10 Computer numbering systems

10.1 Binary numbers

The system of numbers in everyday use is the **denary** or **decimal** system of numbers, using the digits 0 to 9. It has ten different digits (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) and is said to have a **radix** or **base** of 10.

The **binary** system of numbers has a radix of 2 and uses only the digits 0 and 1.

10.2 Conversion of binary to denary

The denary number 234.5 is equivalent to

$$2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0 + 5 \times 10^{-1}$$

i.e. is the sum of terms comprising: (a digit) multiplied by (the base raised to some power).

In the binary system of numbers, the base is 2, so 1101.1 is equivalent to:

$$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1}$$

Thus the denary number equivalent to the binary number 1101.1 is $8 + 4 + 0 + 1 + \frac{1}{2}$, that is 13.5 i.e. $\mathbf{1101.1_2 = 13.5_{10}}$, the suffixes 2 and 10 denoting binary and denary systems of numbers respectively.

Problem 1. Convert 11011_2 to a denary number.

From above:
$$\begin{aligned} 11011_2 &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 \\ &\quad + 1 \times 2^1 + 1 \times 2^0 \\ &= 16 + 8 + 0 + 2 + 1 \\ &= \mathbf{27_{10}} \end{aligned}$$

Problem 2. Convert 0.1011_2 to a denary fraction.

$$\begin{aligned} 0.1011_2 &= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\ &= 1 \times \frac{1}{2} + 0 \times \frac{1}{2^2} + 1 \times \frac{1}{2^3} + 1 \times \frac{1}{2^4} \\ &= \frac{1}{2} + \frac{1}{8} + \frac{1}{16} \\ &= 0.5 + 0.125 + 0.0625 \\ &= \mathbf{0.6875_{10}} \end{aligned}$$

Problem 3. Convert 101.0101_2 to a denary number.

$$\begin{aligned} 101.0101_2 &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} \\ &\quad + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} \\ &= 4 + 0 + 1 + 0 + 0.25 + 0 + 0.0625 \\ &= \mathbf{5.3125_{10}} \end{aligned}$$

Now try the following exercise.

Exercise 42 Further problems on conversion of binary to denary numbers

In Problems 1 to 4, convert the binary numbers given to denary numbers.

1. (a) 110 (b) 1011 (c) 1110 (d) 1001

   [(a) 6_{10} (b) 11_{10} (c) 14_{10} (d) 9_{10}]

2. (a) 10101 (b) 11001 (c) 101101 (d) 110011

   [(a) 21_{10} (b) 25_{10} (c) 45_{10} (d) 51_{10}]

3. (a) 0.1101 (b) 0.11001 (c) 0.00111 (d) 0.01011

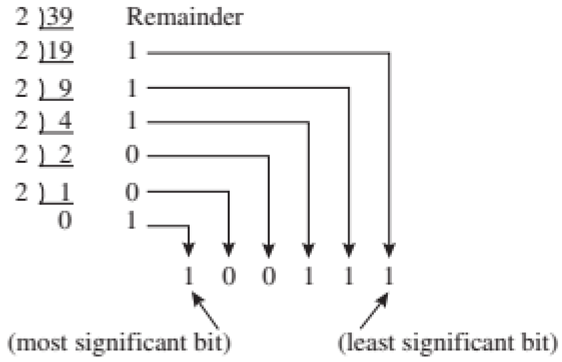
   [(a) 0.8125_{10} (b) 0.78125_{10} (c) 0.21875_{10} (d) 0.34375_{10}]

4. (a) 11010.11 (b) 10111.011 (c) 110101.0111 (d) 11010101.10111

   [(a) 26.75_{10} (b) 23.375_{10} (c) 53.4375_{10} (d) 213.71875_{10}]

10.3 Conversion of denary to binary

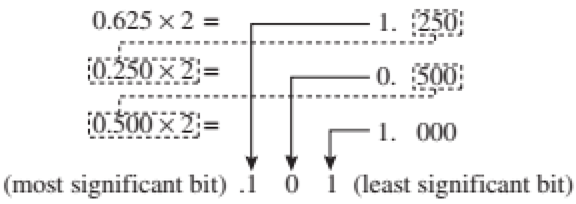
An integer denary number can be converted to a corresponding binary number by repeatedly dividing by 2 and noting the remainder at each stage, as shown below for 39_{10}.



