PUZZLER

Small "black boxes" like this one are commonly used to supply power to electronic devices such as CD players and tape players. Whereas these devices need only about 12 V to operate, wall outlets provide an output of 120 V. What do the black boxes do, and how do they work? *(George Semple)*

chapter

33

Alternating-Current Circuits

Chapter Outline

In this chapter we describe alternating-current (ac) circuits. Every time we turn on a television set, a stereo, or any of a multitude of other electrical appliances, we are calling on alternating currents to provide the power to operate them. We begin our study by investigating the characteristics of simple series circuits that contain resistors, inductors, and capacitors and that are driven by a sinusoidal voltage. We shall find that the maximum alternating current in each element is proportional to the maximum alternating voltage across the element. We shall also find that when the applied voltage is sinusoidal, the current in each element is sinusoidal, too, but not necessarily in phase with the applied voltage. We conclude the chapter with two sections concerning transformers, power transmission, and *RC* filters.

33.1 AC SOURCES AND PHASORS

An ac circuit consists of circuit elements and a generator that provides the alternating current. As you recall from Section 31.5, the basic principle of the ac generator is a direct consequence of Faraday's law of induction. When a conducting loop is rotated in a magnetic field at constant angular frequency ω, a sinusoidal voltage (emf) is induced in the loop. This instantaneous voltage Δv is

$$\Delta v = \Delta V_{\max} \sin \omega t$$

where $\Delta V_{\max}$ is the maximum output voltage of the ac generator, or the **voltage amplitude.** From Equation 13.6, the angular frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where f is the frequency of the generator (the voltage source) and T is the period. The generator determines the frequency of the current in any circuit connected to the generator. Because the output voltage of an ac generator varies sinusoidally with time, the voltage is positive during one half of the cycle and negative during the other half. Likewise, the current in any circuit driven by an ac generator is an alternating current that also varies sinusoidally with time. Commercial electric-power plants in the United States use a frequency of 60 Hz, which corresponds to an angular frequency of 377 rad/s.

The primary aim of this chapter can be summarized as follows: If an ac generator is connected to a series circuit containing resistors, inductors, and capacitors, we want to know the amplitude and time characteristics of the alternating current. To simplify our analysis of circuits containing two or more elements, we use graphical constructions called *phasor diagrams.* In these constructions, alternating (sinusoidal) quantities, such as current and voltage, are represented by rotating vectors called **phasors.** The length of the phasor represents the amplitude (maximum value) of the quantity, and the projection of the phasor onto the vertical axis represents the instantaneous value of the quantity. As we shall see, a phasor diagram greatly simplifies matters when we must combine several sinusoidally varying currents or voltages that have different phases.

33.2 RESISTORS IN AN AC CIRCUIT

Consider a simple ac circuit consisting of a resistor and an ac generator —(~)—, as shown in Figure 33.1. At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule). Therefore, $\Delta v - \Delta v_R = 0$, or[1]

$$\Delta v = \Delta v_R = \Delta V_{\max} \sin \omega t \qquad \textbf{(33.1)}$$

where Δv_R is the **instantaneous voltage across the resistor.** Therefore, the instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{\max}}{R} \sin \omega t = I_{\max} \sin \omega t \qquad \textbf{(33.2)}$$

where $I_{\max}$ is the maximum current:

Maximum current in a resistor

$$I_{\max} = \frac{\Delta V_{\max}}{R}$$

From Equations 33.1 and 33.2, we see that the instantaneous voltage across the resistor is

$$\Delta v_R = I_{\max} R \sin \omega t \qquad \textbf{(33.3)}$$

Let us discuss the current-versus-time curve shown in Figure 33.2a. At point *a*, the current has a maximum value in one direction, arbitrarily called the positive direction. Between points *a* and *b*, the current is decreasing in magnitude but is still in the positive direction. At *b*, the current is momentarily zero; it then begins to increase in the negative direction between points *b* and *c*. At *c*, the current has reached its maximum value in the negative direction.

The current and voltage are in step with each other because they vary identically with time. Because i_R and Δv_R both vary as $\sin \omega t$ and reach their maximum values at the same time, as shown in Figure 33.2a, they are said to be **in phase.** Thus we can say that, for a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.

The current in a resistor is in phase with the voltage

A *phasor diagram* is used to represent current–voltage phase relationships. The lengths of the arrows correspond to $\Delta V_{\max}$ and $I_{\max}$. The projections of the phasor arrows onto the vertical axis give Δv_R and i_R values. As we showed in Section 13.5, if the phasor arrow is imagined to rotate steadily with angular speed ω, its vertical-axis component oscillates sinusoidally in time. In the case of the single-loop resistive circuit of Figure 33.1, the current and voltage phasors lie along the same line, as in Figure 33.2b, because i_R and Δv_R are in phase.

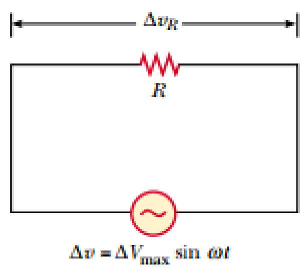


Figure 33.1 A circuit consisting of a resistor of resistance R connected to an ac generator, designated by the symbol

.

Note that **the average value of the current over one cycle is zero.** That is, the current is maintained in the positive direction for the same amount of time and at the same magnitude as it is maintained in the negative direction. However, the direction of the current has no effect on the behavior of the resistor. We can understand this by realizing that collisions between electrons and the fixed atoms of the resistor result in an increase in the temperature of the resistor. Although this temperature increase depends on the magnitude of the current, it is independent of the direction of the current.

We can make this discussion quantitative by recalling that the rate at which electrical energy is converted to internal energy in a resistor is the power $\mathcal{P} = i^2R$, where i is the instantaneous current in the resistor. Because this rate is proportional to the square of the current, it makes no difference whether the current is direct or alternating—that is, whether the sign associated with the current is positive or negative. However, the temperature increase produced by an alternating

[1] The lowercase symbols v and i are used to indicate the instantaneous values of the voltage and the current.

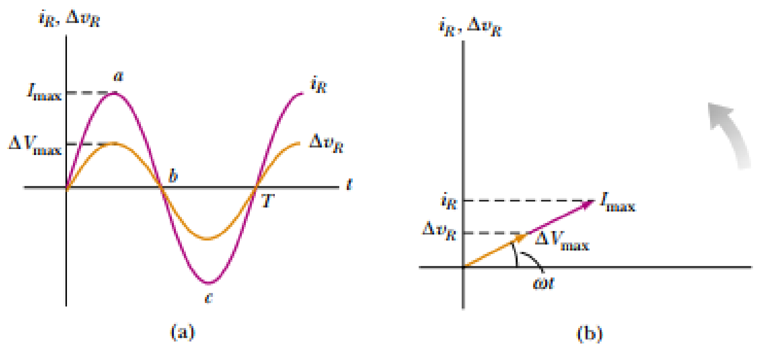


Figure 33.2 (a) Plots of the instantaneous current i_R and instantaneous voltage Δv_R across a resistor as functions of time. The current is in phase with the voltage, which means that the current is zero when the voltage is zero, maximum when the voltage is maximum, and minimum when the voltage is minimum. At time $t = T$, one cycle of the time-varying voltage and current has been completed. (b) Phasor diagram for the resistive circuit showing that the current is in phase with the voltage.

current having a maximum value I_{max} is not the same as that produced by a direct current equal to I_{max}. This is because the alternating current is at this maximum value for only an instant during each cycle (Fig. 33.3a). What is of importance in an ac circuit is an average value of current, referred to as the **rms current.** As we learned in Section 21.1, the notation *rms* stands for *root mean square,* which in this case means the square root of the mean (average) value of the square of the current: $I_{rms} = \sqrt{\overline{i^2}}$. Because i^2 varies as $\sin^2 \omega t$ and because the average value of i^2 is $\frac{1}{2} I_{max}^2$ (see Fig. 33.3b), the rms current is[2]

rms current

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \quad (33.4)$$

This equation states that an alternating current whose maximum value is 2.00 A delivers to a resistor the same power as a direct current that has a value of (0.707)(2.00 A) = 1.41 A. Thus, we can say that the average power delivered to a resistor that carries an alternating current is

Average power delivered to a resistor

$$\mathcal{P}_{av} = I_{rms}^2 R$$

[2] That the square root of the average value of i^2 is equal to $I_{max}/\sqrt{2}$ can be shown as follows: The current in the circuit varies with time according to the expression $i = I_{max} \sin \omega t$, so $i^2 = I_{max}^2 \sin^2 \omega t$. Therefore, we can find the average value of i^2 by calculating the average value of $\sin^2 \omega t$. A graph of $\cos^2 \omega t$ versus time is identical to a graph of $\sin^2 \omega t$ versus time, except that the points are shifted on the time axis. Thus, the time average of $\sin^2 \omega t$ is equal to the time average of $\cos^2 \omega t$ when taken over one or more complete cycles. That is,

$$(\sin^2 \omega t)_{av} = (\cos^2 \omega t)_{av}$$

Using this fact and the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$, we obtain

$$(\sin^2 \omega t)_{av} + (\cos^2 \omega t)_{av} = 2(\sin^2 \omega t)_{av} = 1$$

$$(\sin^2 \omega t)_{av} = \tfrac{1}{2}$$

When we substitute this result in the expression $i^2 = I_{max}^2 \sin^2 \omega t$, we obtain $(i^2)_{av} = \overline{i^2} = I_{rms}^2 = I_{max}^2/2$, or $I_{rms} = I_{max}/\sqrt{2}$. The factor $1/\sqrt{2}$ is valid only for sinusoidally varying currents. Other waveforms, such as sawtooth variations, have different factors.

