PUZZLER

Aurora Borealis, the Northern Lights, photographed near Fairbanks, Alaska. Such beautiful auroral displays are a common sight in far northern or southern latitudes, but they are quite rare in the middle latitudes. What causes these shimmering curtains of light, and why are they usually visible only near the Earth's North and South poles? *(George Lepp/Tony Stone Images)*

chapter

29 Magnetic Fields

Chapter Outline

Many historians of science believe that the compass, which uses a magnetic needle, was used in China as early as the 13th century B.C., its invention being of Arabic or Indian origin. The early Greeks knew about magnetism as early as 800 B.C. They discovered that the stone magnetite (Fe_3O_4) attracts pieces of iron. Legend ascribes the name *magnetite* to the shepherd Magnes, the nails of whose shoes and the tip of whose staff stuck fast to chunks of magnetite while he pastured his flocks.

An electromagnet is used to move tons of scrap metal.

In 1269 a Frenchman named Pierre de Maricourt mapped out the directions taken by a needle placed at various points on the surface of a spherical natural magnet. He found that the directions formed lines that encircled the sphere and passed through two points diametrically opposite each other, which he called the *poles* of the magnet. Subsequent experiments showed that every magnet, regardless of its shape, has two poles, called *north* and *south* poles, that exert forces on other magnetic poles just as electric charges exert forces on one another. That is, like poles repel each other, and unlike poles attract each other.

The poles received their names because of the way a magnet behaves in the presence of the Earth's magnetic field. If a bar magnet is suspended from its midpoint and can swing freely in a horizontal plane, it will rotate until its north pole points to the Earth's geographic North Pole and its south pole points to the Earth's geographic South Pole.[1] (The same idea is used in the construction of a simple compass.)

In 1600 William Gilbert (1540–1603) extended de Maricourt's experiments to a variety of materials. Using the fact that a compass needle orients in preferred directions, he suggested that the Earth itself is a large permanent magnet. In 1750 experimenters used a torsion balance to show that magnetic poles exert attractive or repulsive forces on each other and that these forces vary as the inverse square of the distance between interacting poles. Although the force between two magnetic poles is similar to the force between two electric charges, there is an important difference. Electric charges can be isolated (witness the electron and proton), whereas **a single magnetic pole has never been isolated.** That is, **magnetic poles are always found in pairs.** All attempts thus far to detect an isolated magnetic pole have been unsuccessful. No matter how many times a permanent magnet is cut in two, each piece always has a north and a south pole. (There is some theoretical basis for speculating that magnetic *monopoles*—isolated north or south poles—may exist in nature, and attempts to detect them currently make up an active experimental field of investigation.)

Hans Christian Oersted
Danish physicist (1777–1851)
(North Wind Picture Archives)

The relationship between magnetism and electricity was discovered in 1819 when, during a lecture demonstration, the Danish scientist Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle.[2] Shortly thereafter, André Ampère (1775–1836) formulated quantitative laws for calculating the magnetic force exerted by one current-carrying electrical conductor on another. He also suggested that on the atomic level, electric current loops are responsible for *all* magnetic phenomena.

In the 1820s, further connections between electricity and magnetism were demonstrated by Faraday and independently by Joseph Henry (1797–1878). They

[1] Note that the Earth's geographic North Pole is magnetically a south pole, whereas its geographic South Pole is magnetically a north pole. Because *opposite* magnetic poles attract each other, the pole on a magnet that is attracted to the Earth's geographic North Pole is the magnet's *north* pole and the pole attracted to the Earth's geographic South Pole is the magnet's *south* pole.

[2] The same discovery was reported in 1802 by an Italian jurist, Gian Dominico Romognosi, but was overlooked, probably because it was published in the newspaper *Gazetta de Trentino* rather than in a scholarly journal.

showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field. Years later, theoretical work by Maxwell showed that the reverse is also true: A changing electric field creates a magnetic field.

A similarity between electric and magnetic effects has provided methods of making permanent magnets. In Chapter 23 we learned that when rubber and wool are rubbed together, both become charged—one positively and the other negatively. In an analogous fashion, one can magnetize an unmagnetized piece of iron by stroking it with a magnet. Magnetism can also be induced in iron (and other materials) by other means. For example, if a piece of unmagnetized iron is placed near (but not touching) a strong magnet, the unmagnetized piece eventually becomes magnetized.

This chapter examines the forces that act on moving charges and on current-carrying wires in the presence of a magnetic field. The source of the magnetic field itself is described in Chapter 30.

QuickLab

If iron or steel is left in a weak magnetic field (such as that due to the Earth) long enough, it can become magnetized. Use a compass to see if you can detect a magnetic field near a steel file cabinet, cast iron radiator, or some other piece of ferrous metal that has been in one position for several years.

29.1 THE MAGNETIC FIELD

12.2

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any stationary or moving electric charge. In addition to an electric field, the region of space surrounding any *moving* electric charge also contains a magnetic field, as we shall see in Chapter 30. A magnetic field also surrounds any magnetic substance.

These refrigerator magnets are similar to a series of very short bar magnets placed end to end. If you slide the back of one refrigerator magnet in a circular path across the back of another one, you can feel a vibration as the two series of north and south poles move across each other.

Historically, the symbol **B** has been used to represent a magnetic field, and this is the notation we use in this text. The direction of the magnetic field **B** at any location is the direction in which a compass needle points at that location. Figure 29.1 shows how the magnetic field of a bar magnet can be traced with the aid of a compass. Note that the magnetic field lines outside the magnet point away from north poles and toward south poles. One can display magnetic field patterns of a bar magnet using small iron filings, as shown in Figure 29.2.

We can define a magnetic field **B** at some point in space in terms of the magnetic force $\mathbf{F}_B$ that the field exerts on a test object, for which we use a charged particle moving with a velocity **v**. For the time being, let us assume that no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.

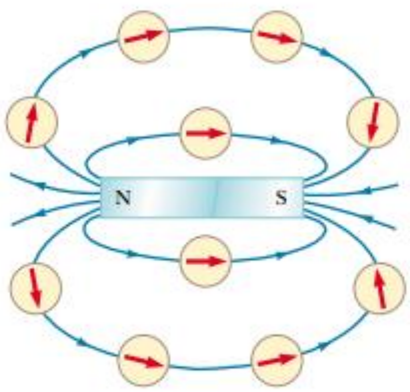


Figure 29.1 Compass needles can be used to trace the magnetic field lines of a bar magnet.

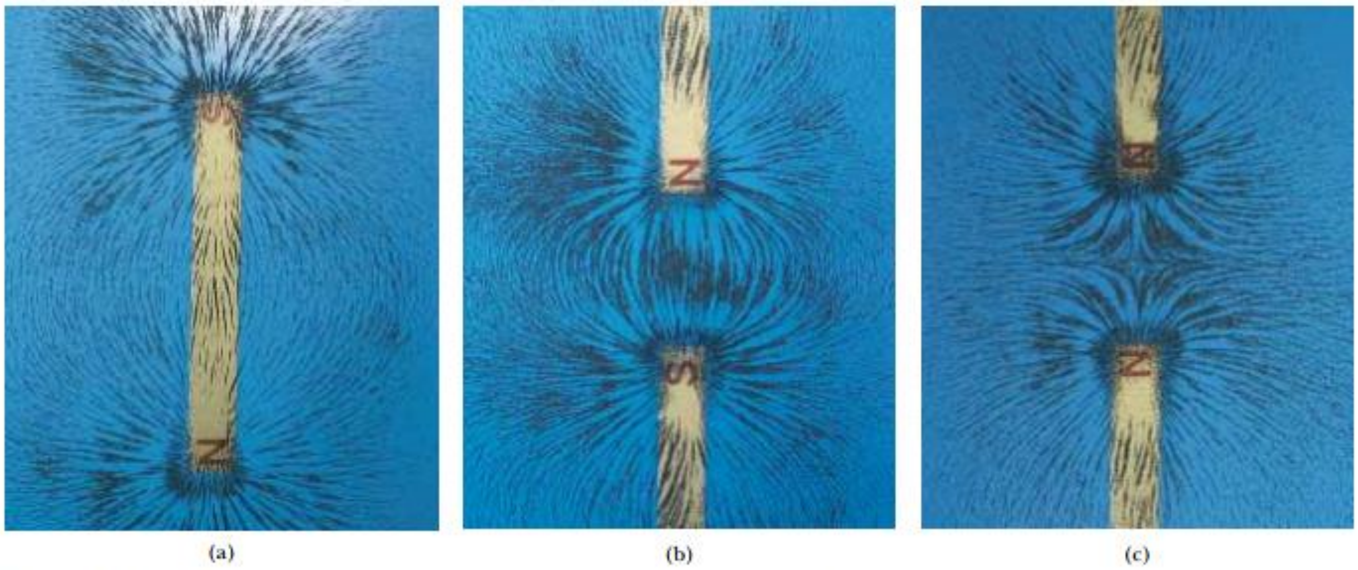


Figure 29.2 (a) Magnetic field pattern surrounding a bar magnet as displayed with iron filings. (b) Magnetic field pattern between *unlike* poles of two bar magnets. (c) Magnetic field pattern between *like* poles of two bar magnets.

Properties of the magnetic force on a charge moving in a magnetic field **B**

- The magnitude and direction of $\mathbf{F}_B$ depend on the velocity of the particle and on the magnitude and direction of the magnetic field **B**.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both **v** and **B**; that is, $\mathbf{F}_B$ is perpendicular to the plane formed by **v** and **B** (Fig. 29.3a).

