PUZZLER

This person is exposed to very bright sunlight at the beach. If he is wearing the wrong kind of sunglasses, he may be causing more permanent harm to his vision than he would be if he took the glasses off and squinted. What determines whether certain types of sunglasses are good for your eyes? *(Ron Chapple/FPG International)*

chapter 34

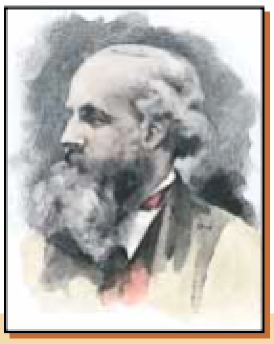
Electromagnetic Waves

Chapter Outline

The waves described in Chapters 16, 17, and 18 are mechanical waves. By definition, the propagation of mechanical disturbances—such as sound waves, water waves, and waves on a string—requires the presence of a medium. This chapter is concerned with the properties of electromagnetic waves, which (unlike mechanical waves) can propagate through empty space.

In Section 31.7 we gave a brief description of Maxwell's equations, which form the theoretical basis of all electromagnetic phenomena. The consequences of Maxwell's equations are far-reaching and dramatic. The Ampère–Maxwell law predicts that a time-varying electric field produces a magnetic field, just as Faraday's law tells us that a time-varying magnetic field produces an electric field. Maxwell's introduction of the concept of displacement current as a new source of a magnetic field provided the final important link between electric and magnetic fields in classical physics.

Astonishingly, Maxwell's equations also predict the existence of electromagnetic waves that propagate through space at the speed of light c. This chapter begins with a discussion of how Heinrich Hertz confirmed Maxwell's prediction when he generated and detected electromagnetic waves in 1887. That discovery has led to many practical communication systems, including radio, television, and radar. On a conceptual level, Maxwell unified the subjects of light and electromagnetism by developing the idea that light is a form of electromagnetic radiation.



James Clerk Maxwell Scottish theoretical physicist (1831–1879) Maxwell developed the electromagnetic theory of light and the kinetic theory of gases, and he explained the nature of color vision and of Saturn's rings. His successful interpretation of the electromagnetic field produced the field equations that bear his name. Formidable mathematical ability combined with great insight enabled Maxwell to lead the way in the study of electromagnetism and kinetic theory. He died of cancer before he was 50. *(North Wind Picture Archives)*

Next, we learn how electromagnetic waves are generated by oscillating electric charges. The waves consist of oscillating electric and magnetic fields that are at right angles to each other and to the direction of wave propagation. Thus, electromagnetic waves are transverse waves. Maxwell's prediction of electromagnetic radiation shows that the amplitudes of the electric and magnetic fields in an electromagnetic wave are related by the expression $E = cB$. The waves radiated from the oscillating charges can be detected at great distances. Furthermore, electromagnetic waves carry energy and momentum and hence can exert pressure on a surface.

The chapter concludes with a look at the wide range of frequencies covered by electromagnetic waves. For example, radio waves (frequencies of about 10^7 Hz) are electromagnetic waves produced by oscillating currents in a radio tower's transmitting antenna. Light waves are a high-frequency form of electromagnetic radiation (about 10^{14} Hz) produced by oscillating electrons in atoms.

34.1 MAXWELL'S EQUATIONS AND HERTZ'S DISCOVERIES

In his unified theory of electromagnetism, Maxwell showed that electromagnetic waves are a natural consequence of the fundamental laws expressed in the following four equations (see Section 31.7):

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0} \tag{34.1}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \tag{34.2}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \tag{34.3}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{34.4}$$

As we shall see in the next section, Equations 34.3 and 34.4 can be combined to obtain a wave equation for both the electric field and the magnetic field. In empty space ($Q = 0$, $I = 0$), the solution to these two equations shows that the speed at which electromagnetic waves travel equals the measured speed of light. This result led Maxwell to predict that light waves are a form of electromagnetic radiation.

The experimental apparatus that Hertz used to generate and detect electromagnetic waves is shown schematically in Figure 34.1. An induction coil is connected to a transmitter made up of two spherical electrodes separated by a narrow gap. The coil provides short voltage surges to the electrodes, making one positive and the other negative. A spark is generated between the spheres when the electric field near either electrode surpasses the dielectric strength for air (3×10^6 V/m; see Table 26.1). In a strong electric field, the acceleration of free electrons provides them with enough energy to ionize any molecules they strike. This ionization provides more electrons, which can accelerate and cause further ionizations. As the air in the gap is ionized, it becomes a much better conductor, and the discharge between the electrodes exhibits an oscillatory behavior at a very high frequency. From an electric-circuit viewpoint, this is equivalent to an LC circuit in which the inductance is that of the coil and the capacitance is due to the spherical electrodes.

Because L and C are quite small in Hertz's apparatus, the frequency of oscillation is very high, ≈ 100 MHz. (Recall from Eq. 32.22 that $\omega = 1/\sqrt{LC}$ for an LC circuit.) Electromagnetic waves are radiated at this frequency as a result of the oscillation (and hence acceleration) of free charges in the transmitter circuit. Hertz was able to detect these waves by using a single loop of wire with its own spark gap (the receiver). Such a receiver loop, placed several meters from the transmitter, has its own effective inductance, capacitance, and natural frequency of oscillation. In Hertz's experiment, sparks were induced across the gap of the receiving electrodes when the frequency of the receiver was adjusted to match that of the transmitter. Thus, Hertz demonstrated that the oscillating current induced in the receiver was produced by electromagnetic waves radiated by the transmitter. His experiment is analogous to the mechanical phenomenon in which a tuning fork responds to acoustic vibrations from an identical tuning fork that is oscillating.

Heinrich Rudolf Hertz German physicist (1857–1894) Hertz made his most important discovery—radio waves—in 1887. After finding that the speed of a radio wave was the same as that of light, he showed that radio waves, like light waves, could be reflected, refracted, and diffracted. Hertz died of blood poisoning at age 36. He made many contributions to science during his short life. The hertz, equal to one complete vibration or cycle per second, is named after him. *(The Bettmann Archive)*

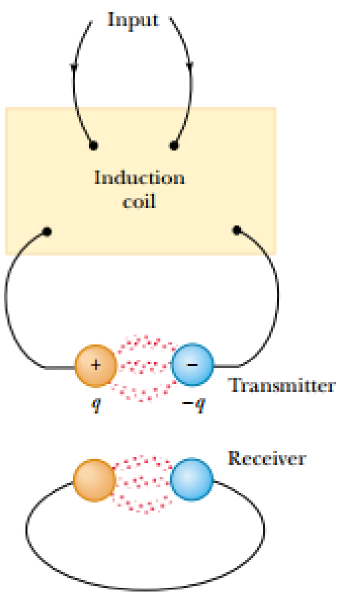
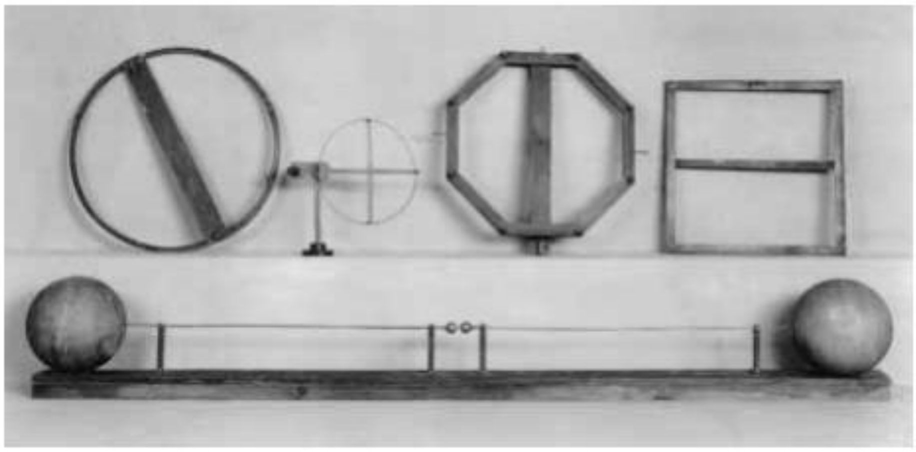


Figure 34.1 Schematic diagram of Hertz's apparatus for generating and detecting electromagnetic waves. The transmitter consists of two spherical electrodes connected to an induction coil, which provides short voltage surges to the spheres, setting up oscillations in the discharge between the electrodes (suggested by the red dots). The receiver is a nearby loop of wire containing a second spark gap.