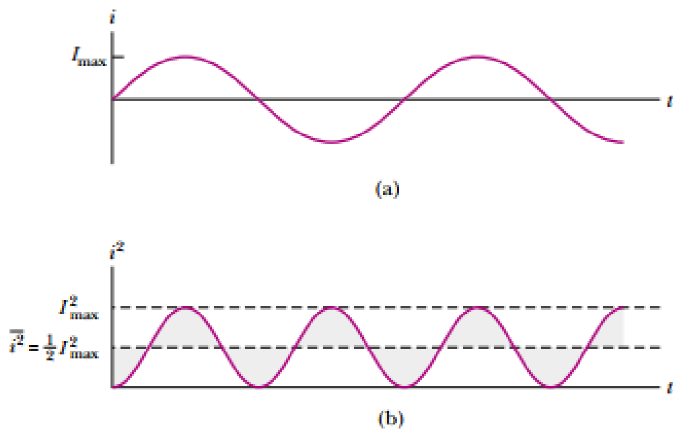


Figure 33.3 (a) Graph of the current in a resistor as a function of time. (b) Graph of the current squared in a resistor as a function of time. Notice that the gray shaded regions *under* the curve and *above* the dashed line for $I^2_{max}/2$ have the same area as the gray shaded regions *above* the curve and *below* the dashed line for $I^2_{max}/2$. Thus, the average value of i^2 is $I^2_{max}/2$.

Alternating voltage also is best discussed in terms of rms voltage, and the relationship is identical to that for current:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\,\Delta V_{max} \quad (33.5)$$

rms voltage

When we speak of measuring a 120-V alternating voltage from an electrical outlet, we are referring to an rms voltage of 120 V. A quick calculation using Equation 33.5 shows that such an alternating voltage has a maximum value of about 170 V. One reason we use rms values when discussing alternating currents and voltages in this chapter is that ac ammeters and voltmeters are designed to read rms values. Furthermore, with rms values, many of the equations we use have the same form as their direct-current counterparts.

Quick Quiz 33.1

Which of the following statements might be true for a resistor connected to an ac generator? (a) $\mathcal{P}_{av} = 0$ and $i_{av} = 0$; (b) $\mathcal{P}_{av} = 0$ and $i_{av} > 0$; (c) $\mathcal{P}_{av} > 0$ and $i_{av} = 0$; (d) $\mathcal{P}_{av} > 0$ and $i_{av} > 0$.

Example 33.1 What Is the rms Current?

The voltage output of a generator is given by $\Delta v = (200\text{ V})\sin \omega t$. Find the rms current in the circuit when this generator is connected to a 100-Ω resistor.

Solution Comparing this expression for voltage output with the general form $\Delta v = \Delta V_{max} \sin \omega t$, we see that $\Delta V_{max} = 200$ V. Thus, the rms voltage is

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{200\text{ V}}{\sqrt{2}} = 141\text{ V}$$

Therefore,

$$I_{rms} = \frac{\Delta V_{rms}}{R} = \frac{141\text{ V}}{100\ \Omega} = 1.41\text{ A}$$

Exercise Find the maximum current in the circuit.

Answer 2.00 A.

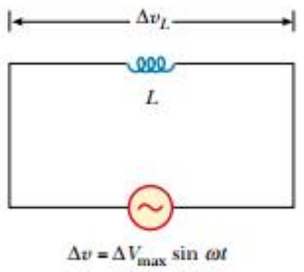


Figure 33.4 A circuit consisting of an inductor of inductance L connected to an ac generator.

33.3 INDUCTORS IN AN AC CIRCUIT

Now consider an ac circuit consisting only of an inductor connected to the terminals of an ac generator, as shown in Figure 33.4. If $\Delta v_L = \mathcal{E}_L = -L(di/dt)$ is the self-induced instantaneous voltage across the inductor (see Eq. 32.1), then Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_L = 0$, or

$$\Delta v - L\frac{di}{dt} = 0$$

When we substitute $\Delta V_{\max} \sin \omega t$ for Δv and rearrange, we obtain

$$L\frac{di}{dt} = \Delta V_{\max} \sin \omega t \qquad \textbf{(33.6)}$$

Solving this equation for di, we find that

$$di = \frac{\Delta V_{\max}}{L} \sin \omega t \, dt$$

Integrating this expression[3] gives the instantaneous current in the inductor as a function of time:

$$i_L = \frac{\Delta V_{\max}}{L} \int \sin \omega t \, dt = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t \qquad \textbf{(33.7)}$$

When we use the trigonometric identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we can express Equation 33.7 as

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \textbf{(33.8)}$$

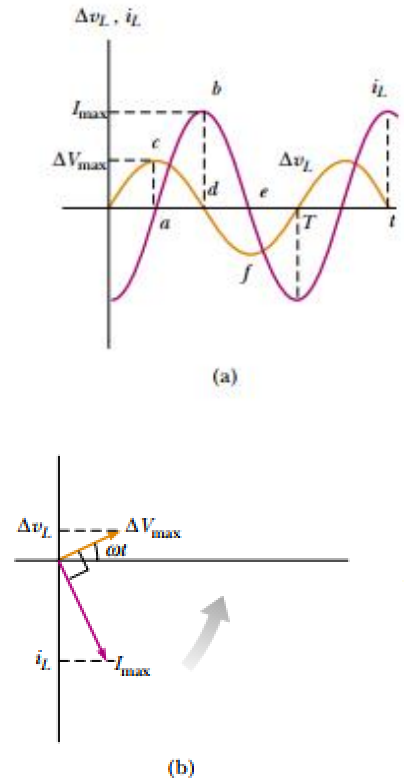


Figure 33.5 (a) Plots of the instantaneous current i_L and instantaneous voltage Δv_L across an inductor as functions of time. The current lags behind the voltage by 90°. (b) Phasor diagram for the inductive circuit, showing that the current lags behind the voltage by 90°.

Comparing this result with Equation 33.6, we see that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $(\pi/2)$ rad = 90°.

In general, inductors in an ac circuit produce a current that is out of phase with the ac voltage. A plot of voltage and current versus time is provided in Figure 33.5a. At point *a*, the current begins to increase in the positive direction. At this instant the rate of change of current is at a maximum, and thus the voltage across the inductor is also at a maximum. As the current increases between points *a* and *b*, di/dt (the slope of the current curve) gradually decreases until it reaches zero at point *b*. As a result, the voltage across the inductor is decreasing during this same time interval, as the curve segment between *c* and *d* indicates. Immediately after point *b*, the current begins to decrease, although it still has the same direction it had during the previous quarter cycle (from *a* to *b*). As the current decreases to zero (from *b* to *e*), a voltage is again induced in the inductor (*d* to *f*), but the polarity of this voltage is opposite that of the voltage induced between *c* and *d* (because back emfs are always directed to oppose the change in the current). Note that the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value. Thus, we see that

The current in an inductor lags the voltage by 90°

for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° (one-quarter cycle in time).

[3] We neglect the constant of integration here because it depends on the initial conditions, which are not important for this situation.

The phasor diagram for the inductive circuit of Figure 33.4 is shown in Figure 33.5b.

From Equation 33.7 we see that the current in an inductive circuit reaches its maximum value when $\cos \omega t = -1$:

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L} = \frac{\Delta V_{\max}}{X_L} \quad (33.9)$$

Maximum current in an inductor

where the quantity X_L, called the **inductive reactance,** is

$$X_L = \omega L \quad (33.10)$$

Inductive reactance

Equation 33.9 indicates that, for a given applied voltage, the maximum current decreases as the inductive reactance increases. The expression for the rms current in an inductor is similar to Equation 33.9, with $I_{\max}$ replaced by I_{rms} and $\Delta V_{\max}$ replaced by ΔV_{rms}.

Inductive reactance, like resistance, has units of ohms. However, unlike resistance, reactance depends on frequency as well as on the characteristics of the inductor. Note that the reactance of an inductor in an ac circuit increases as the frequency of the current increases. This is because at higher frequencies, the instantaneous current must change more rapidly than it does at the lower frequencies; this causes an increase in the maximum induced emf associated with a given maximum current.

Using Equations 33.6 and 33.9, we find that the instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di}{dt} = -\Delta V_{\max} \sin \omega t = -I_{\max} X_L \sin \omega t \quad (33.11)$$

Conceptual Example 33.2

Figure 33.6 shows a circuit consisting of a series combination of an alternating voltage source, a switch, an inductor, and a lightbulb. The switch is thrown closed, and the circuit is allowed to come to equilibrium so that the lightbulb glows steadily. An iron rod is then inserted into the interior of the inductor. What happens to the brightness of the lightbulb, and why?

Solution The bulb gets dimmer. As the rod is inserted, the inductance increases because the magnetic field inside the inductor increases. According to Equation 33.10, this increase in L means that the inductive reactance of the inductor also increases. The voltage across the inductor increases while the voltage across the lightbulb decreases. With less voltage across it, the lightbulb glows more dimly. In theatrical productions of the early 20th century, this method was used to dim the lights in the theater gradually.

