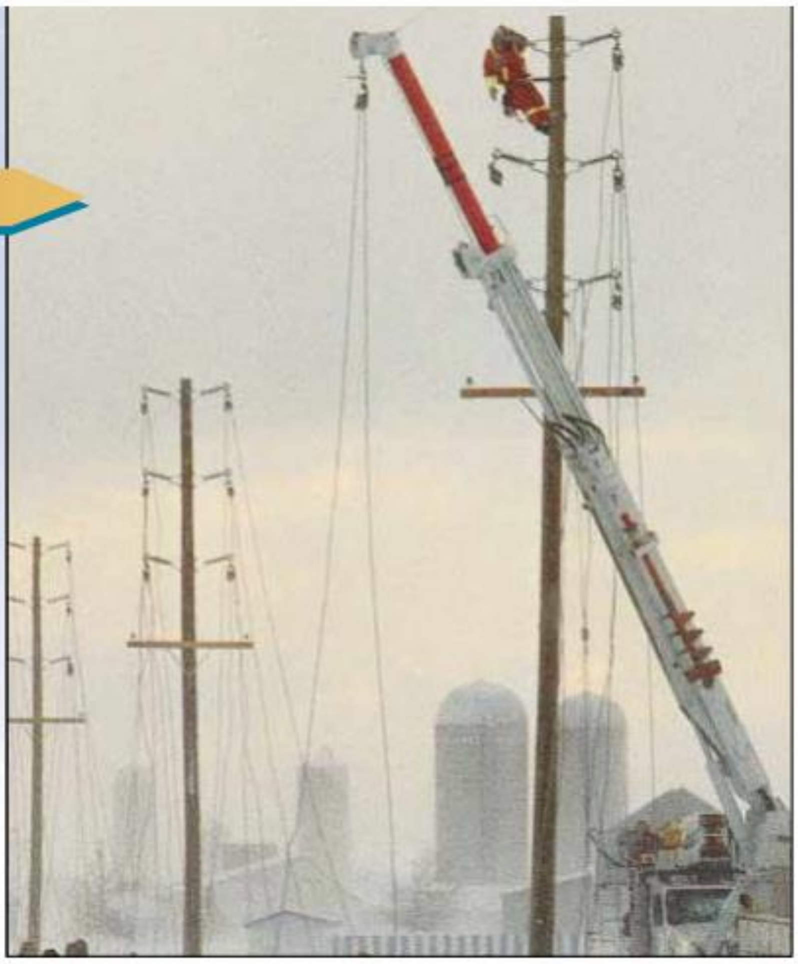
PUZZLER

Electrical workers restoring power to the eastern Ontario town of St. Isadore, which was without power for several days in January 1998 because of a severe ice storm. It is very dangerous to touch fallen power transmission lines because of their high electric potential, which might be hundreds of thousands of volts relative to the ground. Why is such a high potential difference used in power transmission if it is so dangerous, and why aren't birds that perch on the wires electrocuted? *(AP/Wide World Photos/Fred Chartrand)*



chapter

# 27 Current and Resistance

## Chapter Outline

Thus far our treatment of electrical phenomena has been confined to the study of charges at rest, or *electrostatics.* We now consider situations involving electric charges in motion. We use the term *electric current,* or simply *current,* to describe the rate of flow of charge through some region of space. Most practical applications of electricity deal with electric currents. For example, the battery in a flashlight supplies current to the filament of the bulb when the switch is turned on. A variety of home appliances operate on alternating current. In these common situations, the charges flow through a conductor, such as a copper wire. It also is possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

This chapter begins with the definitions of current and current density. A microscopic description of current is given, and some of the factors that contribute to the resistance to the flow of charge in conductors are discussed. A classical model is used to describe electrical conduction in metals, and some of the limitations of this model are cited.

## 27.1 ELECTRIC CURRENT

13.2 It is instructive to draw an analogy between water flow and current. In many localities it is common practice to install low-flow showerheads in homes as a water-conservation measure. We quantify the flow of water from these and similar devices by specifying the amount of water that emerges during a given time interval, which is often measured in liters per minute. On a grander scale, we can characterize a river current by describing the rate at which the water flows past a particular location. For example, the flow over the brink at Niagara Falls is maintained at rates between 1 400 $m^3/s$ and 2 800 $m^3/s$.

Now consider a system of electric charges in motion. Whenever there is a net flow of charge through some region, a **current** is said to exist. To define current more precisely, suppose that the charges are moving perpendicular to a surface of area $A$, as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface.** If $\Delta Q$ is the amount of charge that passes through this area in a time interval $\Delta t$, the **average current** $I_{av}$ is equal to the charge that passes through $A$ per unit time:

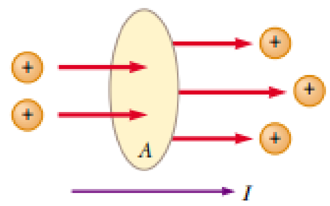



***Figure 27.1*** Charges in motion through an area $A$. The time rate at which charge flows through the area is defined as the current $I$. The direction of the current is the direction in which positive charges flow when free to do so.

$$I_{av} = \frac{\Delta Q}{\Delta t} \tag{27.1}$$

If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** $I$ as the differential limit of average current:

Electric current

$$I \equiv \frac{dQ}{dt} \tag{27.2}$$

The SI unit of current is the **ampere** (A):

$$1\text{ A} = \frac{1\text{ C}}{1\text{ s}} \tag{27.3}$$

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

The direction of the current

The charges passing through the surface in Figure 27.1 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, **the direction of the current is opposite the direction of flow of electrons.** However, if we are considering a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance—the current is the result of the flow of both positive and negative charges.

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential, and hence the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire, and therefore there is no current. The current in the conductor is zero even if the conductor has an excess of charge on it. However, if the ends of the conducting wire are connected to a battery, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the conduction electrons in the wire, causing them to move around the loop and thus creating a current.

It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier.** For example, the mobile charge carriers in a metal are electrons.

## Microscopic Model of Current

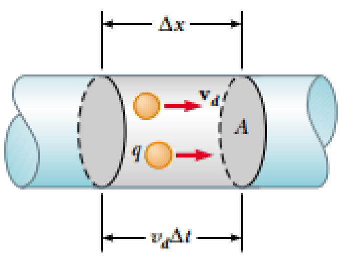



***Figure 27.2*** A section of a uniform conductor of cross-sectional area $A$. The mobile charge carriers move with a speed $v_d$, and the distance they travel in a time $\Delta t$ is $\Delta x = v_d\,\Delta t$. The number of carriers in the section of length $\Delta x$ is $nAv_d\,\Delta t$, where $n$ is the number of carriers per unit volume.

