PUZZLER

You can estimate the distance to an approaching storm by listening carefully to the sound of the thunder. How is this done? Why is the sound that follows a lightning strike sometimes a short, sharp thunderclap and other times a long-lasting rumble? *(Richard Kaylin/Tony Stone Images)*

chapter

17

Sound Waves

Chapter Outline

Sound waves are the most important example of longitudinal waves. They can travel through any material medium with a speed that depends on the properties of the medium. As the waves travel, the particles in the medium vibrate to produce changes in density and pressure along the direction of motion of the wave. These changes result in a series of high-pressure and low-pressure regions. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. We shall find that the mathematical description of sinusoidal sound waves is identical to that of sinusoidal string waves, which was discussed in the previous chapter.

Sound waves are divided into three categories that cover different frequency ranges. (1) *Audible waves* are waves that lie within the range of sensitivity of the human ear. They can be generated in a variety of ways, such as by musical instruments, human vocal cords, and loudspeakers. (2) *Infrasonic waves* are waves having frequencies below the audible range. Elephants can use infrasonic waves to communicate with each other, even when separated by many kilometers. (3) *Ultrasonic waves* are waves having frequencies above the audible range. You may have used a "silent" whistle to retrieve your dog. The ultrasonic sound it emits is easily heard by dogs, although humans cannot detect it at all. Ultrasonic waves are also used in medical imaging.

We begin this chapter by discussing the speed of sound waves and then wave intensity, which is a function of wave amplitude. We then provide an alternative description of the intensity of sound waves that compresses the wide range of intensities to which the ear is sensitive to a smaller range. Finally, we treat effects of the motion of sources and/or listeners.

17.1 SPEED OF SOUND WAVES

Let us describe pictorially the motion of a one-dimensional longitudinal pulse moving through a long tube containing a compressible gas (Fig. 17.1). A piston at the left end can be moved to the right to compress the gas and create the pulse. Before the piston is moved, the gas is undisturbed and of uniform density, as represented by the uniformly shaded region in Figure 17.1a. When the piston is suddenly pushed to the right (Fig. 17.1b), the gas just in front of it is compressed (as represented by the more heavily shaded region); the pressure and density in this region are now higher than they were before the piston moved. When the piston comes to rest (Fig. 17.1c), the compressed region of the gas continues to move to the right, corresponding to a longitudinal pulse traveling through the tube with

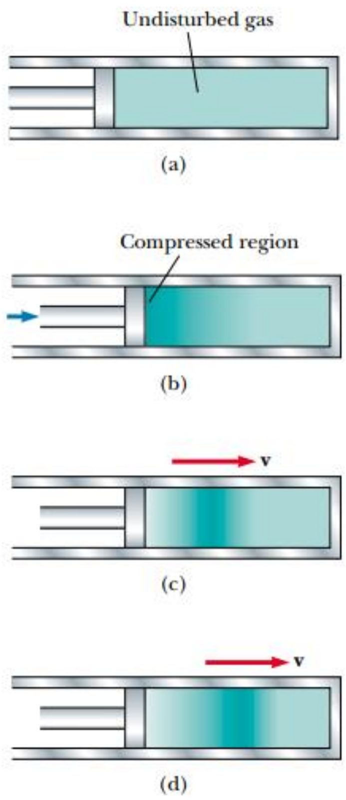


Figure 17.1 Motion of a longitudinal pulse through a compressible gas. The compression (darker region) is produced by the moving piston.

An ultrasound image of a human fetus in the womb after 20 weeks of development, showing the head, body, arms, and legs in profile.

speed v. Note that the piston speed does *not* equal v. Furthermore, the compressed region does not "stay with" the piston as the piston moves, because the speed of the wave may be greater than the speed of the piston.

The speed of sound waves depends on the compressibility and inertia of the medium. If the medium has a bulk modulus B (see Section 12.4) and density ρ, the speed of sound waves in that medium is

$$v = \sqrt{\frac{B}{\rho}} \tag{17.1}$$

Speed of sound

It is interesting to compare this expression with Equation 16.4 for the speed of transverse waves on a string, $v = \sqrt{T/\mu}$. In both cases, the wave speed depends on an elastic property of the medium—bulk modulus B or string tension T—and on an inertial property of the medium—ρ or μ. In fact, the speed of *all mechanical waves* follows an expression of the general form

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

The speed of sound also depends on the temperature of the medium. For sound traveling through air, the relationship between wave speed and medium temperature is

$$v = (331 \text{ m/s}) \sqrt{1 + \frac{T_C}{273°C}}$$

where 331 m/s is the speed of sound in air at 0°C, and T_C is the temperature in degrees Celsius. Using this equation, one finds that at 20°C the speed of sound in air is approximately 343 m/s.

This information provides a convenient way to estimate the distance to a thunderstorm, as demonstrated in the QuickLab. During a lightning flash, the temperature of a long channel of air rises rapidly as the bolt passes through it. This temperature increase causes the air in the channel to expand rapidly, and this expansion creates a sound wave. The channel produces sound throughout its entire length at essentially the same instant. If the orientation of the channel is such that all of its parts are approximately the same distance from you, sounds from the different parts reach you at the same time, and you hear a short, intense thunderclap. However, if the distances between your ear and different portions of the channel vary, sounds from different portions arrive at your ears at different times. If the channel were a straight line, the resulting sound would be a steady roar, but the zigzag shape of the path produces variations in loudness.

QuickLab

The next time a thunderstorm approaches, count the seconds between a flash of lightning (which reaches you almost instantaneously) and the following thunderclap. Divide this time by 3 to determine the approximate number of kilometers (or by 5 to estimate the miles) to the storm.

To learn more about lightning, read E. Williams, "The Electrification of Thunderstorms" *Sci. Am.* 259(5):88–89, 1988.

Quick Quiz 17.1

The speed of sound in air is a function of (a) wavelength, (b) frequency, (c) temperature, (d) amplitude.

Quick Quiz 17.2

As a result of a distant explosion, an observer first senses a ground tremor and then hears the explosion later. Explain.

EXAMPLE 17.1 Speed of Sound in a Solid

If a solid bar is struck at one end with a hammer, a longitudinal pulse propagates down the bar with a speed $v = \sqrt{Y/\rho}$, where Y is the Young's modulus for the material (see Section 12.4). Find the speed of sound in an aluminum bar.