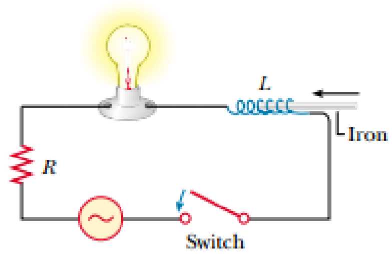


Figure 33.6

Example 33.3 A Purely Inductive ac Circuit

In a purely inductive ac circuit (see Fig. 33.4), $L = 25.0$ mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

Solution Equation 33.10 gives

$$X_L = \omega L = 2\pi f L = 2\pi(60.0 \text{ Hz})(25.0 \times 10^{-3} \text{ H}) = 9.42\ \Omega$$

From a modified version of Equation 33.9, the rms current is

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{150 \text{ V}}{9.42\ \Omega} = 15.9 \text{ A}$$

Exercise Calculate the inductive reactance and rms current in the circuit if the frequency is 6.00 kHz.

Answer 942 Ω, 0.159 A.

Exercise Show that inductive reactance has SI units of ohms.

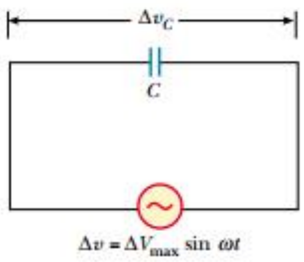


Figure 33.7 A circuit consisting of a capacitor of capacitance C connected to an ac generator.

33.4 CAPACITORS IN AN AC CIRCUIT

Figure 33.7 shows an ac circuit consisting of a capacitor connected across the terminals of an ac generator. Kirchhoff's loop rule applied to this circuit gives $\Delta v - \Delta v_C = 0$, or

$$\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t \tag{33.12}$$

where Δv_C is the instantaneous voltage across the capacitor. We know from the definition of capacitance that $C = q/\Delta v_C$; hence, Equation 33.12 gives

$$q = C\,\Delta V_{max} \sin \omega t \tag{33.13}$$

where q is the instantaneous charge on the capacitor. Because $i = dq/dt$, differentiating Equation 33.13 gives the instantaneous current in the circuit:

$$i_C = \frac{dq}{dt} = \omega C\,\Delta V_{max} \cos \omega t \tag{33.14}$$

Using the trigonometric identity

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

we can express Equation 33.14 in the alternative form

$$i_C = \omega C\,\Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right) \tag{33.15}$$

Comparing this expression with Equation 33.12, we see that the current is $\pi/2$ rad = 90° out of phase with the voltage across the capacitor. A plot of current and voltage versus time (Fig. 33.8a) shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

Looking more closely, we see that the segment of the current curve from a to b indicates that the current starts out at a relatively high value. We can understand this by recognizing that there is no charge on the capacitor at $t = 0$; as a consequence, nothing in the circuit except the resistance of the wires can hinder the flow of charge at this instant. However, the current decreases as the voltage across the capacitor increases (from c to d on the voltage curve), and the capacitor is charging. When the voltage is at point d, the current reverses and begins to increase in the opposite direction (from b to e on the current curve). During this time, the voltage across the capacitor decreases from d to f because the plates are now losing the charge they accumulated earlier. During the second half of the cycle, the current is initially at its maximum value in the opposite direction (point e) and then decreases as the voltage across the capacitor builds up. The phasor diagram in Figure 33.8b also shows that

> for a sinusoidally applied voltage, the current in a capacitor always leads the voltage across the capacitor by 90°.

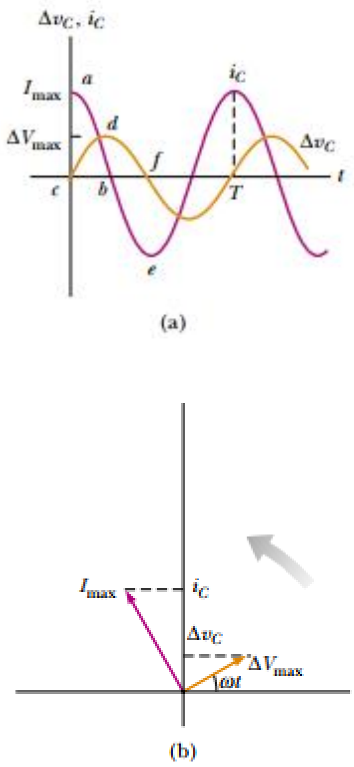


Figure 33.8 (a) Plots of the instantaneous current i_C and instantaneous voltage Δv_C across a capacitor as functions of time. The voltage lags behind the current by 90°. (b) Phasor diagram for the capacitive circuit, showing that the current leads the voltage by 90°.

From Equation 33.14, we see that the current in the circuit reaches its maximum value when $\cos \omega t = 1$:

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{X_C} \tag{33.16}$$

where X_C is called the **capacitive reactance:**

$$X_C = \frac{1}{\omega C} \tag{33.17}$$

Capacitive reactance

Note that capacitive reactance also has units of ohms.

The rms current is given by an expression similar to Equation 33.16, with $I_{\max}$ replaced by I_{rms} and $\Delta V_{\max}$ replaced by ΔV_{rms}.

Combining Equations 33.12 and 33.16, we can express the instantaneous voltage across the capacitor as

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t \tag{33.18}$$

Equations 33.16 and 33.17 indicate that as the frequency of the voltage source increases, the capacitive reactance decreases and therefore the maximum current increases. Again, note that the frequency of the current is determined by the frequency of the voltage source driving the circuit. As the frequency approaches zero, the capacitive reactance approaches infinity, and hence the current approaches zero. This makes sense because the circuit approaches direct-current conditions as ω approaches 0.

EXAMPLE 33.4 A Purely Capacitive ac Circuit

An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz ac generator whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Solution Using Equation 33.17 and the fact that $\omega = 2\pi f = 377 \text{ s}^{-1}$ gives

$$X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ s}^{-1})(8.00 \times 10^{-6} \text{ F})} = \boxed{332\ \Omega}$$

Hence, from a modified Equation 33.16, the rms current is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150 \text{ V}}{332\ \Omega} = \boxed{0.452 \text{ A}}$$

Exercise If the frequency is doubled, what happens to the capacitive reactance and the current?

Answer X_C is halved, and $I_{\max}$ is doubled.

33.5 THE *RLC* SERIES CIRCUIT

13.7 Figure 33.9a shows a circuit that contains a resistor, an inductor, and a capacitor connected in series across an alternating-voltage source. As before, we assume that the applied voltage varies sinusoidally with time. It is convenient to assume that the instantaneous applied voltage is given by

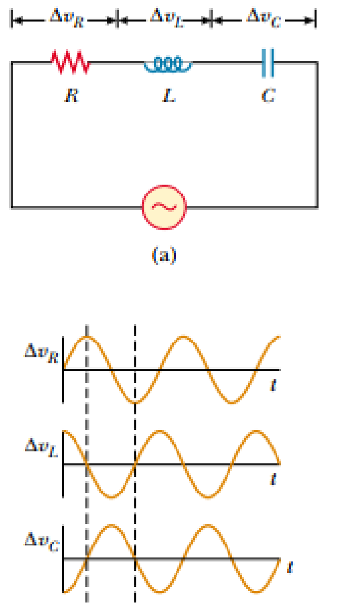
$$\Delta v = \Delta V_{\max} \sin \omega t$$

while the current varies as

$$i = I_{\max} \sin(\omega t - \phi)$$

where ϕ is the **phase angle** between the current and the applied voltage. Our aim

Phase angle ϕ



(b)

Figure 33.9 (a) A series circuit consisting of a resistor, an inductor, and a capacitor connected to an ac generator. (b) Phase relationships for instantaneous voltages in the series *RLC* circuit.

is to determine ϕ and $I_{\max}$. Figure 33.9b shows the voltage versus time across each element in the circuit and their phase relationships.

To solve this problem, we must analyze the phasor diagram for this circuit. First, we note that because the elements are in series, the current everywhere in the circuit must be the same at any instant. That is, **the current at all points in a series ac circuit has the same amplitude and phase.** Therefore, as we found in the preceding sections, the voltage across each element has a different amplitude and phase, as summarized in Figure 33.10. In particular, the voltage across the resistor is in phase with the current, the voltage across the inductor leads the current by 90°, and the voltage across the capacitor lags behind the current by 90°. Using these phase relationships, we can express the instantaneous voltages across the three elements as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t \tag{33.19}$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t \tag{33.20}$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t \tag{33.21}$$

where ΔV_R, ΔV_L, and ΔV_C are the maximum voltage values across the elements:

$$\Delta V_R = I_{\max} R \qquad \Delta V_L = I_{\max} X_L \qquad \Delta V_C = I_{\max} X_C$$

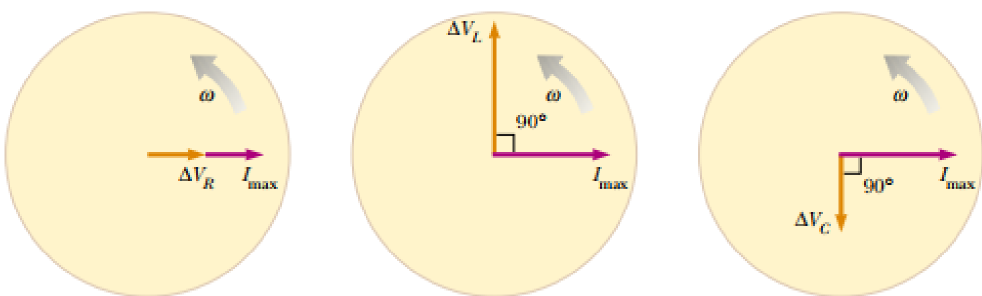
At this point, we could proceed by noting that the instantaneous voltage Δv across the three elements equals the sum

$$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

Quick Quiz 33.2

For the circuit of Figure 33.9a, is the voltage of the ac source equal to (a) the sum of the maximum voltages across the elements, (b) the sum of the instantaneous voltages across the elements, or (c) the sum of the rms voltages across the elements?