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a conductor of cross-sectional area $A$ (Fig. 27.2). The volume of a section of the conductor of length $\Delta x$ (the gray region shown in Fig. 27.2) is $A\,\Delta x$. If $n$ represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is $nA\,\Delta x$. Therefore, the charge $\Delta Q$ in this section is

$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA\,\Delta x)q$$

where $q$ is the charge on each carrier. If the carriers move with a speed $v_d$, the distance they move in a time $\Delta t$ is $\Delta x = v_d\,\Delta t$. Therefore, we can write $\Delta Q$ in the form

$$\Delta Q = (nAv_d\,\Delta t)q$$

If we divide both sides of this equation by $\Delta t$, we see that the average current in the conductor is

Average current in a conductor

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t} = nqv_dA \qquad \textbf{(27.4)}$$

The speed of the charge carriers $v_d$ is an average speed called the **drift speed.** To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated—that is, the potential difference across it is zero—then these electrons undergo random motion that is analogous to the motion of gas molecules. As we discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. However, the electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag (Fig. 27.3). Despite the collisions, the electrons move slowly along the conductor (in a direction opposite that of $\mathbf{E}$) at the drift velocity $\mathbf{v}_d$.

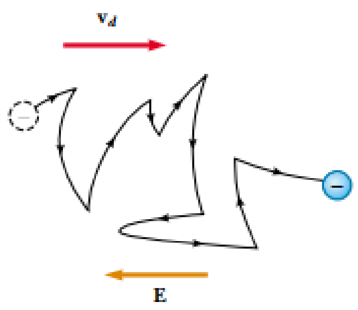



**Figure 27.3** A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. Note that the net motion of the electron is opposite the direction of the electric field. Each section of the zigzag path is a parabolic segment.

We can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

## Quick Quiz 27.1

Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions, from lowest to highest.

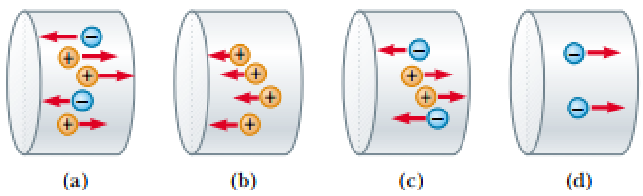



**Figure 27.4**

## Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \mathrm{m}^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ \mathrm{g/cm}^3$.

**Solution** From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms ($6.02 \times 10^{23}$). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \mathrm{g}}{8.95\ \mathrm{g/cm}^3} = 7.09\ \mathrm{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \mathrm{cm}^3}(1.00 \times 10^6\ \mathrm{cm}^3/\mathrm{m}^3)$$

$$= 8.49 \times 10^{28}\ \text{electrons}/\mathrm{m}^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where $q$ is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$

$$= \frac{10.0\ \mathrm{C/s}}{(8.49 \times 10^{28}\ \mathrm{m}^{-3})(1.60 \times 10^{-19}\ \mathrm{C})(3.31 \times 10^{-6}\ \mathrm{m}^2)}$$

$$= \boxed{2.22 \times 10^{-4}\ \mathrm{m/s}}$$

**Exercise** If a copper wire carries a current of 80.0 mA, how many electrons flow past a given cross-section of the wire in 10.0 min?

**Answer** $3.0 \times 10^{20}$ electrons.

Example 27.1 shows that typical drift speeds are very low. For instance, electrons traveling with a speed of $2.46 \times 10^{-4}$ m/s would take about 68 min to travel 1 m! In view of this, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons travels through the conductor with a speed close to that of light. Thus, when you flip on a light switch, the message for the electrons to start moving through the wire (the electric field) reaches them at a speed on the order of $10^8$ m/s.

## 27.2 RESISTANCE AND OHM'S LAW

13.3 In Chapter 24 we found that no electric field can exist inside a conductor. However, this statement is true *only* if the conductor is in static equilibrium. The purpose of this section is to describe what happens when the charges in the conductor are allowed to move.

Charges moving in a conductor produce a current under the action of an electric field, which is maintained by the connection of a battery across the conductor. An electric field can exist in the conductor because the charges in this situation are in motion—that is, this is a *nonelectrostatic* situation.

Consider a conductor of cross-sectional area $A$ carrying a current $I$. The **current density** $J$ in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d \tag{27.5}$$

where $J$ has SI units of A/m$^2$. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area $A$ is perpendicular to the direction of the current. In general, the current density is a vector quantity:

Current density

$$\mathbf{J} = nq\mathbf{v}_d \tag{27.6}$$

From this equation, we see that current density, like current, is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

**A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.** If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

Ohm's law

$$\mathbf{J} = \sigma\mathbf{E} \tag{27.7}$$

where the constant of proportionality $\sigma$ is called the **conductivity** of the conductor.[1] Materials that obey Equation 27.7 are said to follow **Ohm's law,** named after Georg Simon Ohm (1787–1854). More specifically, Ohm's law states that

> for many materials (including most metals), the ratio of the current density to the electric field is a constant $\sigma$ that is independent of the electric field producing the current.

Materials that obey Ohm's law and hence demonstrate this simple relationship between **E** and **J** are said to be *ohmic*. Experimentally, it is found that not all materials have this property, however, and materials that do not obey Ohm's law are said to

[1] Do not confuse conductivity $\sigma$ with surface charge density, for which the same symbol is used.

be *nonohmic*. Ohm's law is not a fundamental law of nature but rather an empirical relationship valid only for certain materials.

**Quick Quiz 27.2**

Suppose that a current-carrying ohmic metal wire has a cross-sectional area that gradually becomes smaller from one end of the wire to the other. How do drift velocity, current density, and electric field vary along the wire? Note that the current must have the same value everywhere in the wire so that charge does not accumulate at any one point.

We can obtain a form of Ohm's law useful in practical applications by considering a segment of straight wire of uniform cross-sectional area $A$ and length $\ell$, as shown in Figure 27.5. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the potential difference is related to the field through the relationship[2]

$$\Delta V = E\ell$$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I$$

The quantity $\ell/\sigma A$ is called the **resistance** $R$ of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current through the conductor:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \qquad \textbf{(27.8)}$$

Resistance of a conductor

From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be 1 **ohm** ($\Omega$):

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}} \qquad \textbf{(27.9)}$$

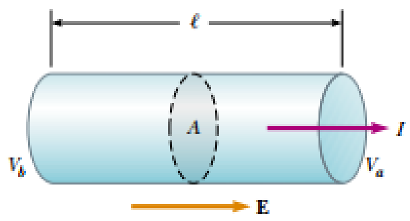



***Figure 27.5*** A uniform conductor of length $\ell$ and cross-sectional area $A$. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field **E**, and this field produces a current $I$ that is proportional to the potential difference.

[2] This result follows from the definition of potential difference:

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s} = E \int_0^\ell dx = E\ell$$

An assortment of resistors used in electric circuits.

This expression shows that if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω. For example, if an electrical appliance connected to a 120-V source of potential difference carries a current of 6 A, its resistance is 20 Ω.

Equation 27.8 solved for potential difference ($\Delta V = I\ell/\sigma A$) explains part of the chapter-opening puzzler: How can a bird perch on a high-voltage power line without being electrocuted? Even though the potential difference between the ground and the wire might be hundreds of thousands of volts, that between the bird's feet (which is what determines how much current flows through the bird) is very small.