A large oscillator (bottom) and circular, octagonal, and square receivers used by Heinrich Hertz.

QuickLab

Some electric motors use commutators that make and break electrical contact, creating sparks reminiscent of Hertz's method for generating electromagnetic waves. Try running an electric shaver or kitchen mixer near an AM radio. What happens to the reception?

Additionally, Hertz showed in a series of experiments that the radiation generated by his spark-gap device exhibited the wave properties of interference, diffraction, reflection, refraction, and polarization, all of which are properties exhibited by light. Thus, it became evident that the radio-frequency waves Hertz was generating had properties similar to those of light waves and differed only in frequency and wavelength. Perhaps his most convincing experiment was the measurement of the speed of this radiation. Radio-frequency waves of known frequency were reflected from a metal sheet and created a standing-wave interference pattern whose nodal points could be detected. The measured distance between the nodal points enabled determination of the wavelength λ. Using the relationship $v = \lambda f$ (Eq. 16.14), Hertz found that v was close to 3×10^8 m/s, the known speed c of visible light.

34.2 PLANE ELECTROMAGNETIC WAVES

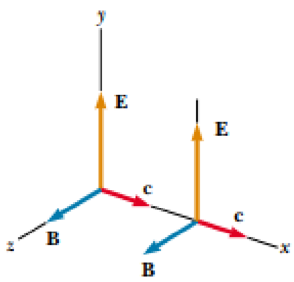


Figure 34.2 An electromagnetic wave traveling at velocity $\mathbf{c}$ in the positive x direction. The electric field is along the y direction, and the magnetic field is along the z direction. These fields depend only on x and t.

The properties of electromagnetic waves can be deduced from Maxwell's equations. One approach to deriving these properties is to solve the second-order differential equation obtained from Maxwell's third and fourth equations. A rigorous mathematical treatment of that sort is beyond the scope of this text. To circumvent this problem, we assume that the vectors for the electric field and magnetic field in an electromagnetic wave have a specific space–time behavior that is simple but consistent with Maxwell's equations.

To understand the prediction of electromagnetic waves more fully, let us focus our attention on an electromagnetic wave that travels in the x direction (the *direction of propagation*). In this wave, the electric field $\mathbf{E}$ is in the y direction, and the magnetic field $\mathbf{B}$ is in the z direction, as shown in Figure 34.2. Waves such as this one, in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes, are said to be **linearly polarized waves.**[1] Furthermore, we assume that at any point P, the magnitudes E and B of the fields depend

[1] Waves having other particular patterns of vibration of the electric and magnetic fields include circularly polarized waves. The most general polarization pattern is elliptical.

upon x and t only, and not upon the y or z coordinate. A collection of such waves from individual sources is called a **plane wave.** A surface connecting points of equal phase on all waves, which we call a **wave front,** would be a geometric plane. In comparison, a point source of radiation sends waves out in all directions. A surface connecting points of equal phase is a sphere for this situation, so we call this a **spherical wave.**

We can relate E and B to each other with Equations 34.3 and 34.4. In empty space, where $Q = 0$ and $I = 0$, Equation 34.3 remains unchanged and Equation 34.4 becomes

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \tag{34.5}$$

Using Equations 34.3 and 34.5 and the plane-wave assumption, we obtain the following differential equations relating E and B. (We shall derive these equations formally later in this section.) For simplicity, we drop the subscripts on the components E_y and B_z:

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \tag{34.6}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \tag{34.7}$$

Note that the derivatives here are partial derivatives. For example, when we evaluate $\partial E/\partial x$, we assume that t is constant. Likewise, when we evaluate $\partial B/\partial t$, x is held constant. Taking the derivative of Equation 34.6 with respect to x and combining the result with Equation 34.7, we obtain

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}} \tag{34.8}$$

In the same manner, taking the derivative of Equation 34.7 with respect to x and combining it with Equation 34.6, we obtain

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}} \tag{34.9}$$

Equations 34.8 and 34.9 both have the form of the general wave equation[2] with the wave speed v replaced by c, where

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} \tag{34.10}$$

Speed of electromagnetic waves

Taking $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ and $\epsilon_0 = 8.854\ 19 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ in Equation 34.10, we find that $c = 2.997\ 92 \times 10^8\ \mathrm{m/s}$. Because this speed is precisely the same as the speed of light in empty space, we are led to believe (correctly) that light is an electromagnetic wave.

[2] The general wave equation is of the form $(\partial^2 y/\partial x^2) = (1/v^2)(\partial^2 y/\partial t^2)$, where v is the speed of the wave and y is the wave function. The general wave equation was introduced as Equation 16.26, and it would be useful for you to review Section 16.9.

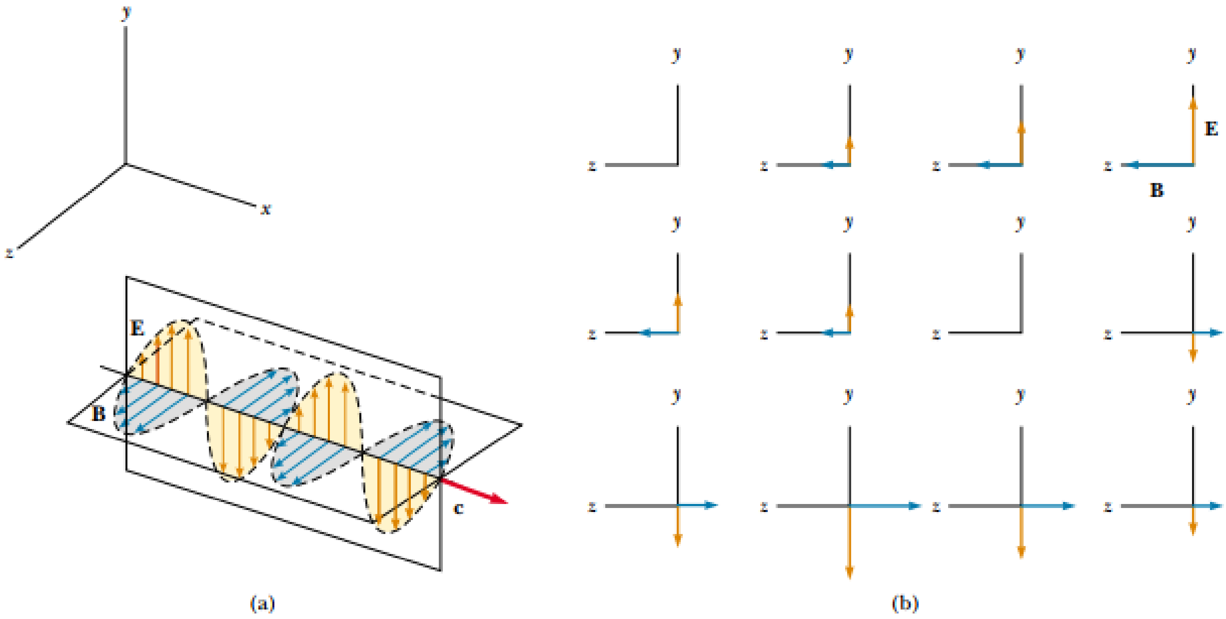


Figure 34.3 Representation of a sinusoidal, linearly polarized plane electromagnetic wave moving in the positive x direction with velocity $\mathbf{c}$. (a) The wave at some instant. Note the sinusoidal variations of E and B with x. (b) A time sequence illustrating the electric and magnetic field vectors present in the yz plane, as seen by an observer looking in the negative x direction. Note the sinusoidal variations of E and B with t.

The simplest solution to Equations 34.8 and 34.9 is a sinusoidal wave, for which the field magnitudes E and B vary with x and t according to the expressions

Sinusoidal electric and magnetic fields

$$E = E_{\max} \cos(kx - \omega t) \tag{34.11}$$

$$B = B_{\max} \cos(kx - \omega t) \tag{34.12}$$

where $E_{\max}$ and $B_{\max}$ are the maximum values of the fields. The angular wave number is the constant $k = 2\pi/\lambda$, where λ is the wavelength. The angular frequency is $\omega = 2\pi f$, where f is the wave frequency. The ratio ω/k equals the speed c:

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

We have used Equation 16.14, $v = c = \lambda f$, which relates the speed, frequency, and wavelength of any continuous wave. Figure 34.3a is a pictorial representation, at one instant, of a sinusoidal, linearly polarized plane wave moving in the positive x direction. Figure 34.3b shows how the electric and magnetic field vectors at a fixed location vary with time.

Quick Quiz 34.1

What is the phase difference between B and E in Figure 34.3?

