the range of motion is small compared with the radius of the Earth (6.4×10^6 m). In effect, this assumption is equivalent to assuming that the Earth is flat over the range of motion considered.

[2] This assumption is generally *not* justified, especially at high velocities. In addition, any spin imparted to a projectile, such as that applied when a pitcher throws a curve ball, can give rise to some very interesting effects associated with aerodynamic forces, which will be discussed in Chapter 15.

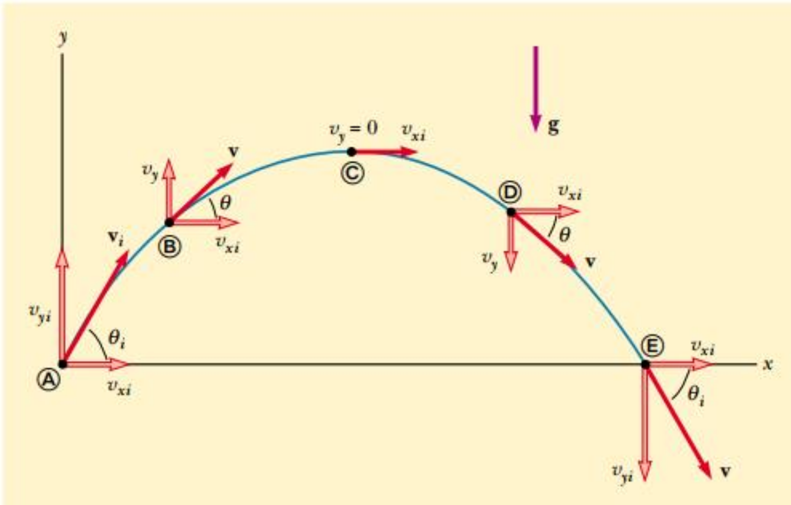
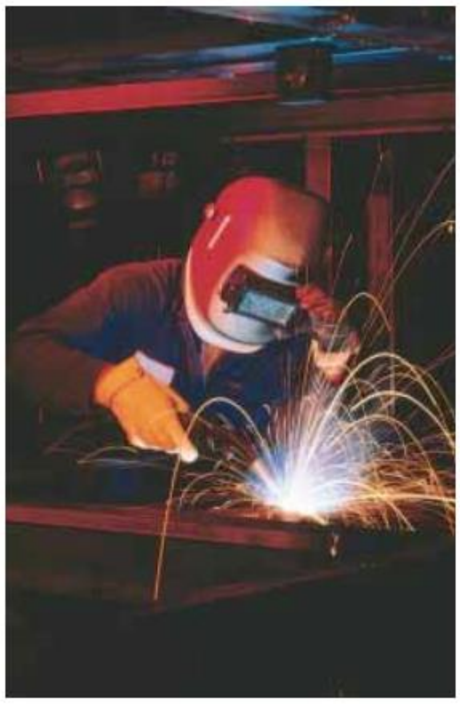


Figure 4.6 The parabolic path of a projectile that leaves the origin with a velocity $\mathbf{v}_i$. The velocity vector $\mathbf{v}$ changes with time in both magnitude and direction. This change is the result of acceleration in the negative y direction. The x component of velocity remains constant in time because there is no acceleration along the horizontal direction. The y component of velocity is zero at the peak of the path.



A welder cuts holes through a heavy metal construction beam with a hot torch. The sparks generated in the process follow parabolic paths.

This equation is valid for launch angles in the range $0 < \theta_i < \pi/2$. We have left the subscripts off the x and y because the equation is valid for any point (x, y) along the path of the projectile. The equation is of the form $y = ax - bx^2$, which is the equation of a parabola that passes through the origin. Thus, we have shown that the trajectory of a projectile is a parabola. Note that the trajectory is completely specified if both the initial speed v_i and the launch angle θ_i are known.

The vector expression for the position vector of the projectile as a function of time follows directly from Equation 4.9, with $\mathbf{r}_i = 0$ and $\mathbf{a} = \mathbf{g}$:

$$\mathbf{r} = \mathbf{v}_i t + \tfrac{1}{2}\mathbf{g}t^2$$

This expression is plotted in Figure 4.7.

QuickLab

Place two tennis balls at the edge of a tabletop. Sharply snap one ball horizontally off the table with one hand while gently tapping the second ball off with your other hand. Compare how long it takes the two to reach the floor. Explain your results.

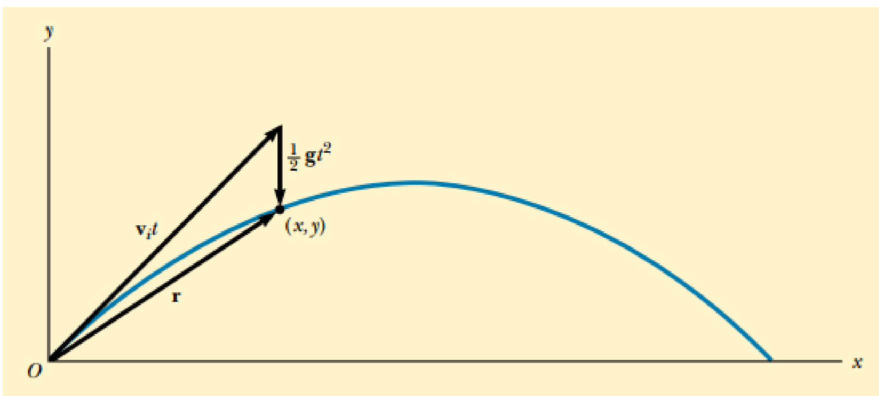


Figure 4.7 The position vector $\mathbf{r}$ of a projectile whose initial velocity at the origin is $\mathbf{v}_i$. The vector $\mathbf{v}_i t$ would be the displacement of the projectile if gravity were absent, and the vector $\frac{1}{2}\mathbf{g}t^2$ is its vertical displacement due to its downward gravitational acceleration.

Multiflash exposure of a tennis player executing a forehand swing. Note that the ball follows a parabolic path characteristic of a projectile. Such photographs can be used to study the quality of sports equipment and the performance of an athlete.

It is interesting to realize that the motion of a particle can be considered the superposition of the term $\mathbf{v}_i t$, the displacement if no acceleration were present, and the term $\frac{1}{2}\mathbf{g}t^2$, which arises from the acceleration due to gravity. In other words, if there were no gravitational acceleration, the particle would continue to move along a straight path in the direction of $\mathbf{v}_i$. Therefore, the vertical distance $\frac{1}{2}\mathbf{g}t^2$ through which the particle "falls" off the straight-line path is the same distance that a freely falling body would fall during the same time interval. We conclude that **projectile motion is the superposition of two motions:** (1) **constant-velocity motion in the horizontal direction** and (2) **free-fall motion in the vertical direction.** Except for t, the time of flight, the horizontal and vertical components of a projectile's motion are completely independent of each other.

EXAMPLE 4.2 Approximating Projectile Motion

A ball is thrown in such a way that its initial vertical and horizontal components of velocity are 40 m/s and 20 m/s, respectively. Estimate the total time of flight and the distance the ball is from its starting point when it lands.

Solution We start by remembering that the two velocity components are independent of each other. By considering the vertical motion first, we can determine how long the ball remains in the air. Then, we can use the time of flight to estimate the horizontal distance covered.

A motion diagram like Figure 4.8 helps us organize what we know about the problem. The acceleration vectors are all the same, pointing downward with a magnitude of nearly 10 m/s^2. The velocity vectors change direction. Their hori-

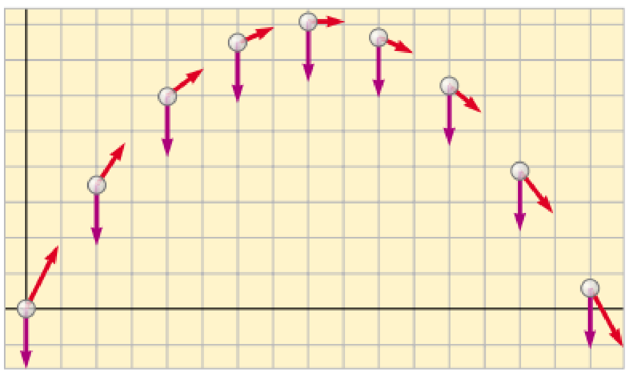


Figure 4.8 Motion diagram for a projectile.

zontal components are all the same: 20 m/s. Because the vertical motion is free fall, the vertical components of the velocity vectors change, second by second, from 40 m/s to roughly 30, 20, and 10 m/s in the upward direction, and then to 0 m/s. Subsequently, its velocity becomes 10, 20, 30, and 40 m/s in the downward direction. Thus it takes the ball about 4 s to go up and another 4 s to come back down, for a total time of flight of approximately 8 s. Because the horizontal component of velocity is 20 m/s, and because the ball travels at this speed for 8 s, it ends up approximately 160 m from its starting point.

Horizontal Range and Maximum Height of a Projectile

Let us assume that a projectile is fired from the origin at $t_i = 0$ with a positive v_{yi} component, as shown in Figure 4.9. Two points are especially interesting to analyze: the peak point Ⓐ, which has cartesian coordinates $(R/2, h)$, and the point Ⓑ, which has coordinates $(R, 0)$. The distance R is called the *horizontal range* of the projectile, and the distance h is its *maximum height.* Let us find h and R in terms of v_i, θ_i, and g.

We can determine h by noting that at the peak, $v_{yA} = 0$. Therefore, we can use Equation 4.8a to determine the time t_A it takes the projectile to reach the peak:

$$v_{yf} = v_{yi} + a_y t$$

$$0 = v_i \sin\theta_i - g t_A$$

$$t_A = \frac{v_i \sin\theta_i}{g}$$

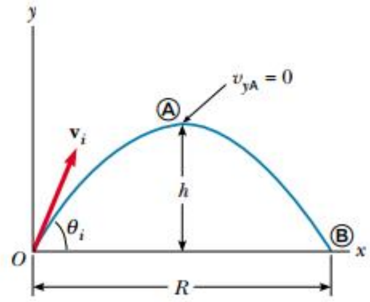


Figure 4.9 A projectile fired from the origin at $t_i = 0$ with an initial velocity $\mathbf{v}_i$. The maximum height of the projectile is h, and the horizontal range is R. At Ⓐ, the peak of the trajectory, the particle has coordinates $(R/2, h)$.

Substituting this expression for t_A into the y part of Equation 4.9a and replacing $y_f = y_A$ with h, we obtain an expression for h in terms of the magnitude and direction of the initial velocity vector:

$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \tfrac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g} \qquad \textbf{(4.13)}$$

Maximum height of projectile

The range R is the horizontal distance that the projectile travels in twice the time it takes to reach its peak, that is, in a time $t_B = 2t_A$. Using the x part of Equation 4.9a, noting that $v_{xi} = v_{xB} = v_i \cos\theta_i$, and setting $R \equiv x_B$ at $t = 2t_A$, we find that

$$R = v_{xi}t_B = (v_i \cos\theta_i)2t_A$$

$$= (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Using the identity $\sin 2\theta = 2\sin\theta\cos\theta$ (see Appendix B.4), we write R in the more compact form

$$R = \frac{v_i^2 \sin 2\theta_i}{g} \qquad \textbf{(4.14)}$$

Range of projectile

Keep in mind that Equations 4.13 and 4.14 are useful for calculating h and R only if v_i and θ_i are known (which means that only $\mathbf{v}_i$ has to be specified) and if the projectile lands at the same height from which it started, as it does in Figure 4.9.

The maximum value of R from Equation 4.14 is $R_{max} = v_i^2/g$. This result follows from the fact that the maximum value of $\sin 2\theta_i$ is 1, which occurs when $2\theta_i = 90°$. Therefore, R is a maximum when $\theta_i = 45°$.