The inverse of conductivity is **resistivity**[3] $\rho$:

Resistivity

$$\rho \equiv \frac{1}{\sigma} \tag{27.10}$$

where $\rho$ has the units ohm-meters ($\Omega \cdot \text{m}$). We can use this definition and Equation 27.8 to express the resistance of a uniform block of material as

Resistance of a uniform conductor

$$R = \rho \frac{\ell}{A} \tag{27.11}$$

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. Additionally, as you can see from Equation 27.11, the resistance of a sample depends on geometry as well as on resistivity. Table 27.1 gives the resistivities of a variety of materials at 20°C. Note the enormous range, from very low values for good conductors such as copper and silver, to very high values for good insulators such as glass and rubber. An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Equation 27.11 shows that the resistance of a given cylindrical conductor is proportional to its length and inversely proportional to its cross-sectional area. If the length of a wire is doubled, then its resistance doubles. If its cross-sectional area is doubled, then its resistance decreases by one half. The situation is analogous to the flow of a liquid through a pipe. As the pipe's length is increased, the

[3] Do not confuse resistivity with mass density or charge density, for which the same symbol is used.

**TABLE 27.1 Resistivities and Temperature Coefficients of Resistivity for Various Materials**

| Material | Resistivity[a] ($\Omega \cdot m$) | Temperature Coefficient $\alpha[(°C)^{-1}]$ |
|---|---|---|
| Silver | $1.59 \times 10^{-8}$ | $3.8 \times 10^{-3}$ |
| Copper | $1.7 \times 10^{-8}$ | $3.9 \times 10^{-3}$ |
| Gold | $2.44 \times 10^{-8}$ | $3.4 \times 10^{-3}$ |
| Aluminum | $2.82 \times 10^{-8}$ | $3.9 \times 10^{-3}$ |
| Tungsten | $5.6 \times 10^{-8}$ | $4.5 \times 10^{-3}$ |
| Iron | $10 \times 10^{-8}$ | $5.0 \times 10^{-3}$ |
| Platinum | $11 \times 10^{-8}$ | $3.92 \times 10^{-3}$ |
| Lead | $22 \times 10^{-8}$ | $3.9 \times 10^{-3}$ |
| Nichrome[b] | $1.50 \times 10^{-6}$ | $0.4 \times 10^{-3}$ |
| Carbon | $3.5 \times 10^{-5}$ | $-0.5 \times 10^{-3}$ |
| Germanium | 0.46 | $-48 \times 10^{-3}$ |
| Silicon | 640 | $-75 \times 10^{-3}$ |
| Glass | $10^{10}$ to $10^{14}$ | |
| Hard rubber | $\approx 10^{13}$ | |
| Sulfur | $10^{15}$ | |
| Quartz (fused) | $75 \times 10^{16}$ | |

[a] All values at 20°C.

[b] A nickel–chromium alloy commonly used in heating elements.

resistance to flow increases. As the pipe's cross-sectional area is increased, more liquid crosses a given cross-section of the pipe per unit time. Thus, more liquid flows for the same pressure differential applied to the pipe, and the resistance to flow decreases.

Most electric circuits use devices called **resistors** to control the current level in the various parts of the circuit. Two common types of resistors are the *composition resistor,* which contains carbon, and the *wire-wound resistor,* which consists of a coil of wire. Resistors' values in ohms are normally indicated by color-coding, as shown in Figure 27.6 and Table 27.2.

Ohmic materials have a linear current–potential difference relationship over a broad range of applied potential differences (Fig. 27.7a). The slope of the $I$-versus-$\Delta V$ curve in the linear region yields a value for $1/R$. Nonohmic materials

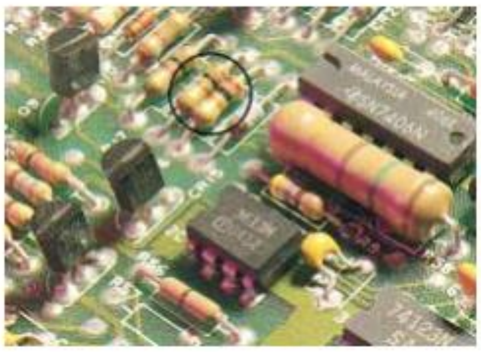


***Figure 27.6*** The colored bands on a resistor represent a code for determining resistance. The first two colors give the first two digits in the resistance value. The third color represents the power of ten for the multiplier of the resistance value. The last color is the tolerance of the resistance value. As an example, the four colors on the circled resistors are red (= 2), black (= 0), orange (= $10^3$), and gold (= 5%), and so the resistance value is $20 \times 10^3\ \Omega$ = 20 k$\Omega$ with a tolerance value of 5% = 1 k$\Omega$. (The values for the colors are from Table 27.2.)

**TABLE 27.2 Color Coding for Resistors**

| Color | Number | Multiplier | Tolerance |
|---|---|---|---|
| Black | 0 | 1 | |
| Brown | 1 | $10^1$ | |
| Red | 2 | $10^2$ | |
| Orange | 3 | $10^3$ | |
| Yellow | 4 | $10^4$ | |
| Green | 5 | $10^5$ | |
| Blue | 6 | $10^6$ | |
| Violet | 7 | $10^7$ | |
| Gray | 8 | $10^8$ | |
| White | 9 | $10^9$ | |
| Gold | | $10^{-1}$ | 5% |
| Silver | | $10^{-2}$ | 10% |
| Colorless | | | 20% |

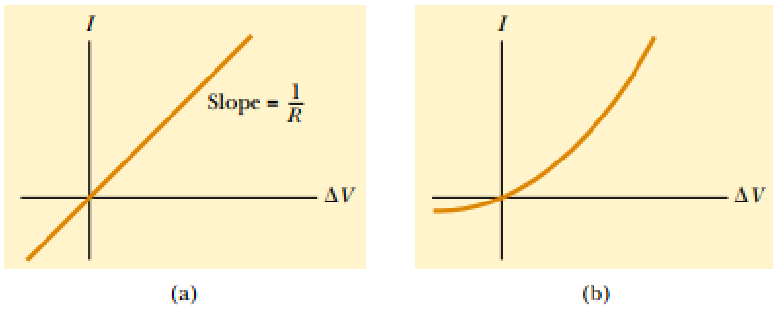



**Figure 27.7** (a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a semiconducting diode. This device does not obey Ohm's law.

have a nonlinear current–potential difference relationship. One common semiconducting device that has nonlinear $I$-versus-$\Delta V$ characteristics is the *junction diode* (Fig. 27.7b). The resistance of this device is low for currents in one direction (positive $\Delta V$) and high for currents in the reverse direction (negative $\Delta V$). In fact, most modern electronic devices, such as transistors, have nonlinear current–potential difference relationships; their proper operation depends on the particular way in which they violate Ohm's law.

**Quick Quiz 27.3**

What does the slope of the curved line in Figure 27.7b represent?

**Quick Quiz 27.4**

Your boss asks you to design an automobile battery jumper cable that has a low resistance. In view of Equation 27.11, what factors would you consider in your design?

## EXAMPLE 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

**Solution** From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m}) \left( \frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m}) \left( \frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

As you might guess from the large difference in resistivities, the resistance of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

Electrical insulators on telephone poles are often made of glass because of its low electrical conductivity.

## EXAMPLE 27.3 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

**Solution** The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega \cdot \text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega \cdot \text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

**Solution** Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

***Exercise*** What is the resistance of a 6.0-m length of 22-gauge Nichrome wire? How much current does the wire carry when connected to a 120-V source of potential difference?