Taking partial derivatives of Equations 34.11 (with respect to x) and 34.12

(with respect to t), we find that

$$\frac{\partial E}{\partial x} = -kE_{\max}\sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{\max}\sin(kx - \omega t)$$

Substituting these results into Equation 34.6, we find that at any instant

$$kE_{\max} = \omega B_{\max}$$

$$\frac{E_{\max}}{B_{\max}} = \frac{\omega}{k} = c$$

Using these results together with Equations 34.11 and 34.12, we see that

$$\boxed{\frac{E_{\max}}{B_{\max}} = \frac{E}{B} = c} \qquad \textbf{(34.13)}$$

That is, **at every instant the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light.**

Finally, note that electromagnetic waves obey the superposition principle (which we discussed in Section 16.4 with respect to mechanical waves) because the differential equations involving E and B are linear equations. For example, we can add two waves with the same frequency simply by adding the magnitudes of the two electric fields algebraically.

Properties of electromagnetic waves

- The solutions of Maxwell's third and fourth equations are wave-like, with both E and B satisfying a wave equation.
- Electromagnetic waves travel through empty space at the speed of light $c = 1/\sqrt{\mu_0\epsilon_0}$.
- The components of the electric and magnetic fields of plane electromagnetic waves are perpendicular to each other and perpendicular to the direction of wave propagation. We can summarize the latter property by saying that electromagnetic waves are transverse waves.
- The magnitudes of $\mathbf{E}$ and $\mathbf{B}$ in empty space are related by the expression $E/B = c$.
- Electromagnetic waves obey the principle of superposition.

EXAMPLE 34.1 An Electromagnetic Wave

A sinusoidal electromagnetic wave of frequency 40.0 MHz travels in free space in the x direction, as shown in Figure 34.4. (a) Determine the wavelength and period of the wave.

Solution Using Equation 16.14 for light waves, $c = \lambda f$, and given that $f = 40.0 \text{ MHz} = 4.00 \times 10^7 \text{ s}^{-1}$, we have

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{4.00 \times 10^7 \text{ s}^{-1}} = \boxed{7.50 \text{ m}}$$

The period T of the wave is the inverse of the frequency:

$$T = \frac{1}{f} = \frac{1}{4.00 \times 10^7 \text{ s}^{-1}} = \boxed{2.50 \times 10^{-8} \text{ s}}$$

(b) At some point and at some instant, the electric field has its maximum value of 750 N/C and is along the y axis. Calculate the magnitude and direction of the magnetic field at this position and time.

Solution From Equation 34.13 we see that

$$B_{\max} = \frac{E_{\max}}{c} = \frac{750 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.50 \times 10^{-6} \text{ T}}$$

Because $\mathbf{E}$ and $\mathbf{B}$ must be perpendicular to each other and perpendicular to the direction of wave propagation (x in this case), we conclude that $\mathbf{B}$ is in the z direction.

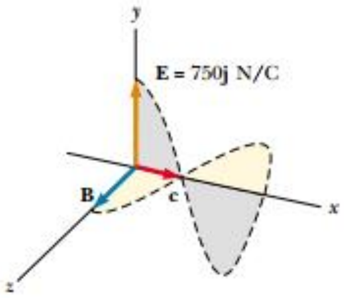


Figure 34.4 At some instant, a plane electromagnetic wave moving in the x direction has a maximum electric field of 750 N/C in the positive y direction. The corresponding magnetic field at that point has a magnitude E/c and is in the z direction.

(c) Write expressions for the space-time variation of the components of the electric and magnetic fields for this wave.

Solution We can apply Equations 34.11 and 34.12 directly:

$$E = E_{\max}\cos(kx - \omega t) = (750\text{ N/C})\cos(kx - \omega t)$$

$$B = B_{\max}\cos(kx - \omega t) = (2.50 \times 10^{-6}\text{ T})\cos(kx - \omega t)$$

where

$$\omega = 2\pi f = 2\pi(4.00 \times 10^{7}\text{ s}^{-1}) = 2.51 \times 10^{8}\text{ rad/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.50\text{ m}} = 0.838\text{ rad/m}$$

Let us summarize the properties of electromagnetic waves as we have described them:

Optional Section

Derivation of Equations 34.6 and 34.7

To derive Equation 34.6, we start with Faraday's law, Equation 34.3:

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

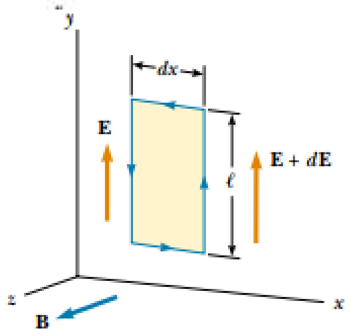


Figure 34.5 As a plane wave passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\mathbf{E}$ to $\mathbf{E} + d\mathbf{E}$. This spatial variation in $\mathbf{E}$ gives rise to a time-varying magnetic field along the z direction, according to Equation 34.6.

Let us again assume that the electromagnetic wave is traveling in the x direction, with the electric field $\mathbf{E}$ in the positive y direction and the magnetic field $\mathbf{B}$ in the positive z direction.

Consider a rectangle of width dx and height ℓ lying in the xy plane, as shown in Figure 34.5. To apply Equation 34.3, we must first evaluate the line integral of $\mathbf{E}\cdot d\mathbf{s}$ around this rectangle. The contributions from the top and bottom of the rectangle are zero because $\mathbf{E}$ is perpendicular to $d\mathbf{s}$ for these paths. We can express the electric field on the right side of the rectangle as

$$E(x + dx, t) \approx E(x, t) + \left.\frac{dE}{dx}\right]_{t\text{ constant}} dx = E(x, t) + \frac{\partial E}{\partial x}\,dx$$

while the field on the left side is simply $E(x, t)$.[3] Therefore, the line integral over this rectangle is approximately

$$\oint \mathbf{E}\cdot d\mathbf{s} = E(x + dx, t)\cdot \ell - E(x, t)\cdot \ell \approx (\partial E/\partial x)\,dx\cdot \ell \qquad \textbf{(34.14)}$$

Because the magnetic field is in the z direction, the magnetic flux through the rectangle of area $\ell\,dx$ is approximately $\Phi_B = B\ell\,dx$. (This assumes that dx is very small compared with the wavelength of the wave.) Taking the time derivative of

[3] Because dE/dx in this equation is expressed as the change in E with x at a given instant t, dE/dx is equivalent to the partial derivative $\partial E/\partial x$. Likewise, dB/dt means the change in B with time at a particular position x, so in Equation 34.15 we can replace dB/dt with $\partial B/\partial t$.

the magnetic flux gives

$$\frac{d\Phi_B}{dt} = \ell\, dx \frac{dB}{dt}\bigg]_{x\ \text{constant}} = \ell\, dx \frac{\partial B}{\partial t} \qquad \textbf{(34.15)}$$

Substituting Equations 34.14 and 34.15 into Equation 34.3, we obtain

$$\left(\frac{\partial E}{\partial x}\right) dx \cdot \ell = -\ell\, dx \frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

This expression is Equation 34.6.

In a similar manner, we can verify Equation 34.7 by starting with Maxwell's fourth equation in empty space (Eq. 34.5). In this case, we evaluate the line integral of $\mathbf{B} \cdot d\mathbf{s}$ around a rectangle lying in the xz plane and having width dx and length ℓ, as shown in Figure 34.6. Noting that the magnitude of the magnetic field changes from $B(x, t)$ to $B(x + dx, t)$ over the width dx, we find the line integral over this rectangle to be approximately

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(x, t) \cdot \ell - B(x + dx, t) \cdot \ell \approx -(\partial B/\partial x)\, dx \cdot \ell \qquad \textbf{(34.16)}$$

The electric flux through the rectangle is $\Phi_E = E\ell\, dx$, which, when differentiated with respect to time, gives

$$\frac{\partial \Phi_E}{\partial t} = \ell\, dx \frac{\partial E}{\partial t} \qquad \textbf{(34.17)}$$

Substituting Equations 34.16 and 34.17 into Equation 34.5 gives

$$-(\partial B/\partial x)\, dx \cdot \ell = \mu_0 \epsilon_0 \ell\, dx(\partial E/\partial t)$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

which is Equation 34.7.

