

## # PUZZLER

After this bottle of champagne was shaken, the cork was popped off and champagne spewed everywhere. Contrary to common belief, shaking a champagne bottle before opening it does not increase the pressure of the carbon dioxide ( $\text{CO}_2$ ) inside. In fact, if you know the trick, you can open a thoroughly shaken bottle without spraying a drop. What's the secret? And why isn't the pressure inside the bottle greater after the bottle is shaken? (Steve Niedorf/The Image Bank)



## chapter

# 19

## Temperature

### Chapter Outline

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| <b>19.1</b> Temperature and the Zeroth Law of Thermodynamics                       | <b>19.4</b> Thermal Expansion of Solids and Liquids |
| <b>19.2</b> Thermometers and the Celsius Temperature Scale                         | <b>19.5</b> Macroscopic Description of an Ideal Gas |
| <b>19.3</b> The Constant-Volume Gas Thermometer and the Absolute Temperature Scale |   |



In our study of mechanics, we carefully defined such concepts as mass, force, and kinetic energy to facilitate our quantitative approach. Likewise, a quantitative description of thermal phenomena requires a careful definition of such important terms as *temperature*, *heat*, and *internal energy*. This chapter begins with a look at these three entities and with a description of one of the laws of thermodynamics (the poetically named “zeroth law”). We then discuss the three most common temperature scales—Celsius, Fahrenheit, and Kelvin.

Next, we consider why the composition of a body is an important factor when we are dealing with thermal phenomena. For example, gases expand appreciably when heated, whereas liquids and solids expand only slightly. If a gas is not free to expand as it is heated, its pressure increases. Certain substances may melt, boil, burn, or explode when they are heated, depending on their composition and structure.

This chapter concludes with a study of ideal gases on the macroscopic scale. Here, we are concerned with the relationships among such quantities as pressure, volume, and temperature. Later on, in Chapter 21, we shall examine gases on a microscopic scale, using a model that represents the components of a gas as small particles.

## 19.1 TEMPERATURE AND THE ZEROTH LAW OF THERMODYNAMICS



We often associate the concept of temperature with how hot or cold an object feels when we touch it. Thus, our senses provide us with a qualitative indication of temperature. However, our senses are unreliable and often mislead us. For example, if we remove a metal ice tray and a cardboard box of frozen vegetables from the freezer, the ice tray feels colder than the box even though both are at the same temperature. The two objects feel different because metal is a better thermal conductor than cardboard is. What we need, therefore, is a reliable and reproducible method for establishing the relative hotness or coldness of bodies. Scientists have developed a variety of thermometers for making such quantitative measurements.

We are all familiar with the fact that two objects at different initial temperatures eventually reach some intermediate temperature when placed in contact with each other. For example, when a scoop of ice cream is placed in a room-temperature glass bowl, the ice cream melts and the temperature of the bowl decreases. Likewise, when an ice cube is dropped into a cup of hot coffee, it melts and the coffee’s temperature decreases.

To understand the concept of temperature, it is useful to define two often-used phrases: *thermal contact* and *thermal equilibrium*. To grasp the meaning of thermal contact, let us imagine that two objects are placed in an insulated container such that they interact with each other but not with the rest of the world. If the objects are at different temperatures, energy is exchanged between them, even if they are initially not in physical contact with each other. **Heat is the transfer of energy from one object to another object as a result of a difference in temperature between the two.** We shall examine the concept of heat in greater detail in Chapter 20. For purposes of the current discussion, we assume that two objects are in **thermal contact** with each other if energy can be exchanged between them. **Thermal equilibrium** is a situation in which two objects in thermal contact with each other cease to exchange energy by the process of heat.

Let us consider two objects A and B, which are not in thermal contact, and a third object C, which is our thermometer. We wish to determine whether A and B



Molten lava flowing down a mountain in Kilauea, Hawaii. The temperature of the hot lava flowing from a central crater decreases until the lava is in thermal equilibrium with its surroundings. At that equilibrium temperature, the lava has solidified and formed the mountains.

### QuickLab

Fill three cups with tap water: one hot, one cold, and one lukewarm. Dip your left index finger into the hot water and your right index finger into the cold water. Slowly count to 20, then quickly dip both fingers into the lukewarm water. What do you feel?

are in thermal equilibrium with each other. The thermometer (object C) is first placed in thermal contact with object A until thermal equilibrium is reached. From that moment on, the thermometer's reading remains constant, and we record this reading. The thermometer is then removed from object A and placed in thermal contact with object B. The reading is again recorded after thermal equilibrium is reached. If the two readings are the same, then object A and object B are in thermal equilibrium with each other.

We can summarize these results in a statement known as the **zeroth law of thermodynamics** (the law of equilibrium):

#### Zeroth law of thermodynamics

If objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

This statement can easily be proved experimentally and is very important because it enables us to define temperature. We can think of **temperature** as the property that determines whether an object is in thermal equilibrium with other objects. **Two objects in thermal equilibrium with each other are at the same temperature.** Conversely, if two objects have different temperatures, then they are not in thermal equilibrium with each other.

## 19.2 THERMOMETERS AND THE CELSIUS TEMPERATURE SCALE

Thermometers are devices that are used to define and measure temperatures. All thermometers are based on the principle that some physical property of a system changes as the system's temperature changes. Some physical properties that change with temperature are (1) the volume of a liquid, (2) the length of a solid, (3) the pressure of a gas at constant volume, (4) the volume of a gas at constant pressure, (5) the electric resistance of a conductor, and (6) the color of an object. For a given substance and a given temperature range, a temperature scale can be established on the basis of any one of these physical properties.