Although this analytical approach is correct, it is simpler to obtain the sum by examining the phasor diagram. Because the current at any instant is the same in all



(a) Resistor (b) Inductor (c) Capacitor

Figure 33.10 Phase relationships between the voltage and current phasors for (a) a resistor, (b) an inductor, and (c) a capacitor connected in series.

elements, we can obtain a phasor diagram for the circuit. We combine the three phasor pairs shown in Figure 33.10 to obtain Figure 33.11a, in which a single phasor $I_{\max}$ is used to represent the current in each element. To obtain the vector sum of the three voltage phasors in Figure 33.11a, we redraw the phasor diagram as in Figure 33.11b. From this diagram, we see that the vector sum of the voltage amplitudes ΔV_R, ΔV_L, and ΔV_C equals a phasor whose length is the maximum applied voltage $\Delta V_{\max}$, where the phasor $\Delta V_{\max}$ makes an angle ϕ with the current phasor $I_{\max}$. Note that the voltage phasors ΔV_L and ΔV_C are in opposite directions along the same line, and hence we can construct the difference phasor $\Delta V_L - \Delta V_C$, which is perpendicular to the phasor ΔV_R. From either one of the right triangles in Figure 33.11b, we see that

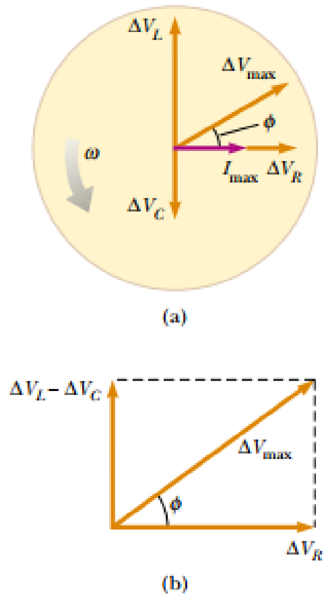


Figure 33.11 (a) Phasor diagram for the series *RLC* circuit shown in Figure 33.9a. The phasor ΔV_R is in phase with the current phasor $I_{\max}$, the phasor ΔV_L leads $I_{\max}$ by 90°, and the phasor ΔV_C lags $I_{\max}$ by 90°. The total voltage $\Delta V_{\max}$ makes an angle ϕ with $I_{\max}$. (b) Simplified version of the phasor diagram shown in (a).

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{\max}R)^2 + (I_{\max}X_L - I_{\max}X_C)^2}$$

$$\Delta V_{\max} = I_{\max}\sqrt{R^2 + (X_L - X_C)^2} \tag{33.22}$$

Therefore, we can express the maximum current as

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The **impedance** Z of the circuit is defined as

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \tag{33.23}$$

where impedance also has units of ohms. Therefore, we can write Equation 33.22 in the form

$$\Delta V_{\max} = I_{\max}Z \tag{33.24}$$

We can regard Equation 33.24 as the ac equivalent of Equation 27.8, which defined *resistance* in a dc circuit as the ratio of the voltage across a conductor to the current in that conductor. Note that the impedance and therefore the current in an ac circuit depend upon the resistance, the inductance, the capacitance, and the frequency (because the reactances are frequency-dependent).

By removing the common factor $I_{\max}$ from each phasor in Figure 33.11a, we can construct the *impedance triangle* shown in Figure 33.12. From this phasor diagram we find that the phase angle ϕ between the current and the voltage is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \tag{33.25}$$

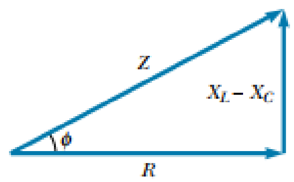
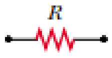
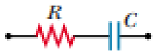
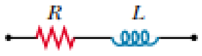
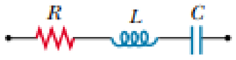


Figure 33.12 An impedance triangle for a series *RLC* circuit gives the relationship $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Also, from Figure 33.12, we see that $\cos\phi = R/Z$. When $X_L > X_C$ (which occurs at high frequencies), the phase angle is positive, signifying that the current lags behind the applied voltage, as in Figure 33.11a. When $X_L < X_C$, the phase angle is negative, signifying that the current leads the applied voltage. When $X_L = X_C$, the phase angle is zero. In this case, the impedance equals the resistance and the current has its maximum value, given by $\Delta V_{\max}/R$. The frequency at which this occurs is called the *resonance frequency;* it is described further in Section 33.7.

Table 33.1 gives impedance values and phase angles for various series circuits containing different combinations of elements.

TABLE 33.1 Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	0°
	X_C	−90°
	X_L	+90°
	$\sqrt{R^2 + X_C^2}$	Negative, between −90° and 0°
	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an ac voltage (not shown) is applied across the elements.

Quick Quiz 33.3

Label each part of Figure 33.13 as being $X_L > X_C$, $X_L = X_C$, or $X_L < X_C$.

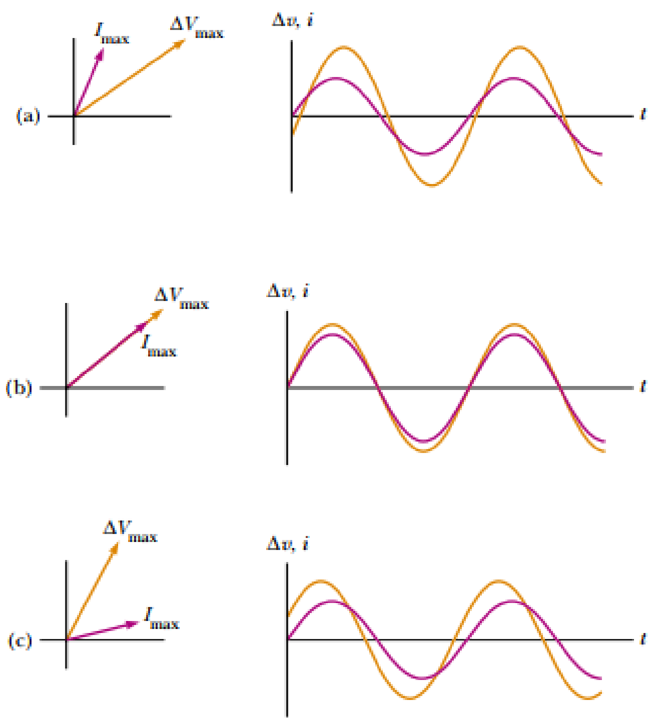


Figure 33.13

EXAMPLE 33.5 Finding *L* from a Phasor Diagram

In a series *RLC* circuit, the applied voltage has a maximum value of 120 V and oscillates at a frequency of 60.0 Hz. The circuit contains an inductor whose inductance can be varied, a 200-Ω resistor, and a 4.00-μF capacitor. What value of *L* should an engineer analyzing the circuit choose such that the voltage across the capacitor lags the applied voltage by 30.0°?

Solution The phase relationships for the drops in voltage across the elements are shown in Figure 33.14. From the figure we see that the phase angle is $\phi = -60.0°$. This is because the phasors representing I_{max} and ΔV_R are in the same direction (they are in phase). From Equation 33.25, we find that

$$X_L = X_C + R \tan \phi$$

Substituting Equations 33.10 and 33.17 (with $\omega = 2\pi f$) into this expression gives

$$2\pi f L = \frac{1}{2\pi f C} + R \tan \phi$$

$$L = \frac{1}{2\pi f}\left[\frac{1}{2\pi f C} + R \tan \phi\right]$$

Substituting the given values into the equation gives $L =$ $\boxed{0.84 \text{ H.}}$

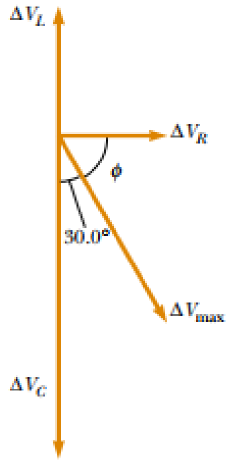


Figure 33.14

EXAMPLE 33.6 Analyzing a Series *RLC* Circuit

A series *RLC* ac circuit has $R = 425\ \Omega$, $L = 1.25$ H, $C = 3.50\ \mu$F, $\omega = 377\ \text{s}^{-1}$, and $\Delta V_{max} = 150$ V. (a) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

Solution The reactances are $X_L = \omega L = \boxed{471\ \Omega}$ and $X_C = 1/\omega C = \boxed{758\ \Omega.}$ The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(425\ \Omega)^2 + (471\ \Omega - 758\ \Omega)^2} = \boxed{513\ \Omega}$$

(b) Find the maximum current in the circuit.