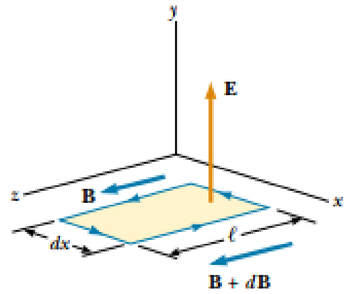


Figure 34.6 As a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\mathbf{B}$ to $\mathbf{B} + d\mathbf{B}$. This spatial variation in $\mathbf{B}$ gives rise to a time-varying electric field along the y direction, according to Equation 34.7.

34.3 ENERGY CARRIED BY ELECTROMAGNETIC WAVES

Electromagnetic waves carry energy, and as they propagate through space they can transfer energy to objects placed in their path. The rate of flow of energy in an electromagnetic wave is described by a vector $\mathbf{S}$, called the **Poynting vector,** which is defined by the expression

Poynting vector

$$\mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \qquad \textbf{(34.18)}$$

The magnitude of the Poynting vector represents the rate at which energy flows through a unit surface area perpendicular to the direction of wave propagation. Thus, the magnitude of the Poynting vector represents *power per unit area.* The direction of the vector is along the direction of wave propagation (Fig. 34.7). The SI units of the Poynting vector are $\text{J/s} \cdot \text{m}^2 = \text{W/m}^2$.

Magnitude of the Poynting vector for a plane wave

As an example, let us evaluate the magnitude of **S** for a plane electromagnetic wave where $|\mathbf{E} \times \mathbf{B}| = EB$. In this case,

$$S = \frac{EB}{\mu_0} \tag{34.19}$$

Because $B = E/c$, we can also express this as

$$S = \frac{E^2}{\mu_0 c} = \frac{c}{\mu_0} B^2$$

These equations for S apply at any instant of time and represent the *instantaneous* rate at which energy is passing through a unit area.

Wave intensity

What is of greater interest for a sinusoidal plane electromagnetic wave is the time average of S over one or more cycles, which is called the ***wave intensity*** I. (We discussed the intensity of sound waves in Chapter 17.) When this average is taken, we obtain an expression involving the time average of $\cos^2(kx - \omega t)$, which equals $\frac{1}{2}$. Hence, the average value of S (in other words, the intensity of the wave) is

$$I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\max}^2 \tag{34.20}$$

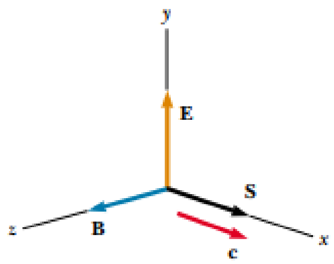


Figure 34.7 The Poynting vector **S** for a plane electromagnetic wave is along the direction of wave propagation.

Recall that the energy per unit volume, which is the instantaneous energy density u_E associated with an electric field, is given by Equation 26.13,

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

and that the instantaneous energy density u_B associated with a magnetic field is given by Equation 32.14:

$$u_B = \frac{B^2}{2\mu_0}$$

Because E and B vary with time for an electromagnetic wave, the energy densities also vary with time. When we use the relationships $B = E/c$ and $c = 1/\sqrt{\mu_0\epsilon_0}$, Equation 32.14 becomes

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0\epsilon_0}{2\mu_0} E^2 = \tfrac{1}{2}\epsilon_0 E^2$$

Comparing this result with the expression for u_E, we see that

$$u_B = u_E = \tfrac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

That is, **for an electromagnetic wave, the instantaneous energy density associated with the magnetic field equals the instantaneous energy density associated with the electric field.** Hence, in a given volume the energy is equally shared by the two fields.

Total instantaneous energy density

The **total instantaneous energy density** u is equal to the sum of the energy densities associated with the electric and magnetic fields:

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

Average energy density of an electromagnetic wave

When this total instantaneous energy density is averaged over one or more cycles of an electromagnetic wave, we again obtain a factor of $\frac{1}{2}$. Hence, for any electromagnetic wave, the total average energy per unit volume is

$$u_{av} = \epsilon_0 (E^2)_{av} = \frac{1}{2}\epsilon_0 E_{max}^2 = \frac{B_{max}^2}{2\mu_0} \qquad \textbf{(34.21)}$$

EXAMPLE 34.2 Fields on the Page

Estimate the maximum magnitudes of the electric and magnetic fields of the light that is incident on this page because of the visible light coming from your desk lamp. Treat the bulb as a point source of electromagnetic radiation that is about 5% efficient at converting electrical energy to visible light.

Solution Recall from Equation 17.8 that the wave intensity I a distance r from a point source is $I = \mathcal{P}_{av}/4\pi r^2$, where $\mathcal{P}_{av}$ is the average power output of the source and $4\pi r^2$ is the area of a sphere of radius r centered on the source. Because the intensity of an electromagnetic wave is also given by Equation 34.20, we have

$$I = \frac{\mathcal{P}_{av}}{4\pi r^2} = \frac{E_{max}^2}{2\mu_0 c}$$

We must now make some assumptions about numbers to enter in this equation. If we have a 60-W lightbulb, its output at 5% efficiency is approximately 3.0 W in the form of visible light. (The remaining energy transfers out of the bulb by conduction and invisible radiation.) A reasonable distance from the bulb to the page might be 0.30 m. Thus, we have

$$E_{max} = \sqrt{\frac{\mu_0 c \mathcal{P}_{av}}{2\pi r^2}}$$

$$= \sqrt{\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(3.00 \times 10^8\ \text{m/s})(3.0\ \text{W})}{2\pi(0.30\ \text{m})^2}}$$

$$= \boxed{45\ \text{V/m}}$$

From Equation 34.13,

$$B_{max} = \frac{E_{max}}{c} = \frac{45\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = \boxed{1.5 \times 10^{-7}\ \text{T}}$$

This value is two orders of magnitude smaller than the Earth's magnetic field, which, unlike the magnetic field in the light wave from your desk lamp, is not oscillating.

Exercise Estimate the energy density of the light wave just before it strikes this page.

Answer $9.0 \times 10^{-9}\ \text{J/m}^3$.

Comparing this result with Equation 34.20 for the average value of S, we see that

$$I = S_{av} = c u_{av} \qquad \textbf{(34.22)}$$

In other words, **the intensity of an electromagnetic wave equals the average energy density multiplied by the speed of light.**

34.4 MOMENTUM AND RADIATION PRESSURE

Electromagnetic waves transport linear momentum as well as energy. It follows that, as this momentum is absorbed by some surface, pressure is exerted on the surface. We shall assume in this discussion that the electromagnetic wave strikes the surface at normal incidence and transports a total energy U to the surface in a time t. Maxwell showed that, if the surface absorbs all the incident energy U in this time (as does a black body, introduced in Chapter 20), the total momentum $\mathbf{p}$ transported to the surface has a magnitude

Momentum transported to a perfectly absorbing surface

$$p = \frac{U}{c} \quad \text{(complete absorption)} \qquad \textbf{(34.23)}$$

The pressure exerted on the surface is defined as force per unit area F/A. Let us combine this with Newton's second law:

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$

Radiation pressure exerted on a perfectly absorbing surface

If we now replace p, the momentum transported to the surface by light, from Equation 34.23, we have

$$P = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{c}\right) = \frac{1}{c}\frac{(dU/dt)}{A}$$

We recognize $(dU/dt)/A$ as the rate at which energy is arriving at the surface per unit area, which is the magnitude of the Poynting vector. Thus, the radiation pressure P exerted on the perfectly absorbing surface is

$$P = \frac{S}{c} \tag{34.24}$$

QuickLab

Using Example 34.2 as a starting point, estimate the total force exerted on this page by the light from your desk lamp. Does it make a difference if the page contains large, dark photographs instead of mostly white space?

Note that Equation 34.24 is an expression for uppercase P, the pressure, while Equation 34.23 is an expression for lowercase p, linear momentum.

If the surface is a perfect reflector (such as a mirror) and incidence is normal, then the momentum transported to the surface in a time t is twice that given by Equation 34.23. That is, the momentum transferred to the surface by the incoming light is $p = U/c$, and that transferred by the reflected light also is $p = U/c$. Therefore,

Radiation pressure exerted on a perfectly reflecting surface

$$p = \frac{2U}{c} \quad \text{(complete reflection)} \tag{34.25}$$

The momentum delivered to a surface having a reflectivity somewhere between these two extremes has a value between U/c and $2U/c$, depending on the properties of the surface. Finally, the radiation pressure exerted on a perfectly reflecting surface for normal incidence of the wave is[4]

$$P = \frac{2S}{c} \tag{34.26}$$

web

Visit **http://pds.jpl.nasa.gov** for more information about missions to the planets. You may also want to read Arthur C. Clarke's 1963 science fiction story *The Wind from the Sun* about a solar yacht race.