A common thermometer in everyday use consists of a mass of liquid—usually mercury or alcohol—that expands into a glass capillary tube when heated (Fig. 19.1). In this case the physical property is the change in volume of a liquid. Any temperature change can be defined as being proportional to the change in length of the liquid column. The thermometer can be calibrated by placing it in thermal contact with some natural systems that remain at constant temperature. One such system is a mixture of water and ice in thermal equilibrium at atmospheric pressure. On the **Celsius temperature scale**, this mixture is defined to have a temperature of zero degrees Celsius, which is written as  $0^{\circ}\text{C}$ ; this temperature is called the *ice point* of water. Another commonly used system is a mixture of water and steam in thermal equilibrium at atmospheric pressure; its temperature is  $100^{\circ}\text{C}$ , which is the *steam point* of water. Once the liquid levels in the thermometer have been established at these two points, the distance between the two points is divided into 100 equal segments to create the Celsius scale. Thus, each segment denotes a change in temperature of one Celsius degree. (This temperature scale used to be called the *centigrade scale* because there are 100 gradations between the ice and steam points of water.)

Thermometers calibrated in this way present problems when extremely accurate readings are needed. For instance, the readings given by an alcohol ther-





**Figure 19.1** As a result of thermal expansion, the level of the mercury in the thermometer rises as the mercury is heated by water in the test tube.

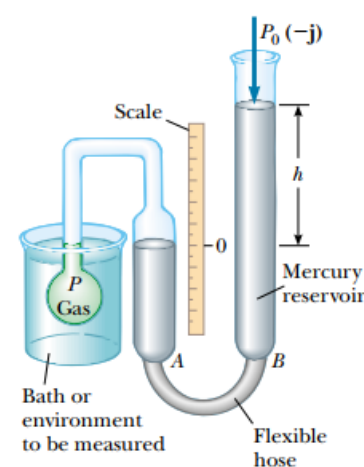
momometer calibrated at the ice and steam points of water might agree with those given by a mercury thermometer only at the calibration points. Because mercury and alcohol have different thermal expansion properties, when one thermometer reads a temperature of, for example,  $50^{\circ}\text{C}$ , the other may indicate a slightly different value. The discrepancies between thermometers are especially large when the temperatures to be measured are far from the calibration points.<sup>1</sup>

An additional practical problem of any thermometer is the limited range of temperatures over which it can be used. A mercury thermometer, for example, cannot be used below the freezing point of mercury, which is  $-39^{\circ}\text{C}$ , and an alcohol thermometer is not useful for measuring temperatures above  $85^{\circ}\text{C}$ , the boiling point of alcohol. To surmount this problem, we need a universal thermometer whose readings are independent of the substance used in it. The gas thermometer, discussed in the next section, approaches this requirement.

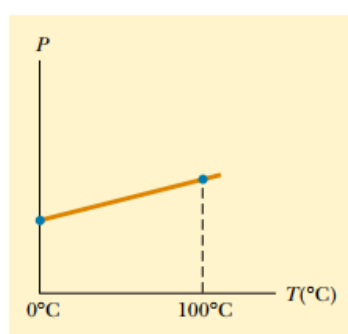
### 19.3 THE CONSTANT-VOLUME GAS THERMOMETER AND THE ABSOLUTE TEMPERATURE SCALE

The temperature readings given by a gas thermometer are nearly independent of the substance used in the thermometer. One version is the constant-volume gas thermometer shown in Figure 19.2. The physical change exploited in this device is the variation of pressure of a fixed volume of gas with temperature. When the constant-volume gas thermometer was developed, it was calibrated by using the ice

<sup>1</sup> Two thermometers that use the same liquid may also give different readings. This is due in part to difficulties in constructing uniform-bore glass capillary tubes.



**Figure 19.2** A constant-volume gas thermometer measures the pressure of the gas contained in the flask immersed in the bath. The volume of gas in the flask is kept constant by raising or lowering reservoir *B* to keep the mercury level in column *A* constant.



**Figure 19.3** A typical graph of pressure versus temperature taken with a constant-volume gas thermometer. The two dots represent known reference temperatures (the ice and steam points of water).

and steam points of water, as follows (a different calibration procedure, which we shall discuss shortly, is now used): The flask was immersed in an ice bath, and mercury reservoir *B* was raised or lowered until the top of the mercury in column *A* was at the zero point on the scale. The height *h*, the difference between the mercury levels in reservoir *B* and column *A*, indicated the pressure in the flask at 0°C.

The flask was then immersed in water at the steam point, and reservoir *B* was readjusted until the top of the mercury in column *A* was again at zero on the scale; this ensured that the gas's volume was the same as it was when the flask was in the ice bath (hence, the designation "constant volume"). This adjustment of reservoir *B* gave a value for the gas pressure at 100°C. These two pressure and temperature values were then plotted, as shown in Figure 19.3. The line connecting the two points serves as a calibration curve for unknown temperatures. If we wanted to measure the temperature of a substance, we would place the gas flask in thermal contact with the substance and adjust the height of reservoir *B* until the top of the mercury column in *A* was at zero on the scale. The height of the mercury column would indicate the pressure of the gas; knowing the pressure, we could find the temperature of the substance using the graph in Figure 19.3.

Now let us suppose that temperatures are measured with gas thermometers containing different gases at different initial pressures. Experiments show that the thermometer readings are nearly independent of the type of gas used, as long as the gas pressure is low and the temperature is well above the point at which the gas liquefies (Fig. 19.4). The agreement among thermometers using various gases improves as the pressure is reduced.

If you extend the curves shown in Figure 19.4 toward negative temperatures, you find, in every case, that the pressure is zero when the temperature is  $-273.15^\circ\text{C}$ . This significant temperature is used as the basis for the **absolute temperature scale**, which sets  $-273.15^\circ\text{C}$  as its zero point. This temperature is often referred to as **absolute zero**. The size of a degree on the absolute temperature scale is identical to the size of a degree on the Celsius scale. Thus, the conversion between these temperatures is

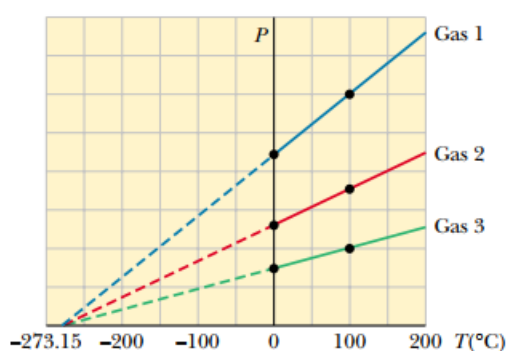
$$T_C = T - 273.15 \quad (19.1)$$

where  $T_C$  is the Celsius temperature and  $T$  is the absolute temperature.