Solution From Table 12.1 we obtain $Y = 7.0 \times 10^{10}\ \text{N/m}^2$ for aluminum, and from Table 1.5 we obtain $\rho = 2.70 \times 10^3\ \text{kg/m}^3$. Therefore,

$$v_{\text{Al}} = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.0 \times 10^{10}\ \text{N/m}^2}{2.70 \times 10^3\ \text{kg/m}^3}} \approx \boxed{5.1\ \text{km/s}}$$

This typical value for the speed of sound in solids is much greater than the speed of sound in gases, as Table 17.1 shows. This difference in speeds makes sense because the molecules of a solid are bound together into a much more rigid structure than those in a gas and hence respond more rapidly to a disturbance.

EXAMPLE 17.2 Speed of Sound in a Liquid

(a) Find the speed of sound in water, which has a bulk modulus of $2.1 \times 10^9\ \text{N/m}^2$ and a density of $1.00 \times 10^3\ \text{kg/m}^3$.

Solution Using Equation 17.1, we find that

$$v_{\text{water}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \times 10^9\ \text{N/m}^2}{1.00 \times 10^3\ \text{kg/m}^3}} = \boxed{1.4\ \text{km/s}}$$

In general, sound waves travel more slowly in liquids than in solids because liquids are more compressible than solids.

(b) Dolphins use sound waves to locate food. Experiments have shown that a dolphin can detect a 7.5-cm target 110 m away, even in murky water. For a bit of "dinner" at that distance, how much time passes between the moment the dolphin emits a sound pulse and the moment the dolphin hears its reflection and thereby detects the distant target?

Solution The total distance covered by the sound wave as it travels from dolphin to target and back is $2 \times 110\ \text{m} = 220\ \text{m}$. From Equation 2.2, we have

$$\Delta t = \frac{\Delta x}{v_x} = \frac{220\ \text{m}}{1\,400\ \text{m/s}} = \boxed{0.16\ \text{s}}$$

Bottle-nosed dolphin. *(Stuart Westmoreland/Tony Stone Images)*

17.2 PERIODIC SOUND WAVES

This section will help you better comprehend the nature of sound waves. You will learn that pressure variations control what we hear—an important fact for understanding how our ears work.

One can produce a one-dimensional periodic sound wave in a long, narrow tube containing a gas by means of an oscillating piston at one end, as shown in Figure 17.2. The darker parts of the colored areas in this figure represent regions where the gas is compressed and thus the density and pressure are above their equilibrium values. A compressed region is formed whenever the piston is pushed into the tube. This compressed region, called a **condensation,** moves through the tube as a pulse, continuously compressing the region just in front of itself. When the piston is pulled back, the gas in front of it expands, and the pressure and density in this region fall below their equilibrium values (represented by the lighter parts of the colored areas in Fig. 17.2). These low-pressure regions, called **rarefactions,** also propagate along the tube, following the condensations. Both regions move with a speed equal to the speed of sound in the medium.

As the piston oscillates sinusoidally, regions of condensation and rarefaction are continuously set up. The distance between two successive condensations (or two successive rarefactions) equals the wavelength λ. As these regions travel through the tube, any small volume of the medium moves with simple harmonic motion parallel to the direction of the wave. If $s(x, t)$ is the displacement of a small volume element from its equilibrium position, we can express this harmonic displacement function as

$$s(x, t) = s_{\max} \cos(kx - \omega t) \tag{17.2}$$

where **$s_{\max}$ is the maximum displacement of the medium from equilibrium** (in other words, the **displacement amplitude** of the wave), k is the angular wavenumber, and ω is the angular frequency of the piston. Note that the displacement of the medium is along x, in the direction of motion of the sound wave, which means we are describing a longitudinal wave.

As we shall demonstrate shortly, the variation in the gas pressure ΔP, measured from the equilibrium value, is also periodic and for the displacement function in Equation 17.2 is given by

$$\Delta P = \Delta P_{\max} \sin(kx - \omega t) \tag{17.3}$$

where **the pressure amplitude $\Delta P_{\max}$**—which is the **maximum change in pres-**

TABLE 17.1
Speeds of Sound in Various Media

Medium	v (m/s)
Gases	
Hydrogen (0°C)	1 286
Helium (0°C)	972
Air (20°C)	343
Air (0°C)	331
Oxygen (0°C)	317
Liquids at 25°C	
Glycerol	1 904
Sea water	1 533
Water	1 493
Mercury	1 450
Kerosene	1 324
Methyl alcohol	1 143
Carbon tetrachloride	926
Solids	
Diamond	12 000
Pyrex glass	5 640
Iron	5 130
Aluminum	5 100
Brass	4 700
Copper	3 560
Gold	3 240
Lucite	2 680
Lead	1 322
Rubber	1 600

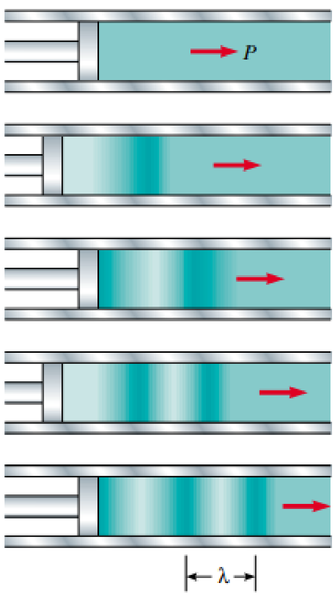


Figure 17.2 A sinusoidal longitudinal wave propagating through a gas-filled tube. The source of the wave is a sinusoidally oscillating piston at the left. The high-pressure and low-pressure regions are colored darkly and lightly, respectively.

sure from the equilibrium value—is given by

Pressure amplitude

$$\Delta P_{\max} = \rho v \omega s_{\max} \tag{17.4}$$

Thus, we see that a sound wave may be considered as either a displacement wave or a pressure wave. A comparison of Equations 17.2 and 17.3 shows that **the pressure wave is 90° out of phase with the displacement wave.** Graphs of these functions are shown in Figure 17.3. Note that the pressure variation is a maximum when the displacement is zero, and the displacement is a maximum when the pressure variation is zero.

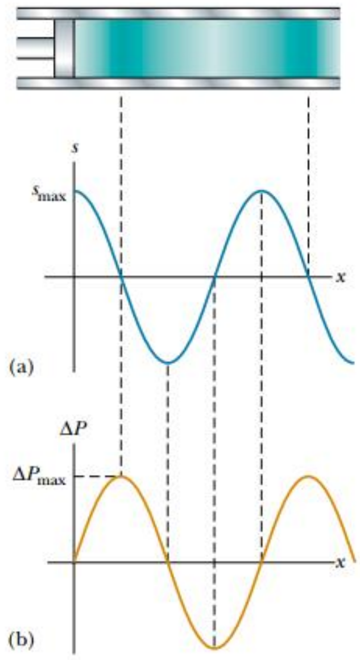


Figure 17.3 (a) Displacement amplitude versus position and (b) pressure amplitude versus position for a sinusoidal longitudinal wave. The displacement wave is 90° out of phase with the pressure wave.