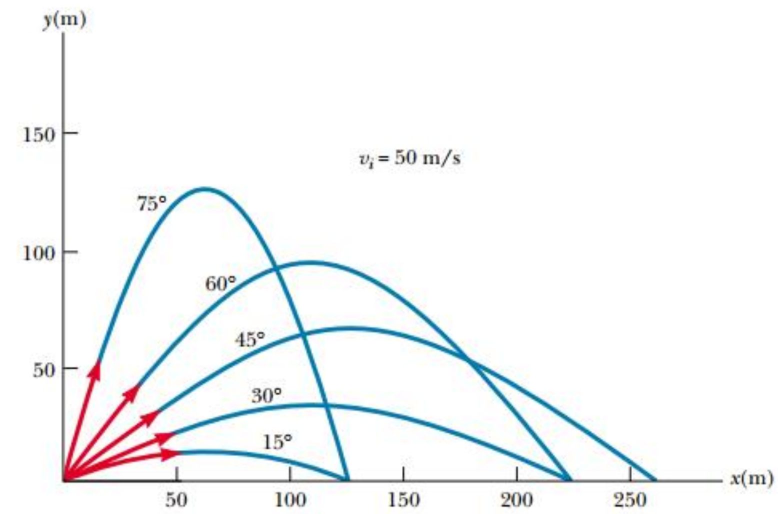


Figure 4.10 A projectile fired from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ_i result in the same value of x (range of the projectile).

QuickLab

To carry out this investigation, you need to be outdoors with a small ball, such as a tennis ball, as well as a wristwatch. Throw the ball straight up as hard as you can and determine the initial speed of your throw and the approximate maximum height of the ball, using only your watch. What happens when you throw the ball at some angle $\theta \neq 90°$? Does this change the time of flight (perhaps because it is easier to throw)? Can you still determine the maximum height and initial speed?

Figure 4.10 illustrates various trajectories for a projectile having a given initial speed but launched at different angles. As you can see, the range is a maximum for $\theta_i = 45°$. In addition, for any θ_i other than 45°, a point having cartesian coordinates $(R, 0)$ can be reached by using either one of two complementary values of θ_i, such as 75° and 15°. Of course, the maximum height and time of flight for one of these values of θ_i are different from the maximum height and time of flight for the complementary value.

Quick Quiz 4.2

As a projectile moves in its parabolic path, is there any point along the path where the velocity and acceleration vectors are (a) perpendicular to each other? (b) parallel to each other? (c) Rank the five paths in Figure 4.10 with respect to time of flight, from the shortest to the longest.

Problem-Solving Hints

Projectile Motion

We suggest that you use the following approach to solving projectile motion problems:

- Select a coordinate system and resolve the initial velocity vector into x and y components.
- Follow the techniques for solving constant-velocity problems to analyze the horizontal motion. Follow the techniques for solving constant-acceleration problems to analyze the vertical motion. The x and y motions share the same time of flight t.

EXAMPLE 4.3 The Long-Jump

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s. (a) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)

Solution Because the initial speed and launch angle are given, the most direct way of solving this problem is to use the range formula given by Equation 4.14. However, it is more instructive to take a more general approach and use Figure 4.9. As before, we set our origin of coordinates at the takeoff point and label the peak as Ⓐ and the landing point as Ⓑ. The horizontal motion is described by Equation 4.10:

$$x_f = x_{\text{B}} = (v_i \cos \theta_i) t_{\text{B}} = (11.0 \text{ m/s})(\cos 20.0°) t_{\text{B}}$$

The value of x_{B} can be found if the total time of the jump is known. We are able to find t_{B} by remembering that $a_y = -g$ and by using the y part of Equation 4.8a. We also note that at the top of the jump the vertical component of velocity $v_{y\text{A}}$ is zero:

$$v_{yf} = v_{y\text{A}} = v_i \sin \theta_i - g t_{\text{A}}$$

$$0 = (11.0 \text{ m/s}) \sin 20.0° - (9.80 \text{ m/s}^2) t_{\text{A}}$$

$$t_{\text{A}} = 0.384 \text{ s}$$

This is the time needed to reach the *top* of the jump. Because of the symmetry of the vertical motion, an identical time interval passes before the jumper returns to the ground. Therefore, the *total time* in the air is $t_{\text{B}} = 2t_{\text{A}} = 0.768$ s. Substituting this value into the above expression for x_f gives

$$x_f = x_{\text{B}} = (11.0 \text{ m/s})(\cos 20.0°)(0.768 \text{ s}) = \boxed{7.94 \text{ m}}$$

This is a reasonable distance for a world-class athlete.

(b) What is the maximum height reached?

Solution We find the maximum height reached by using Equation 4.11:

$$\begin{aligned} y_{\max} = y_{\text{A}} &= (v_i \sin \theta_i) t_{\text{A}} - \tfrac{1}{2} g t_{\text{A}}^2 \\ &= (11.0 \text{ m/s})(\sin 20.0°)(0.384 \text{ s}) \\ &\quad - \tfrac{1}{2}(9.80 \text{ m/s}^2)(0.384 \text{ s})^2 \\ &= \boxed{0.722 \text{ m}} \end{aligned}$$

Treating the long-jumper as a particle is an oversimplification. Nevertheless, the values obtained are reasonable.

Exercise To check these calculations, use Equations 4.13 and 4.14 to find the maximum height and horizontal range.

In a long-jump event, 1993 United States champion Mike Powell can leap horizontal distances of at least 8 m.

EXAMPLE 4.4 A Bull's-Eye Every Time

In a popular lecture demonstration, a projectile is fired at a target in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as shown in Figure 4.11. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.

Solution We can argue that a collision results under the conditions stated by noting that, as soon as they are released, the projectile and the target experience the same acceleration $a_y = -g$. First, note from Figure 4.11b that the initial y coordinate of the target is $x_{\text{T}} \tan \theta_i$ and that it falls through a distance $\frac{1}{2}gt^2$ in a time t. Therefore, the y coordinate of the target at any moment after release is

$$y_{\text{T}} = x_{\text{T}} \tan \theta_i - \tfrac{1}{2} g t^2$$

Now if we use Equation 4.9a to write an expression for the y coordinate of the projectile at any moment, we obtain

$$y_{\text{P}} = x_{\text{P}} \tan \theta_i - \tfrac{1}{2} g t^2$$

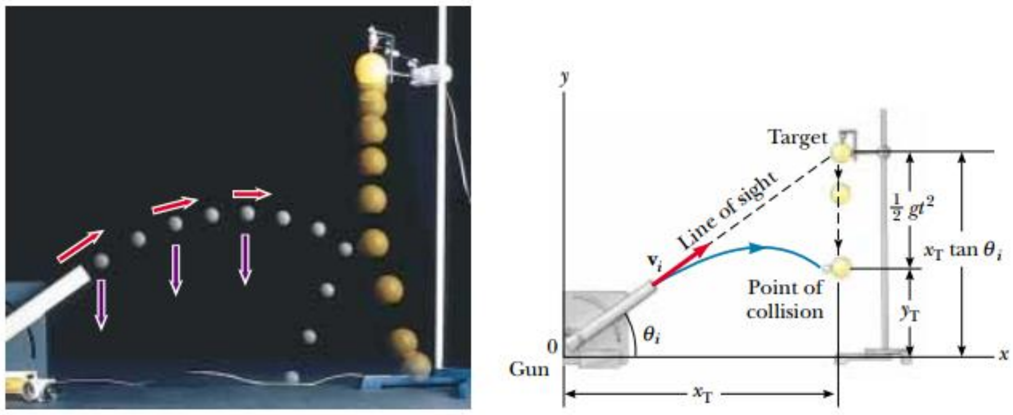


Figure 4.11 (a) Multiflash photograph of projectile–target demonstration. If the gun is aimed directly at the target and is fired at the same instant the target begins to fall, the projectile will hit the target. Note that the velocity of the projectile (red arrows) changes in direction and magnitude, while the downward acceleration (violet arrows) remains constant. *(Central Scientific Company.)* (b) Schematic diagram of the projectile–target demonstration. Both projectile and target fall through the same vertical distance in a time t because both experience the same acceleration $a_y = -g$.

Thus, by comparing the two previous equations, we see that when the y coordinates of the projectile and target are the same, their x coordinates are the same and a collision results. That is, when $y_P = y_T$, $x_P = x_T$. You can obtain the same result, using expressions for the position vectors for the projectile and target.

Note that a collision will *not* always take place owing to a further restriction: A collision can result only when $v_i \sin\theta_i \geq \sqrt{gd/2}$, where d is the initial elevation of the target above the *floor.* If $v_i \sin\theta_i$ is less than this value, the projectile will strike the floor before reaching the target.

EXAMPLE 4.5 That's Quite an Arm!

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as shown in Figure 4.12. If the height of the building is 45.0 m, (a) how long is it before the stone hits the ground?

Solution We have indicated the various parameters in Figure 4.12. When working problems on your own, you should always make a sketch such as this and label it.

The initial x and y components of the stone's velocity are

$$v_{xi} = v_i \cos\theta_i = (20.0 \text{ m/s})(\cos 30.0°) = 17.3 \text{ m/s}$$

$$v_{yi} = v_i \sin\theta_i = (20.0 \text{ m/s})(\sin 30.0°) = 10.0 \text{ m/s}$$

To find t, we can use $y_f = v_{yi}t + \frac{1}{2}a_yt^2$ (Eq. 4.9a) with $y_f = -45.0$ m, $a_y = -g$, and $v_{yi} = 10.0$ m/s (there is a minus sign on the numerical value of y_f because we have chosen the top of the building as the origin):

$$-45.0 \text{ m} = (10.0 \text{ m/s})t - \tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t =$ 4.22 s. Does the negative root have any physical

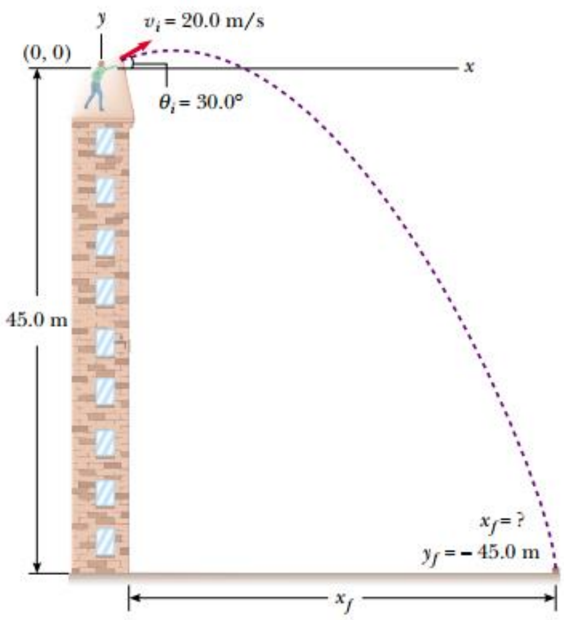


Figure 4.12

meaning? (Can you think of another way of finding t from the information given?)

(b) What is the speed of the stone just before it strikes the ground?

Solution We can use Equation 4.8a, $v_{yf} = v_{yi} + a_y t$, with $t = 4.22$ s to obtain the y component of the velocity just before the stone strikes the ground:

$$v_{yf} = 10.0 \text{ m/s} - (9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.4 \text{ m/s}$$

The negative sign indicates that the stone is moving downward. Because $v_{xf} = v_{xi} = 17.3$ m/s, the required speed is

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m/s} = \boxed{35.9 \text{ m/s}}$$

Exercise Where does the stone strike the ground?

Answer 73.0 m from the base of the building.

EXAMPLE 4.6 The Stranded Explorers

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in Figure 4.13. If the plane is traveling horizontally at 40.0 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it was released?

Solution For this problem we choose the coordinate system shown in Figure 4.13, in which the origin is at the point of release of the package. Consider first the horizontal motion of the package. The only equation available to us for finding the distance traveled along the horizontal direction is $x_f = v_{xi}t$ (Eq. 4.9a). The initial x component of the package velocity is the same as that of the plane when the package is released: 40.0 m/s. Thus, we have

$$x_f = (40.0 \text{ m/s})t$$

If we know t, the length of time the package is in the air, then we can determine x_f, the distance the package travels in the horizontal direction. To find t, we use the equations that describe the vertical motion of the package. We know that at the instant the package hits the ground, its y coordinate is $y_f = -100$ m. We also know that the initial vertical component of the package velocity v_{yi} is zero because at the moment of release, the package had only a horizontal component of velocity.

From Equation 4.9a, we have

$$y_f = -\tfrac{1}{2}gt^2$$

$$-100 \text{ m} = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

$$t = 4.52 \text{ s}$$

Substitution of this value for the time of flight into the equation for the x coordinate gives

$$x_f = (40.0 \text{ m/s})(4.52 \text{ s}) = \boxed{181 \text{ m}}$$

The package hits the ground 181 m to the right of the drop point.