The result is obtained by writing the top digit of the remainder as the least significant bit, (a bit is a **b**inary dig**it** and the least significant bit is the one on the right). The bottom bit of the remainder is the most significant bit, i.e. the bit on the left.

Thus $\mathbf{39_{10} = 100111_2}$

The fractional part of a denary number can be converted to a binary number by repeatedly multiplying by 2, as shown below for the fraction 0.625.

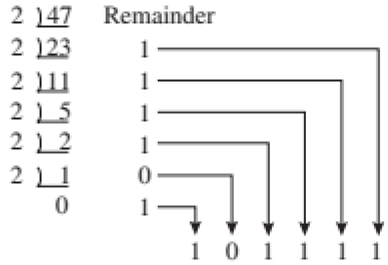


For fractions, the most significant bit of the result is the top bit obtained from the integer part of multiplication by 2. The least significant bit of the result is the bottom bit obtained from the integer part of multiplication by 2.

Thus $\mathbf{0.625_{10} = 0.101_2}$

Problem 4. Convert 47_{10} to a binary number.

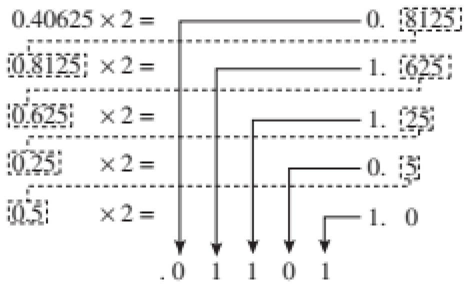
From above, repeatedly dividing by 2 and noting the remainder gives:



Thus $\mathbf{47_{10} = 101111_2}$

Problem 5. Convert 0.40625_{10} to a binary number.

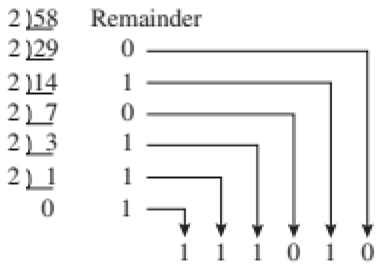
From above, repeatedly multiplying by 2 gives:



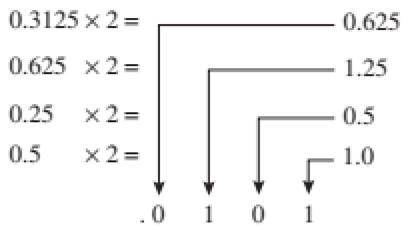
i.e. $\mathbf{0.40625_{10} = 0.01101_2}$

Problem 6. Convert 58.3125_{10} to a binary number.

The integer part is repeatedly divided by 2, giving:



The fractional part is repeatedly multiplied by 2 giving:



Thus $\mathbf{58.3125_{10} = 111010.0101_2}$

Now try the following exercise.

Exercise 43 Further problems on conversion of denary to binary numbers

In Problems 1 to 4, convert the denary numbers given to binary numbers.

1. (a) 5 (b) 15 (c) 19 (d) 29
$$\begin{bmatrix} \text{(a) } 101_2 & \text{(b) } 1111_2 \\ \text{(c) } 10011_2 & \text{(d) } 11101_2 \end{bmatrix}$$

2. (a) 31 (b) 42 (c) 57 (d) 63
$$\begin{bmatrix} \text{(a) } 11111_2 & \text{(b) } 101010_2 \\ \text{(c) } 111001_2 & \text{(d) } 111111_2 \end{bmatrix}$$

3. (a) 0.25 (b) 0.21875 (c) 0.28125 (d) 0.59375
$$\begin{bmatrix} \text{(a) } 0.01_2 & \text{(b) } 0.00111_2 \\ \text{(c) } 0.01001_2 & \text{(d) } 0.10011_2 \end{bmatrix}$$