Solution

$$I_{max} = \frac{V_{max}}{Z} = \frac{150 \text{ V}}{513\ \Omega} = \boxed{0.292 \text{ A}}$$

(c) Find the phase angle between the current and voltage.

Solution

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\ \Omega - 758\ \Omega}{425\ \Omega}\right)$$

$$= \boxed{-34.0°}$$

Because the circuit is more capacitive than inductive, ϕ is negative and the current leads the applied voltage.

(d) Find both the maximum voltage and the instantaneous voltage across each element.

Solution The maximum voltages are

$$\Delta V_R = I_{max} R = (0.292 \text{ A})(425\ \Omega) = \boxed{124 \text{ V}}$$

$$\Delta V_L = I_{max} X_L = (0.292 \text{ A})(471\ \Omega) = \boxed{138 \text{ V}}$$

$$\Delta V_C = I_{max} X_C = (0.292 \text{ A})(758\ \Omega) = \boxed{221 \text{ V}}$$

Using Equations 33.19, 33.20, and 33.21, we find that we can write the instantaneous voltages across the three elements as

$$\Delta v_R = \boxed{(124 \text{ V}) \sin 377t}$$

$$\Delta v_L = \boxed{(138 \text{ V}) \cos 377t}$$

$$\Delta v_C = \boxed{(-221 \text{ V}) \cos 377t}$$

Comments The sum of the maximum voltages across the elements is $\Delta V_R + \Delta V_L + \Delta V_C = 483$ V. Note that this sum is much greater than the maximum voltage of the generator, 150 V. As we saw in Quick Quiz 33.2, the sum of the maximum voltages is a meaningless quantity because when sinusoidally varying quantities are added, *both their amplitudes and their phases* must be taken into account. We know that the maximum voltages across the various elements occur at different times. That is, the voltages must be added in a way that takes account of the different phases. When this is done, Equation 33.22 is satisfied. You should verify this result.

Exercise Construct a phasor diagram to scale, showing the voltages across the elements and the applied voltage. From your diagram, verify that the phase angle is $-34.0°$.

33.6 POWER IN AN AC CIRCUIT

No power losses are associated with pure capacitors and pure inductors in an ac circuit. To see why this is true, let us first analyze the power in an ac circuit containing only a generator and a capacitor.

When the current begins to increase in one direction in an ac circuit, charge begins to accumulate on the capacitor, and a voltage drop appears across it. When this voltage drop reaches its maximum value, the energy stored in the capacitor is $\frac{1}{2}C(\Delta V_{\text{max}})^2$. However, this energy storage is only momentary. The capacitor is charged and discharged twice during each cycle: Charge is delivered to the capacitor during two quarters of the cycle and is returned to the voltage source during the remaining two quarters. Therefore, **the average power supplied by the source is zero.** In other words, **no power losses occur in a capacitor in an ac circuit.**

Similarly, the voltage source must do work against the back emf of the inductor. When the current reaches its maximum value, the energy stored in the inductor is a maximum and is given by $\frac{1}{2}LI_{\text{max}}^2$. When the current begins to decrease in the circuit, this stored energy is returned to the source as the inductor attempts to maintain the current in the circuit.

In Example 28.1 we found that the power delivered by a battery to a dc circuit is equal to the product of the current and the emf of the battery. Likewise, the instantaneous power delivered by an ac generator to a circuit is the product of the generator current and the applied voltage. For the *RLC* circuit shown in Figure 33.9a, we can express the instantaneous power $\mathcal{P}$ as

$$\begin{aligned}\mathcal{P} &= i\,\Delta v = I_{\text{max}} \sin(\omega t - \phi)\,\Delta V_{\text{max}} \sin \omega t \\ &= I_{\text{max}}\,\Delta V_{\text{max}} \sin \omega t \sin(\omega t - \phi)\end{aligned} \tag{33.26}$$

Clearly, this result is a complicated function of time and therefore is not very useful from a practical viewpoint. What is generally of interest is the average power over one or more cycles. Such an average can be computed by first using the trigonometric identity $\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$. Substituting this into Equation 33.26 gives

$$\mathcal{P} = I_{\text{max}}\,\Delta V_{\text{max}} \sin^2 \omega t \cos \phi - I_{\text{max}}\,\Delta V_{\text{max}} \sin \omega t \cos \omega t \sin \phi \tag{33.27}$$

We now take the time average of $\mathcal{P}$ over one or more cycles, noting that I_{max}, ΔV_{max}, ϕ, and ω are all constants. The time average of the first term on the right in Equation 33.27 involves the average value of $\sin^2 \omega t$, which is $\frac{1}{2}$ (as shown in footnote 2). The time average of the second term on the right is identically zero because $\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t$, and the average value of $\sin 2\omega t$ is zero. Therefore, we can express the **average power** $\mathcal{P}_{\text{av}}$ as

$$\mathcal{P}_{\text{av}} = \tfrac{1}{2} I_{\text{max}}\,\Delta V_{\text{max}} \cos \phi \tag{33.28}$$

It is convenient to express the average power in terms of the rms current and rms voltage defined by Equations 33.4 and 33.5:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos \phi \tag{33.29}$$

where the quantity $\cos\phi$ is called the **power factor.** By inspecting Figure 33.11b, we see that the maximum voltage drop across the resistor is given by $\Delta V_R = \Delta V_{max}\cos\phi = I_{max}R$. Using Equation 33.5 and the fact that $\cos\phi = I_{max}R/\Delta V_{max}$, we find that we can express $\mathcal{P}_{av}$ as

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}\cos\phi = I_{rms}\left(\frac{\Delta V_{max}}{\sqrt{2}}\right)\frac{I_{max}R}{\Delta V_{max}} = I_{rms}\frac{I_{max}R}{\sqrt{2}}$$

After making the substitution $I_{max} = \sqrt{2}I_{rms}$ from Equation 33.4, we have

$$\mathcal{P}_{av} = I_{rms}^2 R \tag{33.30}$$

Average power delivered to an *RLC* circuit

In words, the **average power delivered by the generator is converted to internal energy in the resistor,** just as in the case of a dc circuit. **No power loss occurs in an ideal inductor or capacitor.** When the load is purely resistive, then $\phi = 0$, $\cos\phi = 1$, and from Equation 33.29 we see that

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}$$

Equation 33.29 shows that the power delivered by an ac source to any circuit depends on the phase, and this result has many interesting applications. For example, a factory that uses large motors in machines, generators, or transformers has a large inductive load (because of all the windings). To deliver greater power to such devices in the factory without using excessively high voltages, technicians introduce capacitance in the circuits to shift the phase.

EXAMPLE 33.7 Average Power in an *RLC* Series Circuit

Calculate the average power delivered to the series *RLC* circuit described in Example 33.6.

Solution First, let us calculate the rms voltage and rms current, using the values of ΔV_{max} and I_{max} from Example 33.6:

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = \frac{150\text{ V}}{\sqrt{2}} = 106\text{ V}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{0.292\text{ A}}{\sqrt{2}} = 0.206\text{ A}$$

Because $\phi = -34.0°$, the power factor, $\cos\phi$, is 0.829; hence, the average power delivered is

$$\mathcal{P}_{av} = I_{rms}\,\Delta V_{rms}\cos\phi = (0.206\text{ A})(106\text{ V})(0.829)$$

$$= \boxed{18.1\text{ W}}$$

We can obtain the same result using Equation 33.30.

33.7 RESONANCE IN A SERIES *RLC* CIRCUIT

13.7 A series *RLC* circuit is said to be **in resonance** when the current has its maximum value. In general, the rms current can be written

$$I_{rms} = \frac{\Delta V_{rms}}{Z} \tag{33.31}$$

where Z is the impedance. Substituting the expression for Z from Equation 33.23 into 33.31 gives

$$I_{rms} = \frac{\Delta V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}} \tag{33.32}$$

Because the impedance depends on the frequency of the source, the current in the *RLC* circuit also depends on the frequency. The frequency ω_0 at which $X_L - X_C = 0$ is called the **resonance frequency** of the circuit. To find ω_0, we use the condition $X_L = X_C$, from which we obtain $\omega_0 L = 1/\omega_0 C$, or

Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{33.33}$$

Note that this frequency also corresponds to the natural frequency of oscillation of an *LC* circuit (see Section 32.5). Therefore, the current in a series *RLC* circuit reaches its maximum value when the frequency of the applied voltage matches the natural oscillator frequency—which depends only on *L* and *C*. Furthermore, at this frequency the current is in phase with the applied voltage.

Quick Quiz 33.4

What is the impedance of a series *RLC* circuit at resonance? What is the current in the circuit at resonance?

A plot of rms current versus frequency for a series *RLC* circuit is shown in Figure 33.15a. The data assume a constant $\Delta V_{\text{rms}} = 5.0$ mV, that $L = 5.0\ \mu$H, and that $C = 2.0$ nF. The three curves correspond to three values of *R*. Note that in each case the current reaches its maximum value at the resonance frequency ω_0. Furthermore, the curves become narrower and taller as the resistance decreases.