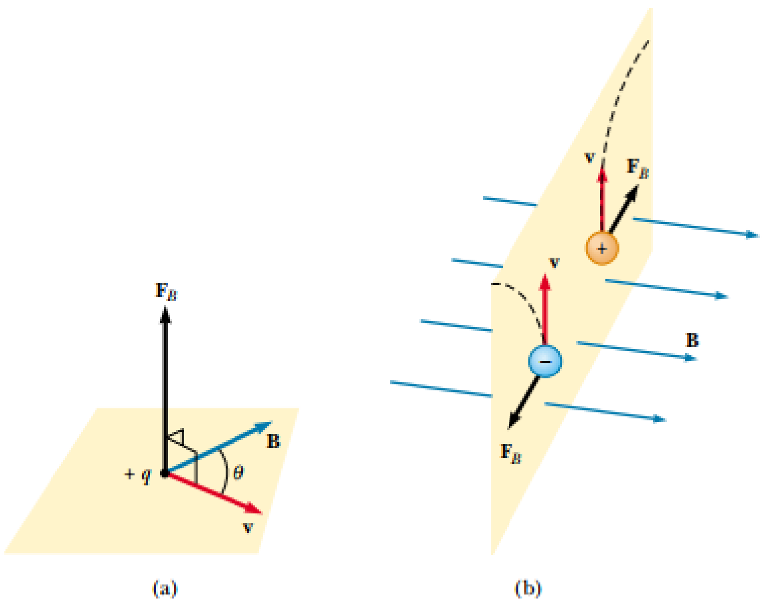


Figure 29.3 The direction of the magnetic force $\mathbf{F}_B$ acting on a charged particle moving with a velocity **v** in the presence of a magnetic field **B**. (a) The magnetic force is perpendicular to both **v** and **B**. (b) Oppositely directed magnetic forces $\mathbf{F}_B$ are exerted on two oppositely charged particles moving at the same velocity in a magnetic field.

The blue-white arc in this photograph indicates the circular path followed by an electron beam moving in a magnetic field. The vessel contains gas at very low pressure, and the beam is made visible as the electrons collide with the gas atoms, which then emit visible light. The magnetic field is produced by two coils (not shown). The apparatus can be used to measure the ratio e/m_e for the electron.

- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. 29.3b).
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle the particle's velocity vector makes with the direction of $\mathbf{B}$.

We can summarize these observations by writing the magnetic force in the form

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \tag{29.1}$$

where the direction of $\mathbf{F}_B$ is in the direction of $\mathbf{v} \times \mathbf{B}$ if q is positive, which by definition of the cross product (see Section 11.2) is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$. We can regard this equation as an operational definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle.

Figure 29.4 reviews the right-hand rule for determining the direction of the cross product $\mathbf{v} \times \mathbf{B}$. You point the four fingers of your right hand along the direction of $\mathbf{v}$ with the palm facing $\mathbf{B}$ and curl them toward $\mathbf{B}$. The extended thumb, which is at a right angle to the fingers, points in the direction of $\mathbf{v} \times \mathbf{B}$. Because

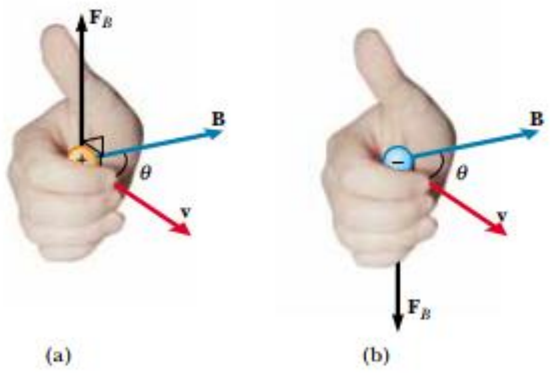


Figure 29.4 The right-hand rule for determining the direction of the magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ acting on a particle with charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. The direction of $\mathbf{v} \times \mathbf{B}$ is the direction in which the thumb points. (a) If q is positive, $\mathbf{F}_B$ is upward. (b) If q is negative, $\mathbf{F}_B$ is downward, antiparallel to the direction in which the thumb points.

$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$, $\mathbf{F}_B$ is in the direction of $\mathbf{v} \times \mathbf{B}$ if q is positive (Fig. 29.4a) and opposite the direction of $\mathbf{v} \times \mathbf{B}$ if q is negative (Fig. 29.4b). (If you need more help understanding the cross product, you should review pages 333 to 334, including Fig. 11.8.)

The magnitude of the magnetic force is

Magnitude of the magnetic force on a charged particle moving in a magnetic field

$$F_B = |q|vB \sin\theta \quad \textbf{(29.2)}$$

where θ is the smaller angle between $\mathbf{v}$ and $\mathbf{B}$. From this expression, we see that F is zero when $\mathbf{v}$ is parallel or antiparallel to $\mathbf{B}$ ($\theta = 0$ or $180°$) and maximum ($F_{B,\max} = |q|vB$) when $\mathbf{v}$ is perpendicular to $\mathbf{B}$ ($\theta = 90°$).

Quick Quiz 29.1

What is the maximum work that a constant magnetic field $\mathbf{B}$ can perform on a charge q moving through the field with velocity $\mathbf{v}$?

There are several important differences between electric and magnetic forces:

Differences between electric and magnetic forces

- The electric force acts in the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced.

From the last statement and on the basis of the work–kinetic energy theorem, we conclude that the kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. In other words,

> when a charged particle moves with a velocity $\mathbf{v}$ through a magnetic field, the field can alter the direction of the velocity vector but cannot change the speed or kinetic energy of the particle.

A magnetic field cannot change the speed of a particle

From Equation 29.2, we see that the SI unit of magnetic field is the newton per coulomb-meter per second, which is called the **tesla** (T):

$$1\ \text{T} = \frac{\text{N}}{\text{C}\cdot\text{m/s}}$$

Because a coulomb per second is defined to be an ampere, we see that

$$1\ \text{T} = 1\ \frac{\text{N}}{\text{A}\cdot\text{m}}$$

A non-SI magnetic-field unit in common use, called the *gauss* (G), is related to the tesla through the conversion $1\ \text{T} = 10^4\ \text{G}$. Table 29.1 shows some typical values of magnetic fields.

Quick Quiz 29.2

The north-pole end of a bar magnet is held near a positively charged piece of plastic. Is the plastic attracted, repelled, or unaffected by the magnet?

TABLE 29.1 Some Approximate Magnetic Field Magnitudes

Source of Field	Field Magnitude (T)
Strong superconducting laboratory magnet	30
Strong conventional laboratory magnet	2
Medical MRI unit	1.5
Bar magnet	10^{-2}
Surface of the Sun	10^{-2}
Surface of the Earth	0.5×10^{-4}
Inside human brain (due to nerve impulses)	10^{-13}

EXAMPLE 29.1 An Electron Moving in a Magnetic Field

An electron in a television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the x axis (Fig. 29.5). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the x axis and lying in the xy plane. Calculate the magnetic force on and acceleration of the electron.

Solution Using Equation 29.2, we can find the magnitude of the magnetic force:

$$F_B = |q|vB\sin\theta$$
$$= (1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^6\ \text{m/s})(0.025\ \text{T})(\sin 60°)$$
$$= \boxed{2.8 \times 10^{-14}\ \text{N}}$$

Because $\mathbf{v} \times \mathbf{B}$ is in the positive z direction (from the right-hand rule) and the charge is negative, $\mathbf{F}_B$ is in the negative z direction.

The mass of the electron is 9.11×10^{-31} kg, and so its acceleration is

$$a = \frac{F_B}{m_e} = \frac{2.8 \times 10^{-14}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{3.1 \times 10^{16}\ \text{m/s}^2}$$

in the negative z direction.

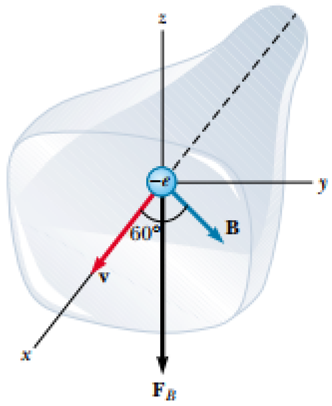


Figure 29.5 The magnetic force $\mathbf{F}_B$ acting on the electron is in the negative z direction when $\mathbf{v}$ and $\mathbf{B}$ lie in the xy plane.

29.2 MAGNETIC FORCE ACTING ON A CURRENT-CARRYING CONDUCTOR

12.3 If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field. This follows from the fact that the current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.

Before we continue our discussion, some explanation of the notation used in this book is in order. To indicate the direction of **B** in illustrations, we sometimes present perspective views, such as those in Figures 29.5, 29.6a, and 29.7. In flat il-

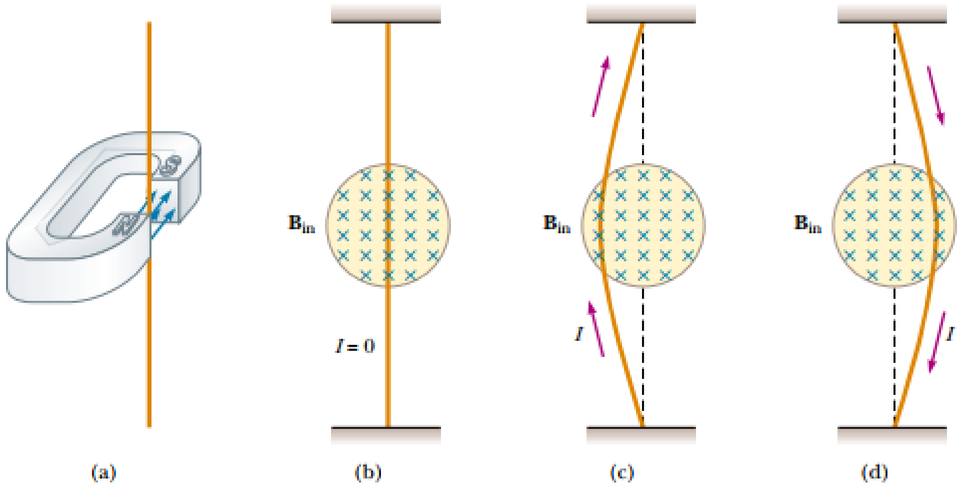


Figure 29.6 (a) A wire suspended vertically between the poles of a magnet. (b) The setup shown in part (a) as seen looking at the south pole of the magnet, so that the magnetic field (blue crosses) is directed into the page. When there is no current in the wire, it remains vertical. (c) When the current is upward, the wire deflects to the left. (d) When the current is downward, the wire deflects to the right.

lustrations, such as in Figure 29.6b to d, we depict a magnetic field directed into the page with blue crosses, which represent the tails of arrows shot perpendicularly and away from you. In this case, we call the field $\mathbf{B}_{\text{in}}$, where the subscript "in" indicates "into the page." If $\mathbf{B}$ is perpendicular and directed out of the page, we use a series of blue dots, which represent the tips of arrows coming toward you (see Fig. P29.56). In this case, we call the field $\mathbf{B}_{\text{out}}$. If $\mathbf{B}$ lies in the plane of the page, we use a series of blue field lines with arrowheads, as shown in Figure 29.8.

