PUZZLER

All three of these commonplace items use magnetism to store information. The cassette can store more than an hour of music, the floppy disk can hold the equivalent of hundreds of pages of information, and many hours of television programming can be recorded on the videotape. How do these devices work? *(George Semple)*

chapter 30

Sources of the Magnetic Field

Chapter Outline

In the preceding chapter, we discussed the magnetic force exerted on a charged particle moving in a magnetic field. To complete the description of the magnetic interaction, this chapter deals with the origin of the magnetic field—moving charges. We begin by showing how to use the law of Biot and Savart to calculate the magnetic field produced at some point in space by a small current element. Using this formalism and the principle of superposition, we then calculate the total magnetic field due to various current distributions. Next, we show how to determine the force between two current-carrying conductors, which leads to the definition of the ampere. We also introduce Ampère's law, which is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

This chapter is also concerned with the complex processes that occur in magnetic materials. All magnetic effects in matter can be explained on the basis of atomic magnetic moments, which arise both from the orbital motion of the electrons and from an intrinsic property of the electrons known as spin.

30.1 THE BIOT–SAVART LAW

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\mathbf{B}$ at a point P associated with a length element $d\mathbf{s}$ of a wire carrying a steady current I (Fig. 30.1):

Properties of the magnetic field created by an electric current

- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude ds of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

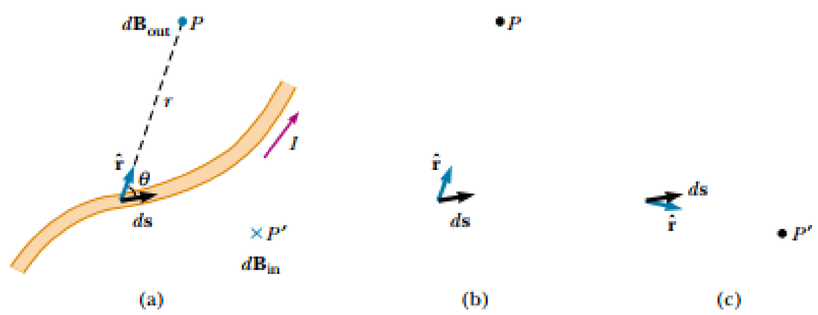


Figure 30.1 (a) The magnetic field $d\mathbf{B}$ at point P due to the current I through a length element $d\mathbf{s}$ is given by the Biot–Savart law. The direction of the field is out of the page at P and into the page at P'. (b) The cross product $d\mathbf{s} \times \hat{\mathbf{r}}$ points out of the page when $\hat{\mathbf{r}}$ points toward P. (c) The cross product $d\mathbf{s} \times \hat{\mathbf{r}}$ points into the page when $\hat{\mathbf{r}}$ points toward P'.

These observations are summarized in the mathematical formula known today as the **Biot–Savart law:**

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \tag{30.1}$$

Biot–Savart law

where μ_0 is a constant called the **permeability of free space:**

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} \tag{30.2}$$

Permeability of free space

It is important to note that the field $d\mathbf{B}$ in Equation 30.1 is the field created by the current in only a small length element $d\mathbf{s}$ of the conductor. To find the total magnetic field $\mathbf{B}$ created at some point by a current of finite size, we must sum up contributions from all current elements $I d\mathbf{s}$ that make up the current. That is, we must evaluate $\mathbf{B}$ by integrating Equation 30.1:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \tag{30.3}$$

where the integral is taken over the entire current distribution. This expression must be handled with special care because the integrand is a cross product and therefore a vector quantity. We shall see one case of such an integration in Example 30.1.

Although we developed the Biot–Savart law for a current-carrying wire, it is also valid for a current consisting of charges flowing through space, such as the electron beam in a television set. In that case, $d\mathbf{s}$ represents the length of a small segment of space in which the charges flow.

Interesting similarities exist between the Biot–Savart law for magnetism and Coulomb's law for electrostatics. The current element produces a magnetic field, whereas a point charge produces an electric field. Furthermore, the magnitude of the magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge. However, the directions of the two fields are quite different. The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\mathbf{s}$ and the unit vector $\hat{\mathbf{r}}$, as described by the cross product in Equation 30.1. Hence, if the conductor lies in the plane of the page, as shown in Figure 30.1, $d\mathbf{B}$ points out of the page at P and into the page at P'.

Another difference between electric and magnetic fields is related to the source of the field. An electric field is established by an isolated electric charge. The Biot–Savart law gives the magnetic field of an isolated current element at some point, but such an isolated current element cannot exist the way an isolated electric charge can. A current element *must* be part of an extended current distribution because we must have a complete circuit in order for charges to flow. Thus, the Biot–Savart law is only the first step in a calculation of a magnetic field; it must be followed by an integration over the current distribution.

In the examples that follow, it is important to recognize that **the magnetic field determined in these calculations is the field created by a current-carrying conductor.** This field is not to be confused with any additional fields that may be present outside the conductor due to other sources, such as a bar magnet placed nearby.

EXAMPLE 30.1 Magnetic Field Surrounding a Thin, Straight Conductor

Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in Figure 30.2. Determine the magnitude and direction of the magnetic field at point P due to this current.

Solution From the Biot–Savart law, we expect that the magnitude of the field is proportional to the current in the wire and decreases as the distance a from the wire to point P increases. We start by considering a length element $d\mathbf{s}$ located a distance r from P. The direction of the magnetic field at point P due to the current in this element is out of the page because $d\mathbf{s} \times \hat{\mathbf{r}}$ is out of the page. In fact, since *all* of the current elements $I\,d\mathbf{s}$ lie in the plane of the page, they all produce a magnetic field directed out of the page at point P. Thus, we have the direction of the magnetic field at point P, and we need only find the magnitude.

Taking the origin at O and letting point P be along the positive y axis, with $\mathbf{k}$ being a unit vector pointing out of the page, we see that

$$d\mathbf{s} \times \hat{\mathbf{r}} = \mathbf{k}\,|d\mathbf{s} \times \hat{\mathbf{r}}| = \mathbf{k}(dx \sin\theta)$$

where, from Chapter 3, $|d\mathbf{s} \times \hat{\mathbf{r}}|$ represents the magnitude of $d\mathbf{s} \times \hat{\mathbf{r}}$. Because $\hat{\mathbf{r}}$ is a unit vector, the unit of the cross product is simply the unit of $d\mathbf{s}$, which is length. Substitution into Equation 30.1 gives

$$d\mathbf{B} = (dB)\,\mathbf{k} = \frac{\mu_0 I}{4\pi}\frac{dx \sin\theta}{r^2}\,\mathbf{k}$$

Because all current elements produce a magnetic field in the $\mathbf{k}$ direction, let us restrict our attention to the magnitude of the field due to one current element, which is

$$(1) \qquad dB = \frac{\mu_0 I}{4\pi}\frac{dx \sin\theta}{r^2}$$

To integrate this expression, we must relate the variables θ, x, and r. One approach is to express x and r in terms of θ. From the geometry in Figure 30.2a, we have

$$(2) \qquad r = \frac{a}{\sin\theta} = a \csc\theta$$

Because $\tan\theta = a/(-x)$ from the right triangle in Figure 30.2a (the negative sign is necessary because $d\mathbf{s}$ is located at a negative value of x), we have

$$x = -a \cot\theta$$

Taking the derivative of this expression gives

$$(3) \qquad dx = a \csc^2\theta\, d\theta$$

Substitution of Equations (2) and (3) into Equation (1) gives

$$(4) \qquad dB = \frac{\mu_0 I}{4\pi}\frac{a \csc^2\theta \sin\theta\, d\theta}{a^2 \csc^2\theta} = \frac{\mu_0 I}{4\pi a}\sin\theta\, d\theta$$

an expression in which the only variable is θ. We can now obtain the magnitude of the magnetic field at point P by integrating Equation (4) over all elements, subtending angles ranging from θ_1 to θ_2 as defined in Figure 30.2b:

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) \qquad \textbf{(30.4)}$$

We can use this result to find the magnetic field of any straight current-carrying wire if we know the geometry and hence the angles θ_1 and θ_2. Consider the special case of an infinitely long, straight wire. If we let the wire in Figure 30.2b become infinitely long, we see that $\theta_1 = 0$ and $\theta_2 = \pi$ for length elements ranging between positions $x = -\infty$ and $x = +\infty$. Because $(\cos\theta_1 - \cos\theta_2) = (\cos 0 - \cos\pi) = 2$, Equation 30.4 becomes

$$B = \frac{\mu_0 I}{2\pi a} \qquad \textbf{(30.5)}$$

Equations 30.4 and 30.5 both show that the magnitude of

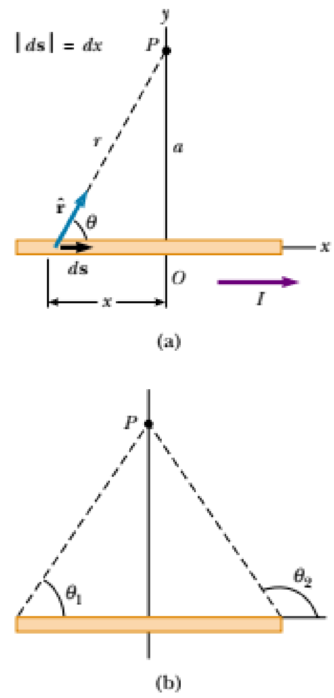


Figure 30.2 (a) A thin, straight wire carrying a current I. The magnetic field at point P due to the current in each element $d\mathbf{s}$ of the wire is out of the page, so the net field at point P is also out of the page. (b) The angles θ_1 and θ_2, used for determining the net field. When the wire is infinitely long, $\theta_1 = 0$ and $\theta_2 = 180°$.

the magnetic field is proportional to the current and decreases with increasing distance from the wire, as we expected. Notice that Equation 30.5 has the same mathematical form as the expression for the magnitude of the electric field due to a long charged wire (see Eq. 24.7).

Exercise Calculate the magnitude of the magnetic field 4.0 cm from an infinitely long, straight wire carrying a current of 5.0 A.

Answer 2.5×10^{-5} T.

The result of Example 30.1 is important because a current in the form of a long, straight wire occurs often. Figure 30.3 is a three-dimensional view of the magnetic field surrounding a long, straight current-carrying wire. Because of the symmetry of the wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire. The magnitude of **B** is constant on any circle of radius *a* and is given by Equation 30.5. A convenient rule for determining the direction of **B** is to grasp the wire with the right hand, positioning the thumb along the direction of the current. The four fingers wrap in the direction of the magnetic field.

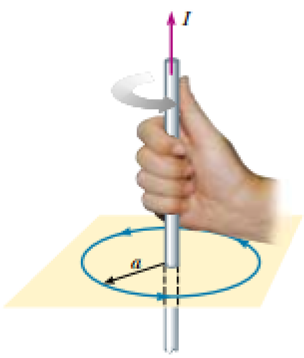


Figure 30.3 The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current. Note that the magnetic field lines form circles around the wire.

EXAMPLE 30.2 Magnetic Field Due to a Curved Wire Segment

Calculate the magnetic field at point *O* for the current-carrying wire segment shown in Figure 30.4. The wire consists of two straight portions and a circular arc of radius *R*, which subtends an angle θ. The arrowheads on the wire indicate the direction of the current.

Solution The magnetic field at *O* due to the current in the straight segments *AA'* and *CC'* is zero because $d\mathbf{s}$ is parallel to $\hat{\mathbf{r}}$ along these paths; this means that $d\mathbf{s} \times \hat{\mathbf{r}} = 0$. Each length element $d\mathbf{s}$ along path *AC* is at the same distance *R* from *O*, and the current in each contributes a field element $d\mathbf{B}$ directed into the page at *O*. Furthermore, at every point on *AC*, $d\mathbf{s}$ is perpendicular to $\hat{\mathbf{r}}$; hence, $|d\mathbf{s} \times \hat{\mathbf{r}}| = ds$. Using this information and Equation 30.1, we can find the magnitude of the field at *O* due to the current in an element of length *ds*:

$$dB = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2}$$

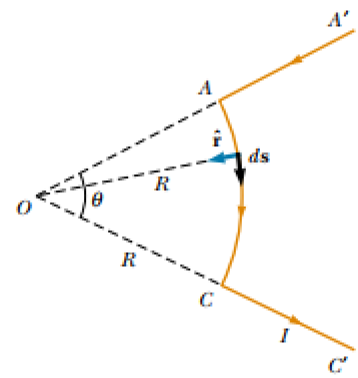


Figure 30.4 The magnetic field at *O* due to the current in the curved segment *AC* is into the page. The contribution to the field at *O* due to the current in the two straight segments is zero.