4. (a) 47.40625 (b) 30.8125 (c) 53.90625 (d) 61.65625
$$\begin{bmatrix} \text{(a) } 101111.01101_2 \\ \text{(b) } 11110.1101_2 \\ \text{(c) } 110101.11101_2 \\ \text{(d) } 111101.10101_2 \end{bmatrix}$$

10.4 Conversion of denary to binary via octal

For denary integers containing several digits, repeatedly dividing by 2 can be a lengthy process. In this case, it is usually easier to convert a denary number to a binary number via the octal system of numbers. This system has a radix of 8, using the digits 0, 1, 2, 3, 4, 5, 6 and 7. The denary number equivalent to the octal number 4317_8 is:

$$4 \times 8^3 + 3 \times 8^2 + 1 \times 8^1 + 7 \times 8^0$$

i.e. $4 \times 512 + 3 \times 64 + 1 \times 8 + 7 \times 1$ or 2255_{10}

An integer denary number can be converted to a corresponding octal number by repeatedly dividing by 8 and noting the remainder at each stage, as shown below for 493_{10}.

$$\begin{array}{rl} 8\,\underline{)493} & \text{Remainder} \\ 8\,\underline{)\ 61} & 5 \\ 8\,\underline{)\ \ 7} & 5 \\ 0 & 7 \end{array} \qquad 7\ \ 5\ \ 5$$

Thus $\mathbf{493_{10} = 755_8}$

The fractional part of a denary number can be converted to an octal number by repeatedly multiplying by 8, as shown below for the fraction 0.4375_{10}

$$\begin{array}{rl} 0.4375 \times 8 = & 3\,.\,\boxed{5} \\ \boxed{0.5} \quad \times 8 = & 4\,.\,0 \end{array} \qquad .3\ \ 4$$

For fractions, the most significant bit is the top integer obtained by multiplication of the denary fraction by 8, thus,

$$0.4375_{10} = 0.34_8$$

The natural binary code for digits 0 to 7 is shown in Table 10.1, and an octal number can be converted to a binary number by writing down the three bits corresponding to the octal digit.

Table 10.1

Octal digit	Natural binary number
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Thus $437_8 = 100\ 011\ 111_2$

and $26.35_8 = 010\ 110.011\ 101_2$

The '0' on the extreme left does not signify anything, thus $26.35_8 = 10\ 110.011\ 101_2$

Conversion of denary to binary via octal is demonstrated in the following worked problems.

Problem 7. Convert 3714_{10} to a binary number, via octal.

Dividing repeatedly by 8, and noting the remainder gives:

$$\begin{array}{rl}
8\,)\underline{3714} & \text{Remainder} \\
8\,)\underline{\ 464} & 2 \\
8\,)\underline{\ \ 58} & 0 \\
8\,)\underline{\ \ \ 7} & 2 \\
0 & 7
\end{array}$$

7 2 0 2

From Table 10.1, $7202_8 = 111\ 010\ 000\ 010_2$

i.e. $\mathbf{3714_{10} = 111\ 010\ 000\ 010_2}$

> Problem 8. Convert 0.59375_{10} to a binary number, via octal.

Multiplying repeatedly by 8, and noting the integer values, gives:

$$\begin{array}{ll}
0.59375 \times 8 = & 4.75 \\
0.75 \quad \times 8 = & 6.00
\end{array}$$

.4 6

Thus $0.59375_{10} = 0.46_8$

From Table 10.1, $0.46_8 = 0.100\ 110_2$

i.e. $\mathbf{0.59375_{10} = 0.100\ 11_2}$

> Problem 9. Convert 5613.90625_{10} to a binary number, via octal.

The integer part is repeatedly divided by 8, noting the remainder, giving:

$$\begin{array}{rl}
8\,)\underline{5613} & \text{Remainder} \\
8\,)\underline{\ 701} & 5 \\
8\,)\underline{\ \ 87} & 5 \\
8\,)\underline{\ \ 10} & 7 \\
8\,)\underline{\ \ \ 1} & 2 \\
0 & 1
\end{array}$$

1 2 7 5 5

This octal number is converted to a binary number, (see Table 10.1).