Although radiation pressures are very small (about 5×10^{-6} N/m^2 for direct sunlight), they have been measured with torsion balances such as the one shown in Figure 34.8. A mirror (a perfect reflector) and a black disk (a perfect absorber) are connected by a horizontal rod suspended from a fine fiber. Normal-incidence light striking the black disk is completely absorbed, so all of the momentum of the

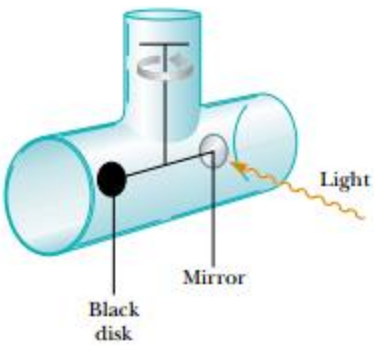


Figure 34.8 An apparatus for measuring the pressure exerted by light. In practice, the system is contained in a high vacuum.

Figure 34.9 Mariner 10 used its solar panels to "sail on sunlight."

[4] For oblique incidence on a perfectly reflecting surface, the momentum transferred is $(2U\cos\theta)/c$ and the pressure is $P = (2S\cos^2\theta)/c$, where θ is the angle between the normal to the surface and the direction of wave propagation.

light is transferred to the disk. Normal-incidence light striking the mirror is totally reflected, and hence the momentum transferred to the mirror is twice as great as that transferred to the disk. The radiation pressure is determined by measuring the angle through which the horizontal connecting rod rotates. The apparatus

Conceptual Example 34.3 Sweeping the Solar System

A great amount of dust exists in interplanetary space. Although in theory these dust particles can vary in size from molecular size to much larger, very little of the dust in our solar system is smaller than about 0.2 μm. Why?

Solution The dust particles are subject to two significant forces—the gravitational force that draws them toward the Sun and the radiation-pressure force that pushes them away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle because it is proportional to the mass and therefore to the volume $4\pi r^3/3$ of the particle. The radiation pressure is proportional to the square of the radius because it depends on the planar cross-section of the particle. For large particles, the gravitational force is greater than the force from radiation pressure. For particles having radii less than about 0.2 μm, the radiation-pressure force is greater than the gravitational force, and as a result these particles are swept out of the Solar System.

Example 34.4 Pressure from a Laser Pointer

Many people giving presentations use a laser pointer to direct the attention of the audience. If a 3.0-mW pointer creates a spot that is 2.0 mm in diameter, determine the radiation pressure on a screen that reflects 70% of the light that strikes it. The power 3.0 mW is a time-averaged value.

Solution We certainly do not expect the pressure to be very large. Before we can calculate it, we must determine the Poynting vector of the beam by dividing the time-averaged power delivered via the electromagnetic wave by the cross-sectional area of the beam:

$$S = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{\pi r^2} = \frac{3.0 \times 10^{-3}\,\text{W}}{\pi\left(\dfrac{2.0 \times 10^{-3}\,\text{m}}{2}\right)^2} = 955\ \text{W/m}^2$$

This is about the same as the intensity of sunlight at the Earth's surface. (Thus, it is not safe to shine the beam of a laser pointer into a person's eyes; that may be more dangerous than looking directly at the Sun.)

Now we can determine the radiation pressure from the laser beam. Equation 34.26 indicates that a completely reflected beam would apply a pressure of $P = 2S/c$. We can model the actual reflection as follows: Imagine that the surface absorbs the beam, resulting in pressure $P = S/c$. Then the surface emits the beam, resulting in additional pressure $P = S/c$. If the surface emits only a fraction f of the beam (so that f is the amount of the incident beam reflected), then the pressure due to the emitted beam is $P = fS/c$. Thus, the total pressure on the surface due to absorption and re-emission (reflection) is

$$P = \frac{S}{c} + f\frac{S}{c} = (1 + f)\,\frac{S}{c}$$

Notice that if $f = 1$, which represents complete reflection, this equation reduces to Equation 34.26. For a beam that is 70% reflected, the pressure is

$$P = (1 + 0.70)\,\frac{955\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = \boxed{5.4 \times 10^{-6}\ \text{N/m}^2}$$

This is an extremely small value, as expected. (Recall from Section 15.2 that atmospheric pressure is approximately 10^5 N/m^2.)

Example 34.5 Solar Energy

As noted in the preceding example, the Sun delivers about 1 000 W/m^2 of energy to the Earth's surface via electromagnetic radiation. (a) Calculate the total power that is incident on a roof of dimensions 8.00 m × 20.0 m.

Solution The magnitude of the Poynting vector for solar radiation at the surface of the Earth is $S = 1\,000$ W/m^2; this represents the power per unit area, or the light intensity. Assuming that the radiation is incident normal to the roof, we obtain

$$\mathcal{P} = SA = (1\,000\ \text{W/m}^2)(8.00 \times 20.0\ \text{m}^2)$$
$$= \boxed{1.60 \times 10^5\ \text{W}}$$

If all of this power could be converted to electrical energy, it would provide more than enough power for the average home. However, solar energy is not easily harnessed, and the prospects for large-scale conversion are not as bright as may appear from this calculation. For example, the efficiency of conversion from solar to electrical energy is typically 10% for photovoltaic cells. Roof systems for converting solar energy to internal energy are approximately 50% efficient; however, solar energy is associated with other practical problems, such as overcast days, geographic location, and methods of energy storage.

(b) Determine the radiation pressure and the radiation force exerted on the roof, assuming that the roof covering is a perfect absorber.

Solution Using Equation 34.24 with $S = 1\,000\ \text{W/m}^2$, we find that the radiation pressure is

$$P = \frac{S}{c} = \frac{1\,000\ \text{W/m}^2}{3.00 \times 10^8\ \text{m/s}} = \boxed{3.33 \times 10^{-6}\ \text{N/m}^2}$$

Because pressure equals force per unit area, this corresponds to a radiation force of

$$F = PA = (3.33 \times 10^{-6}\ \text{N/m}^2)(160\ \text{m}^2) = \boxed{5.33 \times 10^{-4}\ \text{N}}$$

Exercise How much solar energy is incident on the roof in 1 h?

Answer 5.76×10^8 J.

must be placed in a high vacuum to eliminate the effects of air currents.

NASA is exploring the possibility of *solar sailing* as a low-cost means of sending spacecraft to the planets. Large reflective sheets would be used in much the way canvas sheets are used on earthbound sailboats. In 1973 NASA engineers took advantage of the momentum of the sunlight striking the solar panels of Mariner 10 (Fig. 34.9) to make small course corrections when the spacecraft's fuel supply was running low. (This procedure was carried out when the spacecraft was in the vicinity of the planet Mercury. Would it have worked as well near Pluto?)

Optional Section

34.5 RADIATION FROM AN INFINITE CURRENT SHEET

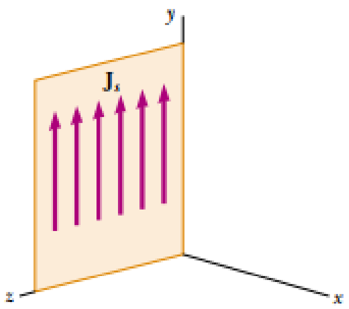


Figure 34.10 A portion of an infinite current sheet lying in the yz plane. The current density is sinusoidal and is given by the expression $J_s = J_{\max} \cos \omega t$. The magnetic field is everywhere parallel to the sheet and lies along z.

In this section, we describe the electric and magnetic fields radiated by a flat conductor carrying a time-varying current. In the symmetric plane geometry employed here, the mathematics is less complex than that required in lower-symmetry situations.

Consider an infinite conducting sheet lying in the yz plane and carrying a surface current in the y direction, as shown in Figure 34.10. The current is distributed across the z direction such that the current per unit length is $\mathbf{J}_s$. Let us assume that J_s varies sinusoidally with time as

$$J_s = J_{\max} \cos \omega t$$

where $J_{\max}$ is the amplitude of the current variation and ω is the angular frequency of the variation. A similar problem concerning the case of a steady current was treated in Example 30.6, in which we found that the magnetic field outside the sheet is everywhere parallel to the sheet and lies along the z axis. The magnetic field was found to have a magnitude

Radiated magnetic field

$$B_z = \mu_0 \frac{J_s}{2}$$

[5] Note that the solution could also be written in the form $\cos(\omega t - kx)$, which is equivalent to $\cos(kx - \omega t)$. That is, $\cos \theta$ is an even function, which means that $\cos(-\theta) = \cos \theta$.