Because I and R are constants, we can easily integrate this expression over the curved path AC:

$$B = \frac{\mu_0 I}{4\pi R^2}\int ds = \frac{\mu_0 I}{4\pi R^2}\, s = \boxed{\frac{\mu_0 I}{4\pi R}\,\theta} \quad \textbf{(30.6)}$$

where we have used the fact that $s = R\theta$ with θ measured in radians. The direction of $\mathbf{B}$ is into the page at O because $d\mathbf{s} \times \hat{\mathbf{r}}$ is into the page for every length element.

Exercise A circular wire loop of radius R carries a current I. What is the magnitude of the magnetic field at its center?

Answer $\mu_0 I/2R$.

EXAMPLE 30.3 Magnetic Field on the Axis of a Circular Current Loop

Consider a circular wire loop of radius R located in the yz plane and carrying a steady current I, as shown in Figure 30.5. Calculate the magnetic field at an axial point P a distance x from the center of the loop.

Solution In this situation, note that every length element $d\mathbf{s}$ is perpendicular to the vector $\hat{\mathbf{r}}$ at the location of the element. Thus, for any element, $d\mathbf{s} \times \hat{\mathbf{r}} = (ds)(1)\sin 90° = ds$. Furthermore, all length elements around the loop are at the same distance r from P, where $r^2 = x^2 + R^2$. Hence, the magnitude of $d\mathbf{B}$ due to the current in any length element $d\mathbf{s}$ is

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\mathbf{s} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(x^2 + R^2)}$$

The direction of $d\mathbf{B}$ is perpendicular to the plane formed by $\hat{\mathbf{r}}$ and $d\mathbf{s}$, as shown in Figure 30.5. We can resolve this vector into a component dB_x along the x axis and a component dB_y perpendicular to the x axis. When the components dB_y are summed over all elements around the loop, the resultant component is zero. That is, by symmetry the current in any element on one side of the loop sets up a perpendicular component of $d\mathbf{B}$ that cancels the perpendicular component set up by the current through the element diametrically opposite it. Therefore, *the resultant field at P must be along the x axis* and we can find it by integrating the components $dB_x = dB\cos\theta$. That is, $\mathbf{B} = B_x\mathbf{i}$, where

$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint\frac{ds\cos\theta}{x^2 + R^2}$$

and we must take the integral over the entire loop. Because θ, x, and R are constants for all elements of the loop and because $\cos\theta = R/(x^2 + R^2)^{1/2}$, we obtain

$$B_x = \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{3/2}}\oint ds = \boxed{\frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}} \quad \textbf{(30.7)}$$

where we have used the fact that $\oint ds = 2\pi R$ (the circumference of the loop).

To find the magnetic field at the center of the loop, we set $x = 0$ in Equation 30.7. At this special point, therefore,

$$B = \frac{\mu_0 I}{2R} \qquad (\text{at } x = 0) \quad \textbf{(30.8)}$$

which is consistent with the result of the exercise in Example 30.2.

It is also interesting to determine the behavior of the magnetic field far from the loop—that is, when x is much greater than R. In this case, we can neglect the term R^2 in the denominator of Equation 30.7 and obtain

$$B \approx \frac{\mu_0 IR^2}{2x^3} \qquad (\text{for } x \gg R) \quad \textbf{(30.9)}$$

Because the magnitude of the magnetic moment μ of the loop is defined as the product of current and loop area (see Eq. 29.10)—$\mu = I(\pi R^2)$ for our circular loop—we can express Equation 30.9 as

$$B \approx \frac{\mu_0}{2\pi}\frac{\mu}{x^3} \quad \textbf{(30.10)}$$

This result is similar in form to the expression for the electric field due to an electric dipole, $E = k_e(2qa/y^3)$ (see Example

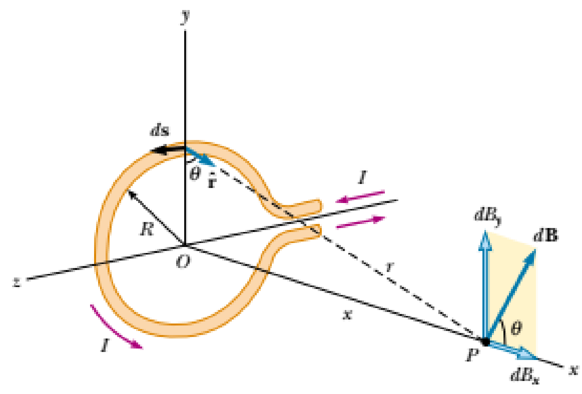


Figure 30.5 Geometry for calculating the magnetic field at a point P lying on the axis of a current loop. By symmetry, the total field $\mathbf{B}$ is along this axis.

23.6), where $2qa = p$ is the electric dipole moment as defined in Equation 26.16.

The pattern of the magnetic field lines for a circular current loop is shown in Figure 30.6a. For clarity, the lines are drawn for only one plane—one that contains the axis of the loop. Note that the field-line pattern is axially symmetric and looks like the pattern around a bar magnet, shown in Figure 30.6c.

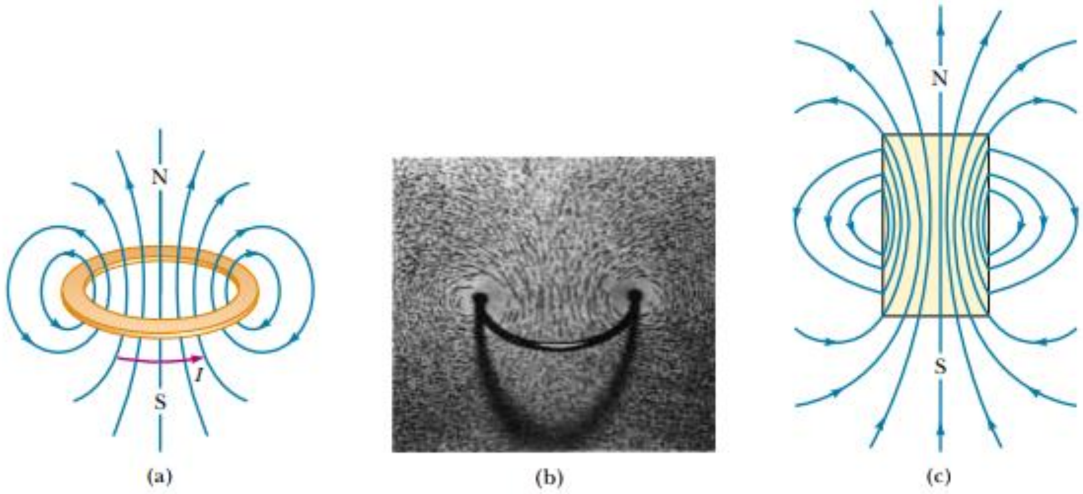


Figure 30.6 (a) Magnetic field lines surrounding a current loop. (b) Magnetic field lines surrounding a current loop, displayed with iron filings *(Education Development Center, Newton, MA)*. (c) Magnetic field lines surrounding a bar magnet. Note the similarity between this line pattern and that of a current loop.

30.2 THE MAGNETIC FORCE BETWEEN TWO PARALLEL CONDUCTORS

In Chapter 29 we described the magnetic force that acts on a current-carrying conductor placed in an external magnetic field. Because a current in a conductor sets up its own magnetic field, it is easy to understand that two current-carrying conductors exert magnetic forces on each other. As we shall see, such forces can be used as the basis for defining the ampere and the coulomb.

Consider two long, straight, parallel wires separated by a distance a and carrying currents I_1 and I_2 in the same direction, as illustrated in Figure 30.7. We can determine the force exerted on one wire due to the magnetic field set up by the other wire. Wire 2, which carries a current I_2, creates a magnetic field $\mathbf{B}_2$ at the location of wire 1. The direction of $\mathbf{B}_2$ is perpendicular to wire 1, as shown in Figure 30.7. According to Equation 29.3, the magnetic force on a length ℓ of wire 1 is $\mathbf{F}_1 = I_1 \boldsymbol{\ell} \times \mathbf{B}_2$. Because $\boldsymbol{\ell}$ is perpendicular to $\mathbf{B}_2$ in this situation, the magnitude of $\mathbf{F}_1$ is $F_1 = I_1 \ell B_2$. Because the magnitude of $\mathbf{B}_2$ is given by Equation 30.5, we see that

$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell \tag{30.11}$$

The direction of $\mathbf{F}_1$ is toward wire 2 because $\boldsymbol{\ell} \times \mathbf{B}_2$ is in that direction. If the field set up at wire 2 by wire 1 is calculated, the force $\mathbf{F}_2$ acting on wire 2 is found to be equal in magnitude and opposite in direction to $\mathbf{F}_1$. This is what we expect be-

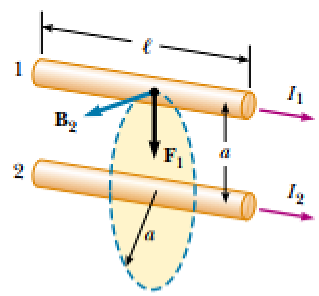


Figure 30.7 Two parallel wires that each carry a steady current exert a force on each other. The field $\mathbf{B}_2$ due to the current in wire 2 exerts a force of magnitude $F_1 = I_1 \ell B_2$ on wire 1. The force is attractive if the currents are parallel (as shown) and repulsive if the currents are antiparallel.

cause Newton's third law must be obeyed.[1] When the currents are in opposite directions (that is, when one of the currents is reversed in Fig. 30.7), the forces are reversed and the wires repel each other. Hence, we find that **parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other.**

Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B. We can rewrite this magnitude in terms of the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \tag{30.12}$$

The force between two parallel wires is used to define the **ampere** as follows:

Definition of the ampere

> When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

web

Visit **http://physics.nist.gov/cuu/Units/ampere.html** for more information.

The value 2×10^{-7} N/m is obtained from Equation 30.12 with $I_1 = I_2 = 1$ A and $a = 1$ m. Because this definition is based on a force, a mechanical measurement can be used to standardize the ampere. For instance, the National Institute of Standards and Technology uses an instrument called a *current balance* for primary current measurements. The results are then used to standardize other, more conventional instruments, such as ammeters.

The SI unit of charge, the **coulomb,** is defined in terms of the ampere:

Definition of the coulomb

> When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross-section of the conductor in 1 s is 1 C.

In deriving Equations 30.11 and 30.12, we assumed that both wires are long compared with their separation distance. In fact, only one wire needs to be long. The equations accurately describe the forces exerted on each other by a long wire and a straight parallel wire of limited length ℓ.

Quick Quiz 30.1

For $I_1 = 2$ A and $I_2 = 6$ A in Figure 30.7, which is true: (a) $F_1 = 3F_2$, (b) $F_1 = F_2/3$, or (c) $F_1 = F_2$?

Quick Quiz 30.2

A loose spiral spring is hung from the ceiling, and a large current is sent through it. Do the coils move closer together or farther apart?

[1] Although the total force exerted on wire 1 is equal in magnitude and opposite in direction to the total force exerted on wire 2, Newton's third law does not apply when one considers two small elements of the wires that are not exactly opposite each other. This apparent violation of Newton's third law and of the law of conservation of momentum is described in more advanced treatments on electricity and magnetism.

30.3 AMPÈRE'S LAW

12.4 Oersted's 1819 discovery about deflected compass needles demonstrates that a current-carrying conductor produces a magnetic field. Figure 30.8a shows how this effect can be demonstrated in the classroom. Several compass needles are placed in a horizontal plane near a long vertical wire. When no current is present in the wire, all the needles point in the same direction (that of the Earth's magnetic field), as expected. When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle, as shown in Figure 30.8b. These observations demonstrate that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule described in Figure 30.3. When the current is reversed, the needles in Figure 30.8b also reverse.

Because the compass needles point in the direction of **B**, we conclude that the lines of **B** form circles around the wire, as discussed in the preceding section. By symmetry, the magnitude of **B** is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire. By varying the current and distance a from the wire, we find that B is proportional to the current and inversely proportional to the distance from the wire, as Equation 30.5 describes.