Quick Quiz 17.3

If you blow across the top of an empty soft-drink bottle, a pulse of air travels down the bottle. At the moment the pulse reaches the bottom of the bottle, compare the displacement of air molecules with the pressure variation.

Derivation of Equation 17.3

From the definition of bulk modulus (see Eq. 12.8), the pressure variation in the gas is

$$\Delta P = -B\frac{\Delta V}{V_i}$$

The volume of gas that has a thickness Δx in the horizontal direction and a cross-sectional area A is $V_i = A\,\Delta x$. The change in volume ΔV accompanying the pressure change is equal to $A\,\Delta s$, where Δs is the difference between the value of s at $x + \Delta x$ and the value of s at x. Hence, we can express ΔP as

$$\Delta P = -B\frac{\Delta V}{V_i} = -B\frac{A\,\Delta s}{A\,\Delta x} = -B\frac{\Delta s}{\Delta x}$$

As Δx approaches zero, the ratio $\Delta s/\Delta x$ becomes $\partial s/\partial x$. (The partial derivative indicates that we are interested in the variation of s with position at a *fixed* time.) Therefore,

$$\Delta P = -B\frac{\partial s}{\partial x}$$

If the displacement is the simple sinusoidal function given by Equation 17.2, we find that

$$\Delta P = -B\frac{\partial}{\partial x}[s_{\max}\cos(kx - \omega t)] = Bks_{\max}\sin(kx - \omega t)$$

Because the bulk modulus is given by $B = \rho v^2$ (see Eq. 17.1), the pressure variation reduces to

$$\Delta P = \rho v^2 k s_{\max}\sin(kx - \omega t)$$

From Equation 16.13, we can write $k = \omega/v$; hence, ΔP can be expressed as

$$\Delta P = \rho v \omega s_{\max}\sin(kx - \omega t)$$

Because the sine function has a maximum value of 1, we see that the maximum value of the pressure variation is $\Delta P_{\max} = \rho v \omega s_{\max}$ (see Eq. 17.4), and we arrive at Equation 17.3:

$$\Delta P = \Delta P_{\max}\sin(kx - \omega t)$$

17.3 INTENSITY OF PERIODIC SOUND WAVES

In the previous chapter, we showed that a wave traveling on a taut string transports energy. The same concept applies to sound waves. Consider a volume of air of mass Δm and width Δx in front of a piston oscillating with a frequency ω, as shown in Figure 17.4. The piston transmits energy to this volume of air in the tube, and the energy is propagated away from the piston by the sound wave.[1] To evaluate the rate of energy transfer for the sound wave, we shall evaluate the kinetic energy of this volume of air, which is undergoing simple harmonic motion. We shall follow a procedure similar to that in Section 16.8, in which we evaluated the rate of energy transfer for a wave on a string.

As the sound wave propagates away from the piston, the displacement of any volume of air in front of the piston is given by Equation 17.2. To evaluate the kinetic energy of this volume of air, we need to know its speed. We find the speed by taking the time derivative of Equation 17.2:

$$v(x, t) = \frac{\partial}{\partial t} s(x, t) = \frac{\partial}{\partial t} [s_{\max} \cos(kx - \omega t)] = \omega s_{\max} \sin(kx - \omega t)$$

Imagine that we take a "snapshot" of the wave at $t = 0$. The kinetic energy of a given volume of air at this time is

$$\begin{aligned} \Delta K &= \tfrac{1}{2}\, \Delta m v^2 = \tfrac{1}{2}\, \Delta m(\omega s_{\max} \sin kx)^2 = \tfrac{1}{2}\rho A\, \Delta x(\omega s_{\max} \sin kx)^2 \\ &= \tfrac{1}{2}\rho A\, \Delta x(\omega s_{\max})^2 \sin^2 kx \end{aligned}$$

where A is the cross-sectional area of the moving air and $A\,\Delta x$ is its volume. Now, as in Section 16.8, we integrate this expression over a full wavelength to find the total kinetic energy in one wavelength. Letting the volume of air shrink to infinitesimal thickness, so that $\Delta x \rightarrow dx$, we have

$$\begin{aligned} K_\lambda = \int dK &= \int_0^\lambda \tfrac{1}{2}\rho A(\omega s_{\max})^2 \sin^2 kx\, dx = \tfrac{1}{2}\rho A(\omega s_{\max})^2 \int_0^\lambda \sin^2 kx\, dx \\ &= \tfrac{1}{2}\rho A(\omega s_{\max})^2 \left(\tfrac{1}{2}\lambda\right) = \tfrac{1}{4}\rho A(\omega s_{\max})^2 \lambda \end{aligned}$$

As in the case of the string wave in Section 16.8, the total potential energy for one wavelength has the same value as the total kinetic energy; thus, the total mechani-

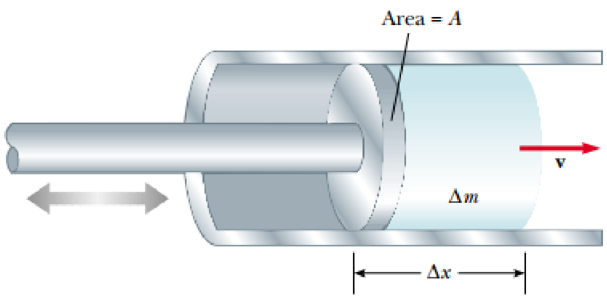


Figure 17.4 An oscillating piston transfers energy to the air in the tube, initially causing the volume of air of width Δx and mass Δm to oscillate with an amplitude $s_{\max}$.

[1] Although it is not proved here, the work done by the piston equals the energy carried away by the wave. For a detailed mathematical treatment of this concept, see Chapter 4 in Frank S. Crawford, Jr., *Waves*, Berkeley Physics Course, vol. 3, New York, McGraw-Hill Book Company, 1968.

cal energy is

$$E_\lambda = K_\lambda + U_\lambda = \tfrac{1}{2}\rho A(\omega s_{\text{max}})^2\lambda$$

As the sound wave moves through the air, this amount of energy passes by a given point during one period of oscillation. Hence, the rate of energy transfer is

$$\mathcal{P} = \frac{E_\lambda}{\Delta t} = \frac{\frac{1}{2}\rho A(\omega s_{\text{max}})^2\lambda}{T} = \tfrac{1}{2}\rho A(\omega s_{\text{max}})^2\left(\frac{\lambda}{T}\right) = \tfrac{1}{2}\rho A v(\omega s_{\text{max}})^2$$

where v is the speed of sound in air.