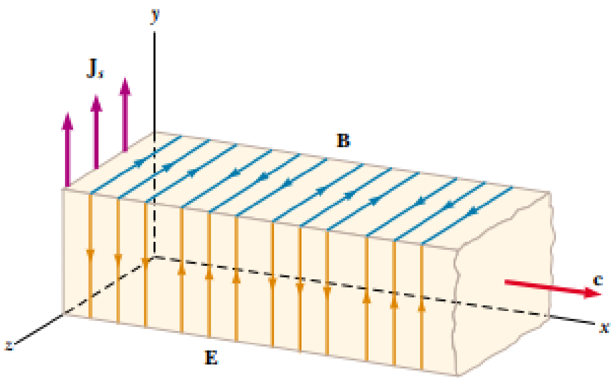


Figure 34.11 Representation of the plane electromagnetic wave radiated by an infinite current sheet lying in the yz plane. The vector $\mathbf{B}$ is in the z direction, the vector $\mathbf{E}$ is in the y direction, and the direction of wave motion is along x. Both vector $\mathbf{B}$ and vector $\mathbf{E}$ behave according to the expression $\cos(kx - \omega t)$. Compare this drawing with Figure 34.3a.

In the present situation, where J_s varies with time, this equation for B_z is valid only for distances close to the sheet. Substituting the expression for J_s, we have

$$B_z = \frac{\mu_0}{2} J_{\max} \cos \omega t \qquad \text{(for small values of } x\text{)}$$

To obtain the expression valid for B_z for arbitrary values of x, we can investigate the solution:[5]

$$B_z = \frac{\mu_0 J_{\max}}{2} \cos(kx - \omega t) \tag{34.27}$$

Radiated electric field

You should note two things about this solution, which is unique to the geometry under consideration. First, when x is very small, it agrees with our original solution. Second, it satisfies the wave equation as expressed in Equation 34.9. We conclude that the magnetic field lies along the z axis, varies with time, and is characterized by a transverse traveling wave having an angular frequency ω and an angular wave number $k = 2\pi/\lambda$.

We can obtain the electric field radiating from our infinite current sheet by using Equation 34.13:

$$E_y = cB_z = \frac{\mu_0 J_{\max} c}{2} \cos(kx - \omega t) \tag{34.28}$$

That is, the electric field is in the y direction, perpendicular to $\mathbf{B}$, and has the same space and time dependencies. These expressions for B_z and E_y show that the radiation field of an infinite current sheet carrying a sinusoidal current is a plane electromagnetic wave propagating with a speed c along the x axis, as shown in Figure 34.11.

We can calculate the Poynting vector for this wave from Equations 34.19,

EXAMPLE 34.6 An Infinite Sheet Carrying a Sinusoidal Current

An infinite current sheet lying in the yz plane carries a sinusoidal current that has a maximum density of 5.00 A/m. (a) Find the maximum values of the radiated magnetic and electric fields.

Solution From Equations 34.27 and 34.28, we see that the maximum values of B_z and E_y are

$$B_{\max} = \frac{\mu_0 J_{\max}}{2} \quad \text{and} \quad E_{\max} = \frac{\mu_0 J_{\max} c}{2}$$

Using the values $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$, $J_{\text{max}} = 5.00\ \text{A/m}$, and $c = 3.00 \times 10^8\ \text{m/s}$, we get

$$B_{\text{max}} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A/m})}{2}$$

$$= \boxed{3.14 \times 10^{-6}\ \text{T}}$$

$$E_{\text{max}} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A/m})(3.00 \times 10^8\ \text{m/s})}{2}$$

$$= \boxed{942\ \text{V/m}}$$

(b) What is the average power incident on a flat surface that is parallel to the sheet and has an area of $3.00\ \text{m}^2$? (The length and width of this surface are both much greater than the wavelength of the radiation.)

Solution The intensity, or power per unit area, radiated in each direction by the current sheet is given by Equation 34.30:

$$I = \frac{\mu_0 J_{\text{max}}^2 c}{8}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A/m})^2(3.00 \times 10^8\ \text{m/s})}{8}$$

$$= 1.18 \times 10^3\ \text{W/m}^2$$

Multiplying this by the area of the surface, we obtain the incident power:

$$\mathcal{P} = IA = (1.18 \times 10^3\ \text{W/m}^2)(3.00\ \text{m}^2)$$

$$= \boxed{3.54 \times 10^3\ \text{W}}$$

The result is independent of the distance from the current sheet because we are dealing with a plane wave.

34.27, and 34.28:

$$S = \frac{EB}{\mu_0} = \frac{\mu_0 J_{\text{max}}^2 c}{4} \cos^2(kx - \omega t) \tag{34.29}$$

The intensity of the wave, which equals the average value of S, is

$$I = S_{\text{av}} = \frac{\mu_0 J_{\text{max}}^2 c}{8} \tag{34.30}$$

Accelerating charges produce electromagnetic radiation

This intensity represents the power per unit area of the outgoing wave on each side of the sheet. The total rate of energy emitted per unit area of the conductor is $2S_{\text{av}} = \mu_0 J_{\text{max}}^2 c/4$.

Optional Section

34.6 PRODUCTION OF ELECTROMAGNETIC WAVES BY AN ANTENNA

Neither stationary charges nor steady currents can produce electromagnetic waves. Whenever the current through a wire changes with time, however, the wire emits

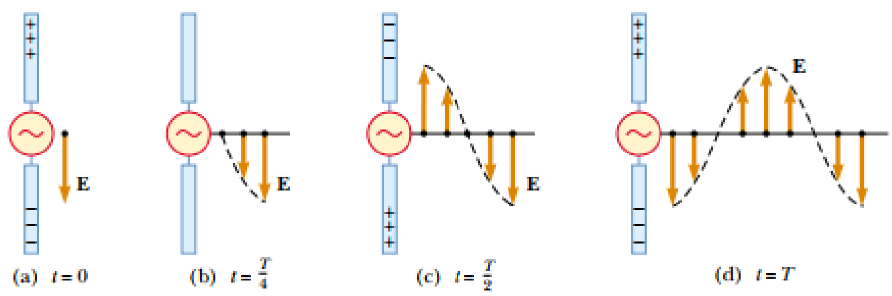


Figure 34.12 The electric field set up by charges oscillating in an antenna. The field moves away from the antenna with the speed of light.

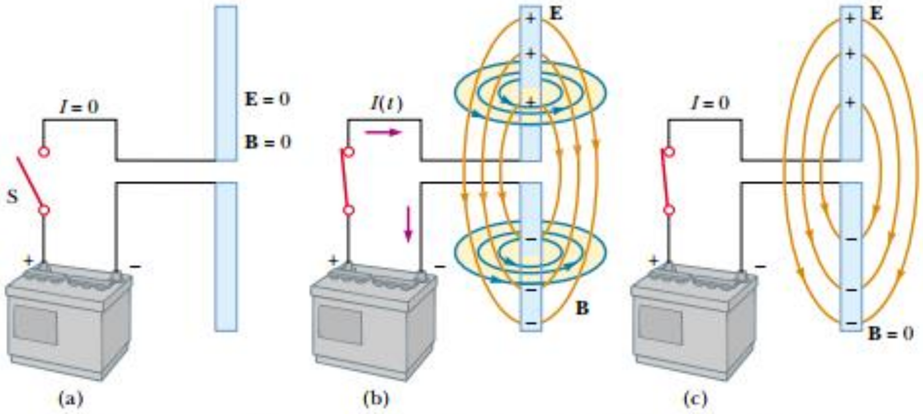


Figure 34.13 A pair of metal rods connected to a battery. (a) When the switch is open and no current exists, the electric and magnetic fields are both zero. (b) Immediately after the switch is closed, the rods are being charged (so a current exists). Because the current is changing, the rods generate changing electric and magnetic fields. (c) When the rods are fully charged, the current is zero, the electric field is a maximum, and the magnetic field is zero.

electromagnetic radiation. **The fundamental mechanism responsible for this radiation is the acceleration of a charged particle. Whenever a charged particle accelerates, it must radiate energy.**

An alternating voltage applied to the wires of an antenna forces an electric charge in the antenna to oscillate. This is a common technique for accelerating charges and is the source of the radio waves emitted by the transmitting antenna of a radio station. Figure 34.12 shows how this is done. Two metal rods are connected to a generator that provides a sinusoidally oscillating voltage. This causes charges to oscillate in the two rods. At $t = 0$, the upper rod is given a maximum positive charge and the bottom rod an equal negative charge, as shown in Figure 34.12a. The electric field near the antenna at this instant is also shown in Figure 34.12a. As the positive and negative charges decrease from their maximum values, the rods become less charged, the field near the rods decreases in strength, and the downward-directed maximum electric field produced at $t = 0$ moves away from the rod. (A magnetic field oscillating in a direction perpendicular to the plane of the diagram in Fig. 34.12 accompanies the oscillating electric field, but it is not shown for the sake of clarity.) When the charges on the rods are momentarily zero (Fig. 34.12b), the electric field at the rod has dropped to zero. This occurs at a time equal to one quarter of the period of oscillation.