Now let us evaluate the product $\mathbf{B} \cdot d\mathbf{s}$ for a small length element $d\mathbf{s}$ on the circular path defined by the compass needles, and sum the products for all elements over the closed circular path. Along this path, the vectors $d\mathbf{s}$ and $\mathbf{B}$ are parallel at each point (see Fig. 30.8b), so $\mathbf{B} \cdot d\mathbf{s} = B\,ds$. Furthermore, the magnitude of $\mathbf{B}$ is constant on this circle and is given by Equation 30.5. Therefore, the sum of the products $B\,ds$ over the closed path, which is equivalent to the line integral of $\mathbf{B} \cdot d\mathbf{s}$, is

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

where $\oint ds = 2\pi r$ is the circumference of the circular path. Although this result was calculated for the special case of a circular path surrounding a wire, it holds

Andre-Marie Ampère **(1775–1836)** Ampère, a Frenchman, is credited with the discovery of electromagnetism—the relationship between electric currents and magnetic fields. Ampère's genius, particularly in mathematics, became evident by the time he was 12 years old; his personal life, however, was filled with tragedy. His father, a wealthy city official, was guillotined during the French Revolution, and his wife died young, in 1803. Ampère died at the age of 61 of pneumonia. His judgment of his life is clear from the epitaph he chose for his gravestone: *Tandem Felix* (Happy at Last). *(AIP Emilio Segre Visual Archive)*

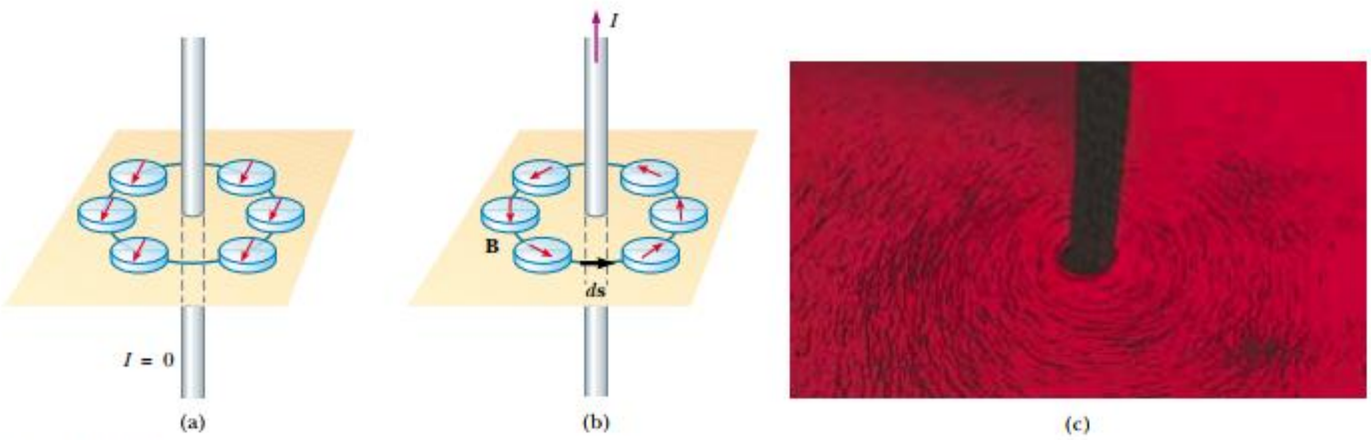


Figure 30.8 (a) When no current is present in the wire, all compass needles point in the same direction (toward the Earth's north pole). (b) When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current. (c) Circular magnetic field lines surrounding a current-carrying conductor, displayed with iron filings.

for a closed path of *any* shape surrounding a *current* that exists in an unbroken circuit. The general case, known as **Ampère's law,** can be stated as follows:

Ampère's law

The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total continuous current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad \textbf{(30.13)}$$

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

Quick Quiz 30.3

Rank the magnitudes of $\oint \mathbf{B} \cdot d\mathbf{s}$ for the closed paths in Figure 30.9, from least to greatest.

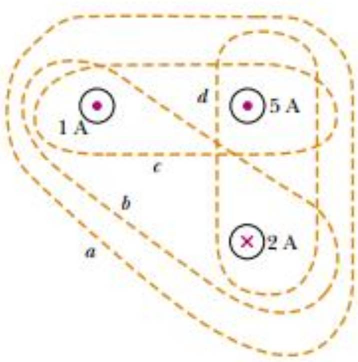


Figure 30.9 Four closed paths around three current-carrying wires.

Quick Quiz 30.4

Rank the magnitudes of $\oint \mathbf{B} \cdot d\mathbf{s}$ for the closed paths in Figure 30.10, from least to greatest.

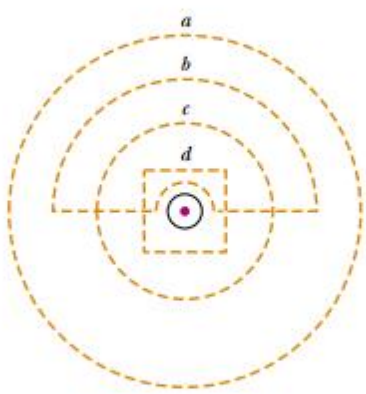


Figure 30.10 Several closed paths near a single current-carrying wire.

EXAMPLE 30.4 The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire (Fig. 30.11). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Solution For the $r \geq R$ case, we should get the same result we obtained in Example 30.1, in which we applied the Biot–Savart law to the same situation. Let us choose for our path of integration circle 1 in Figure 30.11. From symmetry, $\mathbf{B}$ must be constant in magnitude and parallel to $d\mathbf{s}$ at every point on this circle. Because the total current passing through the plane of the circle is I_0, Ampère's law gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R) \qquad \textbf{(30.14)}$$

which is identical in form to Equation 30.5. Note how much easier it is to use Ampère's law than to use the Biot–Savart law. This is often the case in highly symmetric situations.

Now consider the interior of the wire, where $r < R$. Here the current I passing through the plane of circle 2 is less than the total current I_0. Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:[2]

$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2}$$

$$I = \frac{r^2}{R^2} I_0$$

Following the same procedure as for circle 1, we apply Ampère's law to circle 2:

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I = \mu_0 \left(\frac{r^2}{R^2} I_0 \right)$$

$$B = \left(\frac{\mu_0 I_0}{2\pi R^2} \right) r \qquad (\text{for } r < R) \qquad \textbf{(30.15)}$$

This result is similar in form to the expression for the electric field inside a uniformly charged sphere (see Example 24.5). The magnitude of the magnetic field versus r for this configuration is plotted in Figure 30.12. Note that inside the wire, $B \rightarrow 0$ as $r \rightarrow 0$. Note also that Equations 30.14 and 30.15 give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.

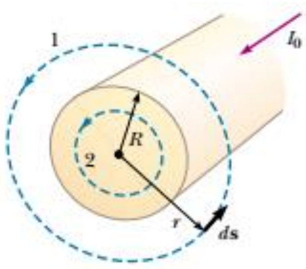


Figure 30.11 A long, straight wire of radius R carrying a steady current I_0 uniformly distributed across the cross-section of the wire. The magnetic field at any point can be calculated from Ampère's law using a circular path of radius r, concentric with the wire.

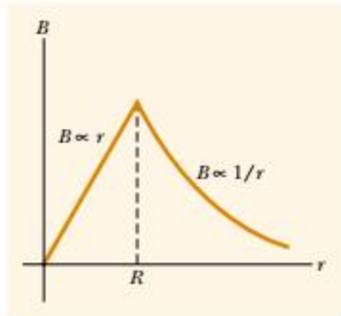


Figure 30.12 Magnitude of the magnetic field versus r for the wire shown in Figure 30.11. The field is proportional to r inside the wire and varies as $1/r$ outside the wire.

EXAMPLE 30.5 The Magnetic Field Created by a Toroid

A device called a *toroid* (Fig. 30.13) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material. For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance r from the center.

[2] Another way to look at this problem is to see that the current enclosed by circle 2 must equal the product of the current density $J = I_0/\pi R^2$ and the area πr^2 of this circle.

Solution To calculate this field, we must evaluate $\oint \mathbf{B} \cdot d\mathbf{s}$ over the circle of radius r in Figure 30.13. By symmetry, we see that the magnitude of the field is constant on this circle and tangent to it, so $\mathbf{B} \cdot d\mathbf{s} = B\,ds$. Furthermore, note that the circular closed path surrounds N loops of wire, each of which carries a current I. Therefore, the right side of Equation 30.13 is $\mu_0 NI$ in this case.

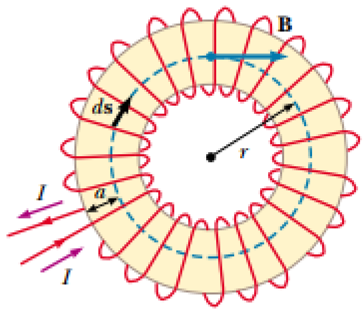


Figure 30.13 A toroid consisting of many turns of wire. If the turns are closely spaced, the magnetic field in the interior of the torus (the gold-shaded region) is tangent to the dashed circle and varies as $1/r$. The field outside the toroid is zero. The dimension a is the cross-sectional radius of the torus.

Ampère's law applied to the circle gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} \qquad \textbf{(30.16)}$$

This result shows that B varies as $1/r$ and hence is nonuniform in the region occupied by the torus. However, if r is very large compared with the cross-sectional radius of the torus, then the field is approximately uniform inside the torus.

For an ideal toroid, in which the turns are closely spaced, the external magnetic field is zero. This can be seen by noting that the net current passing through any circular path lying outside the toroid (including the region of the "hole in the doughnut") is zero. Therefore, from Ampère's law we find that $B = 0$ in the regions exterior to the torus.

EXAMPLE 30.6 Magnetic Field Created by an Infinite Current Sheet

So far we have imagined currents through wires of small cross-section. Let us now consider an example in which a current exists in an extended object. A thin, infinitely large sheet lying in the yz plane carries a current of linear current density $\mathbf{J}_s$. The current is in the y direction, and J_s represents the current per unit length measured along the z axis. Find the magnetic field near the sheet.

Solution This situation brings to mind similar calculations involving Gauss's law (see Example 24.8). You may recall that the electric field due to an infinite sheet of charge does not depend on distance from the sheet. Thus, we might expect a similar result here for the magnetic field.

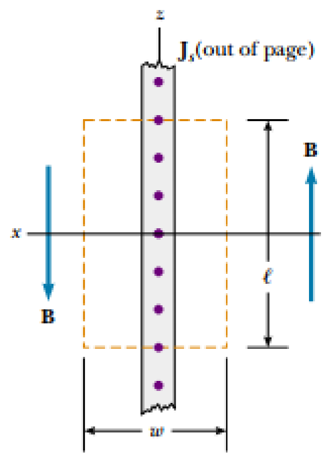


Figure 30.14 End view of an infinite current sheet lying in the yz plane, where the current is in the y direction (out of the page). This view shows the direction of **B** on both sides of the sheet.

To evaluate the line integral in Ampère's law, let us take a rectangular path through the sheet, as shown in Figure 30.14. The rectangle has dimensions ℓ and w, with the sides of length ℓ parallel to the sheet surface. The net current passing through the plane of the rectangle is $J_s\ell$. We apply Ampère's law over the rectangle and note that the two sides of length w do not contribute to the line integral because the component of **B** along the direction of these paths is zero. By symmetry, we can argue that the magnetic field is constant over the sides of length ℓ because every point on the infinitely large sheet is equivalent, and hence the field should not vary from point to point. The only choices of field direction that are reasonable for the symmetry are perpendicular or parallel to the sheet, and a perpendicular field would pass *through* the current, which is inconsistent with the Biot–Savart law. Assuming a field that is constant in magnitude and parallel to the plane of the sheet, we obtain

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I = \mu_0 J_s \ell$$

$$2B\ell = \mu_0 J_s \ell$$

$$B = \mu_0 \frac{J_s}{2}$$

This result shows that *the magnetic field is independent of distance from the current sheet,* as we suspected.

EXAMPLE 30.7 The Magnetic Force on a Current Segment

Wire 1 in Figure 30.15 is oriented along the y axis and carries a steady current I_1. A rectangular loop located to the right of the wire and in the xy plane carries a current I_2. Find the magnetic force exerted by wire 1 on the top wire of length b in the loop, labeled "Wire 2" in the figure.