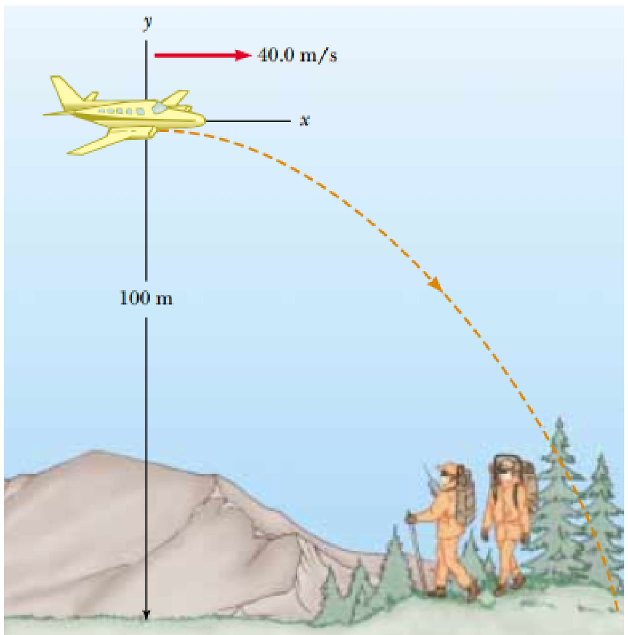


Figure 4.13

Exercise What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

Answer $v_{xf} = 40.0$ m/s; $v_{yf} = -44.3$ m/s.

Exercise Where is the plane when the package hits the ground? (Assume that the plane does not change its speed or course.)

Answer Directly over the package.

EXAMPLE 4.7 The End of the Ski Jump

A ski jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure 4.14. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline?

Solution It is reasonable to expect the skier to be airborne for less than 10 s, and so he will not go farther than 250 m horizontally. We should expect the value of d, the distance traveled along the incline, to be of the same order of magnitude. It is convenient to select the beginning of the jump as the origin ($x_i = 0, y_i = 0$). Because $v_{xi} = 25.0$ m/s and $v_{yi} = 0$, the x and y component forms of Equation 4.9a are

$$(1)\qquad x_f = v_{xi}t = (25.0 \text{ m/s})t$$

$$(2)\qquad y_f = \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

From the right triangle in Figure 4.14, we see that the jumper's x and y coordinates at the landing point are $x_f = d\cos 35.0°$ and $y_f = -d\sin 35.0°$. Substituting these relationships into (1) and (2), we obtain

$$(3)\qquad d\cos 35.0° = (25.0 \text{ m/s})t$$

$$(4)\qquad -d\sin 35.0° = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving (3) for t and substituting the result into (4), we find that $d = 109$ m. Hence, the x and y coordinates of the point at which he lands are

$$x_f = d\cos 35.0° = (109 \text{ m})\cos 35.0° = \boxed{89.3 \text{ m}}$$

$$y_f = -d\sin 35.0° = -(109 \text{ m})\sin 35.0° = \boxed{-62.5 \text{ m}}$$

Exercise Determine how long the jumper is airborne and his vertical component of velocity just before he lands.

Answer 3.57 s; -35.0 m/s.

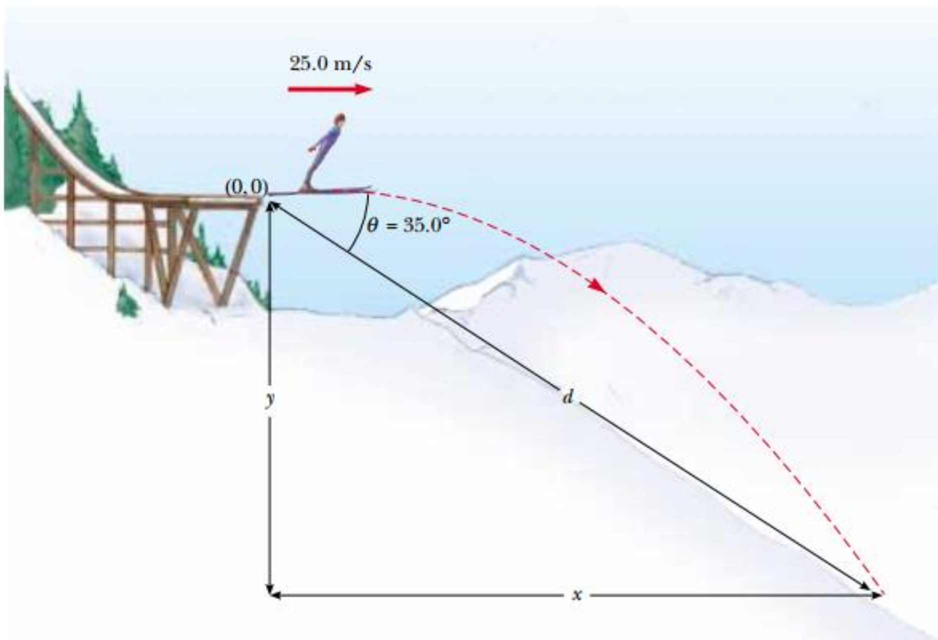


Figure 4.14

What would have occurred if the skier in the last example happened to be carrying a stone and let go of it while in midair? Because the stone has the same initial velocity as the skier, it will stay near him as he moves—that is, it floats alongside him. This is a technique that NASA uses to train astronauts. The plane pictured at the beginning of the chapter flies in the same type of projectile path that the skier and stone follow. The passengers and cargo in the plane fall along-

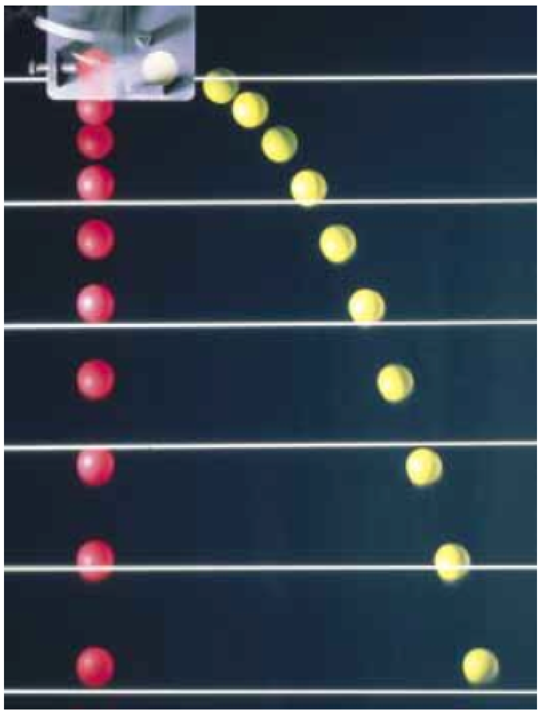


Figure 4.15 This multiflash photograph of two balls released simultaneously illustrates both free fall (red ball) and projectile motion (yellow ball). The yellow ball was projected horizontally, while the red ball was released from rest. *(Richard Megna/Fundamental Photographs)*

QuickLab

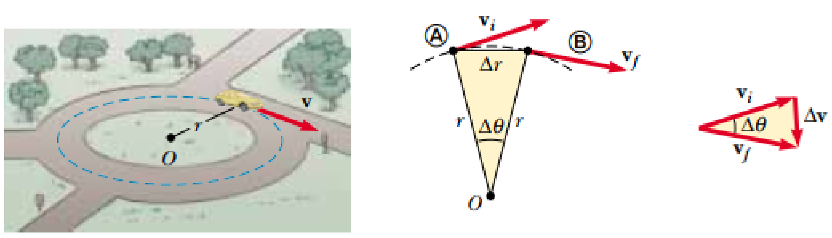
Armed with nothing but a ruler and the knowledge that the time between images was 1/30 s, find the horizontal speed of the yellow ball in Figure 4.15. (*Hint:* Start by analyzing the motion of the red ball. Because you know its vertical acceleration, you can calibrate the distances depicted in the photograph. Then you can find the horizontal speed of the yellow ball.)

side each other; that is, they have the same trajectory. An astronaut can release a piece of equipment and it will float freely alongside her hand. The same thing happens in the space shuttle. The craft and everything in it are falling as they orbit the Earth.

4.4 UNIFORM CIRCULAR MOTION

3.6 Figure 4.16a shows a car moving in a circular path with constant linear speed v. Such motion is called **uniform circular motion.** Because the car's direction of motion changes, the car has an acceleration, as we learned in Section 4.1. For any motion, the velocity vector is tangent to the path. Consequently, when an object moves in a circular path, its velocity vector is perpendicular to the radius of the circle.

We now show that the acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An ac-



(a) (b) (c)

Figure 4.16 (a) A car moving along a circular path at constant speed experiences uniform circular motion. (b) As a particle moves from Ⓐ to Ⓑ, its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. (c) The construction for determining the direction of the change in velocity $\Delta\mathbf{v}$, which is toward the center of the circle for small $\Delta\mathbf{r}$.

celeration of this nature is called a **centripetal** (center-seeking) acceleration, and its magnitude is

$$a_r = \frac{v^2}{r} \tag{4.15}$$

where r is the radius of the circle and the notation a_r is used to indicate that the centripetal acceleration is along the radial direction.

To derive Equation 4.15, consider Figure 4.16b, which shows a particle first at point Ⓐ and then at point Ⓑ. The particle is at Ⓐ at time t_i, and its velocity at that time is $\mathbf{v}_i$. It is at Ⓑ at some later time t_f, and its velocity at that time is $\mathbf{v}_f$. Let us assume here that $\mathbf{v}_i$ and $\mathbf{v}_f$ differ only in direction; their magnitudes (speeds) are the same (that is, $v_i = v_f = v$). To calculate the acceleration of the particle, let us begin with the defining equation for average acceleration (Eq. 4.4):

$$\overline{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta\mathbf{v}}{\Delta t}$$

This equation indicates that we must subtract $\mathbf{v}_i$ from $\mathbf{v}_f$, being sure to treat them as vectors, where $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ is the change in the velocity. Because $\mathbf{v}_i + \Delta\mathbf{v} = \mathbf{v}_f$, we can find the vector $\Delta\mathbf{v}$, using the vector triangle in Figure 4.16c.

Now consider the triangle in Figure 4.16b, which has sides Δr and r. This triangle and the one in Figure 4.16c, which has sides Δv and v, are similar. This fact enables us to write a relationship between the lengths of the sides:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

This equation can be solved for Δv and the expression so obtained substituted into $\overline{a} = \Delta v/\Delta t$ (Eq. 4.4) to give

$$\overline{a} = \frac{v\,\Delta r}{r\,\Delta t}$$

Now imagine that points Ⓐ and Ⓑ in Figure 4.16b are extremely close together. In this case $\Delta\mathbf{v}$ points toward the center of the circular path, and because the acceleration is in the direction of $\Delta\mathbf{v}$, it too points toward the center. Furthermore, as Ⓐ and Ⓑ approach each other, Δt approaches zero, and the ratio $\Delta r/\Delta t$ approaches the speed v. Hence, in the limit $\Delta t \to 0$, the magnitude of the acceleration is

$$a_r = \frac{v^2}{r}$$

Thus, we conclude that in uniform circular motion, the acceleration is directed toward the center of the circle and has a magnitude given by v^2/r, where v is the speed of the particle and r is the radius of the circle. You should be able to show that the dimensions of a_r are L/T^2. We shall return to the discussion of circular motion in Section 6.1.