One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet, as shown in Figure 29.6a. For ease in visualization, part of the horseshoe magnet in part (a) is removed to show the end face of the south pole in parts (b), (c), and (d) of Figure 29.6. The magnetic field is directed into the page and covers the region within the shaded circles. When the current in the wire is zero, the wire remains vertical, as shown in Figure 29.6b. However, when a current directed upward flows in the wire, as shown in Figure 29.6c, the wire deflects to the left. If we reverse the current, as shown in Figure 29.6d, the wire deflects to the right.

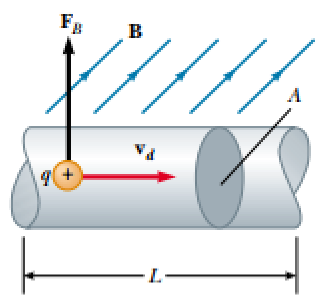


Figure 29.7 A segment of a current-carrying wire located in a magnetic field $\mathbf{B}$. The magnetic force exerted on each charge making up the current is $q\mathbf{v}_d \times \mathbf{B}$, and the net force on the segment of length L is $I\mathbf{L} \times \mathbf{B}$.

Let us quantify this discussion by considering a straight segment of wire of length L and cross-sectional area A, carrying a current I in a uniform magnetic field $\mathbf{B}$, as shown in Figure 29.7. The magnetic force exerted on a charge q moving with a drift velocity $\mathbf{v}_d$ is $q\mathbf{v}_d \times \mathbf{B}$. To find the total force acting on the wire, we multiply the force $q\mathbf{v}_d \times \mathbf{B}$ exerted on one charge by the number of charges in the segment. Because the volume of the segment is AL, the number of charges in the segment is nAL, where n is the number of charges per unit volume. Hence, the total magnetic force on the wire of length L is

$$\mathbf{F}_B = (q\mathbf{v}_d \times \mathbf{B})nAL$$

We can write this expression in a more convenient form by noting that, from Equation 27.4, the current in the wire is $I = nqv_dA$. Therefore,

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B} \qquad (29.3)$$

Force on a segment of a wire in a uniform magnetic field

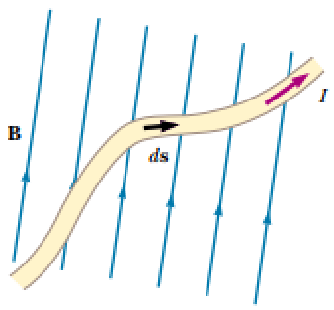


Figure 29.8 A wire segment of arbitrary shape carrying a current I in a magnetic field $\mathbf{B}$ experiences a magnetic force. The force on any segment $d\mathbf{s}$ is $I\,d\mathbf{s} \times \mathbf{B}$ and is directed out of the page. You should use the right-hand rule to confirm this force direction.

where $\mathbf{L}$ is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment. Note that this expression applies only to a straight segment of wire in a uniform magnetic field.

Now let us consider an arbitrarily shaped wire segment of uniform cross-section in a magnetic field, as shown in Figure 29.8. It follows from Equation 29.3 that the magnetic force exerted on a small segment of vector length $d\mathbf{s}$ in the presence of a field $\mathbf{B}$ is

$$d\mathbf{F}_B = I\,d\mathbf{s} \times \mathbf{B} \tag{29.4}$$

where $d\mathbf{F}_B$ is directed out of the page for the directions assumed in Figure 29.8. We can consider Equation 29.4 as an alternative definition of $\mathbf{B}$. That is, we can define the magnetic field $\mathbf{B}$ in terms of a measurable force exerted on a current element, where the force is a maximum when $\mathbf{B}$ is perpendicular to the element and zero when $\mathbf{B}$ is parallel to the element.

To calculate the total force $\mathbf{F}_B$ acting on the wire shown in Figure 29.8, we integrate Equation 29.4 over the length of the wire:

$$\mathbf{F}_B = I \int_a^b d\mathbf{s} \times \mathbf{B} \tag{29.5}$$

where a and b represent the end points of the wire. When this integration is carried out, the magnitude of the magnetic field and the direction the field makes with the vector $d\mathbf{s}$ (in other words, with the orientation of the element) may differ at different points.

Now let us consider two special cases involving Equation 29.5. In both cases, the magnetic field is taken to be constant in magnitude and direction.

Case 1 A curved wire carries a current I and is located in a uniform magnetic field $\mathbf{B}$, as shown in Figure 29.9a. Because the field is uniform, we can take $\mathbf{B}$ outside the integral in Equation 29.5, and we obtain

$$\mathbf{F}_B = I\left(\int_a^b d\mathbf{s}\right) \times \mathbf{B} \tag{29.6}$$

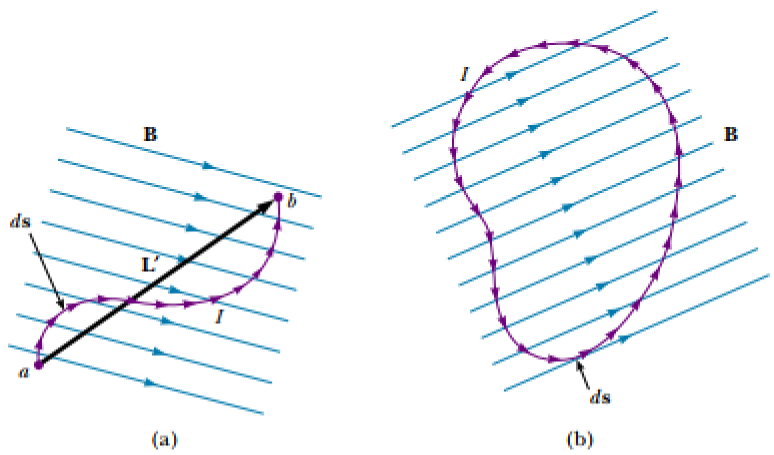


Figure 29.9 (a) A curved wire carrying a current I in a uniform magnetic field. The total magnetic force acting on the wire is equivalent to the force on a straight wire of length L' running between the ends of the curved wire. (b) A current-carrying loop of arbitrary shape in a uniform magnetic field. The net magnetic force on the loop is zero.

But the quantity $\int_a^b d\mathbf{s}$ represents the *vector sum* of all the length elements from a to b. From the law of vector addition, the sum equals the vector $\mathbf{L}'$, directed from a to b. Therefore, Equation 29.6 reduces to

$$\mathbf{F}_B = I\mathbf{L}' \times \mathbf{B} \tag{29.7}$$

Case 2 An arbitrarily shaped closed loop carrying a current I is placed in a uniform magnetic field, as shown in Figure 29.9b. We can again express the force acting on the loop in the form of Equation 29.6, but this time we must take the vector sum of the length elements $d\mathbf{s}$ over the entire loop:

$$\mathbf{F}_B = I\left(\oint d\mathbf{s}\right) \times \mathbf{B}$$

Because the set of length elements forms a closed polygon, the vector sum must be zero. This follows from the graphical procedure for adding vectors by the polygon method. Because $\oint d\mathbf{s} = 0$, we conclude that $\mathbf{F}_B = 0$:

The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

EXAMPLE 29.2 Force on a Semicircular Conductor

A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis, as shown in Figure 29.10. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.

Solution The force $\mathbf{F}_1$ acting on the straight portion has a magnitude $F_1 = ILB = 2IRB$ because $L = 2R$ and the wire is oriented perpendicular to $\mathbf{B}$. The direction of $\mathbf{F}_1$ is out of the page because $\mathbf{L} \times \mathbf{B}$ is along the positive z axis. (That is, $\mathbf{L}$ is to the right, in the direction of the current; thus, according to the rule of cross products, $\mathbf{L} \times \mathbf{B}$ is out of the page in Fig. 29.10.)

To find the force $\mathbf{F}_2$ acting on the curved part, we first write an expression for the force $d\mathbf{F}_2$ on the length element $d\mathbf{s}$ shown in Figure 29.10. If θ is the angle between $\mathbf{B}$ and $d\mathbf{s}$, then the magnitude of $d\mathbf{F}_2$ is

$$dF_2 = I\,|d\mathbf{s} \times \mathbf{B}| = IB\sin\theta\, ds$$

To integrate this expression, we must express ds in terms of θ. Because $s = R\theta$, we have $ds = R\,d\theta$, and we can make this substitution for dF_2:

$$dF_2 = IRB\sin\theta\, d\theta$$

To obtain the total force F_2 acting on the curved portion, we can integrate this expression to account for contributions from all elements $d\mathbf{s}$. Note that the direction of the force on every element is the same: into the page (because $d\mathbf{s} \times \mathbf{B}$ is into the page). Therefore, the resultant force $\mathbf{F}_2$ on the curved wire must also be into the page. Integrating our expression for dF_2 over the limits $\theta = 0$ to $\theta = \pi$ (that is, the entire semicircle) gives

$$F_2 = IRB\int_0^\pi \sin\theta\, d\theta = IRB\Big[-\cos\theta\Big]_0^\pi$$

$$= -IRB(\cos\pi - \cos 0) = -IRB(-1-1) = 2IRB$$

Because $\mathbf{F}_2$, with a magnitude of $2IRB$, is directed into the page and because $\mathbf{F}_1$, with a magnitude of $2IRB$, is directed out of the page, the net force on the closed loop is zero. This result is consistent with Case 2 described earlier.

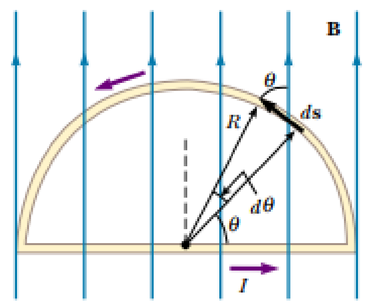


Figure 29.10 The net force acting on a closed current loop in a uniform magnetic field is zero. In the setup shown here, the force on the straight portion of the loop is $2IRB$ and directed out of the page, and the force on the curved portion is $2IRB$ directed into the page.