Solution You may be tempted to use Equation 30.12 to obtain the force exerted on a small segment of length dx of wire 2. However, this equation applies only to two *parallel* wires and cannot be used here. The correct approach is to consider the force exerted by wire 1 on a small segment $d\mathbf{s}$ of wire 2 by using Equation 29.4. This force is given by $d\mathbf{F}_B = I\,d\mathbf{s} \times \mathbf{B}$, where $I = I_2$ and $\mathbf{B}$ is the magnetic field created by the current in wire 1 at the position of $d\mathbf{s}$. From Ampère's law, the field at a distance x from wire 1 (see Eq. 30.14) is

$$\mathbf{B} = \frac{\mu_0 I_1}{2\pi x}(-\mathbf{k})$$

where the unit vector $-\mathbf{k}$ is used to indicate that the field at $d\mathbf{s}$ points into the page. Because wire 2 is along the x axis, $d\mathbf{s} = dx\,\mathbf{i}$, and we find that

$$d\mathbf{F}_B = \frac{\mu_0 I_1 I_2}{2\pi x}[\mathbf{i} \times (-\mathbf{k})]\,dx = \frac{\mu_0 I_1 I_2}{2\pi}\frac{dx}{x}\,\mathbf{j}$$

Integrating over the limits $x = a$ to $x = a + b$ gives

$$\mathbf{F}_B = \frac{\mu_0 I_1 I_2}{2\pi}\ln x\Big]_a^{a+b}\mathbf{j} = \boxed{\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(1 + \frac{b}{a}\right)\mathbf{j}}$$

The force points in the positive y direction, as indicated by the unit vector $\mathbf{j}$ and as shown in Figure 30.15.

Exercise What are the magnitude and direction of the force exerted on the bottom wire of length b?

Answer The force has the same magnitude as the force on wire 2 but is directed downward.

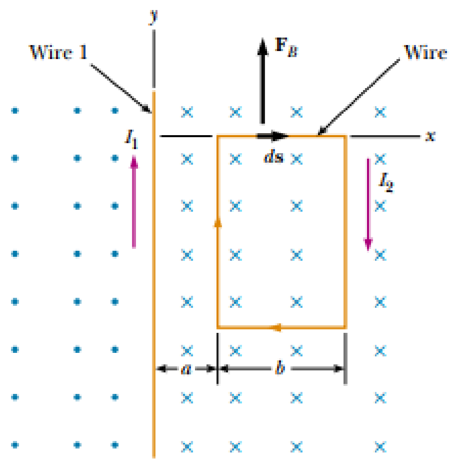


Figure 30.15

Quick Quiz 30.5

Is a net force acting on the current loop in Example 30.7? A net torque?

30.4 THE MAGNETIC FIELD OF A SOLENOID

A **solenoid** is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the *interior* of the solenoid—when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

Figure 30.16 shows the magnetic field lines surrounding a loosely wound solenoid. Note that the field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is uniform and strong. The field lines between current elements on two adjacent turns tend to cancel each other because the field vectors from the two elements are in opposite directions. The field at exterior points such as P is weak because the field due to current elements on the right-hand portion of a turn tends to cancel the field due to current elements on the left-hand portion.

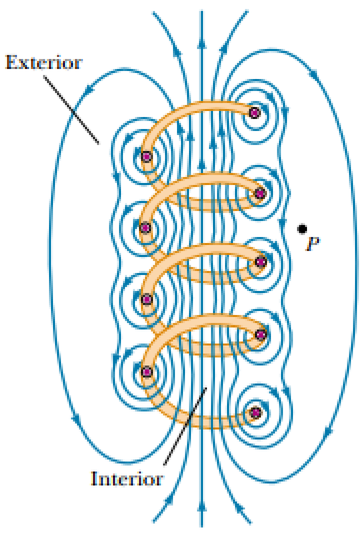


Figure 30.16 The magnetic field lines for a loosely wound solenoid.

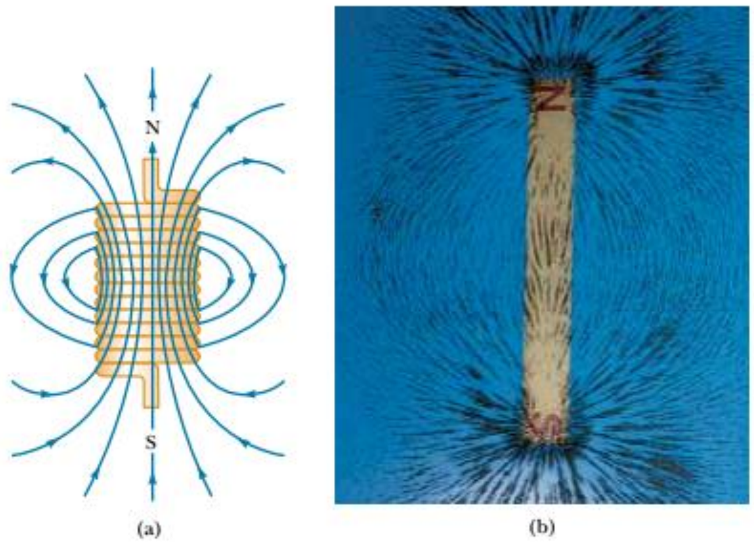
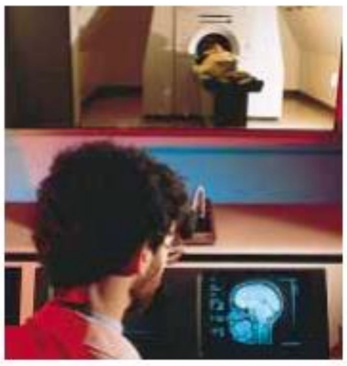


Figure 30.17 (a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is nearly uniform and strong. Note that the field lines resemble those of a bar magnet, meaning that the solenoid effectively has north and south poles. (b) The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper.



A technician studies the scan of a patient's head. The scan was obtained using a medical diagnostic technique known as magnetic resonance imaging (MRI). This instrument makes use of strong magnetic fields produced by superconducting solenoids.

If the turns are closely spaced and the solenoid is of finite length, the magnetic field lines are as shown in Figure 30.17a. This field line distribution is similar to that surrounding a bar magnet (see Fig. 30.17b). Hence, one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker. An *ideal solenoid* is approached when the turns are closely spaced and the length is much greater than the radius of the turns. In this case, the external field is zero, and the interior field is uniform over a great volume.

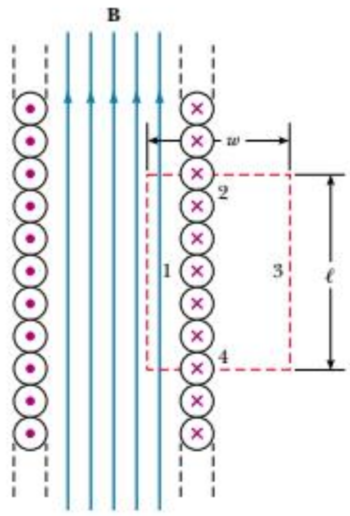


Figure 30.18 Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is zero. Ampère's law applied to the red dashed path can be used to calculate the magnitude of the interior field.

We can use Ampère's law to obtain an expression for the interior magnetic field in an ideal solenoid. Figure 30.18 shows a longitudinal cross-section of part of such a solenoid carrying a current I. Because the solenoid is ideal, **B** in the interior space is uniform and parallel to the axis, and **B** in the exterior space is zero. Consider the rectangular path of length ℓ and width w shown in Figure 30.18. We can apply Ampère's law to this path by evaluating the integral of $\mathbf{B} \cdot d\mathbf{s}$ over each side of the rectangle. The contribution along side 3 is zero because $B = 0$ in this region. The contributions from sides 2 and 4 are both zero because **B** is perpendicular to $d\mathbf{s}$ along these paths. Side 1 gives a contribution $B\ell$ to the integral because along this path **B** is uniform and parallel to $d\mathbf{s}$. The integral over the closed rectangular path is therefore

$$\oint \mathbf{B} \cdot d\mathbf{s} = \int_{\text{path 1}} \mathbf{B} \cdot d\mathbf{s} = B \int_{\text{path 1}} ds = B\ell$$

The right side of Ampère's law involves the total current passing through the area bounded by the path of integration. In this case, the total current through the rectangular path equals the current through each turn multiplied by the number of turns. If N is the number of turns in the length ℓ, the total current through the rectangle is NI. Therefore, Ampère's law applied to this path gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI \qquad (30.17)$$

Magnetic field inside a solenoid

where $n = N/\ell$ is the number of turns per unit length.

We also could obtain this result by reconsidering the magnetic field of a toroid (see Example 30.5). If the radius r of the torus in Figure 30.13 containing N turns is much greater than the toroid's cross-sectional radius a, a short section of the toroid approximates a solenoid for which $n = N/2\pi r$. In this limit, Equation 30.16 agrees with Equation 30.17.

Equation 30.17 is valid only for points near the center (that is, far from the ends) of a very long solenoid. As you might expect, the field near each end is smaller than the value given by Equation 30.17. At the very end of a long solenoid, the magnitude of the field is one-half the magnitude at the center.

QuickLab

Wrap a few turns of wire around a compass, essentially putting the compass inside a solenoid. Hold the ends of the wire to the two terminals of a flashlight battery. What happens to the compass? Is the effect as strong when the compass is outside the turns of wire?

web

For a more detailed discussion of the magnetic field along the axis of a solenoid, visit **www.saunderscollege.com/physics/**

30.5 MAGNETIC FLUX

12.5 The flux associated with a magnetic field is defined in a manner similar to that used to define electric flux (see Eq. 24.3). Consider an element of area dA on an arbitrarily shaped surface, as shown in Figure 30.19. If the magnetic field at this element is **B**, the magnetic flux through the element is $\mathbf{B} \cdot d\mathbf{A}$, where $d\mathbf{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dA. Hence, the total magnetic flux Φ_B through the surface is

$$\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A} \qquad (30.18)$$

Definition of magnetic flux

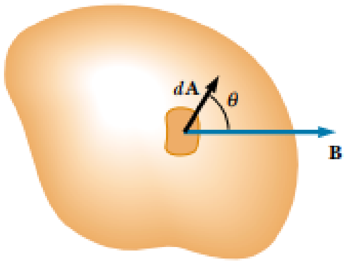


Figure 30.19 The magnetic flux through an area element dA is $\mathbf{B} \cdot d\mathbf{A} = BdA\cos\theta$, where $d\mathbf{A}$ is a vector perpendicular to the surface.

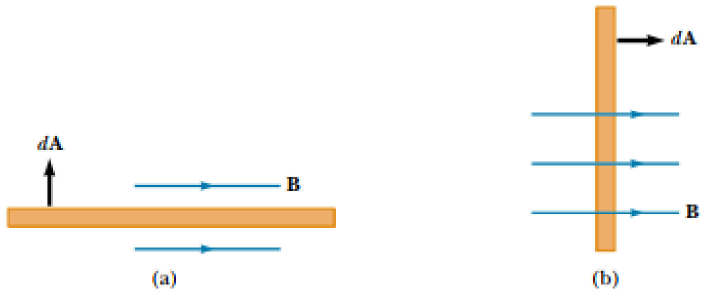


Figure 30.20 Magnetic flux through a plane lying in a magnetic field. (a) The flux through the plane is zero when the magnetic field is parallel to the plane surface. (b) The flux through the plane is a maximum when the magnetic field is perpendicular to the plane.

Consider the special case of a plane of area A in a uniform field $\mathbf{B}$ that makes an angle θ with $d\mathbf{A}$. The magnetic flux through the plane in this case is

$$\Phi_B = BA\cos\theta \tag{30.19}$$

If the magnetic field is parallel to the plane, as in Figure 30.20a, then $\theta = 90°$ and the flux is zero. If the field is perpendicular to the plane, as in Figure 30.20b, then $\theta = 0$ and the flux is BA (the maximum value).

The unit of flux is the $\mathrm{T \cdot m^2}$, which is defined as a *weber* (Wb); $1\ \mathrm{Wb} = 1\ \mathrm{T \cdot m^2}$.