Because the ice and steam points are experimentally difficult to duplicate, an absolute temperature scale based on a single fixed point was adopted in 1954 by the International Committee on Weights and Measures. From a list of fixed points associated with various substances (Table 19.1), the triple point of water was chosen as the reference temperature for this new scale. The **triple point of water** is the single combination of temperature and pressure at which liquid water, gaseous

#### web

For more information about the temperature standard, visit the National Institute of Standards and Technology at <http://www.nist.gov>



**Figure 19.4** Pressure versus temperature for three dilute gases. Note that, for all gases, the pressure extrapolates to zero at the temperature  $-273.15^\circ\text{C}$ .

**TABLE 19.1** Fixed-Point Temperatures<sup>a</sup>

Fixed Point	Temperature (°C)	Temperature (K)
Triple point of hydrogen	− 259.34	13.81
Boiling point of helium	− 268.93	4.215
Boiling point of hydrogen at 33.36 kPa pressure	− 256.108	17.042
Boiling point of hydrogen	− 252.87	20.28
Triple point of neon	− 246.048	27.102
Triple point of oxygen	− 218.789	54.361
Boiling point of oxygen	− 182.962	90.188
Triple point of water	0.01	273.16
Boiling point of water	100.00	373.15
Freezing point of tin	231.968 1	505.118 1
Freezing point of zinc	419.58	692.73
Freezing point of silver	961.93	1 235.08
Freezing point of gold	1 064.43	1 337.58

<sup>a</sup>All values are from National Bureau of Standards Special Publication 420; U. S. Department of Commerce, May 1975. All values are at standard atmospheric pressure except for triple points and as noted.

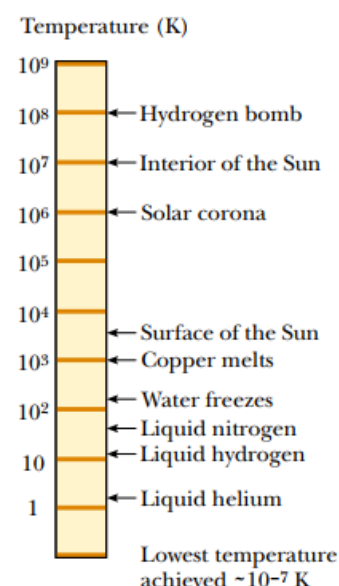
water, and ice (solid water) coexist in equilibrium. This triple point occurs at a temperature of approximately 0.01°C and a pressure of 4.58 mm of mercury. On the new scale, which uses the unit *kelvin*, the temperature of water at the triple point was set at 273.16 kelvin, abbreviated 273.16 K. (*Note:* no degree sign “°” is used with the unit kelvin.) This choice was made so that the old absolute temperature scale based on the ice and steam points would agree closely with the new scale based on the triple point. This new absolute temperature scale (also called the **Kelvin scale**) employs the SI unit of absolute temperature, the **kelvin**, which is defined to be **1/273.16 of the difference between absolute zero and the temperature of the triple point of water**.

Figure 19.5 shows the absolute temperature for various physical processes and structures. The temperature of absolute zero (0 K) cannot be achieved, although laboratory experiments incorporating the laser cooling of atoms have come very close.

What would happen to a gas if its temperature could reach 0 K? As Figure 19.4 indicates, the pressure it exerts on the walls of its container would be zero. In Section 19.5 we shall show that the pressure of a gas is proportional to the average kinetic energy of its molecules. Thus, according to classical physics, the kinetic energy of the gas molecules would become zero at absolute zero, and molecular motion would cease; hence, the molecules would settle out on the bottom of the container. Quantum theory modifies this model and shows that some residual energy, called the *zero-point energy*, would remain at this low temperature.

### The Celsius, Fahrenheit, and Kelvin Temperature Scales<sup>2</sup>

Equation 19.1 shows that the Celsius temperature  $T_C$  is shifted from the absolute (Kelvin) temperature  $T$  by 273.15°. Because the size of a degree is the same on the



**Figure 19.5** Absolute temperatures at which various physical processes occur. Note that the scale is logarithmic.

<sup>2</sup> Named after Anders Celsius (1701–1744), Gabriel Fahrenheit (1686–1736), and William Thomson, Lord Kelvin (1824–1907), respectively.



two scales, a temperature difference of  $5^{\circ}\text{C}$  is equal to a temperature difference of 5 K. The two scales differ only in the choice of the zero point. Thus, the ice-point temperature on the Kelvin scale, 273.15 K, corresponds to  $0.00^{\circ}\text{C}$ , and the Kelvin-scale steam point, 373.15 K, is equivalent to  $100.00^{\circ}\text{C}$ .

A common temperature scale in everyday use in the United States is the **Fahrenheit scale**. This scale sets the temperature of the ice point at  $32^{\circ}\text{F}$  and the temperature of the steam point at  $212^{\circ}\text{F}$ . The relationship between the Celsius and Fahrenheit temperature scales is

$$T_{\text{F}} = \frac{9}{5}T_{\text{C}} + 32^{\circ}\text{F} \quad (19.2)$$

### Quick Quiz 19.1

What is the physical significance of the factor  $\frac{9}{5}$  in Equation 19.2? Why is this factor missing in Equation 19.1?

Extending the ideas considered in Quick Quiz 19.1, we use Equation 19.2 to find a relationship between changes in temperature on the Celsius, Kelvin, and Fahrenheit scales:

$$\Delta T_{\text{C}} = \Delta T = \frac{5}{9}\Delta T_{\text{F}} \quad (19.3)$$

### EXAMPLE 19.1 Converting Temperatures

On a day when the temperature reaches  $50^{\circ}\text{F}$ , what is the temperature in degrees Celsius and in kelvins?