4.5 TANGENTIAL AND RADIAL ACCELERATION

3.6 Now let us consider a particle moving along a curved path where the velocity changes both in direction and in magnitude, as shown in Figure 4.17. As is always the case, the velocity vector is tangent to the path, but now the direction of the ac-

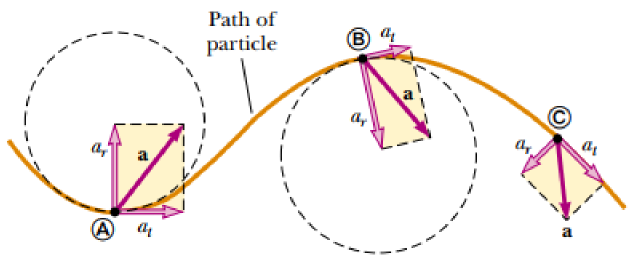


Figure 4.17 The motion of a particle along an arbitrary curved path lying in the xy plane. If the velocity vector **v** (always tangent to the path) changes in direction and magnitude, the component vectors of the acceleration **a** are a tangential component a_t and a radial component a_r.

celeration vector **a** changes from point to point. This vector can be resolved into two component vectors: a radial component vector $\mathbf{a}_r$ and a tangential component vector $\mathbf{a}_t$. Thus, **a** can be written as the vector sum of these component vectors:

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \tag{4.16}$$

Total acceleration

The tangential acceleration causes the change in the speed of the particle. It is parallel to the instantaneous velocity, and its magnitude is

$$a_t = \frac{d|\mathbf{v}|}{dt} \tag{4.17}$$

Tangential acceleration

The radial acceleration arises from the change in direction of the velocity vector as described earlier and has an absolute magnitude given by

$$a_r = \frac{v^2}{r} \tag{4.18}$$

Radial acceleration

where r is the radius of curvature of the path at the point in question. Because $\mathbf{a}_r$ and $\mathbf{a}_t$ are mutually perpendicular component vectors of **a**, it follows that $a = \sqrt{a_r^2 + a_t^2}$. As in the case of uniform circular motion, $\mathbf{a}_r$ in nonuniform circular motion always points toward the center of curvature, as shown in Figure 4.17. Also, at a given speed, a_r is large when the radius of curvature is small (as at points Ⓐ and Ⓑ in Figure 4.17) and small when r is large (such as at point Ⓒ). The direction of $\mathbf{a}_t$ is either in the same direction as **v** (if v is increasing) or opposite **v** (if v is decreasing).

In uniform circular motion, where v is constant, $a_t = 0$ and the acceleration is always completely radial, as we described in Section 4.4. (*Note:* Eq. 4.18 is identical to Eq. 4.15.) In other words, uniform circular motion is a special case of motion along a curved path. Furthermore, if the direction of **v** does not change, then there is no radial acceleration and the motion is one-dimensional (in this case, $a_r = 0$, but a_t may not be zero).

Quick Quiz 4.3

(a) Draw a motion diagram showing velocity and acceleration vectors for an object moving with constant speed counterclockwise around a circle. Draw similar diagrams for an object moving counterclockwise around a circle but (b) slowing down at constant tangential acceleration and (c) speeding up at constant tangential acceleration.

It is convenient to write the acceleration of a particle moving in a circular path in terms of unit vectors. We do this by defining the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ shown in

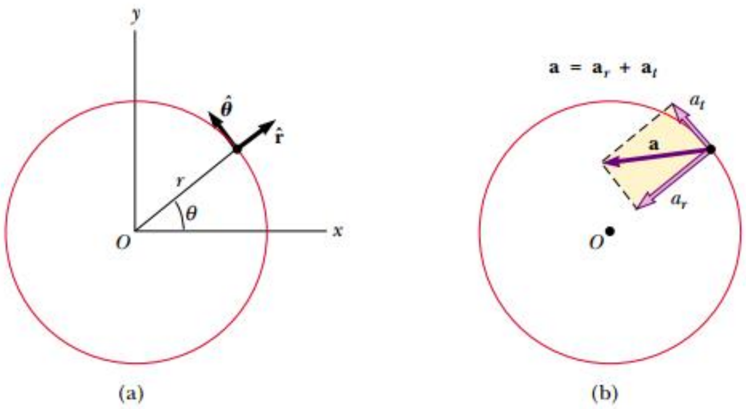


Figure 4.18 (a) Descriptions of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$. (b) The total acceleration **a** of a particle moving along a curved path (which at any instant is part of a circle of radius r) is the sum of radial and tangential components. The radial component is directed toward the center of curvature. If the tangential component of acceleration becomes zero, the particle follows uniform circular motion.

Figure 4.18a, where $\hat{\mathbf{r}}$ is a unit vector lying along the radius vector and directed radially outward from the center of the circle and $\hat{\boldsymbol{\theta}}$ is a unit vector tangent to the circle. The direction of $\hat{\boldsymbol{\theta}}$ is in the direction of increasing θ, where θ is measured counterclockwise from the positive x axis. Note that both $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ "move along with the particle" and so vary in time. Using this notation, we can express the total acceleration as

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r = \frac{d|\mathbf{v}|}{dt}\hat{\boldsymbol{\theta}} - \frac{v^2}{r}\hat{\mathbf{r}} \qquad \textbf{(4.19)}$$

These vectors are described in Figure 4.18b. The negative sign on the v^2/r term in Equation 4.19 indicates that the radial acceleration is always directed radially inward, *opposite* $\hat{\mathbf{r}}$.

Quick Quiz 4.4

Based on your experience, draw a motion diagram showing the position, velocity, and acceleration vectors for a pendulum that, from an initial position 45° to the right of a central vertical line, swings in an arc that carries it to a final position 45° to the left of the central vertical line. The arc is part of a circle, and you should use the center of this circle as the origin for the position vectors.

EXAMPLE 4.8 The Swinging Ball

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as shown in Figure 4.19. When the string makes an angle $\theta = 20°$ with the vertical, the ball has a speed of 1.5 m/s. (a) Find the magnitude of the radial component of acceleration at this instant.

Solution The diagram from the answer to Quick Quiz 4.4 (p. 109) applies to this situation, and so we have a good idea of how the acceleration vector varies during the motion. Figure 4.19 lets us take a closer look at the situation. The radial acceleration is given by Equation 4.18. With $v = 1.5$ m/s and $r = 0.50$ m, we find that

$$a_r = \frac{v^2}{r} = \frac{(1.5\ \text{m/s})^2}{0.50\ \text{m}} = \boxed{4.5\ \text{m/s}^2}$$

(b) What is the magnitude of the tangential acceleration when $\theta = 20°$?

Solution When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude $g \sin \theta$ (the component of **g** tangent to the circle). Therefore, at $\theta = 20°$,

$a_t = g \sin 20° = \boxed{3.4 \text{ m/s}^2}.$

(c) Find the magnitude and direction of the total acceleration **a** at $\theta = 20°$.

Solution Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of **a** at $\theta = 20°$ is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2} \text{ m/s}^2 = \boxed{5.6 \text{ m/s}^2}$$

If ϕ is the angle between **a** and the string, then

$$\phi = \tan^{-1} \frac{a_t}{a_r} = \tan^{-1} \left(\frac{3.4 \text{ m/s}^2}{4.5 \text{ m/s}^2} \right) = \boxed{37°}$$

Note that **a**, $\mathbf{a}_t$, and $\mathbf{a}_r$ all change in direction *and* magnitude as the ball swings through the circle. When the ball is at its lowest elevation ($\theta = 0$), $a_t = 0$ because there is no tangential component of **g** at this angle; also, a_r is a *maximum* because v is a maximum. If the ball has enough speed to reach its highest position ($\theta = 180°$), then a_t is again zero but a_r is a minimum because v is now a minimum. Finally, in the two horizontal positions ($\theta = 90°$ and $270°$), $|\mathbf{a}_t| = g$ and a_r has a value between its minimum and maximum values.

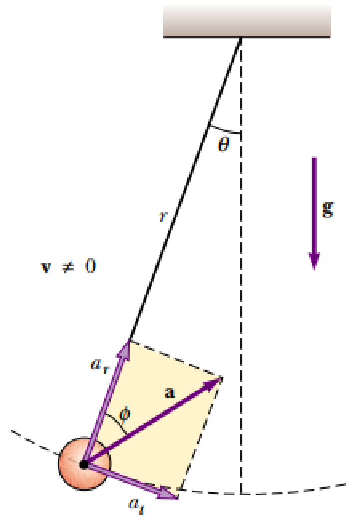


Figure 4.19 Motion of a ball suspended by a string of length r. The ball swings with nonuniform circular motion in a vertical plane, and its acceleration **a** has a radial component a_r and a tangential component a_t.

4.6 RELATIVE VELOCITY AND RELATIVE ACCELERATION

In this section, we describe how observations made by different observers in different frames of reference are related to each other. We find that observers in different frames of reference may measure different displacements, velocities, and accelerations for a given particle. That is, two observers moving relative to each other generally do not agree on the outcome of a measurement.

3.7 For example, suppose two cars are moving in the same direction with speeds of 50 mi/h and 60 mi/h. To a passenger in the slower car, the speed of the faster car is 10 mi/h. Of course, a stationary observer will measure the speed of the faster car to be 60 mi/h, not 10 mi/h. Which observer is correct? They both are! This simple example demonstrates that the velocity of an object depends on the frame of reference in which it is measured.

Suppose a person riding on a skateboard (observer A) throws a ball in such a way that it appears in this person's frame of reference to move first straight upward and then straight downward along the same vertical line, as shown in Figure 4.20a. A stationary observer B sees the path of the ball as a parabola, as illustrated in Figure 4.20b. Relative to observer B, the ball has a vertical component of velocity (resulting from the initial upward velocity and the downward acceleration of gravity) *and* a horizontal component.

Another example of this concept that of is a package dropped from an airplane flying with a constant velocity; this is the situation we studied in Example 4.6. An observer on the airplane sees the motion of the package as a straight line toward the Earth. The stranded explorer on the ground, however, sees the trajectory of the package as a parabola. If, once it drops the package, the airplane con-

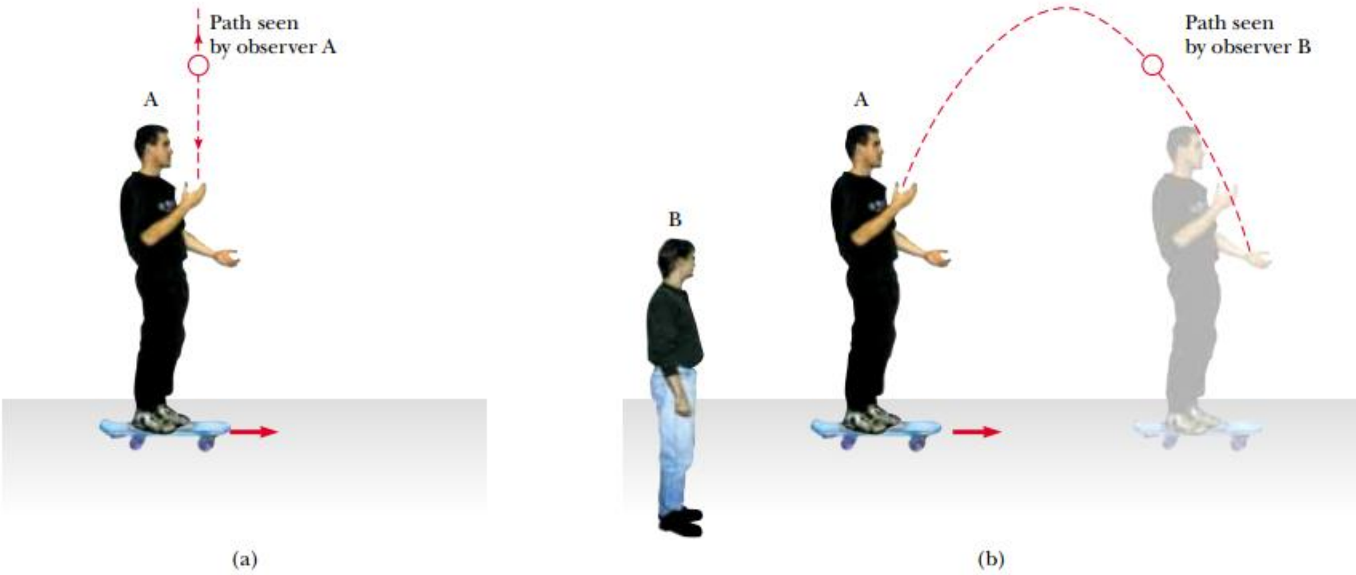


Figure 4.20 (a) Observer A on a moving vehicle throws a ball upward and sees it rise and fall in a straight-line path. (b) Stationary observer B sees a parabolic path for the same ball.

tinues to move horizontally with the same velocity, then the package hits the ground directly beneath the airplane (if we assume that air resistance is neglected)!

In a more general situation, consider a particle located at point Ⓐ in Figure 4.21. Imagine that the motion of this particle is being described by two observers, one in reference frame S, fixed relative to the Earth, and another in reference frame S', moving to the right relative to S (and therefore relative to the Earth) with a constant velocity $\mathbf{v}_0$. (Relative to an observer in S', S moves to the left with a velocity $-\mathbf{v}_0$.) Where an observer stands in a reference frame is irrelevant in this discussion, but for purposes of this discussion let us place each observer at her or his respective origin.