***Answer*** 28 Ω; 4.3 A.

***Exercise*** Calculate the current density and electric field in the wire when it carries a current of 2.2 A.

***Answer*** $6.8 \times 10^6\ \text{A/m}^2$; 10 N/C.

## EXAMPLE 27.4 The Radial Resistance of a Coaxial Cable

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two cylindrical conductors. The gap between the conductors is completely filled with silicon, as shown in Figure 27.8a, and current leakage through the silicon is unwanted. (The cable is designed to conduct current along its length.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer one is $b = 1.75$ cm, and the length of the cable is $L = 15.0$ cm. Calculate the resistance of the silicon between the two conductors.

**Solution** In this type of problem, we must divide the object whose resistance we are calculating into concentric elements of infinitesimal thickness $dr$ (Fig. 27.8b). We start by using the differential form of Equation 27.11, replacing $\ell$ with $r$ for the distance variable: $dR = \rho\, dr/A$, where $dR$ is the resistance of an element of silicon of thickness $dr$ and surface area $A$. In this example, we take as our representative concentric element a hollow silicon cylinder of radius $r$, thickness $dr$, and length $L$, as shown in Figure 27.8. Any current that passes from the inner conductor to the outer one must pass radially through this concentric element, and the area through which this current passes is $A = 2\pi rL$. (This is the curved surface area—circumference multiplied by length—of our hollow silicon cylinder of thickness $dr$.) Hence, we can write the resistance of our hollow cylinder of silicon as

$$dR = \frac{\rho}{2\pi rL}\,dr$$

Because we wish to know the total resistance across the entire thickness of the silicon, we must integrate this expression from $r = a$ to $r = b$:

$$R = \int_a^b dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

Substituting in the values given, and using $\rho = 640\ \Omega\cdot\text{m}$ for silicon, we obtain

$$R = \frac{640\ \Omega\cdot\text{m}}{2\pi(0.150\ \text{m})}\ln\left(\frac{1.75\ \text{cm}}{0.500\ \text{cm}}\right) = \boxed{851\ \Omega}$$

***Exercise*** If a potential difference of 12.0 V is applied between the inner and outer conductors, what is the value of the total current that passes between them?

***Answer*** 14.1 mA.

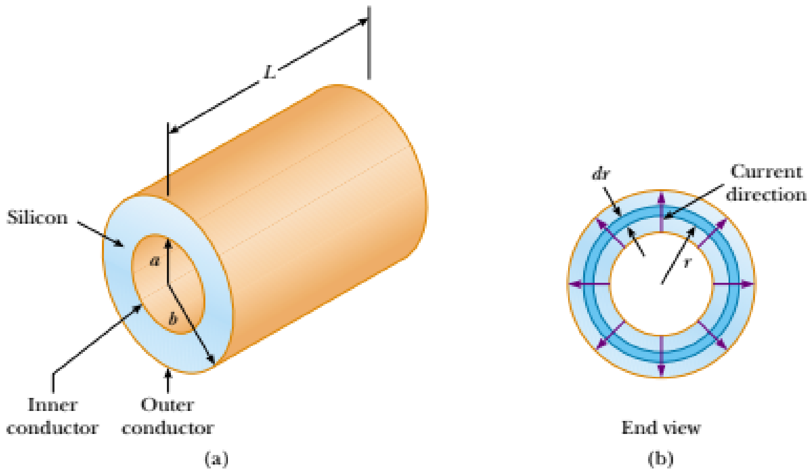



***Figure 27.8*** A coaxial cable. (a) Silicon fills the gap between the two conductors. (b) End view, showing current leakage.

## 27.3 A MODEL FOR ELECTRICAL CONDUCTION

In this section we describe a classical model of electrical conduction in metals that was first proposed by Paul Drude in 1900. This model leads to Ohm's law and shows that resistivity can be related to the motion of electrons in metals. Although the Drude model described here does have limitations, it nevertheless introduces concepts that are still applied in more elaborate treatments.

Consider a conductor as a regular array of atoms plus a collection of free electrons, which are sometimes called *conduction* electrons. The conduction electrons, although bound to their respective atoms when the atoms are not part of a solid, gain mobility when the free atoms condense into a solid. In the absence of an electric field, the conduction electrons move in random directions through the conductor with average speeds of the order of $10^6$ m/s. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an *electron gas*. There is no current through the conductor in the absence of an electric field because the drift velocity of the free electrons is zero. That is, on the average, just as many electrons move in one direction as in the opposite direction, and so there is no net flow of charge.

This situation changes when an electric field is applied. Now, in addition to undergoing the random motion just described, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed $v_d$ that is much smaller (typically $10^{-4}$ m/s) than their average speed between collisions (typically $10^6$ m/s).

Figure 27.9 provides a crude description of the motion of free electrons in a conductor. In the absence of an electric field, there is no net displacement after many collisions (Fig. 27.9a). An electric field **E** modifies the random motion and causes the electrons to drift in a direction opposite that of **E** (Fig. 27.9b). The slight curvature in the paths shown in Figure 27.9b results from the acceleration of the electrons between collisions, which is caused by the applied field.

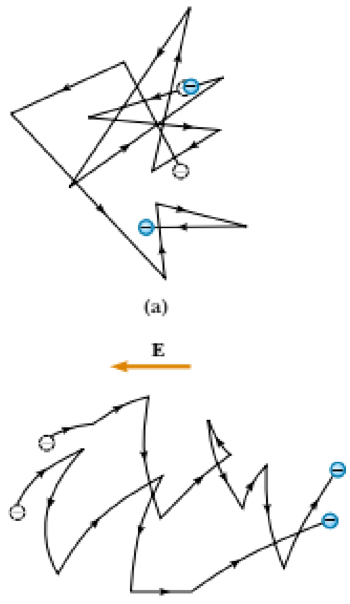



***Figure 27.9*** (a) A schematic diagram of the random motion of two charge carriers in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of the charge carriers in a conductor in the presence of an electric field. Note that the random motion is modified by the field, and the charge carriers have a drift velocity.

In our model, we assume that the motion of an electron after a collision is independent of its motion before the collision. We also assume that the excess energy acquired by the electrons in the electric field is lost to the atoms of the conductor when the electrons and atoms collide. The energy given up to the atoms increases their vibrational energy, and this causes the temperature of the conductor to increase. The temperature increase of a conductor due to resistance is utilized in electric toasters and other familiar appliances.

We are now in a position to derive an expression for the drift velocity. When a free electron of mass $m_e$ and charge $q\ (= -e)$ is subjected to an electric field **E**, it experiences a force $\mathbf{F} = q\mathbf{E}$. Because $\Sigma\mathbf{F} = m_e\mathbf{a}$, we conclude that the acceleration of the electron is

$$\mathbf{a} = \frac{q\mathbf{E}}{m_e} \qquad (27.12)$$

This acceleration, which occurs for only a short time between collisions, enables the electron to acquire a small drift velocity. If $t$ is the time since the last collision and $\mathbf{v}_i$ is the electron's initial velocity the instant after that collision, then the velocity of the electron after a time $t$ is

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \mathbf{v}_i + \frac{q\mathbf{E}}{m_e}t \qquad (27.13)$$

We now take the average value of $\mathbf{v}_f$ over all possible times $t$ and all possible values of $\mathbf{v}_i$. If we assume that the initial velocities are randomly distributed over all possible values, we see that the average value of $\mathbf{v}_i$ is zero. The term $(q\mathbf{E}/m_e)t$ is the velocity added by the field during one trip between atoms. If the electron starts with zero velocity, then the average value of the second term of Equation 27.13 is $(q\mathbf{E}/m_e)\tau$, where $\tau$ is the *average time interval between successive collisions*. Because the average value of $\mathbf{v}_f$ is equal to the drift velocity,[4] we have

Drift velocity

$$\overline{\mathbf{v}}_f = \mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\tau \qquad (27.14)$$

[4] Because the collision process is random, each collision event is *independent* of what happened earlier. This is analogous to the random process of throwing a die. The probability of rolling a particular number on one throw is independent of the result of the previous throw. On average, the particular number comes up every sixth throw, starting at any arbitrary time.