Example 30.8 Magnetic Flux Through a Rectangular Loop

A rectangular loop of width a and length b is located near a long wire carrying a current I (Fig. 30.21). The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

Solution From Equation 30.14, we know that the magnitude of the magnetic field created by the wire at a distance r from the wire is

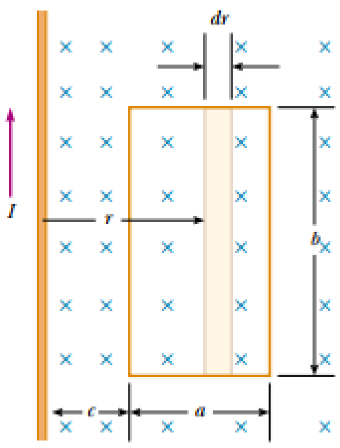


Figure 30.21 The magnetic field due to the wire carrying a current I is not uniform over the rectangular loop.

$$B = \frac{\mu_0 I}{2\pi r}$$

The factor $1/r$ indicates that the field varies over the loop, and Figure 30.21 shows that the field is directed into the page. Because $\mathbf{B}$ is parallel to $d\mathbf{A}$ at any point within the loop, the magnetic flux through an area element dA is

$$\Phi_B = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

(Because B is not uniform but depends on r, it cannot be removed from the integral.)

To integrate, we first express the area element (the tan region in Fig. 30.21) as $dA = b\,dr$. Because r is now the only variable in the integral, we have

$$\Phi_B = \frac{\mu_0 Ib}{2\pi}\int_c^{a+c}\frac{dr}{r} = \frac{\mu_0 Ib}{2\pi}\ln r\Big]_c^{a+c}$$

$$= \frac{\mu_0 Ib}{2\pi}\ln\left(\frac{a+c}{c}\right) = \boxed{\frac{\mu_0 Ib}{2\pi}\ln\left(1+\frac{a}{c}\right)}$$

Exercise Apply the series expansion formula for $\ln(1+x)$ (see Appendix B.5) to this equation to show that it gives a reasonable result when the loop is far from the wire relative to the loop dimensions (in other words, when $c \gg a$).

Answer $\Phi_B \rightarrow 0$.

30.6 GAUSS'S LAW IN MAGNETISM

12.5 In Chapter 24 we found that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gauss's law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This property is based on the fact that electric field lines originate and terminate on electric charges.

The situation is quite different for magnetic fields, which are continuous and form closed loops. In other words, magnetic field lines do not begin or end at any point—as illustrated by the magnetic field lines of the bar magnet in Figure 30.22. Note that for any closed surface, such as the one outlined by the dashed red line in Figure 30.22, the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero. In contrast, for a closed surface surrounding one charge of an electric dipole (Fig. 30.23), the net electric flux is not zero.

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad (30.20)$$

Gauss's law for magnetism

This statement is based on the experimental fact, mentioned in the opening of Chapter 29, that **isolated magnetic poles (monopoles) have never been detected and perhaps do not exist.** Nonetheless, scientists continue the search be-

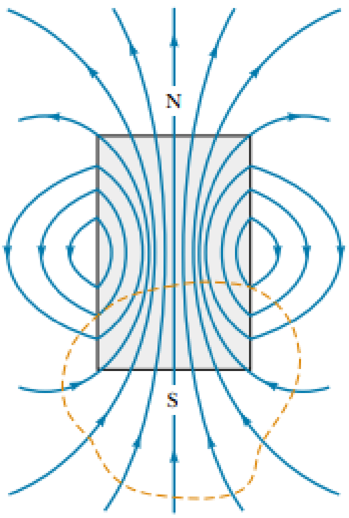


Figure 30.22 The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through the closed surface (dashed red line) surrounding one of the poles (or any other closed surface) is zero.

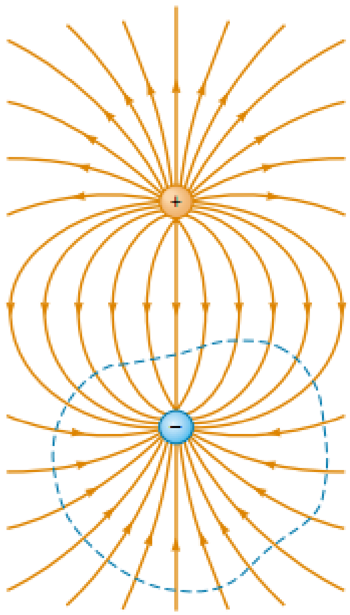


Figure 30.23 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge. The electric flux through a closed surface surrounding one of the charges is not zero.

cause certain theories that are otherwise successful in explaining fundamental physical behavior suggest the possible existence of monopoles.

30.7 DISPLACEMENT CURRENT AND THE GENERAL FORM OF AMPÈRE'S LAW

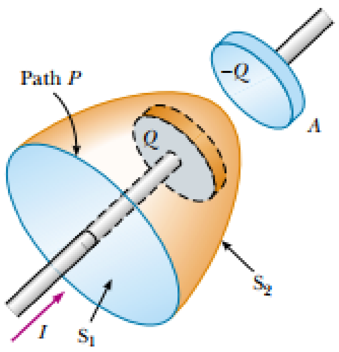


Figure 30.24 Two surfaces S_1 and S_2 near the plate of a capacitor are bounded by the same path P. The conduction current in the wire passes only through S_1. This leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

We have seen that charges in motion produce magnetic fields. When a current-carrying conductor has high symmetry, we can use Ampère's law to calculate the magnetic field it creates. In Equation 30.13, $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$, the line integral is over any closed path through which the conduction current passes, and the conduction current is defined by the expression $I = dq/dt$. (In this section we use the term *conduction current* to refer to the current carried by the wire, to distinguish it from a new type of current that we shall introduce shortly.) We now show that **Ampère's law in this form is valid only if any electric fields present are constant in time.** Maxwell recognized this limitation and modified Ampère's law to include time-varying electric fields.

We can understand the problem by considering a capacitor that is being charged as illustrated in Figure 30.24. When a conduction current is present, the charge on the positive plate changes but *no conduction current passes across the gap between the plates.* Now consider the two surfaces S_1 and S_2 in Figure 30.24, bounded by the same path P. Ampère's law states that $\oint \mathbf{B} \cdot d\mathbf{s}$ around this path must equal $\mu_0 I$, where I is the total current through any surface bounded by the path P.

12.9 When the path P is considered as bounding S_1, $\oint \mathbf{B} \cdot d\mathbf{s}$ is $\mu_0 I$ because the conduction current passes through S_1. When the path is considered as bounding S_2, however, $\oint \mathbf{B} \cdot d\mathbf{s} = 0$ because no conduction current passes through S_2. Thus, we arrive at a contradictory situation that arises from the discontinuity of the current! Maxwell solved this problem by postulating an additional term on the right side of Equation 30.13, which includes a factor called the **displacement current** I_d, defined as[3]

Displacement current

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \quad (30.21)$$

where ϵ_0 is the permittivity of free space (see Section 23.3) and $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ is the electric flux (see Eq. 24.3).

As the capacitor is being charged (or discharged), the changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current in the wire. When the expression for the displacement current given by Equation 30.21 is added to the conduction current on the right side of Ampère's law, the difficulty represented in Figure 30.24 is resolved. No matter which surface bounded by the path P is chosen, either conduction current or displacement current passes through it. With this new term I_d, we can express the general form of Ampère's law (sometimes called the **Ampère–Maxwell law**) as[4]

Ampère–Maxwell law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0(I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad (30.22)$$

[3] *Displacement* in this context does not have the meaning it does in Chapter 2. Despite the inaccurate implications, the word is historically entrenched in the language of physics, so we continue to use it.

[4] Strictly speaking, this expression is valid only in a vacuum. If a magnetic material is present, one must change μ_0 and ϵ_0 on the right-hand side of Equation 30.22 to the permeability μ_m and permittivity ϵ characteristic of the material. Alternatively, one may include a magnetizing current I_m on the righthand side of Equation 30.22 to make Ampère's law fully general. On a microscopic scale, I_m is as real as I.

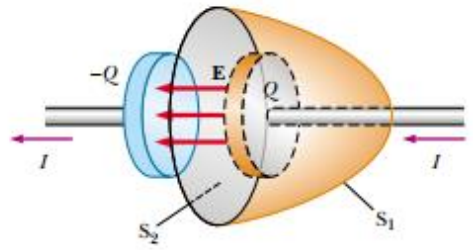


Figure 30.25 Because it exists only in the wires attached to the capacitor plates, the conduction current $I = dQ/dt$ passes through S_1 but not through S_2. Only the displacement current $I_d = \epsilon_0\, d\Phi_E/dt$ passes through S_2. The two currents must be equal for continuity.

We can understand the meaning of this expression by referring to Figure 30.25. The electric flux through surface S_2 is $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = EA$, where A is the area of the capacitor plates and E is the magnitude of the uniform electric field between the plates. If Q is the charge on the plates at any instant, then $E = Q/\epsilon_0 A$ (see Section 26.2). Therefore, the electric flux through S_2 is simply

$$\Phi_E = EA = \frac{Q}{\epsilon_0}$$

Hence, the displacement current through S_2 is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} \qquad \textbf{(30.23)}$$

That is, the displacement current through S_2 is precisely equal to the conduction current I through S_1!

By considering surface S_2, we can identify the displacement current as the source of the magnetic field on the surface boundary. The displacement current has its physical origin in the time-varying electric field. The central point of this formalism, then, is that

magnetic fields are produced both by conduction currents and by time-varying electric fields.

This result was a remarkable example of theoretical work by Maxwell, and it contributed to major advances in the understanding of electromagnetism.

Quick Quiz 30.6

What is the displacement current for a fully charged 3-μF capacitor?

Example 30.9 Displacement Current in a Capacitor

A sinusoidally varying voltage is applied across an 8.00-μF capacitor. The frequency of the voltage is 3.00 kHz, and the voltage amplitude is 30.0 V. Find the displacement current between the plates of the capacitor.

Solution The angular frequency of the source, from Equation 13.6, is $\omega = 2\pi f = 2\pi(3.00 \times 10^3\ \text{Hz}) = 1.88 \times 10^4\ \text{s}^{-1}$. Hence, the voltage across the capacitor in terms of t is

$$\Delta V = \Delta V_{\text{max}} \sin \omega t = (30.0\ \text{V}) \sin(1.88 \times 10^4 t)$$

We can use Equation 30.23 and the fact that the charge on the capacitor is $Q = C\Delta V$ to find the displacement current:

$$I_d = \frac{dQ}{dt} = \frac{d}{dt}(C\Delta V) = C\frac{d}{dt}(\Delta V)$$

$$= (8.00 \times 10^{-6}\ \text{F}) \frac{d}{dt}[(30.0\ \text{V}) \sin(1.88 \times 10^4 t)]$$

$$= \boxed{(4.52\ \text{A}) \cos(1.88 \times 10^4 t)}$$

The displacement current varies sinusoidally with time and has a maximum value of 4.52 A.

Optional Section

30.8 MAGNETISM IN MATTER

The magnetic field produced by a current in a coil of wire gives us a hint as to what causes certain materials to exhibit strong magnetic properties. Earlier we found that a coil like the one shown in Figure 30.17 has a north pole and a south pole. In general, *any* current loop has a magnetic field and thus has a magnetic dipole moment, including the atomic-level current loops described in some models of the atom. Thus, the magnetic moments in a magnetized substance may be described as arising from these atomic-level current loops. For the Bohr model of the atom, these current loops are associated with the movement of electrons around the nucleus in circular orbits. In addition, a magnetic moment is intrinsic to electrons, protons, neutrons, and other particles; it arises from a property called *spin*.

The Magnetic Moments of Atoms

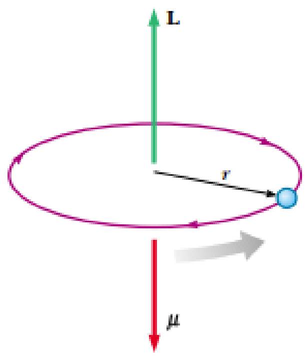


Figure 30.26 An electron moving in a circular orbit of radius r has an angular momentum $\mathbf{L}$ in one direction and a magnetic moment $\boldsymbol{\mu}$ in the opposite direction.

It is instructive to begin our discussion with a classical model of the atom in which electrons move in circular orbits around the much more massive nucleus. In this model, an orbiting electron constitutes a tiny current loop (because it is a moving charge), and the magnetic moment of the electron is associated with this orbital motion. Although this model has many deficiencies, its predictions are in good agreement with the correct theory, which is expressed in terms of quantum physics.