Quick Quiz 29.3

The four wires shown in Figure 29.11 all carry the same current from point A to point B through the same magnetic field. Rank the wires according to the magnitude of the magnetic force exerted on them, from greatest to least.

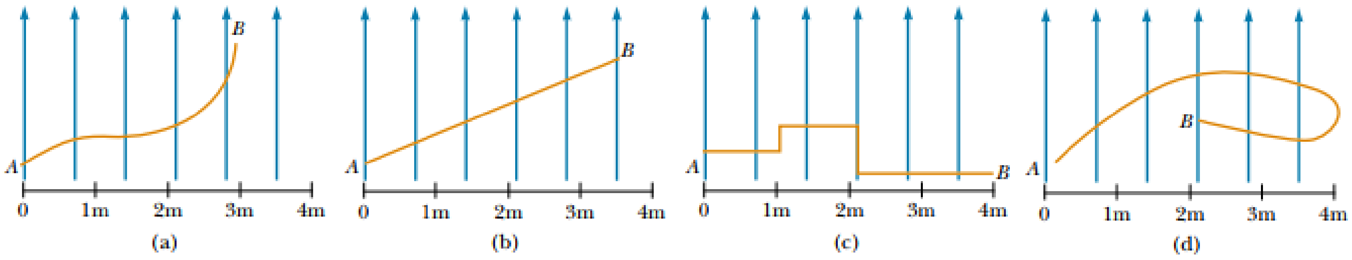


Figure 29.11 Which wire experiences the greatest magnetic force?

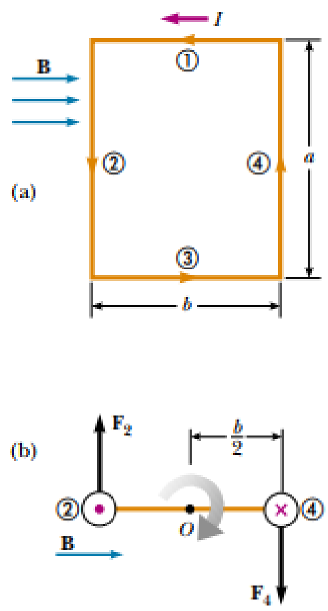


Figure 29. 12 (a) Overhead view of a rectangular current loop in a uniform magnetic field. No forces are acting on sides ① and ③ because these sides are parallel to **B**. Forces are acting on sides ② and ④, however. (b) Edge view of the loop sighting down sides ② and ④ shows that the forces $\mathbf{F}_2$ and $\mathbf{F}_4$ exerted on these sides create a torque that tends to twist the loop clockwise. The purple dot in the left circle represents current in wire ② coming toward you; the purple cross in the right circle represents current in wire ④ moving away from you.

29.3 TORQUE ON A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

In the previous section, we showed how a force is exerted on a current-carrying conductor placed in a magnetic field. With this as a starting point, we now show that a torque is exerted on any current loop placed in a magnetic field. The results of this analysis will be of great value when we discuss motors in Chapter 31.

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field directed parallel to the plane of the loop, as shown in Figure 29.12a. No magnetic forces act on sides ① and ③ because these wires are parallel to the field; hence, $\mathbf{L} \times \mathbf{B} = 0$ for these sides. However, magnetic forces do act on sides ② and ④ because these sides are oriented perpendicular to the field. The magnitude of these forces is, from Equation 29.3,

$$F_2 = F_4 = IaB$$

The direction of $\mathbf{F}_2$, the force exerted on wire ② is out of the page in the view shown in Figure 29.12a, and that of $\mathbf{F}_4$, the force exerted on wire ④, is into the page in the same view. If we view the loop from side ③ and sight along sides ② and ④, we see the view shown in Figure 29.12b, and the two forces $\mathbf{F}_2$ and $\mathbf{F}_4$ are directed as shown. Note that the two forces point in opposite directions but are *not* directed along the same line of action. If the loop is pivoted so that it can rotate about point O, these two forces produce about O a torque that rotates the loop clockwise. The magnitude of this torque $\tau_{\max}$ is

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB$$

where the moment arm about O is $b/2$ for each force. Because the area enclosed by the loop is $A = ab$, we can express the maximum torque as

$$\tau_{\max} = IAB \qquad \textbf{(29.8)}$$

Remember that this maximum-torque result is valid only when the magnetic field is parallel to the plane of the loop. The sense of the rotation is clockwise when viewed from side ③, as indicated in Figure 29.12b. If the current direction were reversed, the force directions would also reverse, and the rotational tendency would be counterclockwise.

Now let us suppose that the uniform magnetic field makes an angle $\theta < 90°$ with a line perpendicular to the plane of the loop, as shown in Figure 29.13a. For convenience, we assume that $\mathbf{B}$ is perpendicular to sides ① and ③. In this case, the magnetic forces $\mathbf{F}_2$ and $\mathbf{F}_4$ exerted on sides ② and ④ cancel each other and produce no torque because they pass through a common origin. However, the forces acting on sides ① and ③, $\mathbf{F}_1$ and $\mathbf{F}_3$, form a couple and hence produce a torque about *any point*. Referring to the end view shown in Figure 29.13b, we note that the moment arm of $\mathbf{F}_1$ about the point O is equal to $(a/2)\sin\theta$. Likewise, the moment arm of $\mathbf{F}_3$ about O is also $(a/2)\sin\theta$. Because $F_1 = F_3 = IbB$, the net torque about O has the magnitude

$$\begin{aligned}\tau &= F_1 \frac{a}{2}\sin\theta + F_3\frac{a}{2}\sin\theta \\ &= IbB\left(\frac{a}{2}\sin\theta\right) + IbB\left(\frac{a}{2}\sin\theta\right) = IabB\sin\theta \\ &= IAB\sin\theta\end{aligned}$$

where $A = ab$ is the area of the loop. This result shows that the torque has its maximum value IAB when the field is perpendicular to the normal to the plane of the loop ($\theta = 90°$), as we saw when discussing Figure 29.12, and that it is zero when the field is parallel to the normal to the plane of the loop ($\theta = 0$). As we see in Figure 29.13, the loop tends to rotate in the direction of decreasing values of θ (that is, such that the area vector $\mathbf{A}$ rotates toward the direction of the magnetic field).

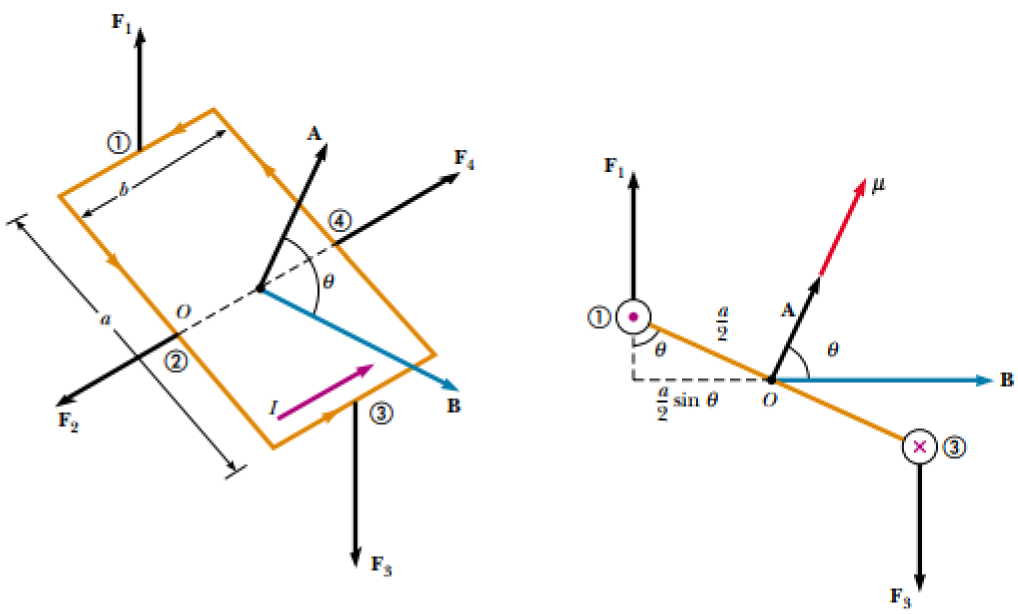


Figure 29.13 (a) A rectangular current loop in a uniform magnetic field. The area vector $\mathbf{A}$ perpendicular to the plane of the loop makes an angle θ with the field. The magnetic forces exerted on sides ② and ④ cancel, but the forces exerted on sides ① and ③ create a torque on the loop. (b) Edge view of the loop sighting down sides ① and ③.

Quick Quiz 29.4

Describe the forces on the rectangular current loop shown in Figure 29.13 if the magnetic field is directed as shown but increases in magnitude going from left to right.

A convenient expression for the torque exerted on a loop placed in a uniform magnetic field **B** is

Torque on a current loop

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B} \tag{29.9}$$

where **A**, the vector shown in Figure 29.13, is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop. We determine the direction of **A** using the right-hand rule described in Figure 29.14. When you curl the fingers of your right hand in the direction of the current in the loop, your thumb points in the direction of **A**. The product $I\mathbf{A}$ is defined to be the **magnetic dipole moment** $\boldsymbol{\mu}$ (often simply called the "magnetic moment") of the loop:

Magnetic dipole moment of a current loop

$$\boldsymbol{\mu} = I\mathbf{A} \tag{29.10}$$

The SI unit of magnetic dipole moment is ampere–meter^2 ($\mathrm{A \cdot m^2}$). Using this definition, we can express the torque exerted on a current-carrying loop in a magnetic field **B** as

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \tag{29.11}$$

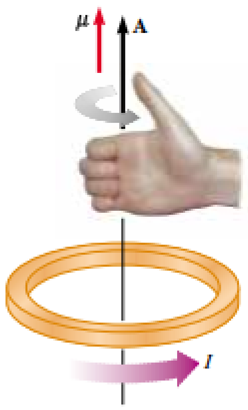


Figure 29.14 Right-hand rule for determining the direction of the vector **A**. The direction of the magnetic moment $\boldsymbol{\mu}$ is the same as the direction of **A**.