We label the position of the particle relative to the S frame with the position vector $\mathbf{r}$ and that relative to the S' frame with the position vector $\mathbf{r}'$, both after some time t. The vectors $\mathbf{r}$ and $\mathbf{r}'$ are related to each other through the expression $\mathbf{r} = \mathbf{r}' + \mathbf{v}_0 t$, or

Galilean coordinate transformation

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t \tag{4.20}$$

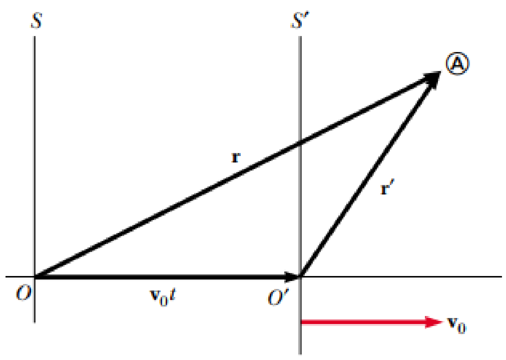
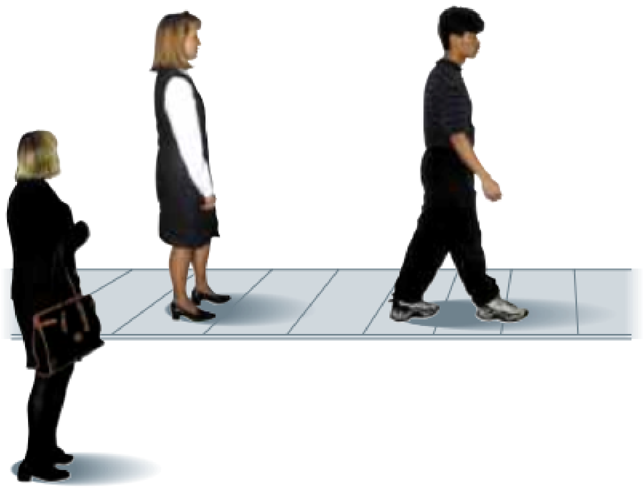


Figure 4.21 A particle located at Ⓐ is described by two observers, one in the fixed frame of reference S, and the other in the frame S', which moves to the right with a constant velocity $\mathbf{v}_0$. The vector $\mathbf{r}$ is the particle's position vector relative to S, and $\mathbf{r}'$ is its position vector relative to S'.



The woman standing on the beltway sees the walking man pass by at a slower speed than the woman standing on the stationary floor does.

That is, after a time t, the S' frame is displaced to the right of the S frame by an amount $\mathbf{v}_0 t$.

If we differentiate Equation 4.20 with respect to time and note that $\mathbf{v}_0$ is constant, we obtain

$$\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{v}_0$$

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0 \qquad \textbf{(4.21)}$$

Galilean velocity transformation

where $\mathbf{v}'$ is the velocity of the particle observed in the S' frame and $\mathbf{v}$ is its velocity observed in the S frame. Equations 4.20 and 4.21 are known as **Galilean transformation equations.** They relate the coordinates and velocity of a particle as measured in a frame fixed relative to the Earth to those measured in a frame moving with uniform motion relative to the Earth.

Although observers in two frames measure different velocities for the particle, they measure the *same acceleration* when $\mathbf{v}_0$ is constant. We can verify this by taking the time derivative of Equation 4.21:

$$\frac{d\mathbf{v}'}{dt} = \frac{d\mathbf{v}}{dt} - \frac{d\mathbf{v}_0}{dt}$$

Because $\mathbf{v}_0$ is constant, $d\mathbf{v}_0/dt = 0$. Therefore, we conclude that $\mathbf{a}' = \mathbf{a}$ because $\mathbf{a}' = d\mathbf{v}'/dt$ and $\mathbf{a} = d\mathbf{v}/dt$. That is, **the acceleration of the particle measured by an observer in the Earth's frame of reference is the same as that measured by any other observer moving with constant velocity relative to the Earth's frame.**

Quick Quiz 4.5

A passenger in a car traveling at 60 mi/h pours a cup of coffee for the tired driver. Describe the path of the coffee as it moves from a Thermos bottle into a cup as seen by (a) the passenger and (b) someone standing beside the road and looking in the window of the car as it drives past. (c) What happens if the car accelerates while the coffee is being poured?

EXAMPLE 4.9 A Boat Crossing a River

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth. Determine the velocity of the boat relative to an observer standing on either bank.

Solution We know $\mathbf{v}_{br}$, the velocity of the *boat* relative to the *river*, and $\mathbf{v}_{rE}$, the velocity of the *river* relative to the *Earth*. What we need to find is $\mathbf{v}_{bE}$, the velocity of the *boat* relative to the *Earth*. The relationship between these three quantities is

$$\mathbf{v}_{bE} = \mathbf{v}_{br} + \mathbf{v}_{rE}$$

The terms in the equation must be manipulated as vector quantities; the vectors are shown in Figure 4.22. The quantity $\mathbf{v}_{br}$ is due north, $\mathbf{v}_{rE}$ is due east, and the vector sum of the two, $\mathbf{v}_{bE}$, is at an angle θ, as defined in Figure 4.22. Thus, we can find the speed v_{bE} of the boat relative to the Earth by using the Pythagorean theorem:

$$v_{bE} = \sqrt{v_{br}^2 + v_{rE}^2} = \sqrt{(10.0)^2 + (5.00)^2}\ \text{km/h}$$

$$= \boxed{11.2\ \text{km/h}}$$

The direction of $\mathbf{v}_{bE}$ is

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6°$$

The boat is moving at a speed of 11.2 km/h in the direction 26.6° east of north relative to the Earth.

Exercise If the width of the river is 3.0 km, find the time it takes the boat to cross it.

Answer 18 min.

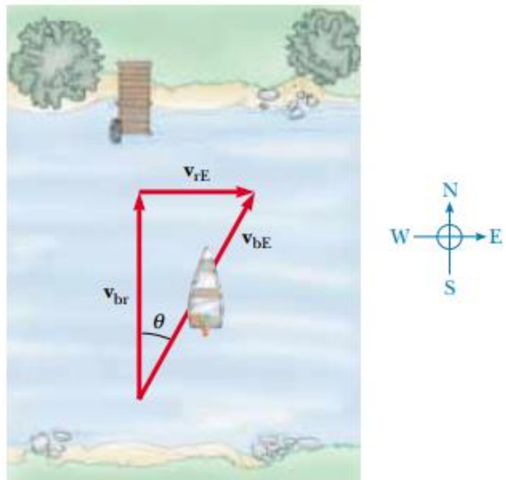


Figure 4.22

EXAMPLE 4.10 Which Way Should We Head?

If the boat of the preceding example travels with the same speed of 10.0 km/h relative to the river and is to travel due north, as shown in Figure 4.23, what should its heading be?

Solution As in the previous example, we know $\mathbf{v}_{rE}$ and the magnitude of the vector $\mathbf{v}_{br}$, and we want $\mathbf{v}_{bE}$ to be directed across the river. Figure 4.23 shows that the boat must head upstream in order to travel directly northward across the river. Note the difference between the triangle in Figure 4.22 and the one in Figure 4.23—specifically, that the hypotenuse in Figure 4.23 is no longer $\mathbf{v}_{bE}$. Therefore, when we use the Pythagorean theorem to find $\mathbf{v}_{bE}$ this time, we obtain

$$v_{bE} = \sqrt{v_{br}^2 - v_{rE}^2} = \sqrt{(10.0)^2 - (5.00)^2}\ \text{km/h} = 8.66\ \text{km/h}$$

Now that we know the magnitude of $\mathbf{v}_{bE}$, we can find the direction in which the boat is heading:

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{bE}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = \boxed{30.0°}$$

The boat must steer a course 30.0° west of north.

Exercise If the width of the river is 3.0 km, find the time it takes the boat to cross it.

Answer 21 min.

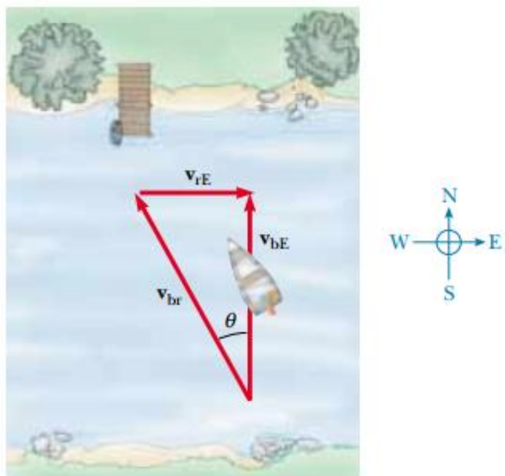


Figure 4.23

SUMMARY

If a particle moves with *constant* acceleration $\mathbf{a}$ and has velocity $\mathbf{v}_i$ and position $\mathbf{r}_i$ at $t = 0$, its velocity and position vectors at some later time t are

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \quad \textbf{(4.8)}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \quad \textbf{(4.9)}$$

For two-dimensional motion in the xy plane under constant acceleration, each of these vector expressions is equivalent to two component expressions—one for the motion in the x direction and one for the motion in the y direction. You should be able to break the two-dimensional motion of any object into these two components.

Projectile motion is one type of two-dimensional motion under constant acceleration, where $a_x = 0$ and $a_y = -g$. It is useful to think of projectile motion as the superposition of two motions: (1) constant-velocity motion in the x direction and (2) free-fall motion in the vertical direction subject to a constant downward acceleration of magnitude $g = 9.80\ \text{m/s}^2$. You should be able to analyze the motion in terms of separate horizontal and vertical components of velocity, as shown in Figure 4.24.

A particle moving in a circle of radius r with constant speed v is in **uniform circular motion**. It undergoes a centripetal (or radial) acceleration $\mathbf{a}_r$ because the direction of $\mathbf{v}$ changes in time. The magnitude of $\mathbf{a}_r$ is

$$a_r = \frac{v^2}{r} \quad \textbf{(4.18)}$$

and its direction is always toward the center of the circle.

If a particle moves along a curved path in such a way that both the magnitude and the direction of $\mathbf{v}$ change in time, then the particle has an acceleration vector that can be described by two component vectors: (1) a radial component vector $\mathbf{a}_r$ that causes the change in direction of $\mathbf{v}$ and (2) a tangential component vector $\mathbf{a}_t$ that causes the change in magnitude of $\mathbf{v}$. The magnitude of $\mathbf{a}_r$ is v^2/r, and the magnitude of $\mathbf{a}_t$ is $d|\mathbf{v}|/dt$. You should be able to sketch motion diagrams for an object following a curved path and show how the velocity and acceleration vectors change as the object's motion varies.

The velocity $\mathbf{v}$ of a particle measured in a fixed frame of reference S can be related to the velocity $\mathbf{v}'$ of the same particle measured in a moving frame of reference S' by

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0 \quad \textbf{(4.21)}$$

where $\mathbf{v}_0$ is the velocity of S' relative to S. You should be able to translate back and forth between different frames of reference.

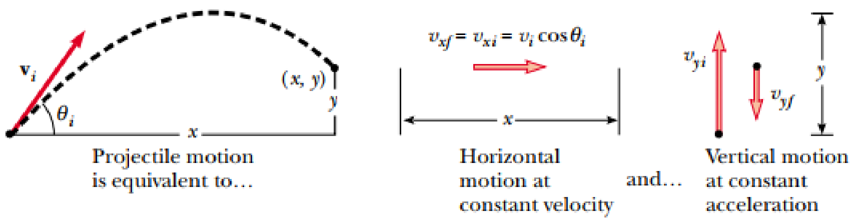


Figure 4.24 Analyzing projectile motion in terms of horizontal and vertical components.