$12755_8 = 001\ 010\ 111\ 101\ 101_2$

i.e. $5613_{10} = 1\ 010\ 111\ 101\ 101_2$

The fractional part is repeatedly multiplied by 8, and noting the integer part, giving:

$$\begin{array}{ll}
0.90625 \times 8 = & 7.25 \\
0.25 \quad \times 8 = & 2.00
\end{array}$$

.7 2

This octal fraction is converted to a binary number, (see Table 10.1).

$0.72_8 = 0.111\ 010_2$

i.e. $0.90625_{10} = 0.111\ 01_2$

Thus, $\mathbf{5613.90625_{10} = 1\ 010\ 111\ 101\ 101.111\ 01_2}$

> Problem 10. Convert $11\ 110\ 011.100\ 01_2$ to a denary number via octal.

Grouping the binary number in three's from the binary point gives: $011\ 110\ 011.100\ 010_2$

Using Table 10.1 to convert this binary number to an octal number gives 363.42_8 and 363.42_8

$= 3 \times 8^2 + 6 \times 8^1 + 3 \times 8^0 + 4 \times 8^{-1} + 2 \times 8^{-2}$

$= 192 + 48 + 3 + 0.5 + 0.03125$

$= \mathbf{243.53125_{10}}$

Now try the following exercise.

Exercise 44 Further problems on conversion between denary and binary numbers via octal

In Problems 1 to 3, convert the denary numbers given to binary numbers, via octal.

1. (a) 343 (b) 572 (c) 1265

$$\left[\begin{array}{l}\text{(a) } 101010111_2 \quad \text{(b) } 1000111100_2 \\ \text{(c) } 10011110001_2\end{array}\right]$$

2. (a) 0.46875 (b) 0.6875 (c) 0.71875

$$\left[\begin{array}{l}\text{(a) } 0.01111_2 \text{ (b) } 0.1011_2 \\ \text{(c) } 0.10111_2\end{array}\right]$$

3. (a) 247.09375 (b) 514.4375 (c) 1716.78125

$$\left[\begin{array}{l}\text{(a) } 11110111.00011_2 \\ \text{(b) } 1000000010.0111_2 \\ \text{(c) } 11010110100.11001_2\end{array}\right]$$

4. Convert the binary numbers given to denary numbers via octal.
(a) 111.011 1 (b) 101 001.01
(c) 1 110 011 011 010.001 1

$$\begin{bmatrix} \text{(a) } 7.4375_{10} \quad \text{(b) } 41.25_{10} \\ \text{(c) } 7386.1875_{10} \end{bmatrix}$$

10.5 Hexadecimal numbers

The complexity of computers requires higher order numbering systems such as octal (base 8) and hexadecimal (base 16) which are merely extensions of the binary system. A **hexadecimal numbering system** has a radix of 16 and uses the following 16 distinct digits:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F

'A' corresponds to 10 in the denary system, B to 11, C to 12, and so on.

To convert from hexadecimal to decimal:

For example

$$1A_{16} = 1 \times 16^1 + A \times 16^0$$
$$= 1 \times 16^1 + 10 \times 1$$
$$= 16 + 10 = 26$$

i.e. $\mathbf{1A_{16} = 26_{10}}$

Similarly, $\mathbf{2E_{16}} = 2 \times 16^1 + E \times 16^0$
$$= 2 \times 16^1 + 14 \times 16^0$$
$$= 32 + 14 = \mathbf{46_{10}}$$

and $\mathbf{1BF_{16}} = 1 \times 16^2 + B \times 16^1 + F \times 16^0$
$$= 1 \times 16^2 + 11 \times 16^1 + 15 \times 16^0$$
$$= 256 + 176 + 15 = \mathbf{447_{10}}$$

Table 10.2 compares decimal, binary, octal and hexadecimal numbers and shows, for example, that $23_{10} = 10111_2 = 27_8 = 17_{16}$

Table 10.2

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17
24	11000	30	18
25	11001	31	19
26	11010	32	1A
27	11011	33	1B
28	11100	34	1C
29	11101	35	1D
30	11110	36	1E
31	11111	37	1F
32	100000	40	20