Note that this result is analogous to Equation 26.18, $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$, for the torque exerted on an electric dipole in the presence of an electric field **E**, where **p** is the electric dipole moment.

Although we obtained the torque for a particular orientation of **B** with respect to the loop, the equation $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ is valid for any orientation. Furthermore, although we derived the torque expression for a rectangular loop, the result is valid for a loop of any shape.

If a coil consists of N turns of wire, each carrying the same current and enclosing the same area, the total magnetic dipole moment of the coil is N times the magnetic dipole moment for one turn. The torque on an N-turn coil is N times that on a one-turn coil. Thus, we write $\boldsymbol{\tau} = N\boldsymbol{\mu}_{\text{loop}} \times \mathbf{B} = \boldsymbol{\mu}_{\text{coil}} \times \mathbf{B}$.

In Section 26.6, we found that the potential energy of an electric dipole in an electric field is given by $U = -\mathbf{p} \cdot \mathbf{E}$. This energy depends on the orientation of the dipole in the electric field. Likewise, the potential energy of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field and is given by

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} \tag{29.12}$$

From this expression, we see that a magnetic dipole has its lowest energy $U_{\text{min}} = -\mu B$ when $\boldsymbol{\mu}$ points in the same direction as **B**. The dipole has its highest energy $U_{\text{max}} = +\mu B$ when $\boldsymbol{\mu}$ points in the direction opposite **B**.

Quick Quiz 29.5

Rank the magnitude of the torques acting on the rectangular loops shown in Figure 29.15, from highest to lowest. All loops are identical and carry the same current.

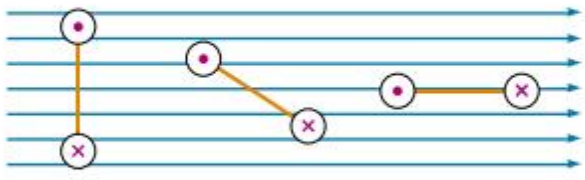


Figure 29.15 Which current loop (seen edge-on) experiences the greatest torque?

EXAMPLE 29.3 The Magnetic Dipole Moment of a Coil

A rectangular coil of dimensions 5.40 cm × 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. A 0.350-T magnetic field is applied parallel to the plane of the loop. (a) Calculate the magnitude of its magnetic dipole moment.

Solution Because the coil has 25 turns, we modify Equation 29.10 to obtain

$$\mu_{\text{coil}} = NIA = (25)(15.0 \times 10^{-3}\ \text{A})(0.054\,0\ \text{m})(0.085\,0\ \text{m})$$
$$= \boxed{1.72 \times 10^{-3}\ \text{A}\cdot\text{m}^2}$$

(b) What is the magnitude of the torque acting on the loop?

Solution Because **B** is perpendicular to $\boldsymbol{\mu}_{\text{coil}}$, Equation 29.11 gives

$$\tau = \mu_{\text{coil}}B = (1.72 \times 10^{-3}\ \text{A}\cdot\text{m}^2)(0.350\ \text{T})$$
$$= \boxed{6.02 \times 10^{-4}\ \text{N}\cdot\text{m}}$$

Exercise Show that the units $\text{A}\cdot\text{m}^2\cdot\text{T}$ reduce to the torque units $\text{N}\cdot\text{m}$.

Exercise Calculate the magnitude of the torque on the coil when the field makes an angle of (a) 60° and (b) 0° with $\boldsymbol{\mu}$.

Answer (a) $5.21 \times 10^{-4}\ \text{N}\cdot\text{m}$; (b) zero.

web

For more information on torquers, visit the Web site of a company that supplies these devices to NASA:
http://www.smad.com

EXAMPLE 29.4 Satellite Attitude Control

Many satellites use coils called *torquers* to adjust their orientation. These devices interact with the Earth's magnetic field to create a torque on the spacecraft in the *x*, *y*, or *z* direction. The major advantage of this type of attitude-control system is that it uses solar-generated electricity and so does not consume any thruster fuel.

If a typical device has a magnetic dipole moment of $250\ \text{A}\cdot\text{m}^2$, what is the maximum torque applied to a satellite when its torquer is turned on at an altitude where the magnitude of the Earth's magnetic field is 3.0×10^{-5} T?

Solution We once again apply Equation 29.11, recognizing that the maximum torque is obtained when the magnetic dipole moment of the torquer is perpendicular to the Earth's magnetic field:

$$\tau_{\text{max}} = \mu B = (250\ \text{A}\cdot\text{m}^2)(3.0 \times 10^{-5}\ \text{T})$$
$$= \boxed{7.5 \times 10^{-3}\ \text{N}\cdot\text{m}}$$

Exercise If the torquer requires 1.3 W of power at a potential difference of 28 V, how much current does it draw when it operates?

Answer 46 mA.

EXAMPLE 29.5 The D'Arsonval Galvanometer

An end view of a D'Arsonval galvanometer (see Section 28.5) is shown in Figure 29.16. When the turns of wire making up the coil carry a current, the magnetic field created by the magnet exerts on the coil a torque that turns it (along with its attached pointer) against the spring. Let us show that the angle of deflection of the pointer is directly proportional to the current in the coil.

Solution We can use Equation 29.11 to find the torque τ_m the magnetic field exerts on the coil. If we assume that the magnetic field through the coil is perpendicular to the normal to the plane of the coil, Equation 29.11 becomes

$$\tau_m = \mu B$$

(This is a reasonable assumption because the circular cross section of the magnet ensures radial magnetic field lines.) This magnetic torque is opposed by the torque due to the spring, which is given by the rotational version of Hooke's law, $\tau_s = -\kappa\varphi$, where κ is the torsional spring constant and φ is the angle through which the spring turns. Because the coil does not have an angular acceleration when the pointer is at rest, the sum of these torques must be zero:

$$(1) \qquad \tau_m + \tau_s = \mu B - \kappa\varphi = 0$$

Equation 29.10 allows us to relate the magnetic moment of the N turns of wire to the current through them:

$$\mu = NIA$$

We can substitute this expression for μ in Equation (1) to obtain

$$(NIA)B - \kappa\varphi = 0$$

$$\boxed{\varphi = \frac{NAB}{\kappa} I}$$

Thus, the angle of deflection of the pointer is directly proportional to the current in the loop. The factor NAB/κ tells us that deflection also depends on the design of the meter.

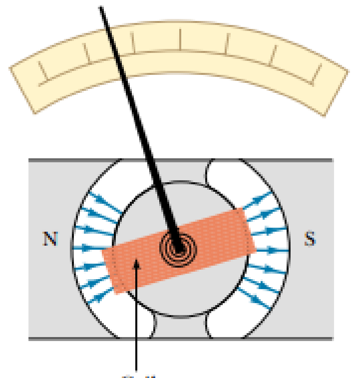


Figure 29.16 End view of a moving-coil galvanometer.

29.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

QuickLab

Move a bar magnet across the screen of a black-and-white television and watch what happens to the picture. The electrons are deflected by the magnetic field as they approach the screen, causing distortion. (WARNING: Do not attempt to do this with a *color* television or computer monitor. These devices typically contain a metallic plate that can become magnetized by the bar magnet. If this happens, a repair shop will need to "degauss" the screen.)

12.3 In Section 29.1 we found that the magnetic force acting on a charged particle moving in a magnetic field is perpendicular to the velocity of the particle and that consequently the work done on the particle by the magnetic force is zero. Let us now consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field. Let us assume that the direction of the magnetic field is into the page. Figure 29.17 shows that the particle moves in a circle in a plane perpendicular to the magnetic field.

The particle moves in this way because the magnetic force $\mathbf{F}_B$ is at right angles to $\mathbf{v}$ and $\mathbf{B}$ and has a constant magnitude qvB. As the force deflects the particle, the directions of $\mathbf{v}$ and $\mathbf{F}_B$ change continuously, as Figure 29.17 shows. Because $\mathbf{F}_B$ always points toward the center of the circle, **it changes only the direction of v and not its magnitude.** As Figure 29.17 illustrates, the rotation is counterclockwise for a positive charge. If q were negative, the rotation would be clockwise. We can use Equation 6.1 to equate this magnetic force to the radial force required to keep the charge moving in a circle:

$$\sum F = ma_r$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} \quad \textbf{(29.13)}$$

That is, the radius of the path is proportional to the linear momentum mv of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field. The angular speed of the particle (from Eq. 10.10) is

$$\omega = \frac{v}{r} = \frac{qB}{m} \quad \textbf{(29.14)}$$

The period of the motion (the time that the particle takes to complete one revolution) is equal to the circumference of the circle divided by the linear speed of the particle:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} \quad \textbf{(29.15)}$$

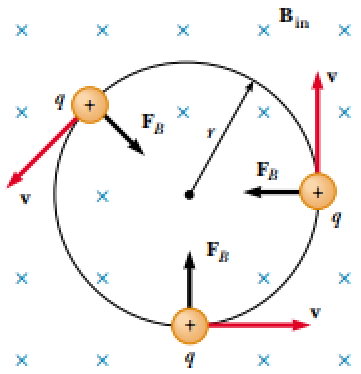


Figure 29.17 When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to **B**. The magnetic force $\mathbf{F}_B$ acting on the charge is always directed toward the center of the circle.

These results show that the angular speed of the particle and the period of the circular motion do not depend on the linear speed of the particle or on the radius of the orbit. The angular speed ω is often referred to as the **cyclotron frequency** because charged particles circulate at this angular speed in the type of accelerator called a *cyclotron*, which is discussed in Section 29.5.

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to **B**, its path is a helix. For example, if the field is directed in the x direction, as shown in Figure 29.18, there is no component of force in the x direction. As a result, $a_x = 0$, and the x component of velocity remains constant. However, the magnetic force $q\mathbf{v} \times \mathbf{B}$ causes the components v_y and v_z to change in time, and the resulting motion is a helix whose axis is parallel to the magnetic field. The projection of the path onto the yz plane (viewed along the x axis) is a circle. (The projections of the path onto the xy and xz planes are sinusoids!) Equations 29.13 to 29.15 still apply provided that v is replaced by $v_\perp = \sqrt{v_y^2 + v_z^2}$.