**Solution** Substituting  $T_{\text{F}} = 50^{\circ}\text{F}$  into Equation 19.2, we obtain

$$T_{\text{C}} = \frac{5}{9}(T_{\text{F}} - 32) = \frac{5}{9}(50 - 32) = 10^{\circ}\text{C}$$

From Equation 19.1, we find that

$$T = T_{\text{C}} + 273.15 = 10^{\circ}\text{C} + 273.15 = 283 \text{ K}$$

A convenient set of weather-related temperature equivalents to keep in mind is that  $0^{\circ}\text{C}$  is (literally) freezing at  $32^{\circ}\text{F}$ ,  $10^{\circ}\text{C}$  is cool at  $50^{\circ}\text{F}$ ,  $30^{\circ}\text{C}$  is warm at  $86^{\circ}\text{F}$ , and  $40^{\circ}\text{C}$  is a hot day at  $104^{\circ}\text{F}$ .

### EXAMPLE 19.2 Heating a Pan of Water

A pan of water is heated from  $25^{\circ}\text{C}$  to  $80^{\circ}\text{C}$ . What is the change in its temperature on the Kelvin scale and on the Fahrenheit scale?

**Solution** From Equation 19.3, we see that the change in temperature on the Celsius scale equals the change on the Kelvin scale. Therefore,

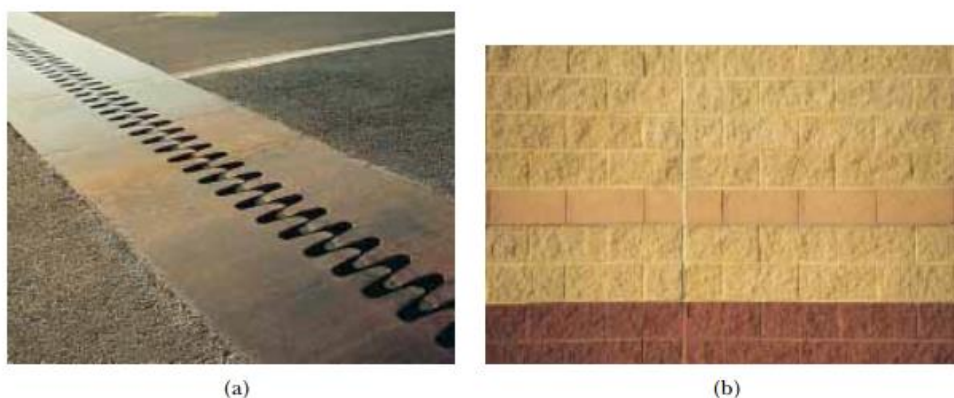
$$\Delta T = \Delta T_{\text{C}} = 80^{\circ}\text{C} - 25^{\circ}\text{C} = 55^{\circ}\text{C} = 55 \text{ K}$$

From Equation 19.3, we also find that

$$\Delta T_{\text{F}} = \frac{9}{5}\Delta T_{\text{C}} = \frac{9}{5}(55^{\circ}\text{C}) = 99^{\circ}\text{F}$$

## 19.4 THERMAL EXPANSION OF SOLIDS AND LIQUIDS

Our discussion of the liquid thermometer made use of one of the best-known changes in a substance: As its temperature increases, its volume almost always increases. (As we shall see shortly, in some substances the volume decreases when the temperature increases.) This phenomenon, known as **thermal expansion**, has



**Figure 19.6** (a) Thermal-expansion joints are used to separate sections of roadways on bridges. Without these joints, the surfaces would buckle due to thermal expansion on very hot days or crack due to contraction on very cold days. (b) The long, vertical joint is filled with a soft material that allows the wall to expand and contract as the temperature of the bricks changes.

an important role in numerous engineering applications. For example, thermal-expansion joints, such as those shown in Figure 19.6, must be included in buildings, concrete highways, railroad tracks, brick walls, and bridges to compensate for dimensional changes that occur as the temperature changes.

Thermal expansion is a consequence of the change in the average separation between the constituent atoms in an object. To understand this, imagine that the atoms are connected by stiff springs, as shown in Figure 19.7. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately  $10^{-11}$  m and a frequency of approximately  $10^{13}$  Hz. The average spacing between the atoms is about  $10^{-10}$  m. As the temperature of the solid increases, the atoms oscillate with greater amplitudes; as a result, the average separation between them increases.<sup>3</sup> Consequently, the object expands.

If thermal expansion is sufficiently small relative to an object's initial dimensions, the change in any dimension is, to a good approximation, proportional to the first power of the temperature change. Suppose that an object has an initial length  $L_i$  along some direction at some temperature and that the length increases by an amount  $\Delta L$  for a change in temperature  $\Delta T$ . Because it is convenient to consider the fractional change in length per degree of temperature change, we define the **average coefficient of linear expansion** as

$$\alpha \equiv \frac{\Delta L/L_i}{\Delta T}$$

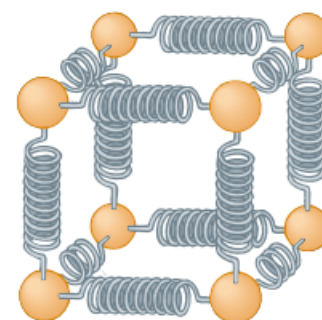
Experiments show that  $\alpha$  is constant for small changes in temperature. For purposes of calculation, this equation is usually rewritten as

$$\Delta L = \alpha L_i \Delta T \quad (19.4)$$

or as

$$L_f - L_i = \alpha L_i (T_f - T_i) \quad (19.5)$$

<sup>3</sup> More precisely, thermal expansion arises from the *asymmetrical* nature of the potential-energy curve for the atoms in a solid. If the oscillators were truly harmonic, the average atomic separations would not change regardless of the amplitude of vibration.



**Figure 19.7** A mechanical model of the atomic configuration in a substance. The atoms (spheres) are imagined to be attached to each other by springs that reflect the elastic nature of the interatomic forces.