Problem 11. Convert the following hexadecimal numbers into their decimal equivalents: (a) $7A_{16}$ (b) $3F_{16}$

(a) $7A_{16} = 7 \times 16^1 + A \times 16^0 = 7 \times 16 + 10 \times 1$
$$= 112 + 10 = 122$$

Thus $\mathbf{7A_{16} = 122_{10}}$

(b) $3F_{16} = 3 \times 16^1 + F \times 16^0 = 3 \times 16 + 15 \times 1$
$$= 48 + 15 = 63$$

Thus $\mathbf{3F_{16} = 63_{10}}$

Problem 12. Convert the following hexadecimal numbers into their decimal equivalents: (a) $C9_{16}$ (b) BD_{16}

(a) $C9_{16} = C \times 16^1 + 9 \times 16^0 = 12 \times 16 + 9 \times 1$
$$= 192 + 9 = 201$$

Thus $\mathbf{C9_{16} = 201_{10}}$

(b) $BD_{16} = B \times 16^1 + D \times 16^0$
$= 11 \times 16 + 13 \times 1 = 176 + 13 = 189$

Thus $\mathbf{BD_{16} = 189_{10}}$

Problem 13. Convert $1A4E_{16}$ into a denary number.

$$\begin{aligned} 1A4E_{16} &= 1 \times 16^3 + A \times 16^2 + 4 \times 16^1 + E \times 16^0 \\ &= 1 \times 16^3 + 10 \times 16^2 + 4 \times 16^1 + 14 \times 16^0 \\ &= 1 \times 4096 + 10 \times 256 + 4 \times 16 + 14 \times 1 \\ &= 4096 + 2560 + 64 + 14 = 6734 \end{aligned}$$

Thus $\mathbf{1A4E_{16} = 6734_{10}}$

To convert from decimal to hexadecimal

This is achieved by repeatedly dividing by 16 and noting the remainder at each stage, as shown below for 26_{10}.

16) 26 Remainder
16) 1 $\quad 10 \equiv A_{16}$
0 $\quad 1 \equiv 1_{16}$

most significant bit → 1 A ← least significant bit

Hence $\mathbf{26_{10} = 1A_{16}}$

Similarly, for 447_{10}

16) 447 Remainder
16) 27 $\quad 15 \equiv F_{16}$
16) 1 $\quad 11 \equiv B_{16}$
0 $\quad 1 \equiv 1_{16}$

1 B F

Thus $\mathbf{447_{10} = 1BF_{16}}$

Problem 14. Convert the following decimal numbers into their hexadecimal equivalents: (a) 37_{10} (b) 108_{10}

(a) 16) 37 Remainder
16) 2 $\quad 5 = 5_{16}$
0 $\quad 2 = 2_{16}$

2 5

most significant bit ↗ ↖ least significant bit

Hence $\mathbf{37_{10} = 25_{16}}$

(b) 16) 108 Remainder
16) 6 $\quad 12 = C_{16}$
0 $\quad 6 = 6_{16}$

6 C

Hence $\mathbf{108_{10} = 6C_{16}}$

Problem 15. Convert the following decimal numbers into their hexadecimal equivalents: (a) 162_{10} (b) 239_{10}

(a) 16) 162 Remainder
16) 10 $\quad 2 = 2_{16}$
0 $\quad 10 = A_{16}$

A 2

Hence $\mathbf{162_{10} = A2_{16}}$

(b) 16) 239 Remainder
16) 14 $\quad 15 = F_{16}$
0 $\quad 14 = E_{16}$

E F

Hence $\mathbf{239_{10} = EF_{16}}$

To convert from binary to hexadecimal:

The binary bits are arranged in groups of four, starting from right to left, and a hexadecimal symbol is assigned to each group. For example, the binary number 1110011110101001 is initially grouped in fours as: $\underbrace{1110}_{E}\ \underbrace{0111}_{7}\ \underbrace{1010}_{A}\ \underbrace{1001}_{9}$ and a hexadecimal symbol assigned to each group as above from Table 10.2.