Consider an electron moving with constant speed v in a circular orbit of radius r about the nucleus, as shown in Figure 30.26. Because the electron travels a distance of $2\pi r$ (the circumference of the circle) in a time T, its orbital speed is $v = 2\pi r/T$. The current I associated with this orbiting electron is its charge e divided by T. Using $T = 2\pi/\omega$ and $\omega = v/r$, we have

$$I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

The magnetic moment associated with this current loop is $\mu = IA$, where $A = \pi r^2$ is the area enclosed by the orbit. Therefore,

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \tfrac{1}{2}evr \qquad \textbf{(30.24)}$$

Because the magnitude of the orbital angular momentum of the electron is $L = m_e vr$ (Eq. 11.16 with $\phi = 90°$), the magnetic moment can be written as

Orbital magnetic moment

$$\mu = \left(\frac{e}{2m_e}\right)L \qquad \textbf{(30.25)}$$

This result demonstrates that **the magnetic moment of the electron is proportional to its orbital angular momentum.** Note that because the electron is negatively charged, the vectors $\boldsymbol{\mu}$ and $\mathbf{L}$ point in opposite directions. Both vectors are perpendicular to the plane of the orbit, as indicated in Figure 30.26.

Angular momentum is quantized

A fundamental outcome of quantum physics is that orbital angular momentum is quantized and is equal to multiples of $\hbar = h/2\pi = 1.05 \times 10^{-34}$ J·s, where h is Planck's constant. The smallest nonzero value of the electron's magnetic moment resulting from its orbital motion is

$$\mu = \sqrt{2}\,\frac{e}{2m_e}\,\hbar \qquad \textbf{(30.26)}$$

We shall see in Chapter 42 how expressions such as Equation 30.26 arise.

Because all substances contain electrons, you may wonder why not all substances are magnetic. The main reason is that in most substances, the magnetic moment of one electron in an atom is canceled by that of another electron orbiting in the opposite direction. The net result is that, for most materials, **the magnetic effect produced by the orbital motion of the electrons is either zero or very small.**

In addition to its orbital magnetic moment, an electron has an intrinsic property called **spin** that also contributes to its magnetic moment. In this regard, the electron can be viewed as spinning about its axis while it orbits the nucleus, as shown in Figure 30.27. (Warning: This classical description should not be taken literally because spin arises from relativistic dynamics that must be incorporated into a quantum-mechanical analysis.) The magnitude of the angular momentum S associated with spin is of the same order of magnitude as the angular momentum L due to the orbital motion. The magnitude of the spin angular momentum predicted by quantum theory is

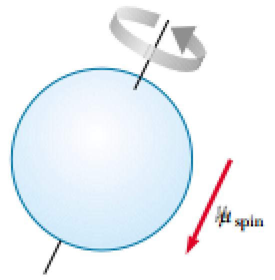


Figure 30.27 Classical model of a spinning electron. This model gives an incorrect magnitude for the magnetic moment, incorrect quantum numbers, and too many degrees of freedom.

Spin angular momentum

$$S = \frac{\sqrt{3}}{2}\hbar$$

The magnetic moment characteristically associated with the spin of an electron has the value

$$\mu_{\text{spin}} = \frac{e\hbar}{2m_e} \tag{30.27}$$

This combination of constants is called the **Bohr magneton:**

Bohr magneton

$$\mu_{\text{B}} = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ \text{J/T} \tag{30.28}$$

Thus, atomic magnetic moments can be expressed as multiples of the Bohr magneton. (Note that $1\ \text{J/T} = 1\ \text{A}\cdot\text{m}^2$.)

In atoms containing many electrons, the electrons usually pair up with their spins opposite each other; thus, the spin magnetic moments cancel. However, atoms containing an odd number of electrons must have at least one unpaired electron and therefore some spin magnetic moment. The total magnetic moment of an atom is the vector sum of the orbital and spin magnetic moments, and a few examples are given in Table 30.1. Note that helium and neon have zero moments because their individual spin and orbital moments cancel.

TABLE 30.1 **Magnetic Moments of Some Atoms and Ions**

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

The nucleus of an atom also has a magnetic moment associated with its constituent protons and neutrons. However, the magnetic moment of a proton or neutron is much smaller than that of an electron and can usually be neglected. We can understand this by inspecting Equation 30.28 and replacing the mass of the electron with the mass of a proton or a neutron. Because the masses of the proton and neutron are much greater than that of the electron, their magnetic moments are on the order of 10^3 times smaller than that of the electron.

Magnetization Vector and Magnetic Field Strength

Magnetization vector **M**

The magnetic state of a substance is described by a quantity called the **magnetization vector M. The magnitude of this vector is defined as the magnetic moment per unit volume of the substance.** As you might expect, the total magnetic field **B** at a point within a substance depends on both the applied (external) field $\mathbf{B}_0$ and the magnetization of the substance.

To understand the problems involved in measuring the total magnetic field **B** in such situations, consider this: Scientists use small probes that utilize the Hall effect (see Section 29.6) to measure magnetic fields. What would such a probe read if it were positioned inside the solenoid mentioned in the QuickLab on page 951 when you inserted the compass? Because the compass is a magnetic material, the probe would measure a total magnetic field **B** that is the sum of the solenoid (external) field $\mathbf{B}_0$ and the (magnetization) field $\mathbf{B}_m$ due to the compass. This tells us that we need a way to distinguish between magnetic fields originating from currents and those originating from magnetic materials. Consider a region in which a magnetic field $\mathbf{B}_0$ is produced by a current-carrying conductor. If we now fill that region with a magnetic substance, the total magnetic field **B** in the region is $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_m$, where $\mathbf{B}_m$ is the field produced by the magnetic substance. We can express this contribution in terms of the magnetization vector of the substance as $\mathbf{B}_m = \mu_0\mathbf{M}$; hence, the total magnetic field in the region becomes

$$\mathbf{B} = \mathbf{B}_0 + \mu_0\mathbf{M} \tag{30.29}$$

Magnetic field strength **H**

When analyzing magnetic fields that arise from magnetization, it is convenient to introduce a field quantity, called the **magnetic field strength H** within the substance. The magnetic field strength represents the effect of the conduction currents in wires on a substance. To emphasize the distinction between the field strength **H** and the field **B**, the latter is often called the *magnetic flux density* or the *magnetic induction*. The magnetic field strength is a vector defined by the relationship $\mathbf{H} = \mathbf{B}_0/\mu_0 = (\mathbf{B}/\mu_0) - \mathbf{M}$. Thus, Equation 30.29 can be written

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \tag{30.30}$$

The quantities **H** and **M** have the same units. In SI units, because **M** is magnetic moment per unit volume, the units are (ampere)(meter)^2/(meter)^3, or amperes per meter.

To better understand these expressions, consider the torus region of a toroid that carries a current I. If this region is a vacuum, $\mathbf{M} = 0$ (because no magnetic material is present), the total magnetic field is that arising from the current alone, and $\mathbf{B} = \mathbf{B}_0 = \mu_0\mathbf{H}$. Because $B_0 = \mu_0 nI$ in the torus region, where n is the number of turns per unit length of the toroid, $H = B_0/\mu_0 = \mu_0 nI/\mu_0$, or

$$H = nI \tag{30.31}$$

In this case, the magnetic field B in the torus region is due only to the current in the windings of the toroid.

If the torus is now made of some substance and the current I is kept constant, **H** in the torus region remains unchanged (because it depends on the current only) and has magnitude nI. The total field **B**, however, is different from that when the torus region was a vacuum. From Equation 30.30, we see that part of **B** arises from the term $\mu_0\mathbf{H}$ associated with the current in the toroid, and part arises from the term $\mu_0\mathbf{M}$ due to the magnetization of the substance of which the torus is made.

Oxygen, a paramagnetic substance, is attracted to a magnetic field. The liquid oxygen in this photograph is suspended between the poles of the magnet.

Classification of Magnetic Substances

Substances can be classified as belonging to one of three categories, depending on their magnetic properties. **Paramagnetic** and **ferromagnetic** materials are those made of atoms that have permanent magnetic moments. **Diamagnetic** materials are those made of atoms that do not have permanent magnetic moments.

For paramagnetic and diamagnetic substances, the magnetization vector **M** is proportional to the magnetic field strength **H**. For these substances placed in an external magnetic field, we can write

$$\mathbf{M} = \chi\mathbf{H} \tag{30.32}$$

TABLE 30.2 Magnetic Susceptibilities of Some Paramagnetic and Diamagnetic Substances at 300 K

Paramagnetic Substance	χ	Diamagnetic Substance	χ
Aluminum	2.3×10^{-5}	Bismuth	-1.66×10^{-5}
Calcium	1.9×10^{-5}	Copper	-9.8×10^{-6}
Chromium	2.7×10^{-4}	Diamond	-2.2×10^{-5}
Lithium	2.1×10^{-5}	Gold	-3.6×10^{-5}
Magnesium	1.2×10^{-5}	Lead	-1.7×10^{-5}
Niobium	2.6×10^{-4}	Mercury	-2.9×10^{-5}
Oxygen	2.1×10^{-6}	Nitrogen	-5.0×10^{-9}
Platinum	2.9×10^{-4}	Silver	-2.6×10^{-5}
Tungsten	6.8×10^{-5}	Silicon	-4.2×10^{-6}

where χ (Greek letter chi) is a dimensionless factor called the **magnetic susceptibility.** For paramagnetic substances, χ is positive and **M** is in the same direction as **H**. For diamagnetic substances, χ is negative and **M** is opposite **H**. (It is important to note that this linear relationship between **M** and **H** does not apply to ferromagnetic substances.) The susceptibilities of some substances are given in Table 30.2.

Magnetic susceptibility χ

Substituting Equation 30.32 for **M** into Equation 30.30 gives

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(\mathbf{H} + \chi\mathbf{H}) = \mu_0(1 + \chi)\mathbf{H}$$

or

$$\mathbf{B} = \mu_m\mathbf{H} \tag{30.33}$$

where the constant μ_m is called the **magnetic permeability** of the substance and is related to the susceptibility by

$$\mu_m = \mu_0(1 + \chi) \tag{30.34}$$

Magnetic permeability μ_m

Substances may be classified in terms of how their magnetic permeability μ_m compares with μ_0 (the permeability of free space), as follows:

Paramagnetic	$\mu_m > \mu_0$
Diamagnetic	$\mu_m < \mu_0$

Because χ is very small for paramagnetic and diamagnetic substances (see Table 30.2), μ_m is nearly equal to μ_0 for these substances. For ferromagnetic substances, however, μ_m is typically several thousand times greater than μ_0 (meaning that χ is very great for ferromagnetic substances).

Although Equation 30.33 provides a simple relationship between **B** and **H**, we must interpret it with care when dealing with ferromagnetic substances. As mentioned earlier, **M** is not a linear function of **H** for ferromagnetic substances. This is because the value of μ_m is not only a characteristic of the ferromagnetic substance but also depends on the previous state of the substance and on the process it underwent as it moved from its previous state to its present one. We shall investigate this more deeply after the following example.

EXAMPLE 30.10 An Iron-Filled Toroid

A toroid wound with 60.0 turns/m of wire carries a current of 5.00 A. The torus is iron, which has a magnetic permeability of $\mu_m = 5\,000\mu_0$ under the given conditions. Find H and B inside the iron.

Solution Using Equations 30.31 and 30.33, we obtain

$$H = nI = \left(60.0 \frac{\text{turns}}{\text{m}}\right)(5.00\text{ A}) = \boxed{300 \frac{\text{A}\cdot\text{turns}}{\text{m}}}$$

$$B = \mu_m H = 5\,000\mu_0 H$$
$$= 5\,000\left(4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(300 \frac{\text{A}\cdot\text{turns}}{\text{m}}\right) = \boxed{1.88\text{ T}}$$

This value of B is 5 000 times the value in the absence of iron!

Exercise Determine the magnitude of the magnetization vector inside the iron torus.

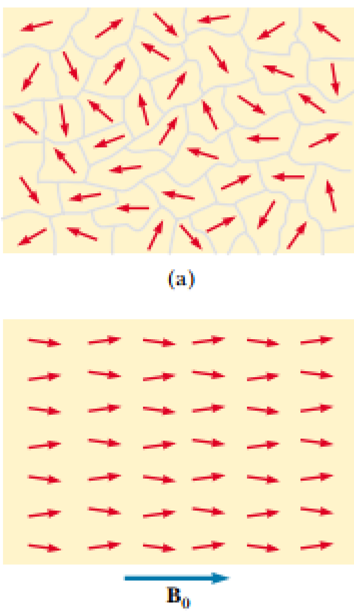
Answer $M = 1.5 \times 10^6$ A/m.