We define the **intensity *I*** of a wave, or the power per unit area, to be the rate at which the energy being transported by the wave flows through a unit area A perpendicular to the direction of travel of the wave.

In the present case, therefore, the intensity is

Intensity of a sound wave

$$I = \frac{\mathcal{P}}{A} = \tfrac{1}{2}\rho v(\omega s_{\text{max}})^2 \qquad \textbf{(17.5)}$$

Thus, we see that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency (as in the case of a periodic string wave). This can also be written in terms of the pressure amplitude ΔP_{max}; in this case, we use Equation 17.4 to obtain

$$I = \frac{\Delta P_{\text{max}}^2}{2\rho v} \qquad \textbf{(17.6)}$$

EXAMPLE 17.3 Hearing Limits

The faintest sounds the human ear can detect at a frequency of 1 000 Hz correspond to an intensity of about 1.00×10^{-12} W/m²—the so-called *threshold of hearing*. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about 1.00 W/m²—the *threshold of pain*. Determine the pressure amplitude and displacement amplitude associated with these two limits.

Solution First, consider the faintest sounds. Using Equation 17.6 and taking $v = 343$ m/s as the speed of sound waves in air and $\rho = 1.20$ kg/m³ as the density of air, we obtain

$$\begin{aligned}\Delta P_{\text{max}} &= \sqrt{2\rho v I}\\ &= \sqrt{2(1.20\ \text{kg/m}^3)(343\ \text{m/s})(1.00 \times 10^{-12}\ \text{W/m}^2)}\\ &= \boxed{2.87 \times 10^{-5}\ \text{N/m}^2}\end{aligned}$$

Because atmospheric pressure is about 10^5 N/m², this result tells us that the ear can discern pressure fluctuations as small as 3 parts in 10^{10}!

We can calculate the corresponding displacement amplitude by using Equation 17.4, recalling that $\omega = 2\pi f$ (see Eqs. 16.10 and 16.12):

$$\begin{aligned}s_{\text{max}} &= \frac{\Delta P_{\text{max}}}{\rho v \omega} = \frac{2.87 \times 10^{-5}\ \text{N/m}^2}{(1.20\ \text{kg/m}^3)(343\ \text{m/s})(2\pi \times 1\,000\ \text{Hz})}\\ &= \boxed{1.11 \times 10^{-11}\ \text{m}}\end{aligned}$$

This is a remarkably small number! If we compare this result for s_{max} with the diameter of a molecule (about 10^{-10} m), we see that the ear is an extremely sensitive detector of sound waves.

In a similar manner, one finds that the loudest sounds the human ear can tolerate correspond to a pressure amplitude of 28.7 N/m² and a displacement amplitude equal to 1.11×10^{-5} m.

Sound Level in Decibels

The example we just worked illustrates the wide range of intensities the human ear can detect. Because this range is so wide, it is convenient to use a logarithmic scale, where the **sound level** β (Greek letter beta) is defined by the equation

$$\beta = 10 \log\left(\frac{I}{I_0}\right) \quad (17.7)$$

The constant I_0 is the *reference intensity,* taken to be at the threshold of hearing ($I_0 = 1.00 \times 10^{-12}$ W/m^2), and I is the intensity, in watts per square meter, at the sound level β, where β is measured in **decibels** (dB).[2] On this scale, the threshold of pain ($I = 1.00$ W/m^2) corresponds to a sound level of $\beta = 10 \log[(1 \text{ W/m}^2)/(10^{-12} \text{ W/m}^2)] = 10 \log(10^{12}) = 120$ dB, and the threshold of hearing corresponds to $\beta = 10 \log[(10^{-12} \text{ W/m}^2)/(10^{-12} \text{ W/m}^2)] = 0$ dB.

Prolonged exposure to high sound levels may seriously damage the ear. Ear plugs are recommended whenever sound levels exceed 90 dB. Recent evidence suggests that "noise pollution" may be a contributing factor to high blood pressure, anxiety, and nervousness. Table 17.2 gives some typical sound-level values.

TABLE 17.2
Sound Levels

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

EXAMPLE 17.4 Sound Levels

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is 2.0×10^{-7} W/m^2. Find the sound level heard by the worker (a) when one machine is operating and (b) when both machines are operating.

Solution (a) The sound level at the location of the worker with one machine operating is calculated from Equation 17.7:

$$\beta_1 = 10 \log\left(\frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2}\right) = 10 \log(2.0 \times 10^5)$$

$$= \boxed{53 \text{ dB}}$$

(b) When both machines are operating, the intensity is doubled to 4.0×10^{-7} W/m^2; therefore, the sound level now is

$$\beta_2 = 10 \log\left(\frac{4.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2}\right) = 10 \log(4.0 \times 10^5)$$

$$= \boxed{56 \text{ dB}}$$

From these results, we see that when the intensity is doubled, the sound level increases by only 3 dB.

Quick Quiz 17.4

A violin plays a melody line and is then joined by nine other violins, all playing at the same intensity as the first violin, in a repeat of the same melody. (a) When all of the violins are playing together, by how many decibels does the sound level increase? (b) If ten more violins join in, how much has the sound level increased over that for the single violin?

[2] The unit *bel* is named after the inventor of the telephone, Alexander Graham Bell (1847–1922). The prefix *deci-* is the SI prefix that stands for 10^{-1}.

17.4 SPHERICAL AND PLANE WAVES

If a spherical body oscillates so that its radius varies sinusoidally with time, a spherical sound wave is produced (Fig. 17.5). The wave moves outward from the source at a constant speed if the medium is uniform.