Hence $\mathbf{1110011110101001_2 = E7A9_{16}}$

To convert from hexadecimal to binary:

The above procedure is reversed, thus, for example,

$$6CF3_{16} = 0110\ 1100\ 1111\ 0011 \quad \text{from Table 10.2}$$

i.e. $\mathbf{6CF3_{16} = 110110011110011_2}$

> Problem 16. Convert the following binary numbers into their hexadecimal equivalents: (a) 11010110_2 (b) 1100111_2

(a) Grouping bits in fours from the right gives: $\underbrace{1101}_{D}\ \underbrace{0110}_{6}$ and assigning hexadecimal symbols to each group gives as above from Table 10.2.

Thus, $\mathbf{11010110_2 = D6_{16}}$

(b) Grouping bits in fours from the right gives: $\underbrace{0110}_{6}\ \underbrace{0111}_{7}$ and assigning hexadecimal symbols to each group gives as above from Table 10.2.

Thus, $\mathbf{1100111_2 = 67_{16}}$

> Problem 17. Convert the following binary numbers into their hexadecimal equivalents: (a) 11001111_2 (b) 110011110_2

(a) Grouping bits in fours from the right gives: $\underbrace{1100}_{C}\ \underbrace{1111}_{F}$ and assigning hexadecimal symbols to each group gives as above from Table 10.2.

Thus, $\mathbf{11001111_2 = CF_{16}}$

(b) Grouping bits in fours from the right gives: $\underbrace{0001}_{1}\ \underbrace{1001}_{9}\ \underbrace{1110}_{E}$ and assigning hexadecimal symbols to each group gives as above from Table 10.2.

Thus, $\mathbf{110011110_2 = 19E_{16}}$

> Problem 18. Convert the following hexadecimal numbers into their binary equivalents: (a) $3F_{16}$ (b) $A6_{16}$

(a) Spacing out hexadecimal digits gives: $\overbrace{0011}^{3}\ \overbrace{1111}^{F}$ and converting each into binary gives as above from Table 10.2.

Thus, $\mathbf{3F_{16} = 111111_2}$

(b) Spacing out hexadecimal digits gives: $\overbrace{1010}^{A}\ \overbrace{0110}^{6}$ and converting each into binary gives as above from Table 10.2.

Thus, $\mathbf{A6_{16} = 10100110_2}$

> Problem 19. Convert the following hexadecimal numbers into their binary equivalents: (a) $7B_{16}$ (b) $17D_{16}$

(a) Spacing out hexadecimal digits gives: $\overbrace{0111}^{7}\ \overbrace{1011}^{B}$ and converting each into binary gives as above from Table 10.2.

Thus, $\mathbf{7B_{16} = 1111011_2}$

(b) Spacing out hexadecimal digits gives: $\overbrace{0001}^{1}\ \overbrace{0111}^{7}\ \overbrace{1101}^{D}$ and converting each into binary gives as above from Table 10.2.

Thus, $\mathbf{17D_{16} = 101111101_2}$

Now try the following exercise.

Exercise 45 Further problems on hexadecimal numbers

In Problems 1 to 4, convert the given hexadecimal numbers into their decimal equivalents.

1. $E7_{16}$ [231_{10}] 2. $2C_{16}$ [44_{10}]
3. 98_{16} [152_{10}] 4. $2F1_{16}$ [753_{10}]

In Problems 5 to 8, convert the given decimal numbers into their hexadecimal equivalents.

5. 54_{10} [36_{16}] 6. 200_{10} [$C8_{16}$]
7. 91_{10} [$5B_{16}$] 8. 238_{10} [EE_{16}]

In Problems 9 to 12, convert the given binary numbers into their hexadecimal equivalents.

9. 11010111_2 [$D7_{16}$]
10. 11101010_2 [EA_{16}]
11. 10001011_2 [$8B_{16}$]
12. 10100101_2 [$A5_{16}$]

In Problems 13 to 16, convert the given hexadecimal numbers into their binary equivalents.

13. 37_{16} [110111_2]
14. ED_{16} [11101101_2]
15. $9F_{16}$ [10011111_2]
16. $A21_{16}$ [101000100001_2]