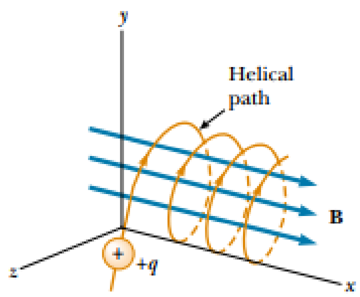


Figure 29.18 A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

EXAMPLE 29.6 A Proton Moving Perpendicular to a Uniform Magnetic Field

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

Solution From Equation 29.13, we have

$$v = \frac{qBr}{m_p} = \frac{(1.60 \times 10^{-19}\ \text{C})(0.35\ \text{T})(14 \times 10^{-2}\ \text{m})}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= \boxed{4.7 \times 10^6\ \text{m/s}}$$

Exercise If an electron moves in a direction perpendicular to the same magnetic field with this same linear speed, what is the radius of its circular orbit?

Answer 7.6×10^{-5} m.

EXAMPLE 29.7 Bending an Electron Beam

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm. (Fig. 29.19 shows such a curved beam of electrons.) If the magnetic field is perpendicular to the beam, (a) what is the magnitude of the field?

Solution First we must calculate the speed of the electrons. We can use the fact that the increase in their kinetic energy must equal the decrease in their potential energy $|e|\Delta V$ (because of conservation of energy). Then we can use Equation 29.13 to find the magnitude of the magnetic field. Because $K_i = 0$ and $K_f = m_e v^2/2$, we have

$$\tfrac{1}{2}m_e v^2 = |e|\Delta V$$

$$v = \sqrt{\frac{2|e|\Delta V}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19}\ \text{C})(350\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= 1.11 \times 10^7\ \text{m/s}$$

$$B = \frac{m_e v}{|e|r} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.11 \times 10^7\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.075\ \text{m})}$$

$$= \boxed{8.4 \times 10^{-4}\ \text{T}}$$

(b) What is the angular speed of the electrons?

Solution Using Equation 29.14, we find that

$$\omega = \frac{v}{r} = \frac{1.11 \times 10^7\ \text{m/s}}{0.075\ \text{m}} = \boxed{1.5 \times 10^8\ \text{rad/s}}$$

Exercise What is the period of revolution of the electrons?

Answer 43 ns.

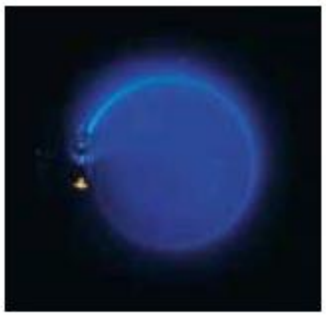


Figure 29.19 The bending of an electron beam in a magnetic field.

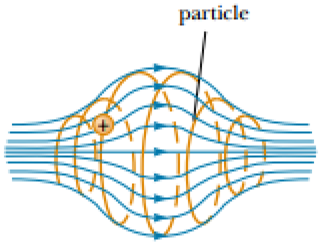


Figure 29.20 A charged particle moving in a nonuniform magnetic field (a magnetic bottle) spirals about the field (red path) and oscillates between the end points. The magnetic force exerted on the particle near either end of the bottle has a component that causes the particle to spiral back toward the center.

When charged particles move in a nonuniform magnetic field, the motion is complex. For example, in a magnetic field that is strong at the ends and weak in the middle, such as that shown in Figure 29.20, the particles can oscillate back and forth between the end points. A charged particle starting at one end spirals along the field lines until it reaches the other end, where it reverses its path and spirals back. This configuration is known as a *magnetic bottle* because charged particles can be trapped within it. The magnetic bottle has been used to confine a *plasma,* a gas consisting of ions and electrons. Such a plasma-confinement scheme could fulfill a crucial role in the control of nuclear fusion, a process that could supply us with an almost endless source of energy. Unfortunately, the magnetic bottle has its problems. If a large number of particles are trapped, collisions between them cause the particles to eventually leak from the system.

The Van Allen radiation belts consist of charged particles (mostly electrons and protons) surrounding the Earth in doughnut-shaped regions (Fig. 29.21). The particles, trapped by the Earth's nonuniform magnetic field, spiral around the field lines from pole to pole, covering the distance in just a few seconds. These particles originate mainly from the Sun, but some come from stars and other heavenly objects. For this reason, the particles are called *cosmic rays.* Most cosmic rays are deflected by the Earth's magnetic field and never reach the atmosphere. However, some of the particles become trapped; it is these particles that make up the Van Allen belts. When the particles are located over the poles, they sometimes collide with atoms in the atmosphere, causing the atoms to emit visible light. Such collisions are the origin of the beautiful Aurora Borealis, or Northern Lights, in the northern hemisphere and the Aurora Australis in the southern hemisphere.

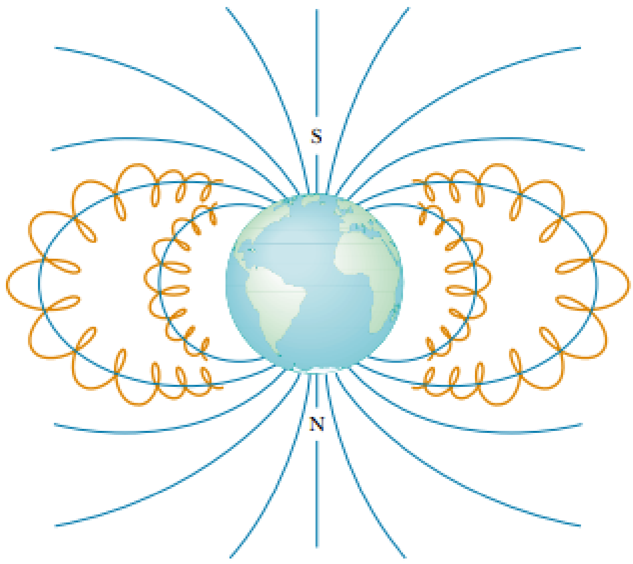
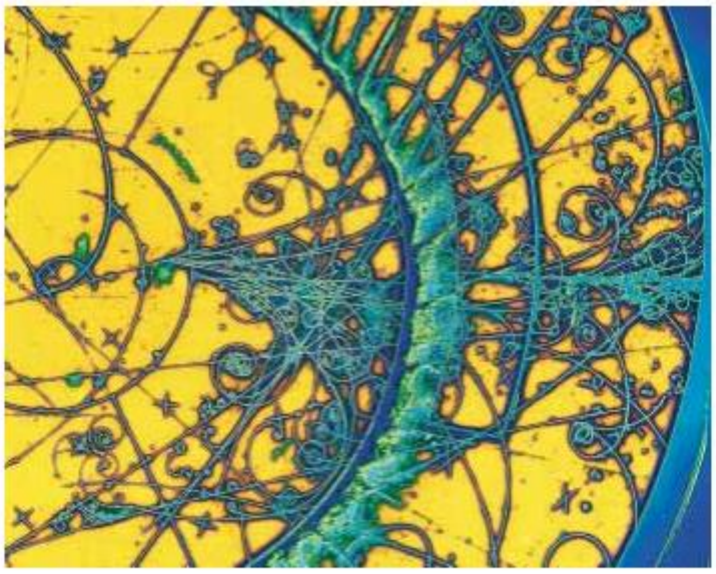


Figure 29.21 The Van Allen belts are made up of charged particles trapped by the Earth's nonuniform magnetic field. The magnetic field lines are in blue and the particle paths in red.

Auroras are usually confined to the polar regions because it is here that the Van Allen belts are nearest the Earth's surface. Occasionally, though, solar activity causes larger numbers of charged particles to enter the belts and significantly distort the normal magnetic field lines associated with the Earth. In these situations an aurora can sometimes be seen at lower latitudes. 12.1 & 12.11



This color-enhanced photograph, taken at CERN, the particle physics laboratory outside Geneva, Switzerland, shows a collection of tracks left by subatomic particles in a bubble chamber. A bubble chamber is a container filled with liquid hydrogen that is superheated, that is, momentarily raised above its normal boiling point by a sudden drop in pressure in the container. Any charged particle passing through the liquid in this state leaves behind a trail of tiny bubbles as the liquid boils in its wake. These bubbles are seen as fine tracks, showing the characteristic paths of different types of particles. The paths are curved because there is an intense applied magnetic field. The tightly wound spiral tracks are due to electrons and positrons.

Optional Section

29.5 APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

A charge moving with a velocity $\mathbf{v}$ in the presence of both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ experiences both an electric force $q\mathbf{E}$ and a magnetic force $q\mathbf{v} \times \mathbf{B}$. The total force (called the Lorentz force) acting on the charge is

Lorentz force

$$\sum \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \quad (29.16)$$

Velocity Selector

In many experiments involving moving charged particles, it is important that the particles all move with essentially the same velocity. This can be achieved by applying a combination of an electric field and a magnetic field oriented as shown in Figure 29.22. A uniform electric field is directed vertically downward (in the plane of the page in Fig. 29.22a), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page in Fig. 29.22a). For q positive, the magnetic force $q\mathbf{v} \times \mathbf{B}$ is upward and the electric force $q\mathbf{E}$ is downward. When the magnitudes of the two fields are chosen so that $qE = qvB$, the particle moves in a straight horizontal line through the region of the fields. From the expression $qE = qvB$, we find that

$$v = \frac{E}{B} \quad (29.17)$$

Only those particles having speed v pass undeflected through the mutually perpendicular electric and magnetic fields. The magnetic force exerted on particles moving at speeds greater than this is stronger than the electric force, and the particles are deflected upward. Those moving at speeds less than this are deflected downward.