Because of this uniformity, we conclude that the energy in a spherical wave propagates equally in all directions. That is, no one direction is preferred over any other. If $\mathcal{P}_{av}$ is the average power emitted by the source, then this power at any distance r from the source must be distributed over a spherical surface of area $4\pi r^2$. Hence, the wave intensity at a distance r from the source is

$$I = \frac{\mathcal{P}_{av}}{A} = \frac{\mathcal{P}_{av}}{4\pi r^2} \qquad \textbf{(17.8)}$$

Because $\mathcal{P}_{av}$ is the same for any spherical surface centered at the source, we see that the intensities at distances r_1 and r_2 are

$$I_1 = \frac{\mathcal{P}_{av}}{4\pi r_1{}^2} \qquad \text{and} \qquad I_2 = \frac{\mathcal{P}_{av}}{4\pi r_2{}^2}$$

Therefore, the ratio of intensities on these two spherical surfaces is

$$\frac{I_1}{I_2} = \frac{r_2{}^2}{r_1{}^2}$$

This inverse-square law states that the intensity decreases in proportion to the square of the distance from the source. Equation 17.5 tells us that the intensity is proportional to s_{max}^2. Setting the right side of Equation 17.5 equal to the right side

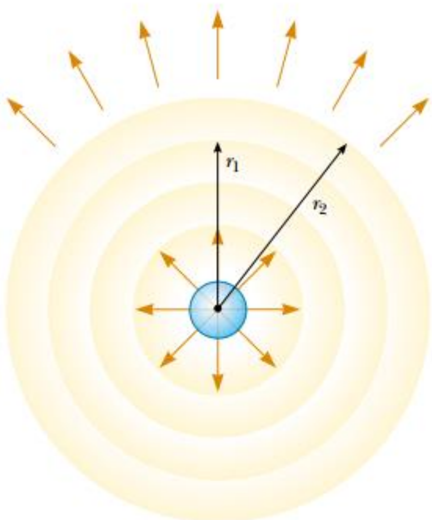


Figure 17.5 A spherical sound wave propagating radially outward from an oscillating spherical body. The intensity of the spherical wave varies as $1/r^2$.

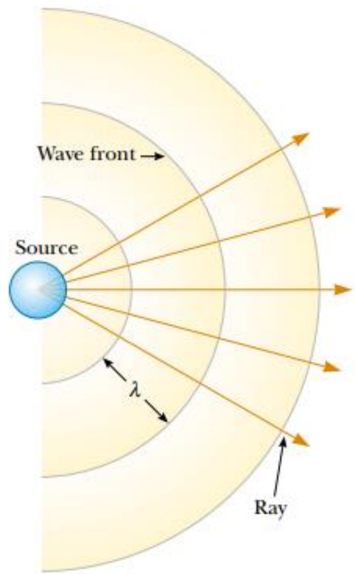


Figure 17.6 Spherical waves emitted by a point source. The circular arcs represent the spherical wave fronts that are concentric with the source. The rays are radial lines pointing outward from the source, perpendicular to the wave fronts.

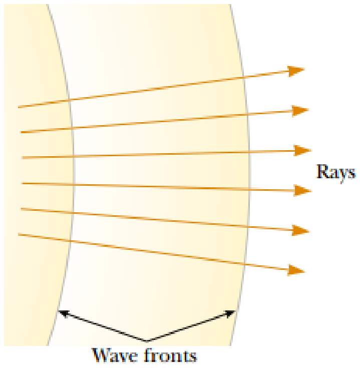


Figure 17.7 Far away from a point source, the wave fronts are nearly parallel planes, and the rays are nearly parallel lines perpendicular to the planes. Hence, a small segment of a spherical wave front is approximately a plane wave.

of Equation 17.8, we conclude that the displacement amplitude s_{max} of a spherical wave must vary as $1/r$. Therefore, we can write the wave function ψ (Greek letter psi) for an outgoing spherical wave in the form

$$\psi(r, t) = \frac{s_0}{r}\sin(kr - \omega t) \tag{17.9}$$

where s_0, the displacement amplitude at unit distance from the source, is a constant parameter characterizing the whole wave.

It is useful to represent spherical waves with a series of circular arcs concentric with the source, as shown in Figure 17.6. Each arc represents a surface over which the phase of the wave is constant. We call such a surface of constant phase a **wave front.** The distance between adjacent wave fronts equals the wavelength λ. The radial lines pointing outward from the source are called **rays.**

Now consider a small portion of a wave front far from the source, as shown in Figure 17.7. In this case, the rays passing through the wave front are nearly parallel to one another, and the wave front is very close to being planar. Therefore, at distances from the source that are great compared with the wavelength, we can approximate a wave front with a plane. Any small portion of a spherical wave far from its source can be considered a **plane wave.**

Figure 17.8 illustrates a plane wave propagating along the x axis, which means that the wave fronts are parallel to the yz plane. In this case, the wave function depends only on x and t and has the form

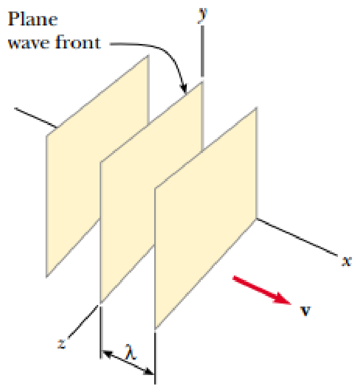


Figure 17.8 A representation of a plane wave moving in the positive x direction with a speed v. The wave fronts are planes parallel to the yz plane.

$$\psi(x, t) = A\sin(kx - \omega t) \tag{17.10}$$

Representation of a plane wave

That is, the wave function for a plane wave is identical in form to that for a one-dimensional traveling wave.

The intensity is the same at all points on a given wave front of a plane wave.

EXAMPLE 17.5 Intensity Variations of a Point Source

A point source emits sound waves with an average power output of 80.0 W. (a) Find the intensity 3.00 m from the source.

Solution A point source emits energy in the form of spherical waves (see Fig. 17.5). At a distance r from the source, the power is distributed over the surface area of a sphere, $4\pi r^2$. Therefore, the intensity at the distance r is given by Equation 17.8:

$$I = \frac{\mathcal{P}_{av}}{4\pi r^2} = \frac{80.0 \text{ W}}{4\pi(3.00 \text{ m})^2} = \boxed{0.707 \text{ W/m}^2}$$

an intensity that is close to the threshold of pain.

(b) Find the distance at which the sound level is 40 dB.

Solution We can find the intensity at the 40-dB sound level by using Equation 17.7 with $I_0 = 1.00 \times 10^{-12}$ W/m^2:

$$10 \log\left(\frac{I}{I_0}\right) = 40 \text{ dB}$$

$$\log I - \log I_0 = \frac{40}{10} = 4$$

$$\log I = 4 + \log 10^{-12}$$

$$\log I = -8$$

$$I = 1.00 \times 10^{-8} \text{ W/m}^2$$

Using this value for I in Equation 17.8 and solving for r, we obtain

$$r = \sqrt{\frac{\mathcal{P}_{av}}{4\pi I}} = \sqrt{\frac{80.0 \text{ W}}{4\pi \times 1.00 \times 10^{-8} \text{ W/m}^2}}$$

$$= \boxed{2.52 \times 10^4 \text{ m}}$$

which equals about 16 miles!

17.5 THE DOPPLER EFFECT

QuickLab

(Before attempting to do this QuickLab, you should check to see whether it is legal to sound a horn in your area.) Sound your car horn while driving toward and away from a friend in a campus parking lot or on a country road. Try this at different speeds while driving toward and past the friend (not *at* the friend). Do the frequencies of the sounds your friend hears agree with what is described in the text?