By inspecting Equation 33.32, we must conclude that, when $R = 0$, the current becomes infinite at resonance. Although the equation predicts this, real circuits always have some resistance, which limits the value of the current.

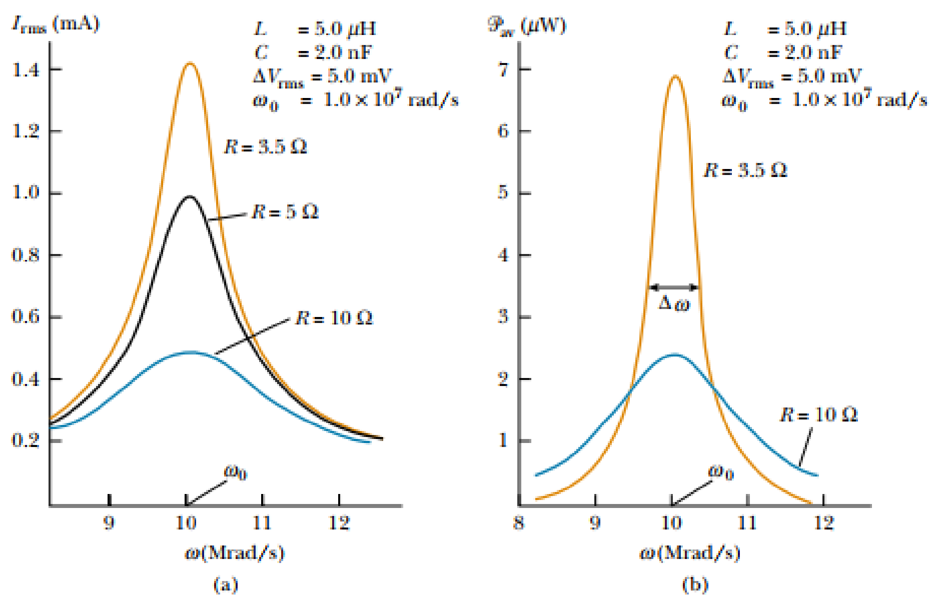


Figure 33.15 (a) The rms current versus frequency for a series *RLC* circuit, for three values of *R*. The current reaches its maximum value at the resonance frequency ω_0. (b) Average power versus frequency for the series *RLC* circuit, for two values of *R*.

It is also interesting to calculate the average power as a function of frequency for a series *RLC* circuit. Using Equations 33.30, 33.31, and 33.23, we find that

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + (X_L - X_C)^2} \tag{33.34}$$

Because $X_L = \omega L$, $X_C = 1/\omega C$, and $\omega_0{}^2 = 1/LC$, we can express the term $(X_L - X_C)^2$ as

$$(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0{}^2)^2$$

Using this result in Equation 33.34 gives

Average power as a function of frequency in an *RLC* circuit

$$\mathcal{P}_{\text{av}} = \frac{(\Delta V_{\text{rms}})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0{}^2)^2} \tag{33.35}$$

This expression shows that at resonance, when $\omega = \omega_0$, **the average power is a maximum** and has the value $(\Delta V_{\text{rms}})^2/R$. Figure 33.15b is a plot of average power versus frequency for two values of *R* in a series *RLC* circuit. As the resistance is made smaller, the curve becomes sharper in the vicinity of the resonance frequency. This curve sharpness is usually described by a dimensionless parameter known as the **quality factor,** denoted by *Q*:[4]

Quality factor

$$Q = \frac{\omega_0}{\Delta\omega}$$

where $\Delta\omega$ is the width of the curve measured between the two values of ω for which $\mathcal{P}_{\text{av}}$ has half its maximum value, called the *half-power points* (see Fig. 33.15b.) It is left as a problem (Problem 70) to show that the width at the half-power points has the value $\Delta\omega = R/L$, so

$$Q = \frac{\omega_0 L}{R} \tag{33.36}$$

QuickLab

Tune a radio to your favorite station. Can you determine what the product of *LC* must be for the radio's tuning circuitry?

The curves plotted in Figure 33.16 show that a high-*Q* circuit responds to only a very narrow range of frequencies, whereas a low-*Q* circuit can detect a much broader range of frequencies. Typical values of *Q* in electronic circuits range from 10 to 100.

The receiving circuit of a radio is an important application of a resonant circuit. One tunes the radio to a particular station (which transmits a specific electromagnetic wave or signal) by varying a capacitor, which changes the resonant frequency of the receiving circuit. When the resonance frequency of the circuit matches that of the incoming electromagnetic wave, the current in the receiving circuit increases. This signal caused by the incoming wave is then amplified and fed to a speaker. Because many signals are often present over a range of frequencies, it is important to design a high-*Q* circuit to eliminate unwanted signals. In this manner, stations whose frequencies are near but not equal to the resonance frequency give signals at the receiver that are negligibly small relative to the signal that matches the resonance frequency.

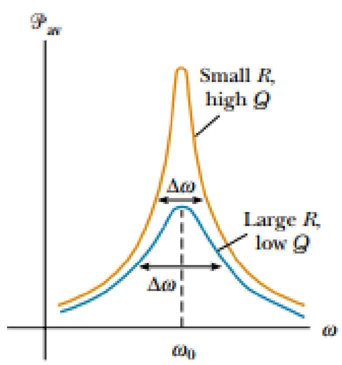


Figure 33.16 Average power versus frequency for a series *RLC* circuit. The width $\Delta\omega$ of each curve is measured between the two points where the power is half its maximum value. The power is a maximum at the resonance frequency ω_0.

[4] The quality factor is also defined as the ratio $2\pi E/\Delta E$, where *E* is the energy stored in the oscillating system and ΔE is the energy lost per cycle of oscillation. The quality factor for a mechanical system can also be defined, as noted in Section 13.7.

Quick Quiz 33.5

An airport metal detector (Fig. 33.17) is essentially a resonant circuit. The portal you step through is an inductor (a large loop of conducting wire) that is part of the circuit. The frequency of the circuit is tuned to the resonant frequency of the circuit when there is no metal in the inductor. Any metal on your body increases the effective inductance of the loop and changes the current in it. If you want the detector to be able to detect a small metallic object, should the circuit have a high quality factor or a low one?

Figure 33.17 When you pass through a metal detector, you become part of a resonant circuit. As you step through the detector, the inductance of the circuit changes, and thus the current in the circuit changes. *(Terry Qing/FPG International)*

EXAMPLE 33.8 A Resonating Series *RLC* Circuit

Consider a series *RLC* circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $\Delta V_{\text{rms}} = 20.0$ V, and $\omega = 5\,000\ \text{s}^{-1}$. Determine the value of the capacitance for which the current is a maximum.

Solution The current has its maximum value at the resonance frequency ω_0, which should be made to match the "driving" frequency of $5\,000\ \text{s}^{-1}$:

$$\omega_0 = 5.00 \times 10^3\ \text{s}^{-1} = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{\omega_0{}^2 L} = \frac{1}{(25.0 \times 10^6\ \text{s}^{-2})(20.0 \times 10^{-3}\ \text{H})}$$

$$= \boxed{2.00\ \mu\text{F}}$$

Exercise Calculate the maximum value of the rms current in the circuit as the frequency is varied.

Answer 0.133 A.

33.8 THE TRANSFORMER AND POWER TRANSMISSION

When electric power is transmitted over great distances, it is economical to use a high voltage and a low current to minimize the I^2R loss in the transmission lines.

Consequently, 350-kV lines are common, and in many areas even higher-voltage (765-kV) lines are under construction. At the receiving end of such lines, the consumer requires power at a low voltage (for safety and for efficiency in design). Therefore, a device is required that can change the alternating voltage and current without causing appreciable changes in the power delivered. The ac transformer is that device.

In its simplest form, the **ac transformer** consists of two coils of wire wound around a core of iron, as illustrated in Figure 33.18. The coil on the left, which is connected to the input alternating voltage source and has N_1 turns, is called the *primary winding* (or the *primary*). The coil on the right, consisting of N_2 turns and connected to a load resistor R, is called the *secondary winding* (or the *secondary*). The purpose of the iron core is to increase the magnetic flux through the coil and to provide a medium in which nearly all the flux through one coil passes through the other coil. Eddy current losses are reduced by using a laminated core. Iron is used as the core material because it is a soft ferromagnetic substance and hence reduces hysteresis losses. Transformation of energy to internal energy in the finite resistance of the coil wires is usually quite small. Typical transformers have power efficiencies from 90% to 99%. In the discussion that follows, we assume an *ideal transformer,* one in which the energy losses in the windings and core are zero.

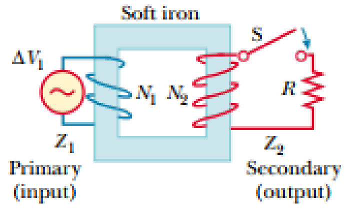


Figure 33.18 An ideal transformer consists of two coils wound on the same iron core. An alternating voltage ΔV_1 is applied to the primary coil, and the output voltage ΔV_2 is across the resistor of resistance R.