Quick Quiz 30.7

A current in a solenoid having air in the interior creates a magnetic field $\mathbf{B} = \mu_0\mathbf{H}$. Describe qualitatively what happens to the magnitude of $\mathbf{B}$ as (a) aluminum, (b) copper, and (c) iron are placed in the interior.

Ferromagnetism

A small number of crystalline substances in which the atoms have permanent magnetic moments exhibit strong magnetic effects called **ferromagnetism.** Some examples of ferromagnetic substances are iron, cobalt, nickel, gadolinium, and dysprosium. These substances contain atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Once the moments are aligned, the substance remains magnetized after the external field is removed. This permanent alignment is due to a strong coupling between neighboring moments, a coupling that can be understood only in quantum-mechanical terms.


(a)

(b)

Figure 30.28 (a) Random orientation of atomic magnetic moments in an unmagnetized substance. (b) When an external field $\mathbf{B}_0$ is applied, the atomic magnetic moments tend to align with the field, giving the sample a net magnetization vector $\mathbf{M}$.

All ferromagnetic materials are made up of microscopic regions called **domains,** regions within which all magnetic moments are aligned. These domains have volumes of about 10^{-12} to 10^{-8} m^3 and contain 10^{17} to 10^{21} atoms. The boundaries between the various domains having different orientations are called **domain walls.** In an unmagnetized sample, the domains are randomly oriented so that the net magnetic moment is zero, as shown in Figure 30.28a. When the sample is placed in an external magnetic field, the magnetic moments of the atoms tend to align with the field, which results in a magnetized sample, as in Figure 30.28b. Observations show that domains initially oriented along the external field grow larger at the expense of the less favorably oriented domains. When the external field is removed, the sample may retain a net magnetization in the direction of the original field. At ordinary temperatures, thermal agitation is not sufficient to disrupt this preferred orientation of magnetic moments.

A typical experimental arrangement that is used to measure the magnetic properties of a ferromagnetic material consists of a torus made of the material wound with N turns of wire, as shown in Figure 30.29, where the windings are represented in black and are referred to as the *primary coil*. This apparatus is sometimes referred to as a **Rowland ring.** A *secondary coil* (the red wires in Fig. 30.29) connected to a galvanometer is used to measure the total magnetic flux through the torus. The magnetic field $\mathbf{B}$ in the torus is measured by increasing the current in the toroid from zero to I. As the current changes, the magnetic flux through the secondary coil changes by an amount BA, where A is the cross-sectional area of the toroid. As we shall find in Chapter 31, because of this changing flux, an emf that is proportional to the rate of change in magnetic flux is induced in the secondary coil. If the galvanometer is properly calibrated, a value for $\mathbf{B}$ corresponding to any value of the current in the primary coil can be obtained. The magnetic field $\mathbf{B}$ is measured first in the absence of the torus and then with the torus in place. The magnetic properties of the torus material are then obtained from a comparison of the two measurements.

Now consider a torus made of unmagnetized iron. If the current in the primary coil is increased from zero to some value I, the magnitude of the magnetic field strength H increases linearly with I according to the expression $H = nI$. Furthermore, the magnitude of the total field B also increases with increasing current, as shown by the curve from point O to point a in Figure 30.30. At point O, the domains in the iron are randomly oriented, corresponding to $B_m = 0$. As the increasing current in the primary coil causes the external field $\mathbf{B}_0$ to increase, the domains become more aligned until all of them are nearly aligned at point a. At this point the iron core is approaching *saturation*, which is the condition in which all domains in the iron are aligned.

Next, suppose that the current is reduced to zero, and the external field is consequently eliminated. The B versus H curve, called a **magnetization curve,** now follows the path ab in Figure 30.30. Note that at point b, $\mathbf{B}$ is not zero even though the external field is $\mathbf{B}_0 = 0$. The reason is that the iron is now magnetized due to the alignment of a large number of its domains (that is, $\mathbf{B} = \mathbf{B}_m$). At this point, the iron is said to have a *remanent* magnetization.

If the current in the primary coil is reversed so that the direction of the external magnetic field is reversed, the domains reorient until the sample is again unmagnetized at point c, where $B = 0$. An increase in the reverse current causes the iron to be magnetized in the opposite direction, approaching saturation at point d in Figure 30.30. A similar sequence of events occurs as the current is reduced to zero and then increased in the original (positive) direction. In this case the magnetization curve follows the path def. If the current is increased sufficiently, the magnetization curve returns to point a, where the sample again has its maximum magnetization.

The effect just described, called **magnetic hysteresis,** shows that the magnetization of a ferromagnetic substance depends on the history of the substance as well as on the magnitude of the applied field. (The word *hysteresis* means "lagging behind.") It is often said that a ferromagnetic substance has a "memory" because it remains magnetized after the external field is removed. The closed loop in Figure 30.30 is referred to as a hysteresis loop. Its shape and size depend on the proper-

QuickLab

You've probably done this experiment before. Magnetize a nail by repeatedly dragging it across a bar magnet. Test the strength of the nail's magnetic field by picking up some paper clips. Now hit the nail several times with a hammer, and again test the strength of its magnetism. Explain what happens in terms of domains in the steel of the nail.

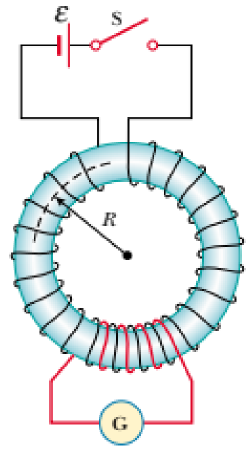


Figure 30.29 A toroidal winding arrangement used to measure the magnetic properties of a material. The torus is made of the material under study, and the circuit containing the galvanometer measures the magnetic flux.

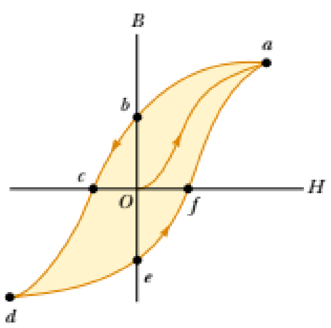


Figure 30.30 Magnetization curve for a ferromagnetic material.

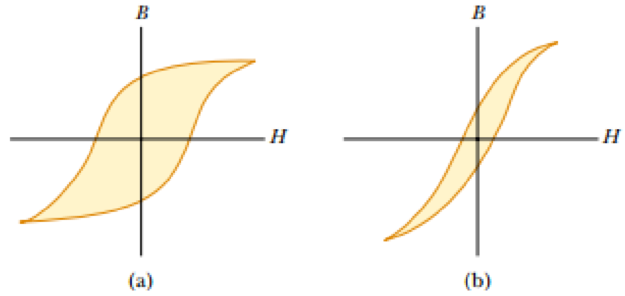


Figure 30.31 Hysteresis loops for (a) a hard ferromagnetic material and (b) a soft ferromagnetic material.

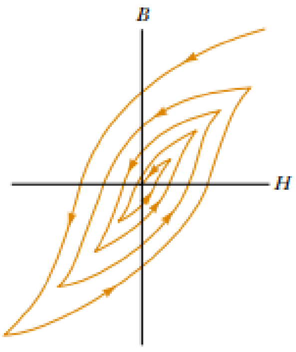


Figure 30.32 Demagnetizing a ferromagnetic material by carrying it through successive hysteresis loops.

ties of the ferromagnetic substance and on the strength of the maximum applied field. The hysteresis loop for "hard" ferromagnetic materials is characteristically wide like the one shown in Figure 30.31a, corresponding to a large remanent magnetization. Such materials cannot be easily demagnetized by an external field. "Soft" ferromagnetic materials, such as iron, have a very narrow hysteresis loop and a small remanent magnetization (Fig. 30.31b.) Such materials are easily magnetized and demagnetized. An ideal soft ferromagnet would exhibit no hysteresis and hence would have no remanent magnetization. A ferromagnetic substance can be demagnetized by being carried through successive hysteresis loops, due to a decreasing applied magnetic field, as shown in Figure 30.32.

Quick Quiz 30.8

Which material would make a better permanent magnet, one whose hysteresis loop looks like Figure 30.31a or one whose loop looks like Figure 30.31b?

The magnetization curve is useful for another reason: **The area enclosed by the magnetization curve represents the work required to take the material through the hysteresis cycle.** The energy acquired by the material in the magnetization process originates from the source of the external field—that is, the emf in the circuit of the toroidal coil. When the magnetization cycle is repeated, dissipative processes within the material due to realignment of the domains result in a transformation of magnetic energy into internal energy, which is evidenced by an increase in the temperature of the substance. For this reason, devices subjected to alternating fields (such as ac adapters for cell phones, power tools, and so on) use cores made of soft ferromagnetic substances, which have narrow hysteresis loops and correspondingly little energy loss per cycle.

Magnetic computer disks store information by alternating the direction of $\mathbf{B}$ for portions of a thin layer of ferromagnetic material. Floppy disks have the layer on a circular sheet of plastic. Hard disks have several rigid platters with magnetic coatings on each side. Audio tapes and videotapes work the same way as floppy disks except that the ferromagnetic material is on a very long strip of plastic. Tiny coils of wire in a recording head are placed close to the magnetic material (which is moving rapidly past the head). Varying the current through the coils creates a magnetic field that magnetizes the recording material. To retrieve the information, the magnetized material is moved past a playback coil. The changing magnetism of the material induces a current in the coil, as we shall discuss in Chapter 31. This current is then amplified by audio or video equipment, or it is processed by computer circuitry.

Paramagnetism

Paramagnetic substances have a small but positive magnetic susceptibility ($0 < \chi \ll 1$) resulting from the presence of atoms (or ions) that have permanent magnetic moments. These moments interact only weakly with each other and are randomly oriented in the absence of an external magnetic field. When a paramagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field. However, this alignment process must compete with thermal motion, which tends to randomize the magnetic moment orientations.

Pierre Curie (1859–1906) and others since him have found experimentally that, under a wide range of conditions, the magnetization of a paramagnetic substance is proportional to the applied magnetic field and inversely proportional to the absolute temperature:

$$M = C\frac{B_0}{T} \tag{30.35}$$

This relationship is known as **Curie's law** after its discoverer, and the constant C is called **Curie's constant.** The law shows that when $B_0 = 0$, the magnetization is zero, corresponding to a random orientation of magnetic moments. As the ratio of magnetic field to temperature becomes great, the magnetization approaches its saturation value, corresponding to a complete alignment of its moments, and Equation 30.35 is no longer valid.

When the temperature of a ferromagnetic substance reaches or exceeds a critical temperature called the **Curie temperature,** the substance loses its residual magnetization and becomes paramagnetic (Fig. 30.33). Below the Curie temperature, the magnetic moments are aligned and the substance is ferromagnetic. Above the Curie temperature, the thermal agitation is great enough to cause a random orientation of the moments, and the substance becomes paramagnetic. Curie temperatures for several ferromagnetic substances are given in Table 30.3.

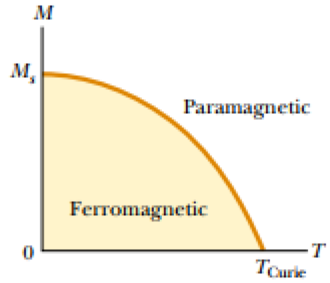


Figure 30.33 Magnetization versus absolute temperature for a ferromagnetic substance. The magnetic moments are aligned below the Curie temperature T_{Curie}, where the substance is ferromagnetic. The substance becomes paramagnetic (magnetic moments unaligned) above T_{Curie}.

TABLE 30.3 Curie Temperatures for Several Ferromagnetic Substances

Substance	T_{Curie} (K)
Iron	1 043
Cobalt	1 394
Nickel	631
Gadolinium	317
Fe_2O_3	893

Diamagnetism

When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field. This causes diamagnetic substances to be weakly repelled by a magnet. Although diamagnetism is present in all matter, its effects are much smaller than those of paramagnetism or ferromagnetism, and are evident only when those other effects do not exist.