QUESTIONS

1. Can an object accelerate if its speed is constant? Can an object accelerate if its velocity is constant?
2. If the average velocity of a particle is zero in some time interval, what can you say about the displacement of the particle for that interval?
3. If you know the position vectors of a particle at two points along its path and also know the time it took to get from one point to the other, can you determine the particle's instantaneous velocity? Its average velocity? Explain.
4. Describe a situation in which the velocity of a particle is always perpendicular to the position vector.
5. Explain whether or not the following particles have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.
6. Correct the following statement: "The racing car rounds the turn at a constant velocity of 90 mi/h."
7. Determine which of the following moving objects have an approximately parabolic trajectory: (a) a ball thrown in an arbitrary direction, (b) a jet airplane, (c) a rocket leaving the launching pad, (d) a rocket whose engines fail a few minutes after launch, (e) a tossed stone moving to the bottom of a pond.
8. A rock is dropped at the same instant that a ball at the same initial elevation is thrown horizontally. Which will have the greater speed when it reaches ground level?
9. A spacecraft drifts through space at a constant velocity. Suddenly, a gas leak in the side of the spacecraft causes a constant acceleration of the spacecraft in a direction perpendicular to the initial velocity. The orientation of the spacecraft does not change, and so the acceleration remains perpendicular to the original direction of the velocity. What is the shape of the path followed by the spacecraft in this situation?
10. A ball is projected horizontally from the top of a building. One second later another ball is projected horizontally from the same point with the same velocity. At what point in the motion will the balls be closest to each other? Will the first ball always be traveling faster than the second ball? How much time passes between the moment the first ball hits the ground and the moment the second one hits the ground? Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?
11. A student argues that as a satellite orbits the Earth in a circular path, the satellite moves with a constant velocity and therefore has no acceleration. The professor claims that the student is wrong because the satellite must have a centripetal acceleration as it moves in its circular orbit. What is wrong with the student's argument?
12. What is the fundamental difference between the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ and the unit vectors $\mathbf{i}$ and $\mathbf{j}$?
13. At the end of its arc, the velocity of a pendulum is zero. Is its acceleration also zero at this point?
14. If a rock is dropped from the top of a sailboat's mast, will it hit the deck at the same point regardless of whether the boat is at rest or in motion at constant velocity?
15. A stone is thrown upward from the top of a building. Does the stone's displacement depend on the location of the origin of the coordinate system? Does the stone's velocity depend on the location of the origin?
16. Is it possible for a vehicle to travel around a curve without accelerating? Explain.
17. A baseball is thrown with an initial velocity of $(10\mathbf{i} + 15\mathbf{j})$ m/s. When it reaches the top of its trajectory, what are (a) its velocity and (b) its acceleration? Neglect the effect of air resistance.
18. An object moves in a circular path with constant speed v. (a) Is the velocity of the object constant? (b) Is its acceleration constant? Explain.
19. A projectile is fired at some angle to the horizontal with some initial speed v_i, and air resistance is neglected. Is the projectile a freely falling body? What is its acceleration in the vertical direction? What is its acceleration in the horizontal direction?
20. A projectile is fired at an angle of 30° from the horizontal with some initial speed. Firing at what other projectile angle results in the same range if the initial speed is the same in both cases? Neglect air resistance.
21. A projectile is fired on the Earth with some initial velocity. Another projectile is fired on the Moon with the same initial velocity. If air resistance is neglected, which projectile has the greater range? Which reaches the greater altitude? (Note that the free-fall acceleration on the Moon is about 1.6 m/s^2.)
22. As a projectile moves through its parabolic trajectory, which of these quantities, if any, remain constant: (a) speed, (b) acceleration, (c) horizontal component of velocity, (d) vertical component of velocity?
23. A passenger on a train that is moving with constant velocity drops a spoon. What is the acceleration of the spoon relative to (a) the train and (b) the Earth?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
□ = paired numerical/symbolic problems

Section 4.1 The Displacement, Velocity, and Acceleration Vectors

WEB 1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the total vector displacement, (b) the average speed, and (c) the average velocity. Use a coordinate system in which east is the positive x axis.

2. Suppose that the position vector for a particle is given as $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, with $x = at + b$ and $y = ct^2 + d$, where $a = 1.00$ m/s, $b = 1.00$ m, $c = 0.125$ m/s^2, and $d = 1.00$ m. (a) Calculate the average velocity during the time interval from $t = 2.00$ s to $t = 4.00$ s. (b) Determine the velocity and the speed at $t = 2.00$ s.

3. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates versus time are given by the following expressions:

$$x = (18.0 \text{ m/s})t$$

and

$$y = (4.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

(a) Write a vector expression for the ball's position as a function of time, using the unit vectors **i** and **j**. By taking derivatives of your results, write expressions for (b) the velocity vector as a function of time and (c) the acceleration vector as a function of time. Now use unit vector notation to write expressions for (d) the position, (e) the velocity, and (f) the acceleration of the ball, all at $t = 3.00$ s.

4. The coordinates of an object moving in the xy plane vary with time according to the equations

$$x = -(5.00 \text{ m}) \sin \omega t$$

and

$$y = (4.00 \text{ m}) - (5.00 \text{ m})\cos \omega t$$

where t is in seconds and ω has units of seconds^{-1}. (a) Determine the components of velocity and components of acceleration at $t = 0$. (b) Write expressions for the position vector, the velocity vector, and the acceleration vector at any time $t > 0$. (c) Describe the path of the object on an xy graph.

Section 4.2 Two-Dimensional Motion with Constant Acceleration

5. At $t = 0$, a particle moving in the xy plane with constant acceleration has a velocity of $\mathbf{v}_i = (3.00\mathbf{i} - 2.00\mathbf{j})$ m/s when it is at the origin. At $t = 3.00$ s, the particle's velocity is $\mathbf{v} = (9.00\mathbf{i} + 7.00\mathbf{j})$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

6. The vector position of a particle varies in time according to the expression $\mathbf{r} = (3.00\mathbf{i} - 6.00t^2\mathbf{j})$ m. (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at $t = 1.00$ s.

7. A fish swimming in a horizontal plane has velocity $\mathbf{v}_i = (4.00\mathbf{i} + 1.00\mathbf{j})$ m/s at a point in the ocean whose displacement from a certain rock is $\mathbf{r}_i = (10.0\mathbf{i} - 4.00\mathbf{j})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\mathbf{v} = (20.0\mathbf{i} - 5.00\mathbf{j})$ m/s. (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to the unit vector **i**? (c) Where is the fish at $t = 25.0$ s if it maintains its original acceleration and in what direction is it moving?

8. A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00\mathbf{j}$ m/s^2 and an initial velocity of $\mathbf{v}_i = 5.00\mathbf{i}$ m/s. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00$ s.

Section 4.3 Projectile Motion

(Neglect air resistance in all problems and take $g = 9.80$ m/s^2.)

WEB 9. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor 1.40 m from the base of the counter. If the height of the counter is 0.860 m, (a) with what velocity did the mug leave the counter and (b) what was the direction of the mug's velocity just before it hit the floor?

10. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor at distance d from the base of the counter. If the height of the counter is h, (a) with what velocity did the mug leave the counter and (b) what was the direction of the mug's velocity just before it hit the floor?

11. One strategy in a snowball fight is to throw a first snowball at a high angle over level ground. While your opponent is watching the first one, you throw a second one at a low angle and timed to arrive at your opponent before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second (low-angle) snowball be thrown if it is to land at the same point as the first? (b) How many seconds later should

the second snowball be thrown if it is to land at the same time as the first?

12. A tennis player standing 12.6 m from the net hits the ball at 3.00° above the horizontal. To clear the net, the ball must rise at least 0.330 m. If the ball just clears the net at the apex of its trajectory, how fast was the ball moving when it left the racket?

13. An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. What are the x and y coordinates of the shell where it explodes, relative to its firing point?

14. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free-fall acceleration on the planet?

15. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection? Give your answer to three significant figures.

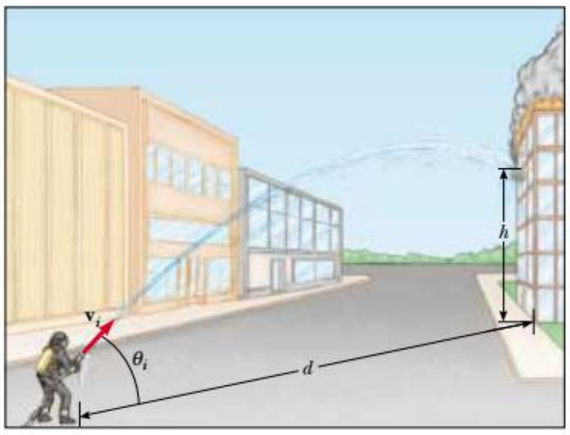
16. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

17. A cannon with a muzzle speed of 1 000 m/s is used to start an avalanche on a mountain slope. The target is 2 000 m from the cannon horizontally and 800 m above the cannon. At what angle, above the horizontal, should the cannon be fired?

18. Consider a projectile that is launched from the origin of an xy coordinate system with speed v_i at initial angle θ_i above the horizontal. Note that at the apex of its trajectory the projectile is moving horizontally, so that the slope of its path is zero. Use the expression for the trajectory given in Equation 4.12 to find the x coordinate that corresponds to the maximum height. Use this x coordinate and the symmetry of the trajectory to determine the horizontal range of the projectile.

WEB **19.** A placekicker must kick a football from a point 36.0 m (about 40 yards) from the goal, and half the crowd hopes the ball will clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of 20.0 m/s at an angle of 53.0° to the horizontal. (a) By how much does the ball clear or fall short of clearing the crossbar? (b) Does the ball approach the crossbar while still rising or while falling?

20. A firefighter 50.0 m away from a burning building directs a stream of water from a fire hose at an angle of 30.0° above the horizontal, as in Figure P4.20. If the speed of the stream is 40.0 m/s, at what height will the water strike the building?



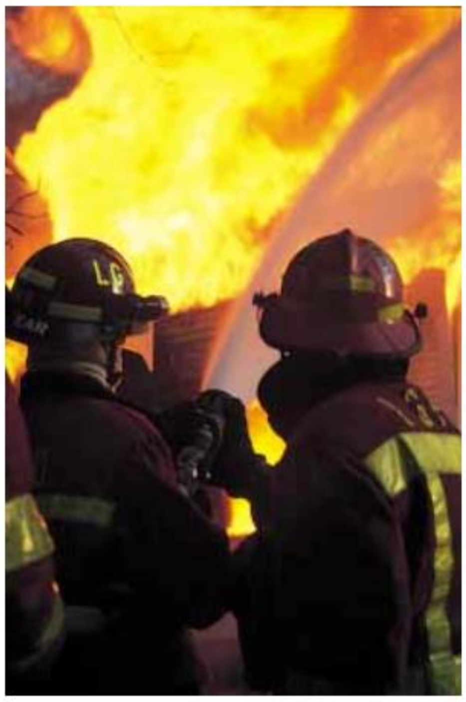


Figure P4.20 Problems 20 and 21. *(Frederick McKinney/FPG International)*

21. A firefighter a distance d from a burning building directs a stream of water from a fire hose at angle θ_i above the horizontal as in Figure P4.20. If the initial speed of the stream is v_i, at what height h does the water strike the building?

22. A soccer player kicks a rock horizontally off a cliff 40.0 m high into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

23. A basketball star covers 2.80 m horizontally in a jump to dunk the ball (Fig. P4.23). His motion through space can be modeled as that of a particle at a point called his center of mass (which we shall define in Chapter 9). His center of mass is at elevation 1.02 m when he leaves the floor. It reaches a maximum height of 1.85 m above the floor and is at elevation 0.900 m when he touches down again. Determine (a) his time of flight (his "hang time"), (b) his horizontal and (c) vertical velocity components at the instant of takeoff, and (d) his takeoff angle. (e) For comparison, determine the hang time of a whitetail deer making a jump with center-of-mass elevations $y_i = 1.20$ m, $y_{max} = 2.50$ m, $y_f = 0.700$ m.

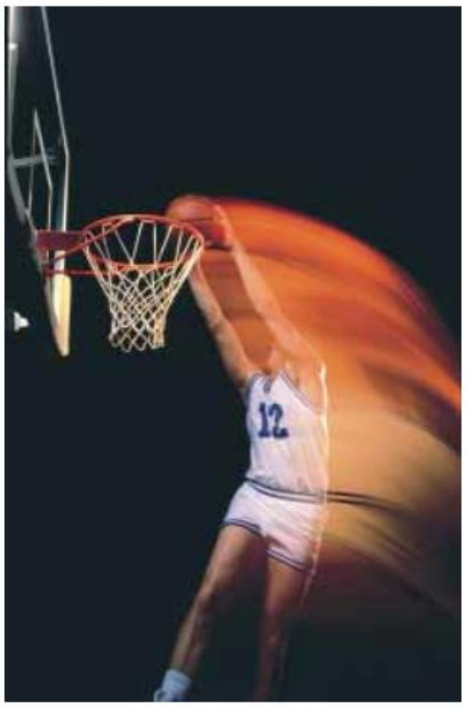


Figure P4.23 *(Top, Ron Chapple/FPG International; bottom, Bill Lea/Dembinsky Photo Associates)*

Section 4.4 Uniform Circular Motion

24. The orbit of the Moon about the Earth is approximately circular, with a mean radius of 3.84×10^8 m. It takes 27.3 days for the Moon to complete one revolution about the Earth. Find (a) the mean orbital speed of the Moon and (b) its centripetal acceleration.