Average coefficient of linear expansion

The change in length of an object is proportional to the change in temperature



where  $L_f$  is the final length,  $T_i$  and  $T_f$  are the initial and final temperatures, and the proportionality constant  $\alpha$  is the average coefficient of linear expansion for a given material and has units of  $^{\circ}\text{C}^{-1}$ .

It may be helpful to think of thermal expansion as an effective magnification or as a photographic enlargement of an object. For example, as a metal washer is heated (Fig. 19.8), all dimensions, including the radius of the hole, increase according to Equation 19.4.

Table 19.2 lists the average coefficient of linear expansion for various materials. Note that for these materials  $\alpha$  is positive, indicating an increase in length with increasing temperature. This is not always the case. Some substances—calcite ( $\text{CaCO}_3$ ) is one example—expand along one dimension (positive  $\alpha$ ) and contract along another (negative  $\alpha$ ) as their temperatures are increased.

Because the linear dimensions of an object change with temperature, it follows that surface area and volume change as well. The change in volume at constant pressure is proportional to the initial volume  $V_i$  and to the change in temperature according to the relationship

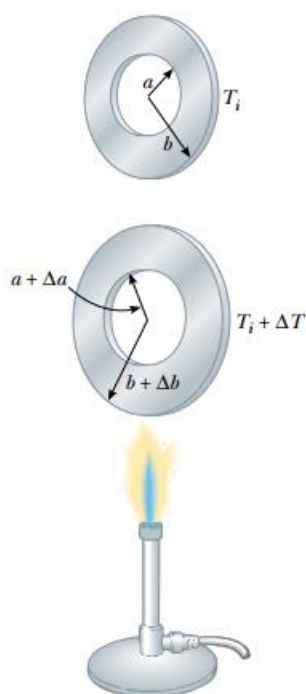
$$\Delta V = \beta V_i \Delta T \quad (19.6)$$

where  $\beta$  is the **average coefficient of volume expansion**. For a solid, the average coefficient of volume expansion is approximately three times the average linear expansion coefficient:  $\beta = 3\alpha$ . (This assumes that the average coefficient of linear expansion of the solid is the same in all directions.)

To see that  $\beta = 3\alpha$  for a solid, consider a box of dimensions  $\ell$ ,  $w$ , and  $h$ . Its volume at some temperature  $T_i$  is  $V_i = \ell wh$ . If the temperature changes to  $T_i + \Delta T$ , its volume changes to  $V_i + \Delta V$ , where each dimension changes according to Equation 19.4. Therefore,

$$\begin{aligned} V_i + \Delta V &= (\ell + \Delta\ell)(w + \Delta w)(h + \Delta h) \\ &= (\ell + \alpha\ell\Delta T)(w + \alpha w\Delta T)(h + \alpha h\Delta T) \\ &= \ell wh(1 + \alpha\Delta T)^3 \\ &= V_i[1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3] \end{aligned}$$

The change in volume of a solid at constant pressure is proportional to the change in temperature



**Figure 19.8** Thermal expansion of a homogeneous metal washer. As the washer is heated, all dimensions increase. (The expansion is exaggerated in this figure.)

**TABLE 19.2** Average Expansion Coefficients for Some Materials Near Room Temperature

Material	Average Linear Expansion Coefficient ( $\alpha$ ) ( $^{\circ}\text{C}^{-1}$ )	Material	Average Volume Expansion Coefficient ( $\beta$ ) ( $^{\circ}\text{C}^{-1}$ )
Aluminum	$24 \times 10^{-6}$	Alcohol, ethyl	$1.12 \times 10^{-4}$
Brass and bronze	$19 \times 10^{-6}$	Benzene	$1.24 \times 10^{-4}$
Copper	$17 \times 10^{-6}$	Acetone	$1.5 \times 10^{-4}$
Glass (ordinary)	$9 \times 10^{-6}$	Glycerin	$4.85 \times 10^{-4}$
Glass (Pyrex)	$3.2 \times 10^{-6}$	Mercury	$1.82 \times 10^{-4}$
Lead	$29 \times 10^{-6}$	Turpentine	$9.0 \times 10^{-4}$
Steel	$11 \times 10^{-6}$	Gasoline	$9.6 \times 10^{-4}$
Invar (Ni-Fe alloy)	$0.9 \times 10^{-6}$	Air at $0^{\circ}\text{C}$	$3.67 \times 10^{-3}$
Concrete	$12 \times 10^{-6}$	Helium	$3.665 \times 10^{-3}$

If we now divide both sides by  $V_i$  and then isolate the term  $\Delta V/V_i$ , we obtain the fractional change in volume:

$$\frac{\Delta V}{V_i} = 3\alpha \Delta T + 3(\alpha \Delta T)^2 + (\alpha \Delta T)^3$$

Because  $\alpha \Delta T \ll 1$  for typical values of  $\Delta T$  ( $< \sim 100^\circ\text{C}$ ), we can neglect the terms  $3(\alpha \Delta T)^2$  and  $(\alpha \Delta T)^3$ . Upon making this approximation, we see that