The Mass Spectrometer

A **mass spectrometer** separates ions according to their mass-to-charge ratio. In one version of this device, known as the *Bainbridge mass spectrometer,* a beam of ions first passes through a velocity selector and then enters a second uniform magnetic field $\mathbf{B}_0$ that has the same direction as the magnetic field in the selector (Fig. 29.23). Upon entering the second magnetic field, the ions move in a semicircle of

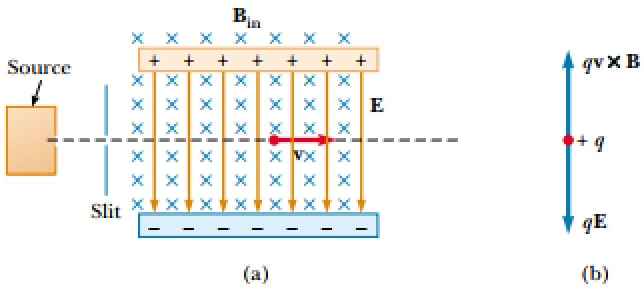


Figure 29.22 (a) A velocity selector. When a positively charged particle is in the presence of a magnetic field directed into the page and an electric field directed downward, it experiences a downward electric force $q\mathbf{E}$ and an upward magnetic force $q\mathbf{v} \times \mathbf{B}$. (b) When these forces balance, the particle moves in a horizontal line through the fields.

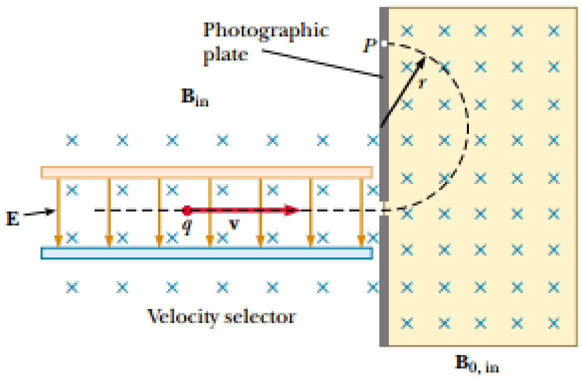


Figure 29.23 A mass spectrometer. Positively charged particles are sent first through a velocity selector and then into a region where the magnetic field $\mathbf{B}_0$ causes the particles to move in a semicircular path and strike a photographic film at P.

radius r before striking a photographic plate at P. If the ions are positively charged, the beam deflects upward, as Figure 29.23 shows. If the ions are negatively charged, the beam would deflect downward. From Equation 29.13, we can express the ratio m/q as

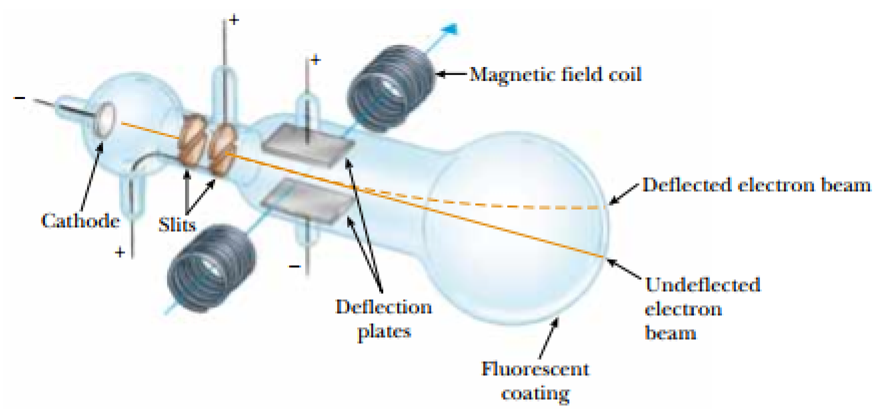
$$\frac{m}{q} = \frac{rB_0}{v}$$

Using Equation 29.17, we find that

$$\frac{m}{q} = \frac{rB_0B}{E} \tag{29.18}$$

Therefore, we can determine m/q by measuring the radius of curvature and knowing the field magnitudes B, B_0, and E. In practice, one usually measures the masses of various isotopes of a given ion, with the ions all carrying the same charge q. In this way, the mass ratios can be determined even if q is unknown.

A variation of this technique was used by J. J. Thomson (1856–1940) in 1897 to measure the ratio e/m_e for electrons. Figure 29.24a shows the basic apparatus he



(a)

(b)

Figure 29.24 (a) Thomson's apparatus for measuring e/m_e. Electrons are accelerated from the cathode, pass through two slits, and are deflected by both an electric field and a magnetic field (directed perpendicular to the electric field). The beam of electrons then strikes a fluorescent screen. (b) J. J. Thomson *(left)* in the Cavendish Laboratory, University of Cambridge. It is interesting to note that the man on the right, Frank Baldwin Jewett, is a distant relative of John W. Jewett, Jr., contributing author of this text.

used. Electrons are accelerated from the cathode and pass through two slits. They then drift into a region of perpendicular electric and magnetic fields. The magnitudes of the two fields are first adjusted to produce an undeflected beam. When the magnetic field is turned off, the electric field produces a measurable beam deflection that is recorded on the fluorescent screen. From the size of the deflection and the measured values of E and B, the charge-to-mass ratio can be determined. The results of this crucial experiment represent the discovery of the electron as a fundamental particle of nature.

Quick Quiz 29.6

When a photographic plate from a mass spectrometer like the one shown in Figure 29.23 is developed, the three patterns shown in Figure 29.25 are observed. Rank the particles that caused the patterns by speed and m/q ratio.

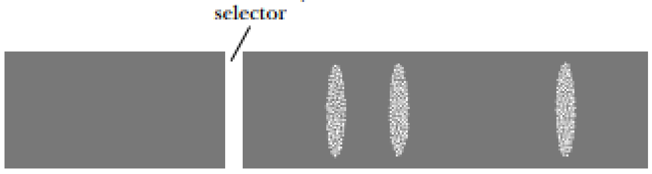


Figure 29.25

The Cyclotron

A **cyclotron** can accelerate charged particles to very high speeds. Both electric and magnetic forces have a key role. The energetic particles produced are used to bombard atomic nuclei and thereby produce nuclear reactions of interest to researchers. A number of hospitals use cyclotron facilities to produce radioactive substances for diagnosis and treatment.

A schematic drawing of a cyclotron is shown in Figure 29.26. The charges move inside two semicircular containers D_1 and D_2, referred to as *dees*. A high-frequency alternating potential difference is applied to the dees, and a uniform magnetic field is directed perpendicular to them. A positive ion released at P near the center of the magnet in one dee moves in a semicircular path (indicated by the dashed red line in the drawing) and arrives back at the gap in a time $T/2$, where T is the time needed to make one complete trip around the two dees, given by Equation 29.15. The frequency of the applied potential difference is adjusted so that the polarity of the dees is reversed in the same time it takes the ion to travel around one dee. If the applied potential difference is adjusted such that D_2 is at a lower electric potential than D_1 by an amount ΔV, the ion accelerates across the gap to D_2 and its kinetic energy increases by an amount $q\Delta V$. It then moves around D_2 in a semicircular path of greater radius (because its speed has increased). After a time $T/2$, it again arrives at the gap between the dees. By this time, the polarity across the dees is again reversed, and the ion is given another "kick" across the gap. The motion continues so that for each half-circle trip around one dee, the ion gains additional kinetic energy equal to $q\,\Delta V$. When the radius of its path is nearly that of the dees, the energetic ion leaves the system through the exit slit. It is important to note that the operation of the cyclotron is

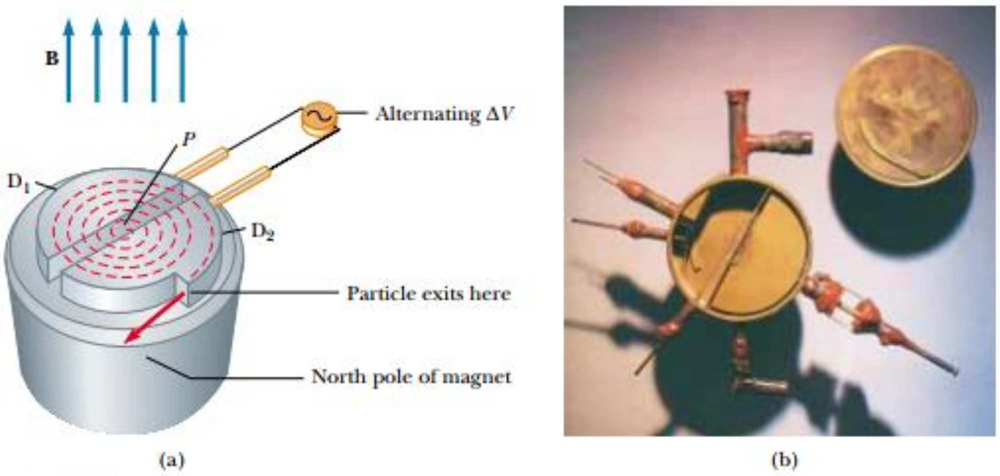


Figure 29.26 (a) A cyclotron consists of an ion source at P, two dees D_1 and D_2 across which an alternating potential difference is applied, and a uniform magnetic field. (The south pole of the magnet is not shown.) The red dashed curved lines represent the path of the particles. (b) The first cyclotron, invented by E.O. Lawrence and M.S. Livingston in 1934.

based on the fact that T is independent of the speed of the ion and of the radius of the circular path.

We can obtain an expression for the kinetic energy of the ion when it exits the cyclotron in terms of the radius R of the dees. From Equation 29.13 we know that $v = qBR/m$. Hence, the kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m} \qquad \textbf{(29.19)}$$

When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play. (Such effects are discussed in Chapter 39.) We observe that T increases and that the moving ions do not remain in phase with the applied potential difference. Some accelerators overcome this problem by modifying the period of the applied potential difference so that it remains in phase with the moving ions.

web

More information on these accelerators is available at
http://www.fnal.gov or
http://www.CERN.ch
The CERN site also discusses the creation of the World Wide Web there by physicists in the mid-1990s.

Optional Section

29.6 THE HALL EFFECT

When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon, first observed by Edwin Hall (1855–1938) in 1879, is known as the *Hall effect*. It arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic force they experience. The Hall effect gives information regarding the sign of the charge carriers and their density; it can also be used to measure the magnitude of magnetic fields.