As the generator charges the rods in the opposite sense from that at the beginning, the upper rod soon obtains a maximum negative charge and the lower rod a maximum positive charge (Fig. 34.12c); this results in an electric field near the rod that is directed upward after a time equal to one-half the period of oscillation. The oscillations continue as indicated in Figure 34.12d. The electric field near the antenna oscillates in phase with the charge distribution. That is, the field points down when the upper rod is positive and up when the upper rod is negative. Furthermore, the magnitude of the field at any instant depends on the amount of charge on the rods at that instant.

As the charges continue to oscillate (and accelerate) between the rods, the

[6] We have neglected the fields caused by the wires leading to the rods. This is a good approximation if the circuit dimensions are much less than the length of the rods.

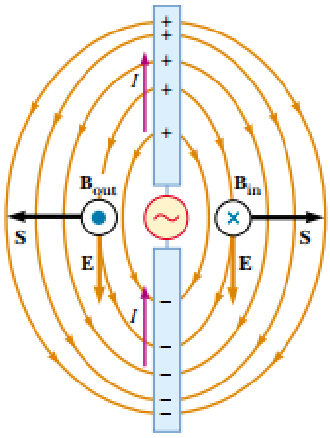


Figure 34.14 A half-wave antenna consists of two metal rods connected to an alternating voltage source. This diagram shows **E** and **B** at an instant when the current is upward. Note that the electric field lines resemble those of a dipole (shown in Fig. 23.21).

electric field they set up moves away from the antenna at the speed of light. As you can see from Figure 34.12, one cycle of charge oscillation produces one wavelength in the electric-field pattern.

Next, consider what happens when two conducting rods are connected to the terminals of a battery (Fig. 34.13). Before the switch is closed, the current is zero, so no fields are present (Fig. 34.13a). Just after the switch is closed, positive charge begins to build up on one rod and negative charge on the other (Fig. 34.13b), a situation that corresponds to a time-varying current. The changing charge distribution causes the electric field to change; this in turn produces a magnetic field around the rods.[6] Finally, when the rods are fully charged, the current is zero; hence, no magnetic field exists at that instant (Fig. 34.13c).

Now let us consider the production of electromagnetic waves by a *half-wave antenna.* In this arrangement, two conducting rods are connected to a source of alternating voltage (such as an *LC* oscillator), as shown in Figure 34.14. The length of each rod is equal to one quarter of the wavelength of the radiation that will be emitted when the oscillator operates at frequency f. The oscillator forces charges to accelerate back and forth between the two rods. Figure 34.14 shows the configuration of the electric and magnetic fields at some instant when the current is upward. The electric field lines resemble those of an electric dipole. (As a result, this type of antenna is sometimes called a *dipole antenna.*) Because these charges are continuously oscillating between the two rods, the antenna can be approximated by an oscillating electric dipole. The magnetic field lines form concentric circles around the antenna and are perpendicular to the electric field lines at all points. The magnetic field is zero at all points along the axis of the antenna. Furthermore, **E** and **B** are 90° out of phase in time because the current is zero when the charges at the outer ends of the rods are at a maximum.

At the two points where the magnetic field is shown in Figure 34.14, the Poynting vector **S** is directed radially outward. This indicates that energy is flowing away from the antenna at this instant. At later times, the fields and the Poynting vector change direction as the current alternates. Because **E** and **B** are 90° out of phase at points near the dipole, the net energy flow is zero. From this, we might conclude (incorrectly) that no energy is radiated by the dipole.

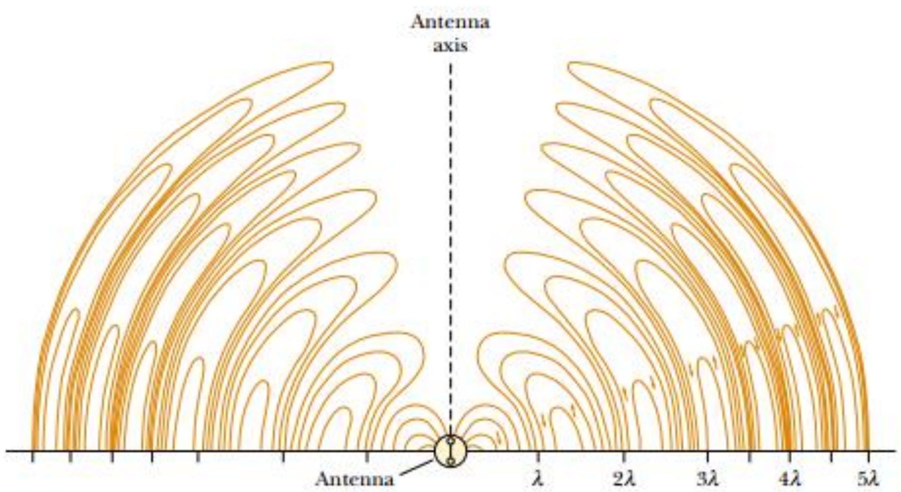


Figure 34.15 Electric field lines surrounding a dipole antenna at a given instant. The radiation fields propagate outward from the antenna with a speed c.

However, we find that energy is indeed radiated. Because the dipole fields fall off as $1/r^3$ (as shown in Example 23.6 for the electric field of a static dipole), they are not important at great distances from the antenna. However, at these great distances, something else causes a type of radiation different from that close to the antenna. The source of this radiation is the continuous induction of an electric field by the time-varying magnetic field and the induction of a magnetic field by the time-varying electric field, predicted by Equations 34.3 and 34.4. The electric and magnetic fields produced in this manner are in phase with each other and vary as $1/r$. The result is an outward flow of energy at all times.

The electric field lines produced by a dipole antenna at some instant are shown in Figure 34.15 as they propagate away from the antenna. Note that the intensity and the power radiated are a maximum in a plane that is perpendicular to the antenna and passing through its midpoint. Furthermore, the power radiated is zero along the antenna's axis. A mathematical solution to Maxwell's equations for the dipole antenna shows that the intensity of the radiation varies as $(\sin^2\theta)/r^2$, where θ is measured from the axis of the antenna. The angular dependence of the radiation intensity is sketched in Figure 34.16.

Electromagnetic waves can also induce currents in a receiving antenna. The response of a dipole receiving antenna at a given position is a maximum when the antenna axis is parallel to the electric field at that point and zero when the axis is perpendicular to the electric field.

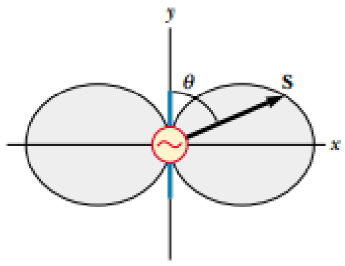


Figure 34.16 Angular dependence of the intensity of radiation produced by an oscillating electric dipole.

QuickLab

Rotate a portable radio (with a telescoping antenna) about a horizontal axis while it is tuned to a weak station. Can you use what you learn from this movement to verify the answer to Quick Quiz 34.2?

Quick Quiz 34.2

If the plane electromagnetic wave in Figure 34.11 represents the signal from a distant radio station, what would be the best orientation for your portable radio antenna—(a) along the x axis, (b) along the y axis, or (c) along the z axis?

34.7 THE SPECTRUM OF ELECTROMAGNETIC WAVES

The various types of electromagnetic waves are listed in Figure 34.17, which shows the **electromagnetic spectrum.** Note the wide ranges of frequencies and wavelengths. No sharp dividing point exists between one type of wave and the next. Remember that **all forms of the various types of radiation are produced by the same phenomenon—accelerating charges.** The names given to the types of waves are simply for convenience in describing the region of the spectrum in which they lie.

Radio waves

Radio waves are the result of charges accelerating through conducting wires. Ranging from more than 10^4 m to about 0.1 m in wavelength, they are generated by such electronic devices as LC oscillators and are used in radio and television communication systems.