$$\frac{\Delta V}{V_i} = 3\alpha \Delta T$$

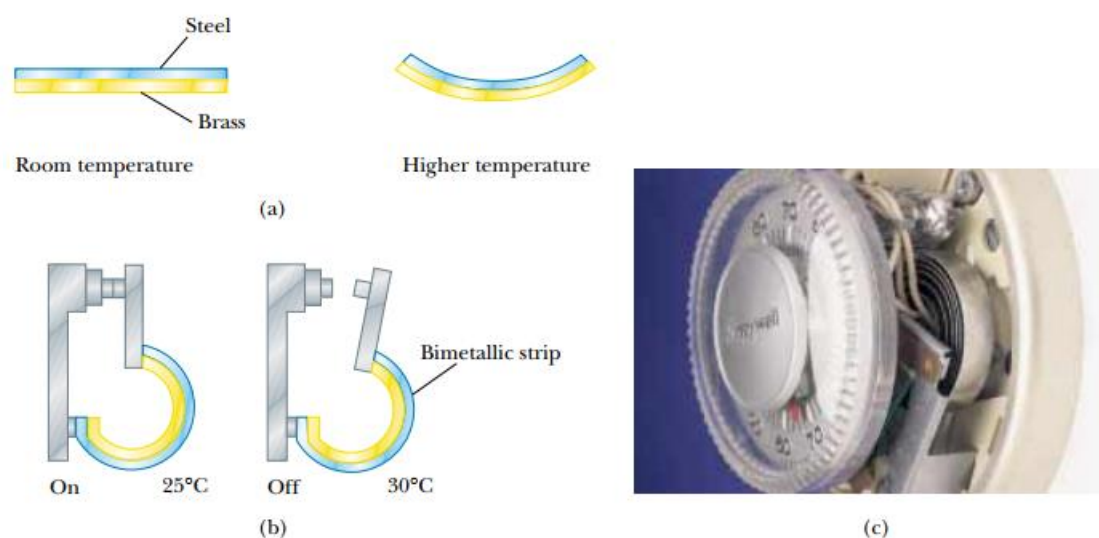
$$3\alpha = \frac{1}{V_i} \frac{\Delta V}{\Delta T}$$

Equation 19.6 shows that the right side of this expression is equal to  $\beta$ , and so we have  $3\alpha = \beta$ , the relationship we set out to prove. In a similar way, you can show that the change in area of a rectangular plate is given by  $\Delta A = 2\alpha A_i \Delta T$  (see Problem 53).

As Table 19.2 indicates, each substance has its own characteristic average coefficient of expansion. For example, when the temperatures of a brass rod and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod does because brass has a greater average coefficient of expansion than steel does. A simple mechanism called a *bimetallic strip* utilizes this principle and is found in practical devices such as thermostats. It consists of two thin strips of dissimilar metals bonded together. As the temperature of the strip increases, the two metals expand by different amounts and the strip bends, as shown in Figure 19.9.

### QuickLab

Tape two plastic straws tightly together along their entire length but with a 2-cm offset. Hold them in a stream of very hot water from a faucet so that water pours through one but not through the other. Quickly hold the straws up and sight along their length. You should be able to see a very slight curvature in the tape caused by the difference in expansion of the two straws. The effect is small, so look closely. Running cold water through the same straw and again sighting along the length will help you see the small change in shape more clearly.



**Figure 19.9** (a) A bimetallic strip bends as the temperature changes because the two metals have different expansion coefficients. (b) A bimetallic strip used in a thermostat to break or make electrical contact. (c) The interior of a thermostat, showing the coiled bimetallic strip. Why do you suppose the strip is coiled?



**Quick Quiz 19.2**

If you quickly plunge a room-temperature thermometer into very hot water, the mercury level will go *down* briefly before going up to a final reading. Why?

**Quick Quiz 19.3**

You are offered a prize for making the most sensitive glass thermometer using the materials in Table 19.2. Which glass and which working liquid would you choose?

**EXAMPLE 19.3** Expansion of a Railroad Track

A steel railroad track has a length of 30.000 m when the temperature is 0.0°C. (a) What is its length when the temperature is 40.0°C?



Thermal expansion: The extreme temperature of a July day in Asbury Park, NJ, caused these railroad tracks to buckle and derail the train in the distance. (AP/Wide World Photos)

**Solution** Making use of Table 19.2 and noting that the change in temperature is 40.0°C, we find that the increase in length is

$$\Delta L = \alpha L_i \Delta T = [11 \times 10^{-6} (\text{°C})^{-1}] (30.000 \text{ m}) (40.0 \text{ °C}) = 0.013 \text{ m}$$

If the track is 30.000 m long at 0.0°C, its length at 40.0°C is 30.013 m.

(b) Suppose that the ends of the rail are rigidly clamped at 0.0°C so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to 40.0°C?

**Solution** From the definition of Young's modulus for a solid (see Eq. 12.6), we have

$$\text{Tensile stress} = \frac{F}{A} = Y \frac{\Delta L}{L_i}$$

Because  $Y$  for steel is  $20 \times 10^{10} \text{ N/m}^2$  (see Table 12.1), we have

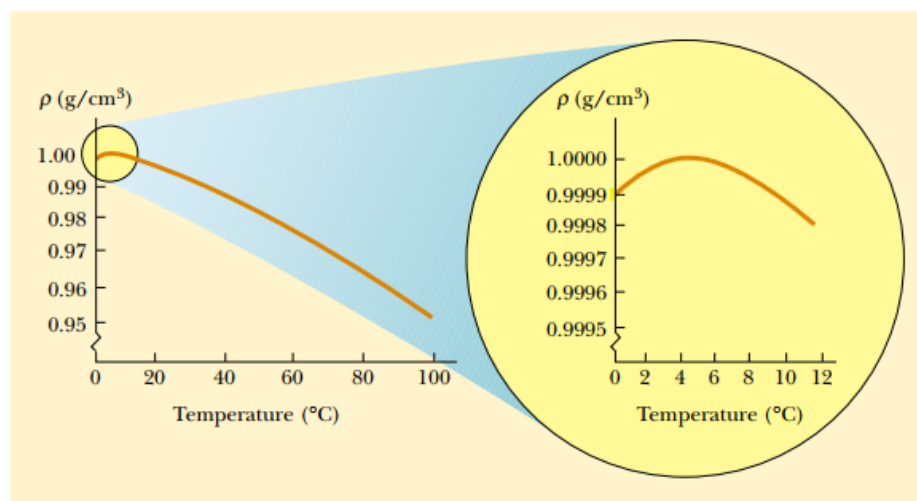
$$\frac{F}{A} = (20 \times 10^{10} \text{ N/m}^2) \left( \frac{0.013 \text{ m}}{30.000 \text{ m}} \right) = 8.7 \times 10^7 \text{ N/m}^2$$

**Exercise** If the rail has a cross-sectional area of 30.0 cm<sup>2</sup>, what is the force of compression in the rail?