We can relate this expression for drift velocity to the current in the conductor. Substituting Equation 27.14 into Equation 27.6, we find that the magnitude of the current density is

Current density

$$J = nqv_d = \frac{nq^2E}{m_e}\tau \tag{27.15}$$

where $n$ is the number of charge carriers per unit volume. Comparing this expression with Ohm's law, $J = \sigma E$, we obtain the following relationships for conductivity and resistivity:

Conductivity

$$\sigma = \frac{nq^2\tau}{m_e} \tag{27.16}$$

Resistivity

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau} \tag{27.17}$$

According to this classical model, conductivity and resistivity do not depend on the strength of the electric field. This feature is characteristic of a conductor obeying Ohm's law.

The average time between collisions $\tau$ is related to the average distance between collisions $\ell$ (that is, the *mean free path;* see Section 21.7) and the average speed $\bar{v}$ through the expression

$$\tau = \frac{\ell}{\bar{v}} \tag{27.18}$$

## EXAMPLE 27.5 Electron Collisions in a Wire

(a) Using the data and results from Example 27.1 and the classical model of electron conduction, estimate the average time between collisions for electrons in household copper wiring.

**Solution** From Equation 27.17, we see that

$$\tau = \frac{m_e}{nq^2\rho}$$

where $\rho = 1.7 \times 10^{-8}\ \Omega\cdot\text{m}$ for copper and the carrier density is $n = 8.49 \times 10^{28}$ electrons/m$^3$ for the wire described in Example 27.1. Substitution of these values into the expression above gives

$$\tau = \frac{(9.11 \times 10^{-31}\text{ kg})}{(8.49 \times 10^{28}\text{ m}^{-3})(1.6 \times 10^{-19}\text{ C})^2(1.7 \times 10^{-8}\ \Omega\cdot\text{m})}$$

$$= \boxed{2.5 \times 10^{-14}\text{ s}}$$

(b) Assuming that the average speed for free electrons in copper is $1.6 \times 10^6$ m/s and using the result from part (a), calculate the mean free path for electrons in copper.

**Solution**

$$\ell = \bar{v}\tau = (1.6 \times 10^6\text{ m/s})(2.5 \times 10^{-14}\text{ s})$$

$$= \boxed{4.0 \times 10^{-8}\text{ m}}$$

which is equivalent to 40 nm (compared with atomic spacings of about 0.2 nm). Thus, although the time between collisions is very short, an electron in the wire travels about 200 atomic spacings between collisions.

Although this classical model of conduction is consistent with Ohm's law, it is not satisfactory for explaining some important phenomena. For example, classical values for $\bar{v}$ calculated on the basis of an ideal-gas model (see Section 21.6) are smaller than the true values by about a factor of ten. Furthermore, if we substitute $\ell/\bar{v}$ for $\tau$ in Equation 27.17 and rearrange terms so that $\bar{v}$ appears in the numerator, we find that the resistivity $\rho$ is proportional to $\bar{v}$. According to the ideal-gas model, $\bar{v}$ is proportional to $\sqrt{T}$; hence, it should also be true that $\rho \propto \sqrt{T}$. This is in disagreement with the fact that, for pure metals, resistivity depends linearly on temperature. We are able to account for the linear dependence only by using a quantum mechanical model, which we now describe briefly.

According to quantum mechanics, electrons have wave-like properties. If the array of atoms in a conductor is regularly spaced (that is, it is periodic), then the wave-like character of the electrons enables them to move freely through the conductor, and a collision with an atom is unlikely. For an idealized conductor, no collisions would occur, the mean free path would be infinite, and the resistivity would be zero. Electron waves are scattered only if the atomic arrangement is irregular (not periodic) as a result of, for example, structural defects or impurities. At low temperatures, the resistivity of metals is dominated by scattering caused by collisions between electrons and defects or impurities. At high temperatures, the resistivity is dominated by scattering caused by collisions between electrons and atoms of the conductor, which are continuously displaced from the regularly spaced array as a result of thermal agitation. The thermal motion of the atoms causes the structure to be irregular (compared with an atomic array at rest), thereby reducing the electron's mean free path.

## 27.4 RESISTANCE AND TEMPERATURE

Over a limited temperature range, the resistivity of a metal varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad \text{(27.19)}$$

Variation of $\rho$ with temperature

where $\rho$ is the resistivity at some temperature $T$ (in degrees Celsius), $\rho_0$ is the resistivity at some reference temperature $T_0$ (usually taken to be 20°C), and $\alpha$ is the **temperature coefficient of resistivity.** From Equation 27.19, we see that the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \quad \text{(27.20)}$$

Temperature coefficient of resistivity

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The temperature coefficients of resistivity for various materials are given in Table 27.1. Note that the unit for $\alpha$ is degrees Celsius$^{-1}$ $[(°\text{C})^{-1}]$. Because resistance is proportional to resistivity (Eq. 27.11), we can write the variation of resistance as

$$R = R_0[1 + \alpha(T - T_0)] \quad \text{(27.21)}$$

Use of this property enables us to make precise temperature measurements, as shown in the following example.

### EXAMPLE 27.6 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

**Solution** Solving Equation 27.21 for $\Delta T$ and using the $\alpha$ value for platinum given in Table 27.1, we obtain

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (°\text{C})^{-1}](50.0\ \Omega)} = 137°\text{C}$$

Because $T_0 = 20.0°\text{C}$, we find that $T$, the temperature of the melting indium sample, is 157°C.

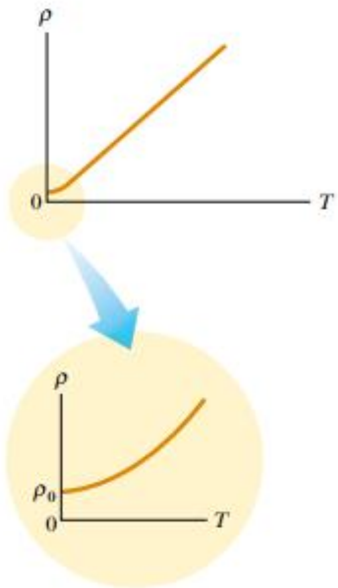



**Figure 27.10** Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures, and $\rho$ increases with increasing temperature. As $T$ approaches absolute zero (inset), the resistivity approaches a finite value $\rho_0$.