Perhaps you have noticed how the sound of a vehicle's horn changes as the vehicle moves past you. The frequency of the sound you hear as the vehicle approaches you is higher than the frequency you hear as it moves away from you (see QuickLab). This is one example of the **Doppler effect.**[3]

To see what causes this apparent frequency change, imagine you are in a boat that is lying at anchor on a gentle sea where the waves have a period of $T = 3.0$ s. This means that every 3.0 s a crest hits your boat. Figure 17.9a shows this situation, with the water waves moving toward the left. If you set your watch to $t = 0$ just as one crest hits, the watch reads 3.0 s when the next crest hits, 6.0 s when the third crest hits, and so on. From these observations you conclude that the wave frequency is $f = 1/T = (1/3.0)$ Hz. Now suppose you start your motor and head directly into the oncoming waves, as shown in Figure 17.9b. Again you set your watch to $t = 0$ as a crest hits the front of your boat. Now, however, because you are moving toward the next wave crest as it moves toward you, it hits you less than 3.0 s after the first hit. In other words, the period you observe is shorter than the 3.0-s period you observed when you were stationary. Because $f = 1/T$, you observe a higher wave frequency than when you were at rest.

If you turn around and move in the same direction as the waves (see Fig. 17.9c), you observe the opposite effect. You set your watch to $t = 0$ as a crest hits the back of the boat. Because you are now moving away from the next crest, more than 3.0 s has elapsed on your watch by the time that crest catches you. Thus, you observe a lower frequency than when you were at rest.

These effects occur because the relative speed between your boat and the waves depends on the direction of travel and on the speed of your boat. When you are moving toward the right in Figure 17.9b, this relative speed is higher than that of the wave speed, which leads to the observation of an increased frequency. When you turn around and move to the left, the relative speed is lower, as is the observed frequency of the water waves.

Let us now examine an analogous situation with sound waves, in which the water waves become sound waves, the water becomes the air, and the person on the boat becomes an observer listening to the sound. In this case, an observer O is moving and a sound source S is stationary. For simplicity, we assume that the air is also stationary and that the observer moves directly toward the source. The observer moves with a speed v_O toward a stationary point source ($v_S = 0$) (Fig. 17.10). In general, *at rest* means at rest with respect to the medium, air.

[3] Named after the Austrian physicist Christian Johann Doppler (1803–1853), who discovered the effect for light waves.

(a)

(b)

(c)

Figure 17.9 (a) Waves moving toward a stationary boat. The waves travel to the left, and their source is far to the right of the boat, out of the frame of the drawing. (b) The boat moving toward the wave source. (c) The boat moving away from the wave source.

We take the frequency of the source to be f, the wavelength to be λ, and the speed of sound to be v. If the observer were also stationary, he or she would detect f wave fronts per second. (That is, when $v_O = 0$ and $v_S = 0$, the observed frequency equals the source frequency.) When the observer moves toward the source,

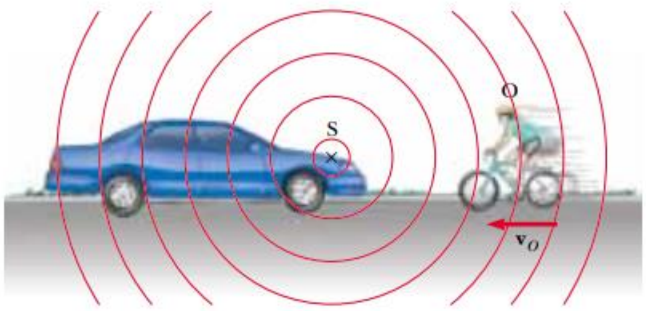


Figure 17.10 An observer O (the cyclist) moves with a speed v_O toward a stationary point source S, the horn of a parked car. The observer hears a frequency f' that is greater than the source frequency.

the speed of the waves relative to the observer is $v' = v + v_O$, as in the case of the boat, but the wavelength λ is unchanged. Hence, using Equation 16.14, $v = \lambda f$, we can say that the frequency heard by the observer is *increased* and is given by

$$f' = \frac{v'}{\lambda} = \frac{v + v_O}{\lambda}$$

Because $\lambda = v/f$, we can express f' as

$$f' = \left(1 + \frac{v_O}{v}\right)f \qquad \text{(observer moving toward source)} \tag{17.11}$$

If the observer is moving away from the source, the speed of the wave relative to the observer is $v' = v - v_O$. The frequency heard by the observer in this case is *decreased* and is given by

$$f' = \left(1 - \frac{v_O}{v}\right)f \qquad \text{(observer moving away from source)} \tag{17.12}$$

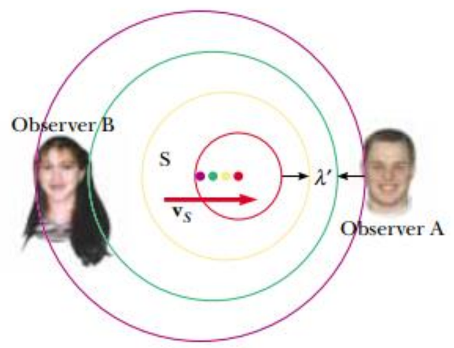
In general, whenever an observer moves with a speed v_O relative to a stationary source, the frequency heard by the observer is

Frequency heard with an observer in motion

$$f' = \left(1 \pm \frac{v_O}{v}\right)f \tag{17.13}$$

where the positive sign is used when the observer moves toward the source and the negative sign is used when the observer moves away from the source.

Now consider the situation in which the source is in motion and the observer is at rest. If the source moves directly toward observer A in Figure 17.11a, the wave fronts heard by the observer are closer together than they would be if the source were not moving. As a result, the wavelength λ' measured by observer A is shorter than the wavelength λ of the source. During each vibration, which lasts for a time T (the period), the source moves a distance $v_ST = v_S/f$ and the wavelength is



(a)

(b)