WEB $\boxed{25.}$ The athlete shown in Figure P4.25 rotates a 1.00-kg discus along a circular path of radius 1.06 m. The maximum speed of the discus is 20.0 m/s. Determine the magnitude of the maximum radial acceleration of the discus.

Figure P4.25 *(Sam Sargent/Liaison International)*

26. From information on the endsheets of this book, compute, for a point located on the surface of the Earth at the equator, the radial acceleration due to the rotation of the Earth about its axis.

27. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge). (*Hint:* In one revolution, the stone travels a distance equal to the circumference of its path, $2\pi r$.)

28. During liftoff, Space Shuttle astronauts typically feel accelerations up to $1.4g$, where $g = 9.80$ m/s^2. In their training, astronauts ride in a device where they experience such an acceleration as a centripetal acceleration. Specifically, the astronaut is fastened securely at the end of a mechanical arm that then turns at constant speed in a horizontal circle. Determine the rotation rate, in revolutions per second, required to give an astronaut a centripetal acceleration of $1.40g$ while the astronaut moves in a circle of radius 10.0 m.

29. Young David who slew Goliath experimented with slings before tackling the giant. He found that he could revolve a sling of length 0.600 m at the rate of 8.00 rev/s. If he increased the length to 0.900 m, he could revolve the sling only 6.00 times per second. (a) Which rate of rotation gives the greater speed for the stone at the end of the sling? (b) What is the centripetal acceleration of the stone at 8.00 rev/s? (c) What is the centripetal acceleration at 6.00 rev/s?

30. The astronaut orbiting the Earth in Figure P4.30 is preparing to dock with a Westar VI satellite. The satellite is in a circular orbit 600 km above the Earth's surface, where the free-fall acceleration is 8.21 m/s^2. The radius of the Earth is 6 400 km. Determine the speed of the satellite and the time required to complete one orbit around the Earth.

Figure P4.30 *(Courtesy of NASA)*

Section 4.5 Tangential and Radial Acceleration

31. A train slows down as it rounds a sharp horizontal curve, slowing from 90.0 km/h to 50.0 km/h in the 15.0 s that it takes to round the curve. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume that the train slows down at a uniform rate during the 15.0-s interval.

32. An automobile whose speed is increasing at a rate of 0.600 m/s^2 travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find (a) the tangential acceleration component, (b) the radial acceleration component, and (c) the magnitude and direction of the total acceleration.

33. Figure P4.33 shows the total acceleration and velocity of a particle moving clockwise in a circle of radius 2.50 m at a given instant of time. At this instant, find (a) the radial acceleration, (b) the speed of the particle, and (c) its tangential acceleration.

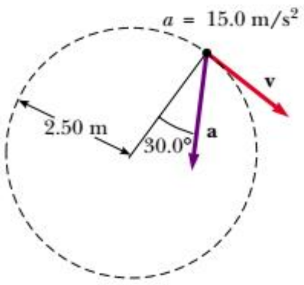


Figure P4.33

34. A student attaches a ball to the end of a string 0.600 m in length and then swings the ball in a vertical circle. The speed of the ball is 4.30 m/s at its highest point and 6.50 m/s at its lowest point. Find the acceleration of the ball when the string is vertical and the ball is at (a) its highest point and (b) its lowest point.

35. A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point and on its way up, its total acceleration is $(-22.5\mathbf{i} + 20.2\mathbf{j})$ m/s^2. At that instant, (a) sketch a vector diagram showing the components of this acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

Section 4.6 Relative Velocity and Relative Acceleration

36. Heather in her Corvette accelerates at the rate of $(3.00\mathbf{i} - 2.00\mathbf{j})$ m/s^2, while Jill in her Jaguar accelerates at $(1.00\mathbf{i} + 3.00\mathbf{j})$ m/s^2. They both start from rest at the origin of an *xy* coordinate system. After 5.00 s, (a) what is Heather's speed with respect to Jill, (b) how far apart are they, and (c) what is Heather's acceleration relative to Jill?

37. A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

38. How long does it take an automobile traveling in the left lane at 60.0 km/h to pull alongside a car traveling in the right lane at 40.0 km/h if the cars' front bumpers are initially 100 m apart?

39. The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. If there is a wind of 30.0 km/h toward the north, find the velocity of the airplane relative to the ground.

40. Two swimmers, Alan and Beth, start at the same point in a stream that flows with a speed v. Both move at the same speed c ($c > v$) relative to the stream. Alan swims downstream a distance L and then upstream the same distance. Beth swims such that her motion relative to the ground is perpendicular to the banks of the stream. She swims a distance L in this direction and then back. The result of the motions of Alan and Beth is that they both return to the starting point. Which swimmer returns first? (*Note:* First guess at the answer.)

41. A child in danger of drowning in a river is being carried downstream by a current that has a speed of 2.50 km/h. The child is 0.600 km from shore and 0.800 km upstream of a boat landing when a rescue boat sets out. (a) If the boat proceeds at its maximum speed of 20.0 km/h relative to the water, what heading relative to the shore should the pilot take? (b) What angle does the boat velocity make with the shore? (c) How long does it take the boat to reach the child?

42. A bolt drops from the ceiling of a train car that is accelerating northward at a rate of 2.50 m/s^2. What is the acceleration of the bolt relative to (a) the train car and (b) the Earth?

43. A science student is riding on a flatcar of a train traveling along a straight horizontal track at a constant speed of 10.0 m/s. The student throws a ball into the air along a path that he judges to make an initial angle of 60.0° with the horizontal and to be in line with the track. The student's professor, who is standing on the ground nearby, observes the ball to rise vertically. How high does she see the ball rise?

ADDITIONAL PROBLEMS

44. A ball is thrown with an initial speed v_i at an angle θ_i with the horizontal. The horizontal range of the ball is R, and the ball reaches a maximum height $R/6$. In terms of R and g, find (a) the time the ball is in motion, (b) the ball's speed at the peak of its path, (c) the initial vertical component of its velocity, (d) its initial speed, and (e) the angle θ_i. (f) Suppose the ball is thrown at the same initial speed found in part (d) but at the angle appropriate for reaching the maximum height. Find this height. (g) Suppose the ball is thrown at the same initial speed but at the angle necessary for maximum range. Find this range.

45. As some molten metal splashes, one droplet flies off to the east with initial speed v_i at angle θ_i above the horizontal, and another droplet flies off to the west with the same speed at the same angle above the horizontal, as in Figure P4.45. In terms of v_i and θ_i, find the distance between the droplets as a function of time.

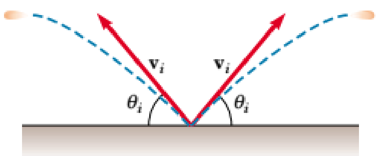


Figure P4.45

46. A ball on the end of a string is whirled around in a horizontal circle of radius 0.300 m. The plane of the circle is 1.20 m above the ground. The string breaks and the ball lands 2.00 m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the radial acceleration of the ball during its circular motion.

47. A projectile is fired up an incline (incline angle ϕ) with an initial speed v_i at an angle θ_i with respect to the horizontal ($\theta_i > \phi$), as shown in Figure P4.47. (a) Show that the projectile travels a distance d up the incline, where

$$d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$$

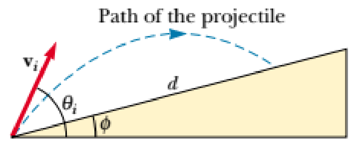


Figure P4.47

(b) For what value of θ_i is d a maximum, and what is that maximum value of d?

48. A student decides to measure the muzzle velocity of the pellets from his BB gun. He points the gun horizontally. On a vertical wall a distance x away from the gun, a target is placed. The shots hit the target a vertical distance y below the gun. (a) Show that the vertical displacement component of the pellets when traveling through the air is given by $y = Ax^2$, where A is a constant. (b) Express the constant A in terms of the initial velocity and the free-fall acceleration. (c) If x = 3.00 m and y = 0.210 m, what is the initial speed of the pellets?

49. A home run is hit in such a way that the baseball just clears a wall 21.0 m high, located 130 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Find (a) the initial speed of the ball, (b) the time it takes the ball to reach the wall, and (c) the velocity components and the speed of the ball when it reaches the wall. (Assume the ball is hit at a height of 1.00 m above the ground.)

50. An astronaut standing on the Moon fires a gun so that the bullet leaves the barrel initially moving in a horizontal direction. (a) What must be the muzzle speed of the bullet so that it travels completely around the Moon and returns to its original location? (b) How long does this trip around the Moon take? Assume that the free-fall acceleration on the Moon is one-sixth that on the Earth.

51. A pendulum of length 1.00 m swings in a vertical plane (Fig. 4.19). When the pendulum is in the two horizontal positions θ = 90° and θ = 270°, its speed is 5.00 m/s. (a) Find the magnitude of the radial acceleration and tangential acceleration for these positions. (b) Draw a vector diagram to determine the direction of the total acceleration for these two positions. (c) Calculate the magnitude and direction of the total acceleration.

52. A basketball player who is 2.00 m tall is standing on the floor 10.0 m from the basket, as in Figure P4.52. If he shoots the ball at a 40.0° angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is 3.05 m.

53. A particle has velocity components

$$v_x = +4\text{ m/s} \qquad v_y = -(6\text{ m/s}^2)t + 4\text{ m/s}$$

Calculate the speed of the particle and the direction $\theta = \tan^{-1}(v_y/v_x)$ of the velocity vector at t = 2.00 s.

54. When baseball players throw the ball in from the outfield, they usually allow it to take one bounce before it reaches the infielder on the theory that the ball arrives

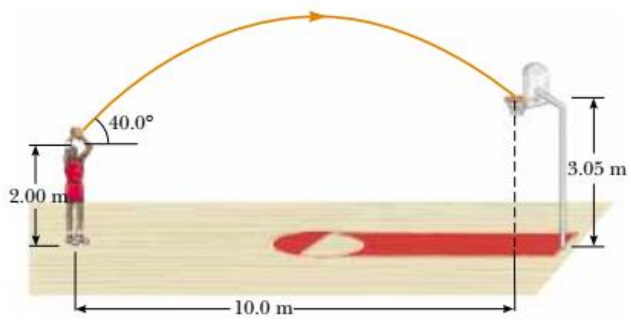


Figure P4.52

sooner that way. Suppose that the angle at which a bounced ball leaves the ground is the same as the angle at which the outfielder launched it, as in Figure P4.54, but that the ball's speed after the bounce is one half of what it was before the bounce. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the ball be thrown in order to go the same distance D with one bounce (blue path) as a ball thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.

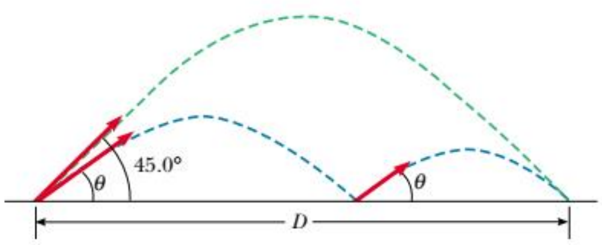


Figure P4.54

55. A boy can throw a ball a maximum horizontal distance of 40.0 m on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

56. A boy can throw a ball a maximum horizontal distance of R on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

57. A stone at the end of a sling is whirled in a vertical circle of radius 1.20 m at a constant speed v_i = 1.50 m/s as in Figure P4.57. The center of the string is 1.50 m above the ground. What is the range of the stone if it is released when the sling is inclined at 30.0° with the horizontal (a) at A? (b) at B? What is the acceleration of the stone (c) just before it is released at A? (d) just after it is released at A?

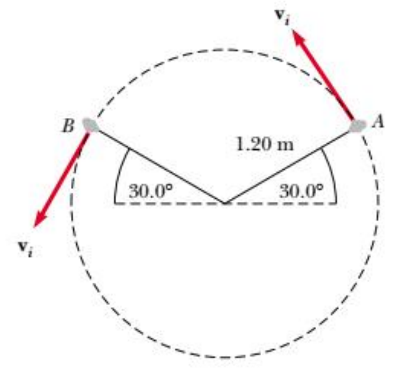


Figure P4.57

58. A quarterback throws a football straight toward a receiver with an initial speed of 20.0 m/s, at an angle of 30.0° above the horizontal. At that instant, the receiver is 20.0 m from the quarterback. In what direction and with what constant speed should the receiver run to catch the football at the level at which it was thrown?