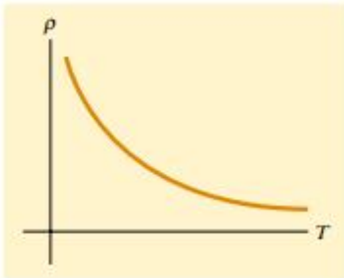



**Figure 27.11** Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.

For metals like copper, resistivity is nearly proportional to temperature, as shown in Figure 27.10. However, a nonlinear region always exists at very low temperatures, and the resistivity usually approaches some finite value as the temperature nears absolute zero. This residual resistivity near absolute zero is caused primarily by the collision of electrons with impurities and imperfections in the metal. In contrast, high-temperature resistivity (the linear region) is predominantly characterized by collisions between electrons and metal atoms.

Notice that three of the $\alpha$ values in Table 27.1 are negative; this indicates that the resistivity of these materials decreases with increasing temperature (Fig. 27.11). This behavior is due to an increase in the density of charge carriers at higher temperatures.

Because the charge carriers in a semiconductor are often associated with impurity atoms, the resistivity of these materials is very sensitive to the type and concentration of such impurities. We shall return to the study of semiconductors in Chapter 43 of the extended version of this text.

### Quick Quiz 27.5

When does a lightbulb carry more current—just after it is turned on and the glow of the metal filament is increasing, or after it has been on for a few milliseconds and the glow is steady?

*Optional Section*

## 27.5 SUPERCONDUCTORS

There is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature $T_c$, known as the *critical temperature*. These materials are known as **superconductors.** The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures above $T_c$ (Fig. 27.12). When the temperature is at or below $T_c$, the resistivity drops suddenly to zero. This phenomenon was discovered in 1911 by the Dutch physicist Heike Kamerlingh-Onnes (1853–1926) as he worked with mercury, which is a superconductor below 4.2 K. Recent measurements have shown that the resistivities of superconductors below their $T_c$ values are less than $4 \times 10^{-25}\ \Omega \cdot \text{m}$—around $10^{17}$ times smaller than the resistivity of copper and in practice considered to be zero.

Today thousands of superconductors are known, and as Figure 27.13 illustrates, the critical temperatures of recently discovered superconductors are substantially higher than initially thought possible. Two kinds of superconductors are recognized. The more recently identified ones, such as $YBa_2Cu_3O_7$, are essentially ceramics with high critical temperatures, whereas superconducting materials such

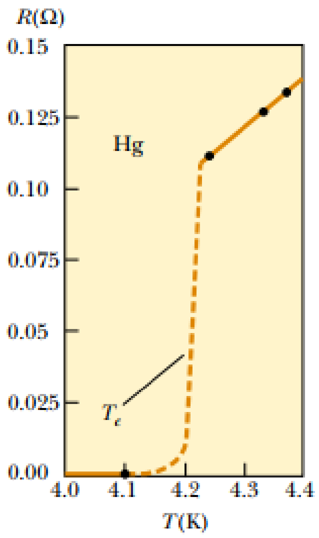



**Figure 27.12** Resistance versus temperature for a sample of mercury (Hg). The graph follows that of a normal metal above the critical temperature $T_c$. The resistance drops to zero at $T_c$, which is 4.2 K for mercury.

A small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$, which is at 77 K.

as those observed by Kamerlingh-Onnes are metals. If a room-temperature superconductor is ever identified, its impact on technology could be tremendous.

The value of $T_c$ is sensitive to chemical composition, pressure, and molecular structure. It is interesting to note that copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

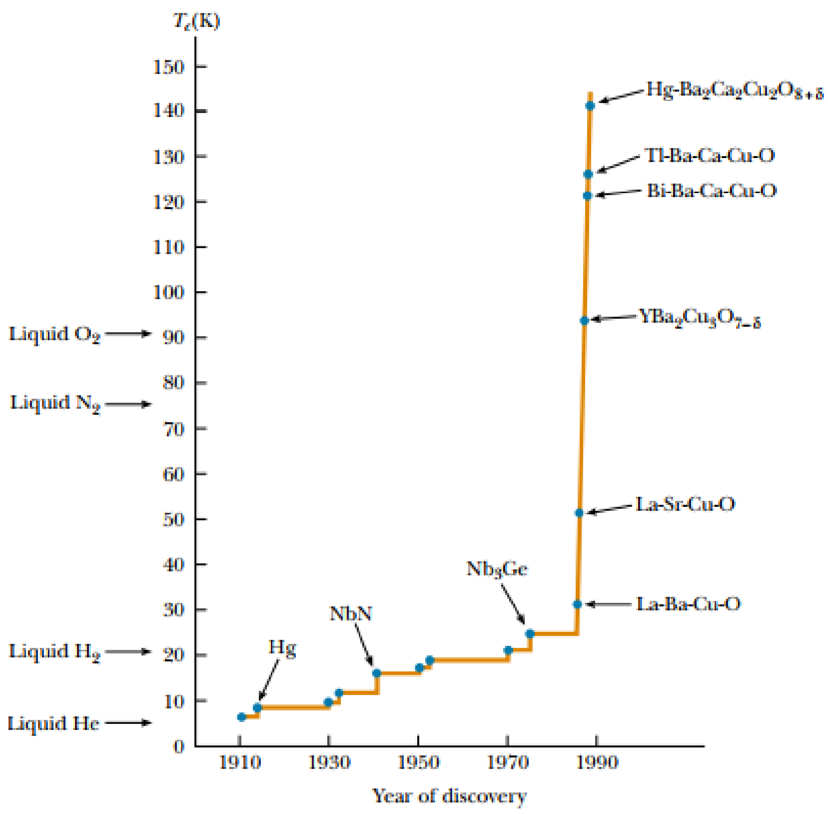



**Figure 27.13** Evolution of the superconducting critical temperature since the discovery of the phenomenon.

One of the truly remarkable features of superconductors is that once a current is set up in them, it persists *without any applied potential difference* (because $R = 0$). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important and useful application of superconductivity is in the development of superconducting magnets, in which the magnitudes of the magnetic field are about ten times greater than those produced by the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. Superconducting magnets are currently used in medical magnetic resonance imaging (MRI) units, which produce high-quality images of internal organs without the need for excessive exposure of patients to x-rays or other harmful radiation.

For further information on superconductivity, see Section 43.8.

## 27.6 ELECTRICAL ENERGY AND POWER

If a battery is used to establish an electric current in a conductor, the chemical energy stored in the battery is continuously transformed into kinetic energy of the charge carriers. In the conductor, this kinetic energy is quickly lost as a result of collisions between the charge carriers and the atoms making up the conductor, and this leads to an increase in the temperature of the conductor. In other words, the chemical energy stored in the battery is continuously transformed to internal energy associated with the temperature of the conductor.

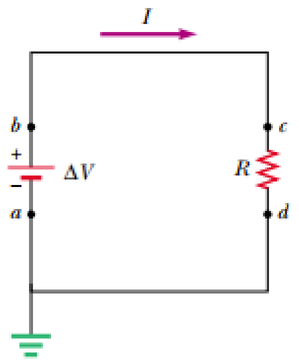
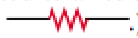
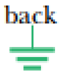



**Figure 27.14** A circuit consisting of a resistor of resistance $R$ and a battery having a potential difference $\Delta V$ across its terminals. Positive charge flows in the clockwise direction. Points $a$ and $d$ are grounded.