**Answer**  $2.6 \times 10^5 \text{ N} = 58\,000 \text{ lb!}$

**The Unusual Behavior of Water**

Liquids generally increase in volume with increasing temperature and have average coefficients of volume expansion about ten times greater than those of solids. Water is an exception to this rule, as we can see from its density-versus-temperature curve shown in Figure 19.10. As the temperature increases from 0°C to 4°C, water contracts and thus its density increases. Above 4°C, water expands with increasing temperature, and so its density decreases. In other words, the density of water reaches a maximum value of 1 000 kg/m<sup>3</sup> at 4°C.



**Figure 19.10** How the density of water at atmospheric pressure changes with temperature. The inset at the right shows that the maximum density of water occurs at 4°C.

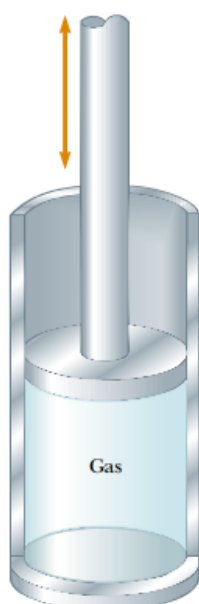
We can use this unusual thermal-expansion behavior of water to explain why a pond begins freezing at the surface rather than at the bottom. When the atmospheric temperature drops from, for example, 7°C to 6°C, the surface water also cools and consequently decreases in volume. This means that the surface water is denser than the water below it, which has not cooled and decreased in volume. As a result, the surface water sinks, and warmer water from below is forced to the surface to be cooled. When the atmospheric temperature is between 4°C and 0°C, however, the surface water expands as it cools, becoming less dense than the water below it. The mixing process stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water. The ice continues to build up at the surface, while water near the bottom remains at 4°C. If this were not the case, then fish and other forms of marine life would not survive.

## 19.5 MACROSCOPIC DESCRIPTION OF AN IDEAL GAS

**10.5** In this section we examine the properties of a gas of mass  $m$  confined to a container of volume  $V$  at a pressure  $P$  and a temperature  $T$ . It is useful to know how these quantities are related. In general, the equation that interrelates these quantities, called the *equation of state*, is very complicated. However, if the gas is maintained at a very low pressure (or low density), the equation of state is quite simple and can be found experimentally. Such a low-density gas is commonly referred to as an *ideal gas*.<sup>4</sup>

<sup>4</sup> To be more specific, the assumption here is that the temperature of the gas must not be too low (the gas must not condense into a liquid) or too high, and that the pressure must be low. In reality, an ideal gas does not exist. However, the concept of an ideal gas is very useful in view of the fact that real gases at low pressures behave as ideal gases do. The concept of an ideal gas implies that the gas molecules do not interact except upon collision, and that the molecular volume is negligible compared with the volume of the container.





**Figure 19.11** An ideal gas confined to a cylinder whose volume can be varied by means of a movable piston.

The universal gas constant

It is convenient to express the amount of gas in a given volume in terms of the number of moles  $n$ . As we learned in Section 1.3, one mole of any substance is that amount of the substance that contains Avogadro's number  $N_A = 6.022 \times 10^{23}$  of constituent particles (atoms or molecules). The number of moles  $n$  of a substance is related to its mass  $m$  through the expression

$$n = \frac{m}{M} \quad (19.7)$$

where  $M$  is the molar mass of the substance (see Section 1.3), which is usually expressed in units of grams per mole (g/mol). For example, the molar mass of oxygen ( $O_2$ ) is 32.0 g/mol. Therefore, the mass of one mole of oxygen is 32.0 g.

Now suppose that an ideal gas is confined to a cylindrical container whose volume can be varied by means of a movable piston, as shown in Figure 19.11. If we assume that the cylinder does not leak, the mass (or the number of moles) of the gas remains constant. For such a system, experiments provide the following information: First, when the gas is kept at a constant temperature, its pressure is inversely proportional to its volume (Boyle's law). Second, when the pressure of the gas is kept constant, its volume is directly proportional to its temperature (the law of Charles and Gay-Lussac). These observations are summarized by the **equation of state for an ideal gas**:

$$PV = nRT \quad (19.8)$$

In this expression, known as the **ideal gas law**,  $R$  is a universal constant that is the same for all gases and  $T$  is the absolute temperature in kelvins. Experiments on numerous gases show that as the pressure approaches zero, the quantity  $PV/nT$  approaches the same value  $R$  for all gases. For this reason,  $R$  is called the **universal gas constant**. In SI units, in which pressure is expressed in pascals ( $1 \text{ Pa} = 1 \text{ N/m}^2$ ) and volume in cubic meters, the product  $PV$  has units of newton·meters, or joules, and  $R$  has the value

$$R = 8.315 \text{ J/mol} \cdot \text{K} \quad (19.9)$$

If the pressure is expressed in atmospheres and the volume in liters ( $1 \text{ L} = 10^3 \text{ cm}^3 = 10^{-3} \text{ m}^3$ ), then  $R$  has the value

$$R = 0.08214 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$$

Using this value of  $R$  and Equation 19.8, we find that the volume occupied by 1 mol of any gas at atmospheric pressure and at  $0^\circ\text{C}$  (273 K) is 22.4 L.

Now that we have presented the equation of state, we are ready for a formal definition of an ideal gas: **An ideal gas is one for which  $PV/nT$  is constant at all pressures.**

The ideal gas law states that if the volume and temperature of a fixed amount of gas do not change, then the pressure also remains constant. Consider the bottle of champagne shown at the beginning of this chapter. Because the temperature of the bottle and its contents remains constant, so does the pressure, as can be shown by replacing the cork with a pressure gauge. Shaking the bottle displaces some carbon dioxide gas from the "head space" to form bubbles within the liquid, and these bubbles become attached to the inside of the bottle. (No new gas is generated by shaking.) When the bottle is opened, the pressure is reduced; this causes the volume of the bubbles to increase suddenly. If the bubbles are attached to the bottle (beneath the liquid surface), their rapid expansion expels liquid from the

### QuickLab

Vigorously shake a can of soda pop and then thoroughly tap its bottom and sides to dislodge any bubbles trapped there. You should be able to open the can without spraying its contents all over.

bottle. If the sides and bottom of the bottle are first tapped until no bubbles remain beneath the surface, then when the champagne is opened, the drop in pressure will not force liquid from the bottle. Try the QuickLab, but practice before demonstrating to a friend!