First, let us consider what happens in the primary circuit when the switch in the secondary circuit is open. If we assume that the resistance of the primary is negligible relative to its inductive reactance, then the primary circuit is equivalent to a simple circuit consisting of an inductor connected to an ac generator. Because the current is 90° out of phase with the voltage, the power factor $\cos\phi$ is zero, and hence the average power delivered from the generator to the primary circuit is zero. Faraday's law states that the voltage ΔV_1 across the primary is

$$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt} \tag{33.37}$$

$$\Delta V_2 = -N_2 \frac{d\Phi_B}{dt} \tag{33.38}$$

Solving Equation 33.37 for $d\Phi_B/dt$ and substituting the result into Equation 33.38, we find that

$$\Delta V_2 = \frac{N_2}{N_1}\Delta V_1 \tag{33.39}$$

When $N_2 > N_1$, the output voltage ΔV_2 exceeds the input voltage ΔV_1. This setup is referred to as a *step-up transformer.* When $N_2 < N_1$, the output voltage is less than the input voltage, and we have a *step-down transformer.*

When the switch in the secondary circuit is thrown closed, a current I_2 is induced in the secondary. If the load in the secondary circuit is a pure resistance, the induced current is in phase with the induced voltage. The power supplied to the secondary circuit must be provided by the ac generator connected to the primary circuit, as shown in Figure 33.19. In an ideal transformer, where there are no losses, the power $I_1\,\Delta V_1$ supplied by the generator is equal to the power $I_2\,\Delta V_2$ in

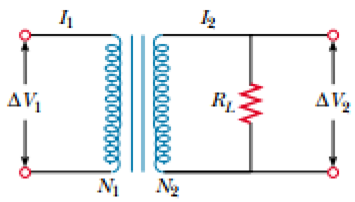


Figure 33.19 Circuit diagram for a transformer.

This cylindrical step-down transformer drops the voltage from 4 000 V to 220 V for delivery to a group of residences. *(George Semple)*

the secondary circuit. That is,

$$I_1 \Delta V_1 = I_2 \Delta V_2 \tag{33.40}$$

The value of the load resistance R_L determines the value of the secondary current because $I_2 = \Delta V_2 / R_L$. Furthermore, the current in the primary is $I_1 = \Delta V_1 / R_{\text{eq}}$, where

$$R_{\text{eq}} = \left(\frac{N_1}{N_2}\right)^2 R_L \tag{33.41}$$

is the equivalent resistance of the load resistance when viewed from the primary side. From this analysis we see that a transformer may be used to match resistances between the primary circuit and the load. In this manner, maximum power transfer can be achieved between a given power source and the load resistance. For example, a transformer connected between the 1-kΩ output of an audio amplifier and an 8-Ω speaker ensures that as much of the audio signal as possible is transferred into the speaker. In stereo terminology, this is called *impedance matching.*

We can now also understand why transformers are useful for transmitting power over long distances. Because the generator voltage is stepped up, the current in the transmission line is reduced, and hence I^2R losses are reduced. In practice, the voltage is stepped up to around 230 000 V at the generating station, stepped down to around 20 000 V at a distributing station, then to 4 000 V for delivery to residential areas, and finally to 120–240 V at the customer's site. The power is supplied by a three-wire cable. In the United States, two of these wires are "hot," with voltages of 120 V with respect to a common ground wire. Home appliances operating on 120 V are connected in parallel between one of the hot wires and ground. Larger appliances, such as electric stoves and clothes dryers, require 240 V. This is obtained across the two hot wires, which are 180° out of phase so that the voltage difference between them is 240 V.

There is a practical upper limit to the voltages that can be used in transmission lines. Excessive voltages could ionize the air surrounding the transmission lines, which could result in a conducting path to ground or to other objects in the vicinity. This, of course, would present a serious hazard to any living creatures. For this reason, a long string of insulators is used to keep high-voltage wires away from their supporting metal towers. Other insulators are used to maintain separation between wires.

Nikola Tesla (1856–1943) Tesla was born in Croatia but spent most of his professional life as an inventor in the United States. He was a key figure in the development of alternating-current electricity, high-voltage transformers, and the transport of electric power via ac transmission lines. Tesla's viewpoint was at odds with the ideas of Thomas Edison, who committed himself to the use of direct current in power transmission. Tesla's ac approach won out. *(UPI/Bettmann)*

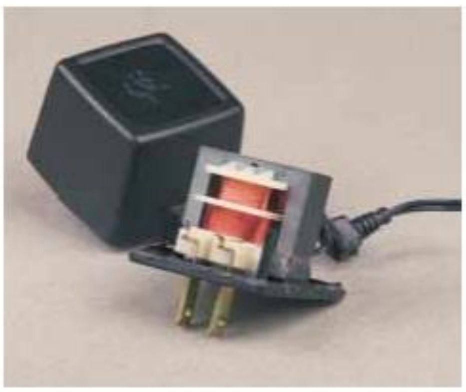


Figure 33.20 The primary winding in this transformer is directly attached to the prongs of the plug. The secondary winding is connected to the wire on the right, which runs to an electronic device. Many of these power-supply transformers also convert alternating current to direct current. *(George Semple)*

Many common household electronic devices require low voltages to operate properly. A small transformer that plugs directly into the wall, like the one illustrated in the photograph at the beginning of this chapter, can provide the proper voltage. Figure 33.20 shows the two windings wrapped around a common iron core that is found inside all these little "black boxes." This particular transformer converts the 120-V ac in the wall socket to 12.5-V ac. (Can you determine the ratio of the numbers of turns in the two coils?) Some black boxes also make use of diodes to convert the alternating current to direct current (see Section 33.9).

web

For information on how small transformers and hundreds of other everyday devices operate, visit **http://www.howstuffworks.com**

EXAMPLE 33.9 The Economics of ac Power

An electricity-generating station needs to deliver 20 MW of power to a city 1.0 km away. (a) If the resistance of the wires is 2.0 Ω and the electricity costs about 10¢/kWh, estimate what it costs the utility company to send the power to the city for one day. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

Solution The power losses in the transmission line are the result of the resistance of the line. We can determine the loss from Equation 27.23, $\mathcal{P} = I^2R$. Because this is an estimate, we can use dc equations and calculate I from Equation 27.22:

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{20 \times 10^6\ \mathrm{W}}{230 \times 10^3\ \mathrm{V}} = 87\ \mathrm{A}$$

Therefore,

$$\mathcal{P} = I^2R = (87\ \mathrm{A})^2(2.0\ \Omega) = 15\ \mathrm{kW}$$

Over the course of a day, the energy loss due to the resistance of the wires is $(15\ \mathrm{kW})(24\ \mathrm{h}) = 360\ \mathrm{kWh}$, at a cost of $\boxed{\$36.}$

(b) Repeat the calculation for the situation in which the power plant delivers the electricity at its original voltage of 22 kV.

Solution

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{20 \times 10^6\ \mathrm{W}}{22 \times 10^3\ \mathrm{V}} = 910\ \mathrm{A}$$

$$\mathcal{P} = I^2R = (910\ \mathrm{A})^2(2.0\ \Omega) = 1.7 \times 10^3\ \mathrm{kW}$$

$$\text{Cost per day} = (1.7 \times 10^3\ \mathrm{kW})(24\ \mathrm{h})(\$0.10/\mathrm{kWh})$$

$$= \boxed{\$4\,100}$$

The tremendous savings that are possible through the use of transformers and high-voltage transmission lines, along with the efficiency of using alternating current to operate motors, led to the universal adoption of alternating current instead of direct current for commercial power grids.

Optional Section

33.9 RECTIFIERS AND FILTERS

Portable electronic devices such as radios and compact disc (CD) players are often powered by direct current supplied by batteries. Many devices come with ac–dc converters that provide a readily available direct-current source if the batteries are low. Such a converter contains a transformer that steps the voltage down from 120 V to typically 9 V and a circuit that converts alternating current to direct current. The process of converting alternating current to direct current is called **rectification,** and the converting device is called a **rectifier.**

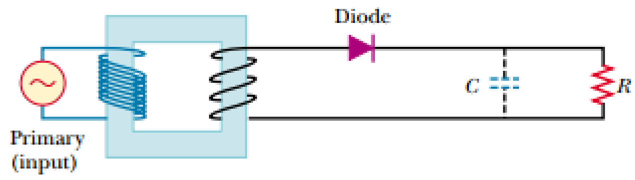
The most important element in a rectifier circuit is a **diode,** a circuit element that conducts current in one direction but not the other. Most diodes used in modern electronics are semiconductor devices. The circuit symbol for a diode is —▶|—, where the arrow indicates the direction of the current through the diode. A diode has low resistance to current in one direction (the direction of the arrow) and high resistance to current in the opposite direction. We can understand how a diode rectifies a current by considering Figure 33.21a, which shows a diode and a resistor connected to the secondary of a transformer. The transformer reduces the voltage from 120-V ac to the lower voltage that is needed for the device having a resistance R (the load resistance). Because current can pass through the diode in only one direction, the alternating current in the load resistor is reduced to the form shown by the solid curve in Figure 33.21b. The diode conducts current only when the side of the symbol containing the arrowhead has a positive potential relative to the other side. In this situation, the diode acts as a *half-wave rectifier* because current is present in the circuit during only half of each cycle.

When a capacitor is added to the circuit, as shown by the dashed lines and the capacitor symbol in Figure 33.21a, the circuit is a simple dc power supply. The time variation in the current in the load resistor (the dashed curve in Fig. 33.21b) is close to being zero, as determined by the RC time constant of the circuit.