Consider a simple circuit consisting of a battery whose terminals are connected to a resistor, as shown in Figure 27.14. (Resistors are designated by the symbol .) Now imagine following a positive quantity of charge $\Delta Q$ that is moving clockwise around the circuit from point $a$ through the battery and resistor back to point $a$. Points $a$ and $d$ are *grounded* (ground is designated by the symbol ); that is, we take the electric potential at these two points to be zero. As the charge moves from $a$ to $b$ through the battery, its electric potential energy $U$ *increases* by an amount $\Delta V \Delta Q$ (where $\Delta V$ is the potential difference between $b$ and $a$), while the chemical potential energy in the battery *decreases* by the same amount. (Recall from Eq. 25.9 that $\Delta U = q\Delta V$.) However, as the charge moves from $c$ to $d$ through the resistor, it *loses* this electric potential energy as it collides with atoms in the resistor, thereby producing internal energy. If we neglect the resistance of the connecting wires, no loss in energy occurs for paths $bc$ and $da$. When the charge arrives at point $a$, it must have the same electric potential energy (zero) that it had at the start.[5] Note that because charge cannot build up at any point, the current is the same everywhere in the circuit.

The rate at which the charge $\Delta Q$ loses potential energy in going through the resistor is

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta V = I\Delta V$$

where $I$ is the current in the circuit. In contrast, the charge regains this energy when it passes through the battery. Because the rate at which the charge loses energy equals the power $\mathcal{P}$ delivered to the resistor (which appears as internal energy), we have

Power

$$\mathcal{P} = I\Delta V \tag{27.22}$$

[5] Note that once the current reaches its steady-state value, there is *no* change in the kinetic energy of the charge carriers creating the current.

In this case, the power is supplied to a resistor by a battery. However, we can use Equation 27.22 to determine the power transferred to *any* device carrying a current $I$ and having a potential difference $\Delta V$ between its terminals.

Using Equation 27.22 and the fact that $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \tag{27.23}$$

Power delivered to a resistor

When $I$ is expressed in amperes, $\Delta V$ in volts, and $R$ in ohms, the SI unit of power is the watt, as it was in Chapter 7 in our discussion of mechanical power. The power lost as internal energy in a conductor of resistance $R$ is called *joule heating*[6]; this transformation is also often referred to as an $I^2R$ loss.

A battery, a device that supplies electrical energy, is called either a *source of electromotive force* or, more commonly, an *emf source.* The concept of emf is discussed in greater detail in Chapter 28. (The phrase *electromotive force* is an unfortunate choice because it describes not a force but rather a potential difference in volts.) **When the internal resistance of the battery is neglected, the potential difference between points $a$ and $b$ in Figure 27.14 is equal to the emf $\mathcal{E}$ of the battery**—that is, $\Delta V = V_b - V_a = \mathcal{E}$. This being true, we can state that the current in the circuit is $I = \Delta V/R = \mathcal{E}/R$. Because $\Delta V = \mathcal{E}$, the power supplied by the emf source can be expressed as $\mathcal{P} = I\mathcal{E}$, which equals the power delivered to the resistor, $I^2R$.

When transporting electrical energy through power lines, such as those shown in Figure 27.15, utility companies seek to minimize the power transformed to internal energy in the lines and maximize the energy delivered to the consumer. Because $\mathcal{P} = I\Delta V$, the same amount of power can be transported either at high currents and low potential differences or at low currents and high potential differences. Utility companies choose to transport electrical energy at low currents and high potential differences primarily for economic reasons. Copper wire is very expensive, and so it is cheaper to use high-resistance wire (that is, wire having a small cross-sectional area; see Eq. 27.11). Thus, in the expression for the power delivered to a resistor, $\mathcal{P} = I^2R$, the resistance of the wire is fixed at a relatively high value for economic considerations. The $I^2R$ loss can be reduced by keeping the current $I$ as low as possible. In some instances, power is transported at potential differences as great as 765 kV. Once the electricity reaches your city, the potential difference is usually reduced to 4 kV by a device called a *transformer.* Another transformer drops the potential difference to 240 V before the electricity finally reaches your home. Of course, each time the potential difference decreases, the current increases by the same factor, and the power remains the same. We shall discuss transformers in greater detail in Chapter 33.

***Figure 27.15*** Power companies transfer electrical energy at high potential differences.

## Quick Quiz 27.6

The same potential difference is applied to the two lightbulbs shown in Figure 27.16. Which one of the following statements is true?

(a) The 30-W bulb carries the greater current and has the higher resistance.
(b) The 30-W bulb carries the greater current, but the 60-W bulb has the higher resistance.

## QuickLab

If you have access to an ohmmeter, verify your answer to Quick Quiz 27.6 by testing the resistance of a few lightbulbs.

[6] It is called *joule heating* even though the process of heat does not occur. This is another example of incorrect usage of the word *heat* that has become entrenched in our language.

**Figure 27.16** These lightbulbs operate at their rated power only when they are connected to a 120-V source.

(c) The 30-W bulb has the higher resistance, but the 60-W bulb carries the greater current.
(d) The 60-W bulb carries the greater current and has the higher resistance.

**QuickLab**

From the labels on household appliances such as hair dryers, televisions, and stereos, estimate the annual cost of operating them.

**Quick Quiz 27.7**

For the two lightbulbs shown in Figure 27.17, rank the current values at points *a* through *f*, from greatest to least.

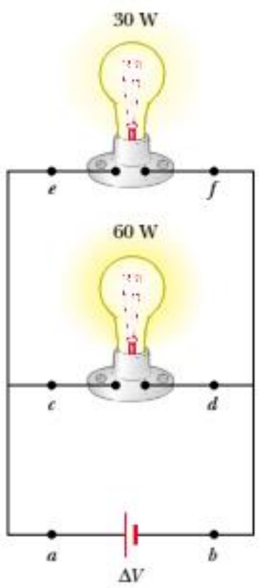



**Figure 27.17** Two lightbulbs connected across the same potential difference. The bulbs operate at their rated power only if they are connected to a 120-V battery.

### EXAMPLE 27.7 Power in an Electric Heater

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

**Solution** Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{8.00\ \Omega} = \boxed{15.0\text{ A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\text{ A})^2(8.00\ \Omega) = \boxed{1.80\text{ kW}}$$

If we doubled the applied potential difference, the current would double but the power would quadruple because $\mathcal{P} = (\Delta V)^2/R$.

### EXAMPLE 27.8 The Cost of Making Dinner

Estimate the cost of cooking a turkey for 4 h in an oven that operates continuously at 20.0 A and 240 V.

**Solution** The power used by the oven is

$$\mathcal{P} = I\Delta V = (20.0\ \text{A})(240\ \text{V}) = 4\,800\ \text{W} = 4.80\ \text{kW}$$

Because the energy consumed equals power × time, the amount of energy for which you must pay is

$$\text{Energy} = \mathcal{P}t = (4.80\ \text{kW})(4\ \text{h}) = 19.2\ \text{kWh}$$

If the energy is purchased at an estimated price of 8.00¢ per kilowatt hour, the cost is

$$\text{Cost} = (19.2\ \text{kWh})(\$0.080/\text{kWh}) = \boxed{\$1.54}$$

Demands on our dwindling energy supplies have made it necessary for us to be aware of the energy requirements of our electrical devices. Every electrical appliance carries a label that contains the information you need to calculate the appliance's power requirements. In many cases, the power consumption in watts is stated directly, as it is on a lightbulb. In other cases, the amount of current used by the device and the potential difference at which it operates are given. This information and Equation 27.22 are sufficient for calculating the operating cost of any electrical device.