We can attain some understanding of diamagnetism by considering a classical model of two atomic electrons orbiting the nucleus in opposite directions but with the same speed. The electrons remain in their circular orbits because of the attractive electrostatic force exerted by the positively charged nucleus. Because the magnetic moments of the two electrons are equal in magnitude and opposite in direction, they cancel each other, and the magnetic moment of the atom is zero. When an external magnetic field is applied, the electrons experience an additional force $q\mathbf{v} \times \mathbf{B}$. This added force combines with the electrostatic force to increase the orbital speed of the electron whose magnetic moment is antiparallel to the field and to decrease the speed of the electron whose magnetic moment is parallel to the field. As a result, the two magnetic moments of the electrons no longer cancel, and the substance acquires a net magnetic moment that is opposite the applied field.

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Visit **www.exploratorium.edu/snacks/diamagnetism_www/index.html** for an experiment showing that grapes are repelled by magnets!

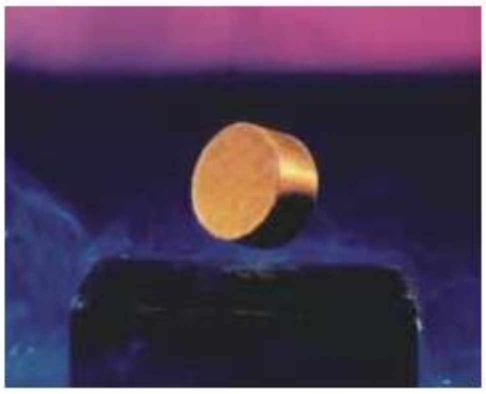

Figure 30.34 A small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$ cooled to liquid nitrogen temperature (77 K).

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For a more detailed description of the unusual properties of superconductors, visit **www.saunderscollege.com/physics/**

As you recall from Chapter 27, a superconductor is a substance in which the electrical resistance is zero below some critical temperature. Certain types of superconductors also exhibit perfect diamagnetism in the superconducting state. As a result, an applied magnetic field is expelled by the superconductor so that the field is zero in its interior. This phenomenon of flux expulsion is known as the **Meissner effect.** If a permanent magnet is brought near a superconductor, the two objects repel each other. This is illustrated in Figure 30.34, which shows a small permanent magnet levitated above a superconductor maintained at 77 K.

EXAMPLE 30.11 Saturation Magnetization

Estimate the saturation magnetization in a long cylinder of iron, assuming one unpaired electron spin per atom.

Solution The saturation magnetization is obtained when all the magnetic moments in the sample are aligned. If the sample contains n atoms per unit volume, then the saturation magnetization M_s has the value

$$M_s = n\mu$$

where μ is the magnetic moment per atom. Because the molar mass of iron is 55 g/mol and its density is 7.9 g/cm^3, the value of n for iron is 8.6×10^{28} atoms/m^3. Assuming that each atom contributes one Bohr magneton (due to one unpaired spin) to the magnetic moment, we obtain

$$M_s = \left(8.6 \times 10^{28}\ \frac{\text{atoms}}{\text{m}^3}\right)\left(9.27 \times 10^{-24}\ \frac{\text{A}\cdot\text{m}^2}{\text{atom}}\right)$$

$$= \boxed{8.0 \times 10^5\ \text{A/m}}$$

This is about one-half the experimentally determined saturation magnetization for iron, which indicates that actually two unpaired electron spins are present per atom.

Optional Section

30.9 THE MAGNETIC FIELD OF THE EARTH

When we speak of a compass magnet having a north pole and a south pole, we should say more properly that it has a "north-seeking" pole and a "south-seeking" pole. By this we mean that one pole of the magnet seeks, or points to, the north geographic pole of the Earth. Because the north pole of a magnet is attracted toward the north geographic pole of the Earth, we conclude that **the Earth's south magnetic pole is located near the north geographic pole, and the Earth's north magnetic pole is located near the south geographic pole.** In fact, the configuration of the Earth's magnetic field, pictured in Figure 30.35, is very much like the one that would be achieved by burying a gigantic bar magnet deep in the interior of the Earth.

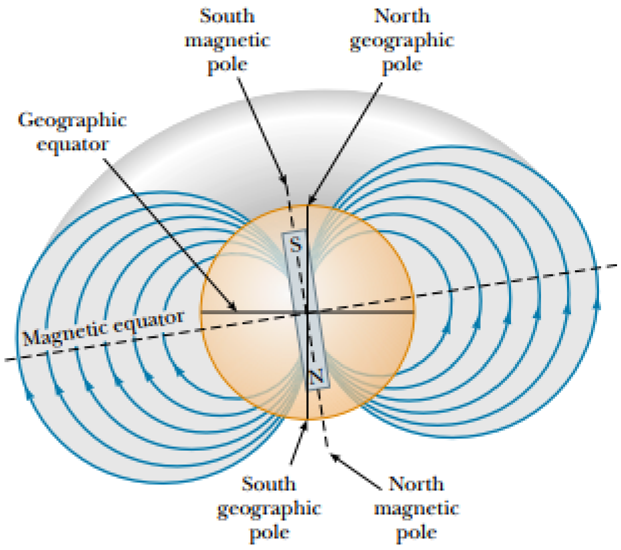
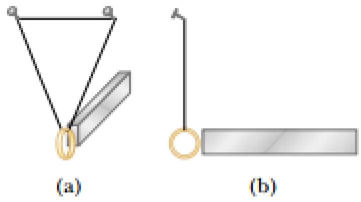


Figure 30.35 The Earth's magnetic field lines. Note that a south magnetic pole is near the north geographic pole, and a north magnetic pole is near the south geographic pole.

QuickLab

A gold ring is very weakly repelled by a magnet. To see this, suspend a 14- or 18-karat gold ring on a long loop of thread, as shown in (a). Gently tap the ring and estimate its period of oscillation. Now bring the ring to rest, letting it hang for a few moments so that you can verify that it is not moving. Quickly bring a very strong magnet to within a few millimeters of the ring, taking care not to bump it, as shown in (b). Now pull the magnet away. Repeat this action many times, matching the oscillation period you estimated earlier. This is just like pushing a child on a swing. A small force applied at the resonant frequency results in a large-amplitude oscillation. If you have a platinum ring, you will be able to see a similar effect except that platinum is weakly attracted to a magnet because it is paramagnetic.



If a compass needle is suspended in bearings that allow it to rotate in the vertical plane as well as in the horizontal plane, the needle is horizontal with respect to the Earth's surface only near the equator. As the compass is moved northward, the needle rotates so that it points more and more toward the surface of the Earth. Finally, at a point near Hudson Bay in Canada, the north pole of the needle points directly downward. This site, first found in 1832, is considered to be the location of the south magnetic pole of the Earth. It is approximately 1 300 mi from the Earth's geographic North Pole, and its exact position varies slowly with time. Similarly, the north magnetic pole of the Earth is about 1 200 mi away from the Earth's geographic South Pole.

Because of this distance between the north geographic and south magnetic poles, it is only approximately correct to say that a compass needle points north. The difference between true north, defined as the geographic North Pole, and north indicated by a compass varies from point to point on the Earth, and the difference is referred to as *magnetic declination*. For example, along a line through Florida and the Great Lakes, a compass indicates true north, whereas in Washington state, it aligns 25° east of true north.

The north end of a compass needle points to the *south* magnetic pole of the Earth. The "north" compass direction varies from true geographic north depending on the magnetic declination at that point on the Earth's surface.

Quick Quiz 30.9

If we wanted to cancel the Earth's magnetic field by running an enormous current loop around the equator, which way would the current have to flow: east to west or west to east?

Although the magnetic field pattern of the Earth is similar to the one that would be set up by a bar magnet deep within the Earth, it is easy to understand why the source of the Earth's magnetic field cannot be large masses of permanently magnetized material. The Earth does have large deposits of iron ore deep beneath its surface, but the high temperatures in the Earth's core prevent the iron from retaining any permanent magnetization. Scientists consider it more likely that the true source of the Earth's magnetic field is charge-carrying convection currents in the Earth's core. Charged ions or electrons circulating in the liquid interior could produce a magnetic field just as a current loop does. There is also strong evidence that the magnitude of a planet's magnetic field is related to the planet's rate of rotation. For example, Jupiter rotates faster than the Earth, and space probes indicate that Jupiter's magnetic field is stronger than ours. Venus, on the other hand, rotates more slowly than the Earth, and its magnetic field is found to be weaker. Investigation into the cause of the Earth's magnetism is ongoing.

There is an interesting sidelight concerning the Earth's magnetic field. It has been found that the direction of the field has been reversed several times during the last million years. Evidence for this is provided by basalt, a type of rock that contains iron and that forms from material spewed forth by volcanic activity on the ocean floor. As the lava cools, it solidifies and retains a picture of the Earth's magnetic field direction. The rocks are dated by other means to provide a timeline for these periodic reversals of the magnetic field.

SUMMARY

The **Biot–Savart law** says that the magnetic field $d\mathbf{B}$ at a point P due to a length element $d\mathbf{s}$ that carries a steady current I is

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2} \tag{30.1}$$

where $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$ is the **permeability of free space,** r is the distance from the element to the point P, and $\hat{\mathbf{r}}$ is a unit vector pointing from $d\mathbf{s}$ to point P. We find the total field at P by integrating this expression over the entire current distribution.

The magnetic field at a distance a from a long, straight wire carrying an electric current I is

$$B = \frac{\mu_0 I}{2\pi a} \tag{30.5}$$

The field lines are circles concentric with the wire.

The magnetic force per unit length between two parallel wires separated by a distance a and carrying currents I_1 and I_2 has a magnitude

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \tag{30.12}$$

The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

Ampère's law says that the line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \tag{30.13}$$

Using Ampère's law, one finds that the fields inside a toroid and solenoid are

$$B = \frac{\mu_0 NI}{2\pi r} \qquad \text{(toroid)} \tag{30.16}$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI \qquad \text{(solenoid)} \tag{30.17}$$

where N is the total number of turns.

The **magnetic flux Φ_B** through a surface is defined by the surface integral

$$\Phi_B \equiv \int \mathbf{B} \cdot d\mathbf{A} \tag{30.18}$$

Gauss's law of magnetism states that the net magnetic flux through any closed surface is zero.

The general form of Ampère's law, which is also called the **Ampère-Maxwell law,** is

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{30.22}$$

This law describes the fact that magnetic fields are produced both by conduction currents and by changing electric fields.

QUESTIONS

1. Is the magnetic field created by a current loop uniform? Explain.

2. A current in a conductor produces a magnetic field that can be calculated using the Biot–Savart law. Because current is defined as the rate of flow of charge, what can you conclude about the magnetic field produced by stationary charges? What about that produced by moving charges?

3. Two parallel wires carry currents in opposite directions. Describe the nature of the magnetic field created by the two wires at points (a) between the wires and (b) outside the wires, in a plane containing them.

4. Explain why two parallel wires carrying currents in opposite directions repel each other.

5. When an electric circuit is being assembled, a common practice is to twist together two wires carrying equal currents in opposite directions. Why does this technique reduce stray magnetic fields?

6. Is Ampère's law valid for all closed paths surrounding a conductor? Why is it not useful for calculating $\mathbf{B}$ for all such paths?

7. Compare Ampère's law with the Biot–Savart law. Which is more generally useful for calculating $\mathbf{B}$ for a current-carrying conductor?

8. Is the magnetic field inside a toroid uniform? Explain.

9. Describe the similarities between Ampère's law in magnetism and Gauss's law in electrostatics.

10. A hollow copper tube carries a current along its length. Why does $\mathbf{B} = 0$ inside the tube? Is $\mathbf{B}$ nonzero outside the tube?

11. Why is $\mathbf{B}$ nonzero outside a solenoid? Why does $\mathbf{B} = 0$ outside a toroid? (Remember that the lines of $\mathbf{B}$ must form closed paths.)

12. Describe the change in the magnetic field in the interior of a solenoid carrying a steady current I (a) if the length of the solenoid is doubled but the number of turns remains the same and (b) if the number of turns is doubled but the length remains the same.

13. A flat conducting loop is positioned in a uniform magnetic field directed along the x axis. For what orientation of the loop is the flux through it a maximum? A minimum?

14. What new concept does Maxwell's general form of Ampère's law include?

15. Many loops of wire are wrapped around a nail and then connected to a battery. Identify the source of $\mathbf{M}$, of $\mathbf{H}$, and of $\mathbf{B}$.