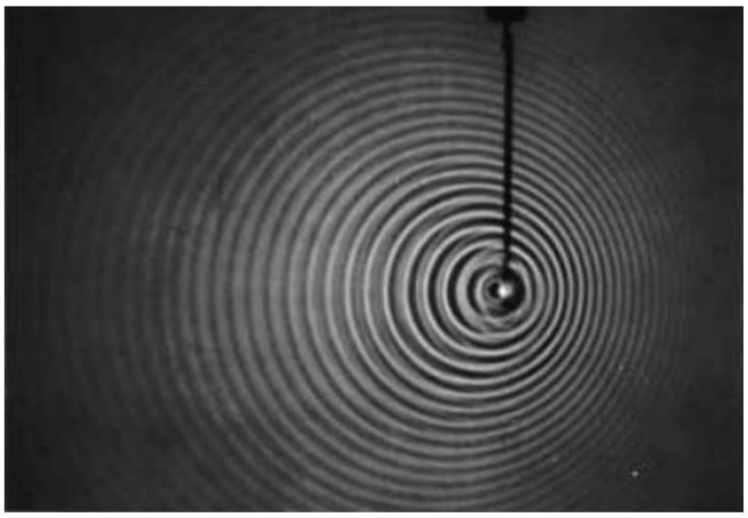


Figure 17.11 (a) A source S moving with a speed v_S toward a stationary observer A and away from a stationary observer B. Observer A hears an increased frequency, and observer B hears a decreased frequency. (b) The Doppler effect in water, observed in a ripple tank. A point source is moving to the right with speed v_S.

shortened by this amount. Therefore, the observed wavelength λ' is

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_S}{f}$$

Because $\lambda = v/f$, the frequency heard by observer A is

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - \frac{v_S}{f}} = \frac{v}{\frac{v}{f} - \frac{v_S}{f}}$$

$$f' = \left(\frac{1}{1 - \frac{v_S}{v}}\right) f \qquad \textbf{(17.14)}$$

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

That is, the observed frequency is *increased* whenever the source is moving toward the observer.

When the source moves away from a stationary observer, as is the case for observer B in Figure 17.11a, the observer measures a wavelength λ' that is *greater* than λ and hears a *decreased* frequency:

$$f' = \left(\frac{1}{1 + \frac{v_S}{v}}\right) f \qquad \textbf{(17.15)}$$

Combining Equations 17.14 and 17.15, we can express the general relationship for the observed frequency when a source is moving and an observer is at rest as

Frequency heard with source in motion

$$f' = \left(\frac{1}{1 \mp \frac{v_S}{v}}\right) f \qquad \textbf{(17.16)}$$

Finally, if both source and observer are in motion, we find the following general relationship for the observed frequency:

Frequency heard with observer and source in motion

$$f' = \left(\frac{v \pm v_O}{v \mp v_S}\right) f \qquad \textbf{(17.17)}$$

In this expression, the upper signs ($+ v_O$ and $- v_S$) refer to motion of one toward the other, and the lower signs ($- v_O$ and $+ v_S$) refer to motion of one away from the other.

A convenient rule concerning signs for you to remember when working with all Doppler-effect problems is as follows:

The word *toward* is associated with an *increase* in observed frequency. The words *away from* are associated with a *decrease* in observed frequency.

Although the Doppler effect is most typically experienced with sound waves, it is a phenomenon that is common to all waves. For example, the relative motion of source and observer produces a frequency shift in light waves. The Doppler effect is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the speeds of stars, galaxies, and other celestial objects relative to the Earth.

EXAMPLE 17.6 The Noisy Siren

As an ambulance travels east down a highway at a speed of 33.5 m/s (75 mi/h), its siren emits sound at a frequency of 400 Hz. What frequency is heard by a person in a car traveling west at 24.6 m/s (55 mi/h) (a) as the car approaches the ambulance and (b) as the car moves away from the ambulance?

Solution (a) We can use Equation 17.17 in both cases, taking the speed of sound in air to be $v = 343$ m/s. As the ambulance and car approach each other, the person in the car hears the frequency

$$f' = \left(\frac{v + v_O}{v - v_S}\right)f = \left(\frac{343 \text{ m/s} + 24.6 \text{ m/s}}{343 \text{ m/s} - 33.5 \text{ m/s}}\right)(400 \text{ Hz})$$

$$= \boxed{475 \text{ Hz}}$$

(b) As the vehicles recede from each other, the person hears the frequency

$$f' = \left(\frac{v - v_O}{v + v_S}\right)f = \left(\frac{343 \text{ m/s} - 24.6 \text{ m/s}}{343 \text{ m/s} + 33.5 \text{ m/s}}\right)(400 \text{ Hz})$$

$$= \boxed{338 \text{ Hz}}$$

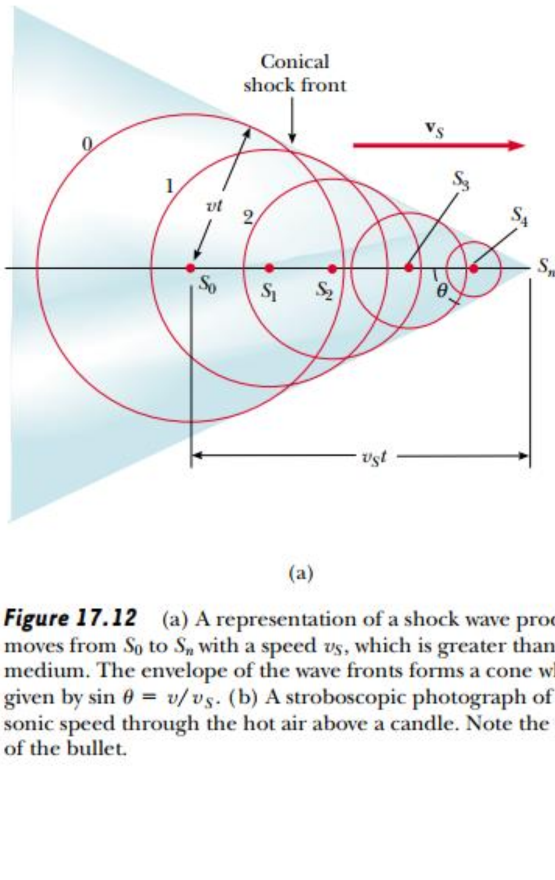
The *change* in frequency detected by the person in the car is $475 - 338 = 137$ Hz, which is more than 30% of the true frequency.

Exercise Suppose the car is parked on the side of the highway as the ambulance speeds by. What frequency does the person in the car hear as the ambulance (a) approaches and (b) recedes?

Answer (a) 443 Hz. (b) 364 Hz.