59. A bomber is flying horizontally over level terrain, with a speed of 275 m/s relative to the ground, at an altitude of 3 000 m. Neglect the effects of air resistance. (a) How far will a bomb travel horizontally between its release from the plane and its impact on the ground? (b) If the plane maintains its original course and speed, where will it be when the bomb hits the ground? (c) At what angle from the vertical should the telescopic bombsight be set so that the bomb will hit the target seen in the sight at the time of release?

60. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\mathbf{v}_i$ as in Figure P4.60. (a) What must be its minimum initial speed if the ball is never to hit the rock after it is kicked? (b) With this initial speed, how far from the base of the rock does the ball hit the ground?

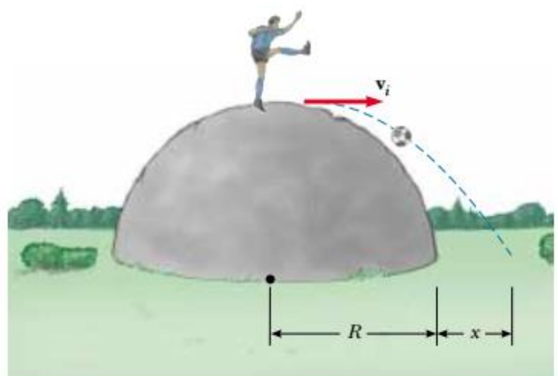


Figure P4.60

61. A hawk is flying horizontally at 10.0 m/s in a straight line, 200 m above the ground. A mouse it has been carrying struggles free from its grasp. The hawk continues on its path at the same speed for 2.00 s before attempting to retrieve its prey. To accomplish the retrieval, it dives in a straight line at constant speed and recaptures the mouse 3.00 m above the ground. (a) Assuming no air resistance, find the diving speed of the hawk. (b) What angle did the hawk make with the horizontal during its descent? (c) For how long did the mouse "enjoy" free fall?

62. A truck loaded with cannonball watermelons stops suddenly to avoid running over the edge of a washed-out bridge (Fig. P4.62). The quick stop causes a number of melons to fly off the truck. One melon rolls over the edge with an initial speed $v_i = 10.0$ m/s in the horizontal direction. A cross-section of the bank has the shape of the bottom half of a parabola with its vertex at the edge of the road, and with the equation $y^2 = 16x$, where x and y are measured in meters. What are the x and y coordinates of the melon when it splatters on the bank?

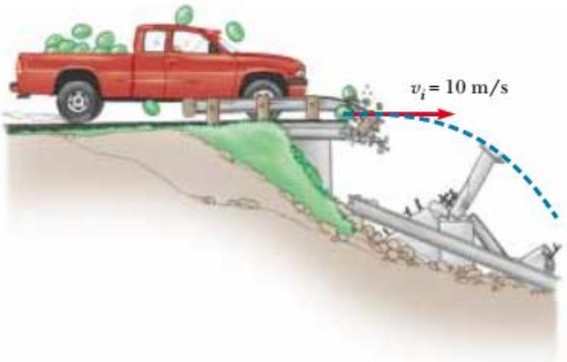


Figure P4.62

63. A catapult launches a rocket at an angle of 53.0° above the horizontal with an initial speed of 100 m/s. The rocket engine immediately starts a burn, and for 3.00 s the rocket moves along its initial line of motion with an acceleration of 30.0 m/s^2. Then its engine fails, and the rocket proceeds to move in free fall. Find (a) the maximum altitude reached by the rocket, (b) its total time of flight, and (c) its horizontal range.

64. A river flows with a uniform velocity **v**. A person in a motorboat travels 1.00 km upstream, at which time she passes a log floating by. Always with the same throttle setting, the boater continues to travel upstream for another 60.0 min and then returns downstream to her starting point, which she reaches just as the same log does. Find the velocity of the river. (*Hint:* The time of travel of the boat after it meets the log equals the time of travel of the log.)

WEB 65. A car is parked on a steep incline overlooking the ocean, where the incline makes an angle of 37.0° below the horizontal. The negligent driver leaves the car in neutral, and the parking brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s^2, traveling 50.0 m to the edge of a vertical cliff. The cliff is 30.0 m above the ocean. Find (a) the speed of the car when it reaches the edge of the cliff and the time it takes to get there, (b) the velocity of the car when it lands in the ocean, (c) the total time the car is in motion, and (d) the position of the car when it lands in the ocean, relative to the base of the cliff.

66. The determined coyote is out once more to try to capture the elusive roadrunner. The coyote wears a pair of Acme jet-powered roller skates, which provide a constant horizontal acceleration of 15.0 m/s^2 (Fig. P4.66). The coyote starts off at rest 70.0 m from the edge of a cliff at the instant the roadrunner zips past him in the direction of the cliff. (a) If the roadrunner moves with constant speed, determine the minimum speed he must have to reach the cliff before the coyote. At the brink of the cliff, the roadrunner escapes by making a sudden turn, while the coyote continues straight ahead. (b) If the cliff is 100 m above the floor of a canyon, determine where the coyote lands in the canyon (assume his skates remain horizontal and continue to operate when he is in "flight"). (c) Determine the components of the coyote's impact velocity.

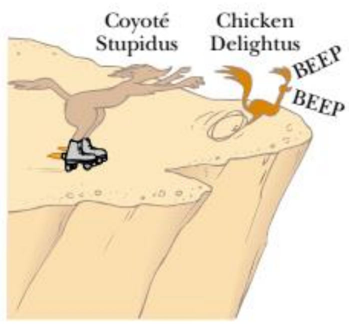


Figure P4.66

67. A skier leaves the ramp of a ski jump with a velocity of 10.0 m/s, 15.0° above the horizontal, as in Figure P4.67. The slope is inclined at 50.0°, and air resistance is negligible. Find (a) the distance from the ramp to where the jumper lands and (b) the velocity components just before the landing. (How do you think the results might be affected if air resistance were included? Note that jumpers lean forward in the shape of an airfoil, with their hands at their sides, to increase their distance. Why does this work?)

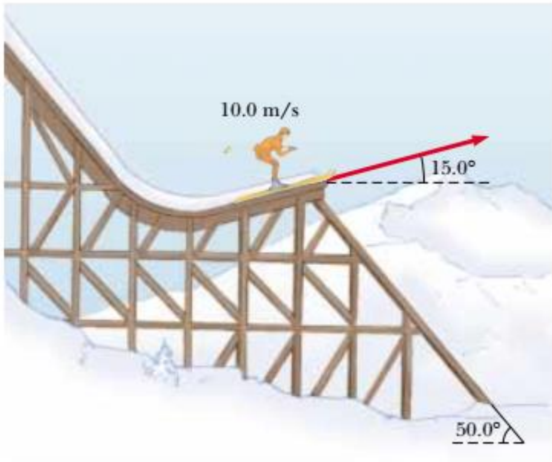


Figure P4.67

68. Two soccer players, Mary and Jane, begin running from nearly the same point at the same time. Mary runs in an easterly direction at 4.00 m/s, while Jane takes off in a direction 60.0° north of east at 5.40 m/s. (a) How long is it before they are 25.0 m apart? (b) What is the velocity of Jane relative to Mary? (c) How far apart are they after 4.00 s?

69. Do not hurt yourself; do not strike your hand against anything. Within these limitations, describe what you do to give your hand a large acceleration. Compute an order-of-magnitude estimate of this acceleration, stating the quantities you measure or estimate and their values.

70. An enemy ship is on the western side of a mountain island, as shown in Figure P4.70. The enemy ship can maneuver to within 2 500 m of the 1 800-m-high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the eastern shoreline is horizontally 300 m from the peak, what are the distances from the eastern shore at which a ship can be safe from the bombardment of the enemy ship?

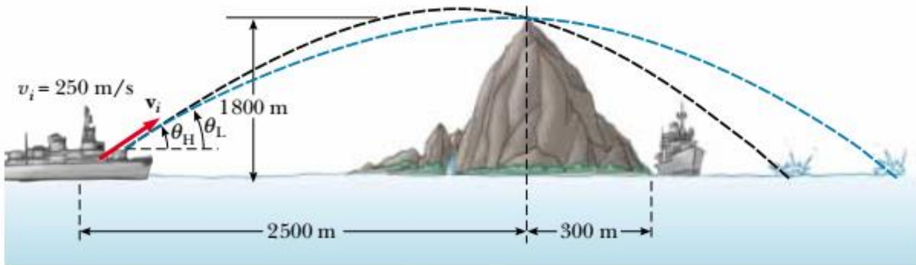


Figure P4.70

Answers to Quick Quizzes

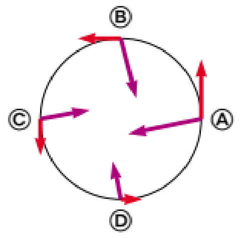
4.1 (a) Because acceleration occurs whenever the velocity changes in any way—with an increase or decrease in speed, a change in direction, or both—the brake pedal can also be considered an accelerator because it causes the car to slow down. The steering wheel is also an accelerator because it changes the direction of the velocity vector. (b) When the car is moving with constant speed, the gas pedal is not causing an acceleration; it is an accelerator only when it causes a change in the speedometer reading.

4.2 (a) At only one point—the peak of the trajectory—are the velocity and acceleration vectors perpendicular to each other. (b) If the object is thrown straight up or down, **v** and **a** are parallel to each other throughout the downward motion. Otherwise, the velocity and acceleration vectors are never parallel to each other. (c) The greater the maximum height, the longer it takes the projectile to reach that altitude and then fall back down from it. So, as the angle increases from 0° to 90°, the time of flight increases. Therefore, the 15° angle gives the shortest time of flight, and the 75° angle gives the longest.

4.3 (a) Because the object is moving with a constant speed, the velocity vector is always the same length; because the motion is circular, this vector is always tangent to the circle. The only acceleration is that which changes the direction of the velocity vector; it points radially inward.

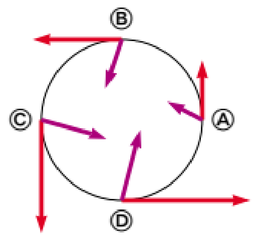
(a)

(b) Now there is a component of the acceleration vector that is tangent to the circle and points in the direction opposite the velocity. As a result, the acceleration vector does not point toward the center. The object is slowing down, and so the velocity vectors become shorter and shorter.



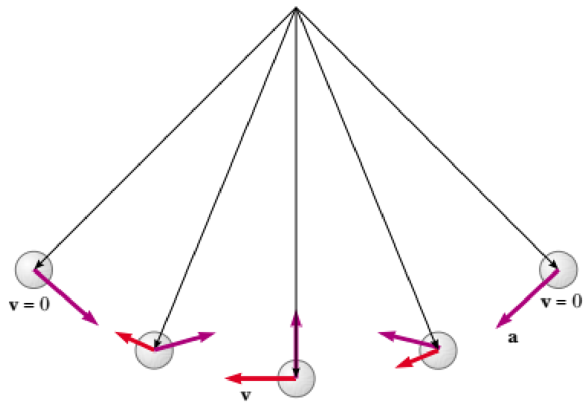
(b)

(c) Now the tangential component of the acceleration points in the same direction as the velocity. The object is speeding up, and so the velocity vectors become longer and longer.



(c)

4.4 The motion diagram is as shown below. Note that each position vector points from the pivot point at the center of the circle to the position of the ball.



4.5 (a) The passenger sees the coffee pouring nearly vertically into the cup, just as if she were standing on the ground pouring it. (b) The stationary observer sees the coffee moving in a parabolic path with a constant horizontal velocity of 60 mi/h (= 88 ft/s) and a downward acceleration of $-g$. If it takes the coffee 0.10 s to reach the cup, the stationary observer sees the coffee moving 8.8 ft horizontally before it hits the cup! (c) If the car slows suddenly, the coffee reaches the place where the cup *would have been* had there been no change in velocity and continues falling because the cup has not yet reached that location. If the car rapidly speeds up, the coffee falls behind the cup. If the car accelerates sideways, the coffee again ends up somewhere other than the cup.