The ideal gas law is often expressed in terms of the total number of molecules  $N$ . Because the total number of molecules equals the product of the number of moles  $n$  and Avogadro's number  $N_A$ , we can write Equation 19.8 as

$$PV = nRT = \frac{N}{N_A} RT$$

$$PV = Nk_B T \quad (19.10)$$

where  $k_B$  is **Boltzmann's constant**, which has the value

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \quad (19.11)$$

Boltzmann's constant

It is common to call quantities such as  $P$ ,  $V$ , and  $T$  the **thermodynamic variables** of an ideal gas. If the equation of state is known, then one of the variables can always be expressed as some function of the other two.

#### EXAMPLE 19.4 How Many Gas Molecules in a Container?

An ideal gas occupies a volume of  $100 \text{ cm}^3$  at  $20^\circ\text{C}$  and  $100 \text{ Pa}$ . Find the number of moles of gas in the container.

**Solution** The quantities given are volume, pressure, and temperature:  $V = 100 \text{ cm}^3 = 1.00 \times 10^{-4} \text{ m}^3$ ,  $P = 100 \text{ Pa}$ , and  $T = 20^\circ\text{C} = 293 \text{ K}$ . Using Equation 19.8, we find that

$$n = \frac{PV}{RT} = \frac{(100 \text{ Pa})(10^{-4} \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 4.10 \times 10^{-6} \text{ mol}$$

**Exercise** How many molecules are in the container?

**Answer**  $2.47 \times 10^{18}$  molecules.

#### EXAMPLE 19.5 Filling a Scuba Tank

A certain scuba tank is designed to hold  $66 \text{ ft}^3$  of air when it is at atmospheric pressure at  $22^\circ\text{C}$ . When this volume of air is compressed to an absolute pressure of  $3000 \text{ lb/in.}^2$  and stored in a 10-L ( $0.35\text{-ft}^3$ ) tank, the air becomes so hot that the tank must be allowed to cool before it can be used. If the air does not cool, what is its temperature? (Assume that the air behaves like an ideal gas.)

**Solution** If no air escapes from the tank during filling, then the number of moles  $n$  remains constant; therefore, using  $PV = nRT$ , and with  $n$  and  $R$  being constant, we obtain for the initial and final values:

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

The initial pressure of the air is  $14.7 \text{ lb/in.}^2$ , its final pressure is  $3000 \text{ lb/in.}^2$ , and the air is compressed from an initial volume of  $66 \text{ ft}^3$  to a final volume of  $0.35 \text{ ft}^3$ . The initial temperature, converted to SI units, is  $295 \text{ K}$ . Solving for  $T_f$ , we obtain

$$T_f = \left( \frac{P_f V_f}{P_i V_i} \right) T_i = \frac{(3000 \text{ lb/in.}^2)(0.35 \text{ ft}^3)}{(14.7 \text{ lb/in.}^2)(66 \text{ ft}^3)} (295 \text{ K})$$

$$= 319 \text{ K}$$

**Exercise** What is the air temperature in degrees Celsius and in degrees Fahrenheit?

**Answer**  $45.9^\circ\text{C}$ ;  $115^\circ\text{F}$ .



**Quick Quiz 19.4**

In the previous example we used SI units for the temperature in our calculation step but not for the pressures or volumes. When working with the ideal gas law, how do you decide when it is necessary to use SI units and when it is not?

**EXAMPLE 19.6** Heating a Spray Can

A spray can containing a propellant gas at twice atmospheric pressure (202 kPa) and having a volume of 125 cm<sup>3</sup> is at 22°C. It is then tossed into an open fire. When the temperature of the gas in the can reaches 195°C, what is the pressure inside the can? Assume any change in the volume of the can is negligible.

**Solution** We employ the same approach we used in Example 19.5, starting with the expression

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

Because the initial and final volumes of the gas are assumed to be equal, this expression reduces to

$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$

Solving for  $P_f$  gives

$$P_f = \left(\frac{T_f}{T_i}\right)(P_i) = \left(\frac{468 \text{ K}}{295 \text{ K}}\right)(202 \text{ kPa}) = 320 \text{ kPa}$$

Obviously, the higher the temperature, the higher the pressure exerted by the trapped gas. Of course, if the pressure increases high enough, the can will explode. Because of this possibility, you should never dispose of spray cans in a fire.

**SUMMARY**

Two bodies are in **thermal equilibrium** with each other if they have the same temperature.

The **zeroth law of thermodynamics** states that if objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

The SI unit of absolute temperature is the **kelvin**, which is defined to be the fraction 1/273.16 of the temperature of the triple point of water.

When the temperature of an object is changed by an amount  $\Delta T$ , its length changes by an amount  $\Delta L$  that is proportional to  $\Delta T$  and to its initial length  $L_i$ :

$$\Delta L = \alpha L_i \Delta T \quad (19.4)$$

where the constant  $\alpha$  is the **average coefficient of linear expansion**. The **average volume expansion coefficient**  $\beta$  for a solid is approximately equal to  $3\alpha$ .

An **ideal gas** is one for which  $PV/nT$  is constant at all pressures. An ideal gas is described by the **equation of state**,

$$PV = nRT \quad (19.8)$$

where  $n$  equals the number of moles of the gas,  $V$  is its volume,  $R$  is the universal gas constant (8.315 J/mol·K), and  $T$  is the absolute temperature. A real gas behaves approximately as an ideal gas if it is far from liquefaction.