Shock Waves

Now let us consider what happens when the speed v_S of a source *exceeds* the wave speed v. This situation is depicted graphically in Figure 17.12a. The circles represent spherical wave fronts emitted by the source at various times during its motion. At $t = 0$, the source is at S_0, and at a later time t, the source is at S_n. In the time t,



(a)

(b)

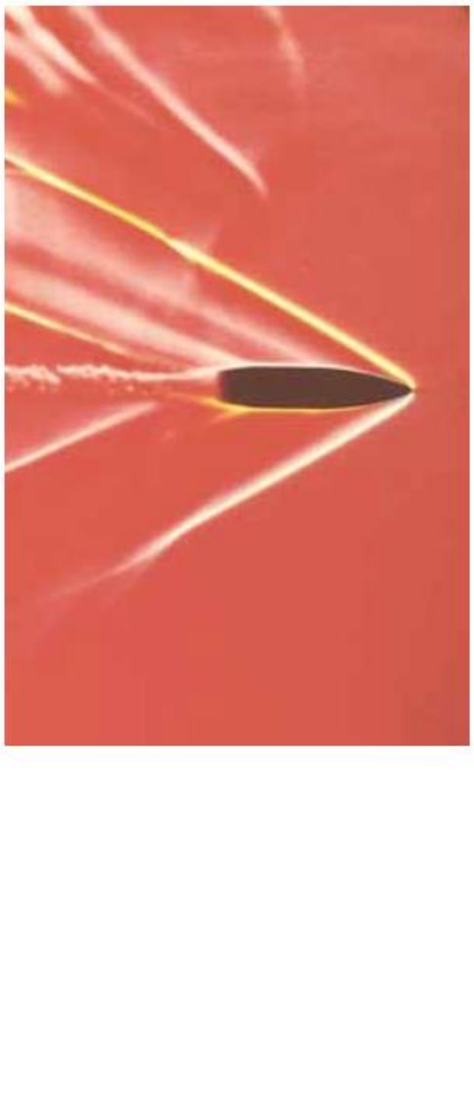


Figure 17.12 (a) A representation of a shock wave produced when a source moves from S_0 to S_n with a speed v_S, which is greater than the wave speed v in the medium. The envelope of the wave fronts forms a cone whose apex half-angle is given by $\sin\theta = v/v_S$. (b) A stroboscopic photograph of a bullet moving at supersonic speed through the hot air above a candle. Note the shock wave in the vicinity of the bullet.

Figure 17.13 The V-shaped bow wave of a boat is formed because the boat speed is greater than the speed of the water waves. A bow wave is analogous to a shock wave formed by an airplane traveling faster than sound.

the wave front centered at S_0 reaches a radius of vt. In this same amount of time, the source travels a distance v_St to S_n. At the instant the source is at S_n, waves are just beginning to be generated at this location, and hence the wave front has zero radius at this point. The tangent line drawn from S_n to the wave front centered on S_0 is tangent to all other wave fronts generated at intermediate times. Thus, we see that the envelope of these wave fronts is a cone whose apex half-angle θ is given by

$$\sin\theta = \frac{vt}{v_St} = \frac{v}{v_S}$$

The ratio v_S/v is referred to as the *Mach number*, and the conical wave front produced when $v_S > v$ (supersonic speeds) is known as a *shock wave*. An interesting analogy to shock waves is the V-shaped wave fronts produced by a boat (the *bow wave*) when the boat's speed exceeds the speed of the surface-water waves (Fig. 17.13).

Jet airplanes traveling at supersonic speeds produce shock waves, which are responsible for the loud "sonic boom" one hears. The shock wave carries a great deal of energy concentrated on the surface of the cone, with correspondingly great pressure variations. Such shock waves are unpleasant to hear and can cause damage to buildings when aircraft fly supersonically at low altitudes. In fact, an airplane flying at supersonic speeds produces a double boom because two shock fronts are formed, one from the nose of the plane and one from the tail (Fig. 17.14). People near the path of the space shuttle as it glides toward its landing point often report hearing what sounds like two very closely spaced cracks of thunder.

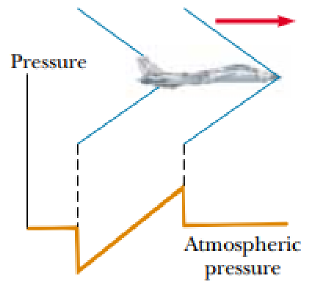


Figure 17.14 The two shock waves produced by the nose and tail of a jet airplane traveling at supersonic speeds.

Quick Quiz 17.5

An airplane flying with a constant velocity moves from a cold air mass into a warm air mass. Does the Mach number increase, decrease, or stay the same?

Quick Quiz 17.6

Suppose that an observer and a source of sound are both at rest and that a strong wind blows from the source toward the observer. Describe the effect of the wind (if any) on

(a) the observed frequency of the sound waves, (b) the observed wave speed, and (c) the observed wavelength.

SUMMARY

Sound waves are longitudinal and travel through a compressible medium with a speed that depends on the compressibility and inertia of that medium. The speed of sound in a medium having a bulk modulus B and density ρ is

$$v = \sqrt{\frac{B}{\rho}} \quad \textbf{(17.1)}$$

With this formula you can determine the speed of a sound wave in many different materials.

For sinusoidal sound waves, the variation in the displacement is given by

$$s(x, t) = s_{\text{max}} \cos(kx - \omega t) \quad \textbf{(17.2)}$$

and the variation in pressure from the equilibrium value is

$$\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t) \quad \textbf{(17.3)}$$

where ΔP_{max} is the **pressure amplitude.** The pressure wave is 90° out of phase with the displacement wave. The relationship between s_{max} and ΔP_{max} is given by

$$\Delta P_{\text{max}} = \rho v \omega s_{\text{max}} \quad \textbf{(17.4)}$$

The intensity of a periodic sound wave, which is the power per unit area, is

$$I = \tfrac{1}{2}\rho v(\omega s_{\text{max}})^2 = \frac{\Delta P_{\text{max}}^2}{2\rho v} \quad \textbf{(17.5, 17.6)}$$

The sound level of a sound wave, in decibels, is given by

$$\beta = 10 \log\left(\frac{I}{I_0}\right) \quad \textbf{(17.7)}$$

The constant I_0 is a reference intensity, usually taken to be at the threshold of hearing ($1.00 \times 10^{-12}\ \text{W/m}^2$), and I is the intensity of the sound wave in watts per square meter.

The intensity of a spherical wave produced by a point source is proportional to the average power emitted and inversely proportional to the square of the distance from the source:

$$I = \frac{\mathcal{P}_{\text{av}}}{4\pi r^2} \quad \textbf{(17.8)}$$

The change in frequency heard by an observer whenever there is relative motion between a source of sound waves and the observer is called the **Doppler effect.** The observed frequency is

$$f' = \left(\frac{v \pm v_O}{v \mp v_S}\right) f \quad \textbf{(17.17)}$$

The upper signs ($+v_O$ and $-v_S$) are used with motion of one toward the other, and the lower signs ($-v_O$ and $+v_S$) are used with motion of one away from the other. You can also use this formula when v_O or v_S is zero.