Microwaves

Microwaves have wavelengths ranging from approximately 0.3 m to 10^{-4} m and are also generated by electronic devices. Because of their short wavelengths, they are well suited for radar systems and for studying the atomic and molecular properties of matter. Microwave ovens (in which the wavelength of the radiation is $\lambda = 0.122$ m) are an interesting domestic application of these waves. It has been suggested that solar energy could be harnessed by beaming microwaves to the Earth from a solar collector in space.[7]

Infrared waves

Visible light waves

[7] P. Glaser, "Solar Power from Satellites," *Phys. Today,* February 1977, p. 30.

Satellite-dish television antennas receive television-station signals from satellites in orbit around the Earth.

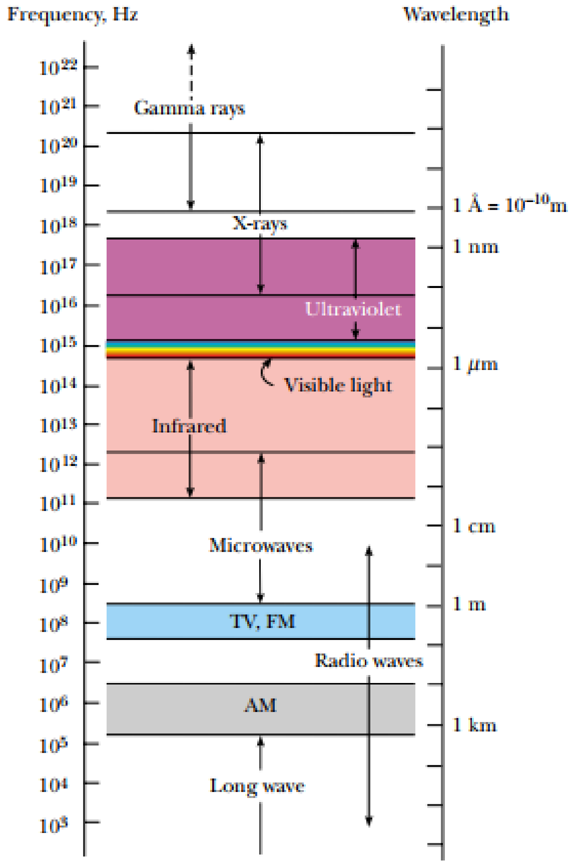


Figure 34.17 The electromagnetic spectrum. Note the overlap between adjacent wave types.

Infrared waves have wavelengths ranging from 10^{-3} m to the longest wavelength of visible light, 7×10^{-7} m. These waves, produced by molecules and room-temperature objects, are readily absorbed by most materials. The infrared (IR) energy absorbed by a substance appears as internal energy because the energy agitates the atoms of the object, increasing their vibrational or translational motion, which results in a temperature increase. Infrared radiation has practical and scientific applications in many areas, including physical therapy, IR photography, and vibrational spectroscopy.

Visible light, the most familiar form of electromagnetic waves, is the part of the electromagnetic spectrum that the human eye can detect. Light is produced by the rearrangement of electrons in atoms and molecules. The various wavelengths of visible light, which correspond to different colors, range from red ($\lambda \approx 7 \times 10^{-7}$ m) to violet ($\lambda \approx 4 \times 10^{-7}$ m). The sensitivity of the human eye is a function of wavelength, being a maximum at a wavelength of about 5.5×10^{-7} m. With this in mind, why do you suppose tennis balls often have a yellow-green color?

Ultraviolet waves

Ultraviolet waves cover wavelengths ranging from approximately 4×10^{-7} m to 6×10^{-10} m. The Sun is an important source of ultraviolet (UV) light, which is the main cause of sunburn. Sunscreen lotions are transparent to visible light but absorb most UV light. The higher a sunscreen's solar protection factor (SPF), the greater the percentage of UV light absorbed. Ultraviolet rays have also been implicated in the formation of cataracts, a clouding of the lens inside the eye. Wearing sunglasses that do not block UV light is worse for your eyes than wearing no sunglasses. The lenses of any sunglasses absorb some visible light, thus causing the wearer's pupils to dilate. If the glasses do not also block UV light, then more damage may be done to the lens of the eye because of the dilated pupils. If you wear no sunglasses at all, your pupils are contracted, you squint, and a lot less UV light enters your eyes. High-quality sunglasses block nearly all the eye-damaging UV light.

Most of the UV light from the Sun is absorbed by ozone (O_3) molecules in the Earth's upper atmosphere, in a layer called the stratosphere. This ozone shield converts lethal high-energy UV radiation to infrared radiation, which in turn warms the stratosphere. Recently, a great deal of controversy has arisen concerning the possible depletion of the protective ozone layer as a result of the chemicals emitted from aerosol spray cans and used as refrigerants.

X-rays

X-rays have wavelengths in the range from approximately 10^{-8} m to 10^{-12} m. The most common source of x-rays is the deceleration of high-energy electrons bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure or overexposure. X-rays are also used in the study of crystal structure because x-ray wavelengths are comparable to the atomic separation distances in solids (about 0.1 nm).

EXAMPLE 34.7 A Half-Wave Antenna

A half-wave antenna works on the principle that the optimum length of the antenna is one-half the wavelength of the radiation being received. What is the optimum length of a car antenna when it receives a signal of frequency 94.0 MHz?

Solution Equation 16.14 tells us that the wavelength of the signal is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{9.40 \times 10^7 \text{ Hz}} = 3.19 \text{ m}$$

Thus, to operate most efficiently, the antenna should have a length of $(3.19 \text{ m})/2 = 1.60$ m. For practical reasons, car antennas are usually one-quarter wavelength in size.

Gamma rays

Gamma rays are electromagnetic waves emitted by radioactive nuclei (such as ^{60}Co and ^{137}Cs) and during certain nuclear reactions. High-energy gamma rays are a component of cosmic rays that enter the Earth's atmosphere from space. They have wavelengths ranging from approximately 10^{-10} m to less than 10^{-14} m. They are highly penetrating and produce serious damage when absorbed by living tissues. Consequently, those working near such dangerous radiation must be protected with heavily absorbing materials, such as thick layers of lead.

Quick Quiz 34.3

The *AM* in *AM radio* stands for ***amplitude modulation,*** and *FM* stands for ***frequency modulation.*** (The word ***modulate*** means "to change.") If our eyes could see the electromagnetic waves from a radio antenna, how could you tell an AM wave from an FM wave?

SUMMARY

Electromagnetic waves, which are predicted by Maxwell's equations, have the following properties:

- The electric field and the magnetic field each satisfy a wave equation. These two wave equations, which can be obtained from Maxwell's third and fourth equations, are

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \tag{34.8}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \tag{34.9}$$

- The waves travel through a vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.00 \times 10^8 \text{ m/s} \tag{34.10}$$

- The electric and magnetic fields are perpendicular to each other and perpendicular to the direction of wave propagation. (Hence, electromagnetic waves are transverse waves.)
- The instantaneous magnitudes of $\mathbf{E}$ and $\mathbf{B}$ in an electromagnetic wave are related by the expression

$$\frac{E}{B} = c \tag{34.13}$$

- The waves carry energy. The rate of flow of energy crossing a unit area is described by the Poynting vector $\mathbf{S}$, where

$$\mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \tag{34.18}$$

- They carry momentum and hence exert pressure on surfaces. If an electromagnetic wave whose Poynting vector is $\mathbf{S}$ is completely absorbed by a surface upon which it is normally incident, the radiation pressure on that surface is

$$P = \frac{S}{c} \quad \text{(complete absorption)} \tag{34.24}$$

If the surface totally reflects a normally incident wave, the pressure is doubled.

The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x direction can be written

$$E = E_{\text{max}} \cos(kx - \omega t) \tag{34.11}$$

$$B = B_{\text{max}} \cos(kx - \omega t) \tag{34.12}$$

where ω is the angular frequency of the wave and k is the angular wave number. These equations represent special solutions to the wave equations for E and B. Be-

Questions

1. For a given incident energy of an electromagnetic wave, why is the radiation pressure on a perfectly reflecting surface twice as great as that on a perfectly absorbing surface?
2. Describe the physical significance of the Poynting vector.
3. Do all current-carrying conductors emit electromagnetic waves? Explain.
4. What is the fundamental cause of electromagnetic radiation?
5. Electrical engineers often speak of the radiation resistance of an antenna. What do you suppose they mean by this phrase?
6. If a high-frequency current is passed through a solenoid containing a metallic core, the core warms up by induc-