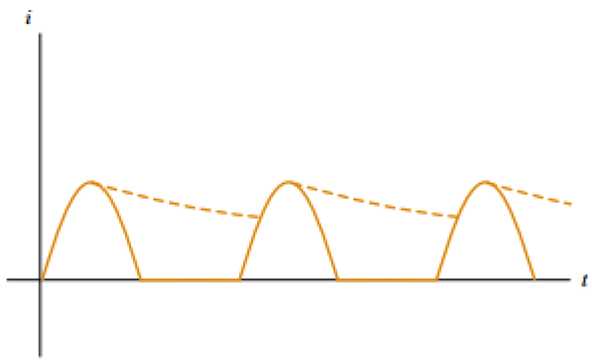
The RC circuit in Figure 33.21a is one example of a **filter circuit,** which is used to smooth out or eliminate a time-varying signal. For example, radios are usually powered by a 60-Hz alternating voltage. After rectification, the voltage still contains a small ac component at 60 Hz (sometimes called *ripple*), which must be filtered. By "filtered," we mean that the 60-Hz ripple must be reduced to a value much less than that of the audio signal to be amplified, because without filtering, the resulting audio signal includes an annoying hum at 60 Hz.

To understand how a filter works, let us consider the simple series RC circuit shown in Figure 33.22a. The input voltage is across the two elements and is represented by $\Delta V_{\max} \sin \omega t$. Because we are interested only in maximum values, we can use Equation 33.24, taking $X_L = 0$ and substituting $X_C = 1/\omega C$. This shows that the maximum input voltage is related to the maximum current by

$$\Delta V_{\text{in}} = I_{\max} Z = I_{\max} \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$



(a)



(b)

Figure 33.21 (a) A half-wave rectifier with an optional filter capacitor. (b) Current versus time in the resistor. The solid curve represents the current with no filter capacitor, and the dashed curve is the current when the circuit includes the capacitor.

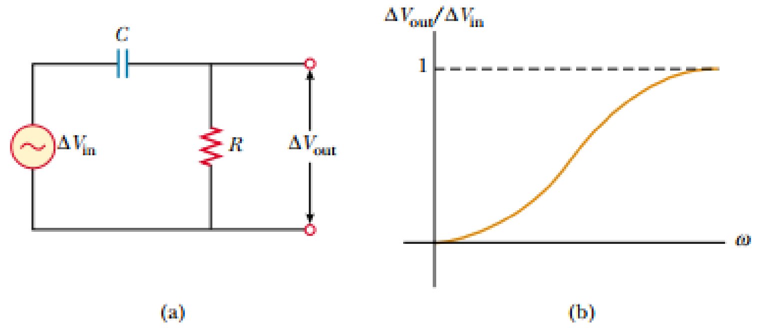


Figure 33.22 (a) A simple RC high-pass filter. (b) Ratio of output voltage to input voltage for an RC high-pass filter as a function of the angular frequency of the circuit.

If the voltage across the resistor is considered to be the output voltage, then the maximum output voltage is

$$\Delta V_{out} = I_{max}R$$

Therefore, the ratio of the output voltage to the input voltage is

$$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \tag{33.42}$$

High-pass filter

A plot of this ratio as a function of angular frequency (see Fig. 33.22b) shows that at low frequencies ΔV_{out} is much smaller than ΔV_{in}, whereas at high frequencies the two voltages are equal. Because the circuit preferentially passes signals of higher frequency while blocking low-frequency signals, the circuit is called an RC high-pass filter. Physically, a high-pass filter works because a capacitor "blocks out" direct current and ac current at low frequencies.

Now let us consider the circuit shown in Figure 33.23a, where the output voltage is taken across the capacitor. In this case, the maximum voltage equals the voltage across the capacitor. Because the impedance across the capacitor is $X_C = 1/\omega C$, we have

$$\Delta V_{out} = I_{max}X_C = \frac{I_{max}}{\omega C}$$

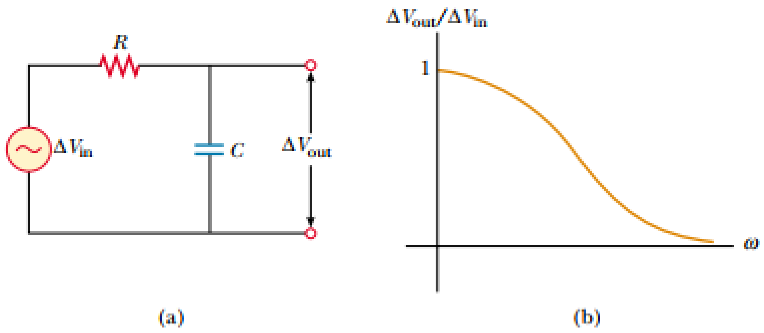


Figure 33.23 (a) A simple RC low-pass filter. (b) Ratio of output voltage to input voltage for an RC low-pass filter as a function of the angular frequency of the circuit.

Therefore, the ratio of the output voltage to the input voltage is

Low-pass filter

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}} \tag{33.43}$$

This ratio, plotted as a function of ω in Figure 33.23b, shows that in this case the circuit preferentially passes signals of low frequency. Hence, the circuit is called an *RC* low-pass filter.

You may be familiar with crossover networks, which are an important part of the speaker systems for high-fidelity audio systems. These networks utilize low-pass filters to direct low frequencies to a special type of speaker, the "woofer," which is designed to reproduce the low notes accurately. The high frequencies are sent to the "tweeter" speaker.

Quick Quiz 33.6

Suppose you are designing a high-fidelity system containing both large loudspeakers (woofers) and small loudspeakers (tweeters). (a) What circuit element would you place in series with a woofer, which passes low-frequency signals? (b) What circuit element would you place in series with a tweeter, which passes high-frequency signals?

Summary

If an ac circuit consists of a generator and a resistor, the current is in phase with the voltage. That is, the current and voltage reach their maximum values at the same time.

The **rms current** and **rms voltage** in an ac circuit in which the voltages and current vary sinusoidally are given by the expressions

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max} \tag{33.4}$$

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = 0.707 \Delta V_{\max} \tag{33.5}$$

where $I_{\max}$ and $\Delta V_{\max}$ are the maximum values.

If an ac circuit consists of a generator and an inductor, the current lags behind the voltage by 90°. That is, the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value.

If an ac circuit consists of a generator and a capacitor, the current leads the voltage by 90°. That is, the current reaches its maximum value one quarter of a period before the voltage reaches its maximum value.

In ac circuits that contain inductors and capacitors, it is useful to define the **inductive reactance** X_L and the **capacitive reactance** X_C as

$$X_L = \omega L \tag{33.10}$$

$$X_C = \frac{1}{\omega C} \tag{33.17}$$

where ω is the angular frequency of the ac generator. The SI unit of reactance is the ohm.

The **impedance** Z of an RLC series ac circuit, which also has the ohm as its unit, is

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \quad \textbf{(33.23)}$$

This expression illustrates that we cannot simply add the resistance and reactances in a circuit. We must account for the fact that the applied voltage and current are out of phase, with the **phase angle** ϕ between the current and voltage being

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad \textbf{(33.25)}$$

The sign of ϕ can be positive or negative, depending on whether X_L is greater or less than X_C. The phase angle is zero when $X_L = X_C$.

The **average power** delivered by the generator in an RLC ac circuit is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}\,\Delta V_{\text{rms}} \cos\phi \quad \textbf{(33.29)}$$

An equivalent expression for the average power is

$$\mathcal{P}_{\text{av}} = I_{\text{rms}}^2 R \quad \textbf{(33.30)}$$

The average power delivered by the generator results in increasing internal energy in the resistor. No power loss occurs in an ideal inductor or capacitor.

The rms current in a series RLC circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \textbf{(33.32)}$$

A series RLC circuit is in resonance when the inductive reactance equals the capacitive reactance. When this condition is met, the current given by Equation 33.32 reaches its maximum value. When $X_L = X_C$ in a circuit, the **resonance frequency** ω_0 of the circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \textbf{(33.33)}$$

The current in a series RLC circuit reaches its maximum value when the frequency of the generator equals ω_0—that is, when the "driving" frequency matches the resonance frequency.

Transformers allow for easy changes in alternating voltage. Because energy (and therefore power) are conserved, we can write

$$I_1\,\Delta V_1 = I_2\,\Delta V_2 \quad \textbf{(33.40)}$$

to relate the currents and voltages in the primary and secondary windings of a transformer.

QUESTIONS

1. Fluorescent lights flicker on and off 120 times every second. Explain what causes this. Why can't you see it happening?
2. Why does a capacitor act as a short circuit at high frequencies? Why does it act as an open circuit at low frequencies?
3. Explain how the acronyms in the mnemonic "ELI the ICE man" can be used to recall whether current leads voltage or voltage leads current in RLC circuits. (Note that "E" represents voltage.)
4. Why is the sum of the maximum voltages across the elements in a series RLC circuit usually greater than the maximum applied voltage? Doesn't this violate Kirchhoff's second rule?
5. Does the phase angle depend on frequency? What is the phase angle when the inductive reactance equals the capacitive reactance?
6. Energy is delivered to a series RLC circuit by a generator. This energy appears as internal energy in the resistor. What is the source of this energy?