***Exercise*** What does it cost to operate a 100-W lightbulb for 24 h if the power company charges \$0.08/kWh?

***Answer*** \$0.19.

### EXAMPLE 27.9 Current in an Electron Beam

In a certain particle accelerator, electrons emerge with an energy of 40.0 MeV ($1\ \text{MeV} = 1.60 \times 10^{-13}\ \text{J}$). The electrons emerge not in a steady stream but rather in pulses at the rate of 250 pulses/s. This corresponds to a time between pulses of 4.00 ms (Fig. 27.18). Each pulse has a duration of 200 ns, and the electrons in the pulse constitute a current of 250 mA. The current is zero between pulses. (a) How many electrons are delivered by the accelerator per pulse?

**Solution** We use Equation 27.2 in the form $dQ = I\,dt$ and integrate to find the charge per pulse. While the pulse is on, the current is constant; thus,

$$Q_{\text{pulse}} = I\int dt = I\Delta t = (250 \times 10^{-3}\ \text{A})(200 \times 10^{-9}\ \text{s})$$
$$= 5.00 \times 10^{-8}\ \text{C}$$

Dividing this quantity of charge per pulse by the electronic charge gives the number of electrons per pulse:

$$\text{Electrons per pulse} = \frac{5.00 \times 10^{-8}\ \text{C/pulse}}{1.60 \times 10^{-19}\ \text{C/electron}}$$
$$= \boxed{3.13 \times 10^{11}\ \text{electrons/pulse}}$$

(b) What is the average current per pulse delivered by the accelerator?

**Solution** Average current is given by Equation 27.1, $I_{\text{av}} = \Delta Q/\Delta t$. Because the time interval between pulses is 4.00 ms, and because we know the charge per pulse from part (a), we obtain

$$I_{\text{av}} = \frac{Q_{\text{pulse}}}{\Delta t} = \frac{5.00 \times 10^{-8}\ \text{C}}{4.00 \times 10^{-3}\ \text{s}} = \boxed{12.5\ \mu\text{A}}$$

This represents only 0.005% of the peak current, which is 250 mA.

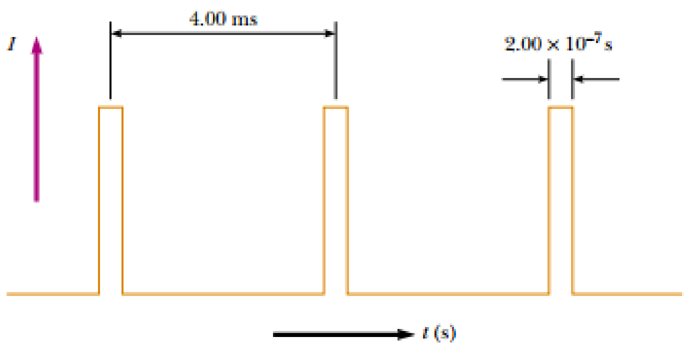



***Figure 27.18*** Current versus time for a pulsed beam of electrons.

(c) What is the maximum power delivered by the electron beam?

**Solution** By definition, power is energy delivered per unit time. Thus, the maximum power is equal to the energy delivered by a pulse divided by the pulse duration:

$$\mathcal{P} = \frac{E}{\Delta t}$$

$$= \frac{(3.13 \times 10^{11} \text{ electrons/pulse})(40.0 \text{ MeV/electron})}{2.00 \times 10^{-7} \text{ s/pulse}}$$

$$= (6.26 \times 10^{19} \text{ MeV/s})(1.60 \times 10^{-13} \text{ J/MeV})$$

$$= 1.00 \times 10^{7} \text{ W} = \boxed{10.0 \text{ MW}}$$

We could also compute this power directly. We assume that each electron had zero energy before being accelerated. Thus, by definition, each electron must have gone through a potential difference of 40.0 MV to acquire a final energy of 40.0 MeV. Hence, we have

$$\mathcal{P} = I\Delta V = (250 \times 10^{-3} \text{ A})(40.0 \times 10^{6} \text{ V}) = \boxed{10.0 \text{ MW}}$$

## SUMMARY

The **electric current** $I$ in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \qquad \textbf{(27.2)}$$

where $dQ$ is the charge that passes through a cross-section of the conductor in a time $dt$. The SI unit of current is the **ampere** (A), where 1 A = 1 C/s.

The average current in a conductor is related to the motion of the charge carriers through the relationship

$$I_{\text{av}} = nqv_d A \qquad \textbf{(27.4)}$$

where $n$ is the density of charge carriers, $q$ is the charge on each carrier, $v_d$ is the drift speed, and $A$ is the cross-sectional area of the conductor.

The magnitude of the **current density** $J$ in a conductor is the current per unit area:

$$J \equiv \frac{I}{A} = nqv_d \qquad \textbf{(27.5)}$$

The current density in a conductor is proportional to the electric field according to the expression

$$\mathbf{J} = \sigma\mathbf{E} \qquad \textbf{(27.7)}$$

The proportionality constant $\sigma$ is called the **conductivity** of the material of which the conductor is made. The inverse of $\sigma$ is known as **resistivity** $\rho$ ($\rho = 1/\sigma$). Equation 27.7 is known as **Ohm's law,** and a material is said to obey this law if the ratio of its current density $\mathbf{J}$ to its applied electric field $\mathbf{E}$ is a constant that is independent of the applied field.

The **resistance** $R$ of a conductor is defined either in terms of the length of the conductor or in terms of the potential difference across it:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \qquad \textbf{(27.8)}$$

where $\ell$ is the length of the conductor, $\sigma$ is the conductivity of the material of which it is made, $A$ is its cross-sectional area, $\Delta V$ is the potential difference across it, and $I$ is the current it carries.

The SI unit of resistance is volts per ampere, which is defined to be 1 **ohm** ($\Omega$); that is, $1\ \Omega = 1$ V/A. If the resistance is independent of the applied potential difference, the conductor obeys Ohm's law.

In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a **drift velocity** $\mathbf{v}_d$ that is opposite the electric field and given by the expression

$$\mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\tau \tag{27.14}$$

where $\tau$ is the average time between electron–atom collisions, $m_e$ is the mass of the electron, and $q$ is its charge. According to this model, the resistivity of the metal is

$$\rho = \frac{m_e}{nq^2\tau} \tag{27.17}$$

where $n$ is the number of free electrons per unit volume.

The resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \tag{27.19}$$

where $\alpha$ is the **temperature coefficient of resistivity** and $\rho_0$ is the resistivity at some reference temperature $T_0$.

If a potential difference $\Delta V$ is maintained across a resistor, the **power,** or rate at which energy is supplied to the resistor, is

$$\mathcal{P} = I\Delta V \tag{27.22}$$

Because the potential difference across a resistor is given by $\Delta V = IR$, we can express the power delivered to a resistor in the form

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \tag{27.23}$$

The electrical energy supplied to a resistor appears in the form of internal energy in the resistor.