The arrangement for observing the Hall effect consists of a flat conductor carrying a current I in the x direction, as shown in Figure 29.27. A uniform magnetic field $\mathbf{B}$ is applied in the y direction. If the charge carriers are electrons moving in the negative x direction with a drift velocity $\mathbf{v}_d$, they experience an upward mag-

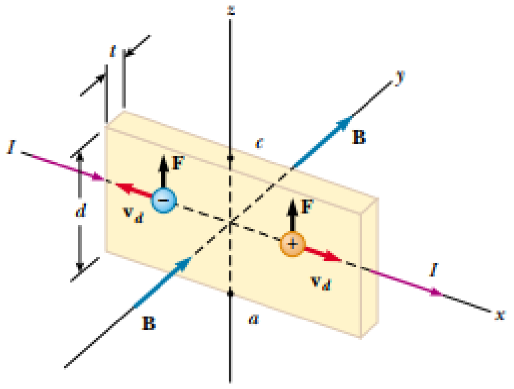


Figure 29.27 To observe the Hall effect, a magnetic field is applied to a current-carrying conductor. When I is in the x direction and $\mathbf{B}$ in the y direction, both positive and negative charge carriers are deflected upward in the magnetic field. The Hall voltage is measured between points a and c.

netic force $\mathbf{F}_B = q\mathbf{v}_d \times \mathbf{B}$, are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge (Fig. 29.28a). This accumulation of charge at the edges increases until the electric force resulting from the charge separation balances the magnetic force acting on the carriers. When this equilibrium condition is reached, the electrons are no longer deflected upward. A sensitive voltmeter or potentiometer connected across the sample, as shown in Figure 29.28, can measure the potential difference—known as the **Hall voltage** ΔV_H—generated across the conductor.

If the charge carriers are positive and hence move in the positive x direction, as shown in Figures 29.27 and 29.28b, they also experience an upward magnetic force $q\mathbf{v}_d \times \mathbf{B}$. This produces a buildup of positive charge on the upper edge and leaves an excess of negative charge on the lower edge. Hence, the sign of the Hall voltage generated in the sample is opposite the sign of the Hall voltage resulting from the deflection of electrons. The sign of the charge carriers can therefore be determined from a measurement of the polarity of the Hall voltage.

In deriving an expression for the Hall voltage, we first note that the magnetic force exerted on the carriers has magnitude qv_dB. In equilibrium, this force is balanced by the electric force qE_H, where E_H is the magnitude of the electric field due to the charge separation (sometimes referred to as the *Hall field*). Therefore,

$$qv_dB = qE_H$$
$$E_H = v_dB$$

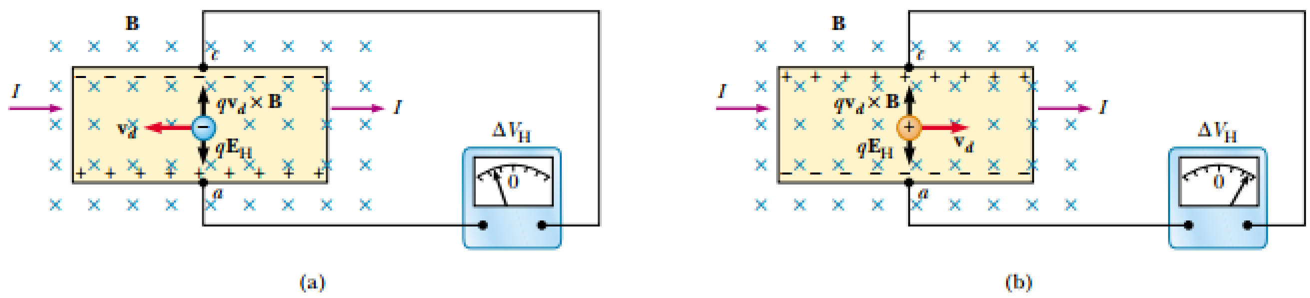


Figure 29.28 (a) When the charge carriers in a Hall effect apparatus are negative, the upper edge of the conductor becomes negatively charged, and c is at a lower electric potential than a. (b) When the charge carriers are positive, the upper edge becomes positively charged, and c is at a higher potential than a. In either case, the charge carriers are no longer deflected when the edges become fully charged, that is, when there is a balance between the electrostatic force qE_H and the magnetic deflection force qvB.

If d is the width of the conductor, the Hall voltage is

$$\Delta V_H = E_H d = v_d B d \quad (29.20)$$

Thus, the measured Hall voltage gives a value for the drift speed of the charge carriers if d and B are known.

We can obtain the charge carrier density n by measuring the current in the sample. From Equation 27.4, we can express the drift speed as

$$v_d = \frac{I}{nqA} \quad (29.21)$$

where A is the cross-sectional area of the conductor. Substituting Equation 29.21 into Equation 29.20, we obtain

$$\Delta V_H = \frac{IBd}{nqA} \quad (29.22)$$

Because $A = td$, where t is the thickness of the conductor, we can also express Equation 29.22 as

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t} \quad (29.23)$$

The Hall voltage

where $R_H = 1/nq$ is the **Hall coefficient.** This relationship shows that a properly calibrated conductor can be used to measure the magnitude of an unknown magnetic field.

Because all quantities in Equation 29.23 other than nq can be measured, a value for the Hall coefficient is readily obtainable. The sign and magnitude of R_H give the sign of the charge carriers and their number density. In most metals, the charge carriers are electrons, and the charge carrier density determined from Hall-effect measurements is in good agreement with calculated values for such metals as lithium (Li), sodium (Na), copper (Cu), and silver (Ag), whose atoms each give up one electron to act as a current carrier. In this case, n is approximately equal to the number of conducting electrons per unit volume. However, this classical model is not valid for metals such as iron (Fe), bismuth (Bi), and cadmium (Cd) or for semiconductors. These discrepancies can be explained only by using a model based on the quantum nature of solids.

web

In 1980, Klaus von Klitzing discovered that the Hall voltage is quantized. He won the Nobel Prize for this discovery in 1985. For a discussion of the quantum Hall effect and some of its consequences, visit our Web site at **http://www.saunderscollege.com/physics/**

EXAMPLE 29.8 The Hall Effect for Copper

A rectangular copper strip 1.5 cm wide and 0.10 cm thick carries a current of 5.0 A. Find the Hall voltage for a 1.2-T magnetic field applied in a direction perpendicular to the strip.

Solution If we assume that one electron per atom is available for conduction, we can take the charge carrier density to be $n = 8.49 \times 10^{28}$ electrons/m^3 (see Example 27.1). Substituting this value and the given data into Equation 29.23 gives

$$\Delta V_H = \frac{IB}{nqt}$$
$$= \frac{(5.0\ \text{A})(1.2\ \text{T})}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})(0.001\ 0\ \text{m})}$$

$$\Delta V_H = \boxed{0.44\ \mu\text{V}}$$

Such an extremely small Hall voltage is expected in good conductors. (Note that the width of the conductor is not needed in this calculation.)

In semiconductors, n is much smaller than it is in metals that contribute one electron per atom to the current; hence, the Hall voltage is usually greater because it varies as the inverse of n. Currents of the order of 0.1 mA are generally used for such materials. Consider a piece of silicon that has the same dimensions as the copper strip in this example and whose value for $n = 1.0 \times 10^{20}$ electrons/m^3. Taking $B = 1.2$ T and $I = 0.10$ mA, we find that $\Delta V_H = 7.5$ mV. A potential difference of this magnitude is readily measured.

SUMMARY

The magnetic force that acts on a charge q moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \tag{29.1}$$

The direction of this magnetic force is perpendicular both to the velocity of the particle and to the magnetic field. The magnitude of this force is

$$F_B = |q|vB \sin\theta \tag{29.2}$$

where θ is the smaller angle between $\mathbf{v}$ and $\mathbf{B}$. The SI unit of $\mathbf{B}$ is the **tesla** (T), where $1\ \mathrm{T} = 1\ \mathrm{N/A \cdot m}$.

When a charged particle moves in a magnetic field, the work done by the magnetic force on the particle is zero because the displacement is always perpendicular to the direction of the force. The magnetic field can alter the direction of the particle's velocity vector, but it cannot change its speed.

If a straight conductor of length L carries a current I, the force exerted on that conductor when it is placed in a uniform magnetic field $\mathbf{B}$ is

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B} \tag{29.3}$$

where the direction of $\mathbf{L}$ is in the direction of the current and $|\mathbf{L}| = L$.

If an arbitrarily shaped wire carrying a current I is placed in a magnetic field, the magnetic force exerted on a very small segment $d\mathbf{s}$ is

$$d\mathbf{F}_B = I\, d\mathbf{s} \times \mathbf{B} \tag{29.4}$$

To determine the total magnetic force on the wire, one must integrate Equation 29.4, keeping in mind that both $\mathbf{B}$ and $d\mathbf{s}$ may vary at each point. Integration gives for the force exerted on a current-carrying conductor of arbitrary shape in a uniform magnetic field

$$\mathbf{F}_B = I\mathbf{L}' \times \mathbf{B} \tag{29.7}$$

where $\mathbf{L}'$ is a vector directed from one end of the conductor to the opposite end. Because integration of Equation 29.4 for a closed loop yields a zero result, the net magnetic force on any closed loop carrying a current in a uniform magnetic field is zero.

The **magnetic dipole moment** $\boldsymbol{\mu}$ of a loop carrying a current I is

$$\boldsymbol{\mu} = I\mathbf{A} \tag{29.10}$$

where the area vector $\mathbf{A}$ is perpendicular to the plane of the loop and $|\mathbf{A}|$ is equal to the area of the loop. The SI unit of $\boldsymbol{\mu}$ is $\mathrm{A \cdot m^2}$.

The torque $\boldsymbol{\tau}$ on a current loop placed in a uniform magnetic field $\mathbf{B}$ is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \tag{29.11}$$

and the potential energy of a magnetic dipole in a magnetic field is

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} \tag{29.12}$$

If a charged particle moves in a uniform magnetic field so that its initial velocity is perpendicular to the field, the particle moves in a circle, the plane of which is perpendicular to the magnetic field. The radius of the circular path is

$$r = \frac{mv}{qB} \tag{29.13}$$

where m is the mass of the particle and q is its charge. The angular speed of the charged particle is

$$\omega = \frac{qB}{m} \tag{29.14}$$

QUESTIONS

1. At a given instant, a proton moves in the positive x direction in a region where a magnetic field is directed in the negative z direction. What is the direction of the magnetic force? Does the proton continue to move in the positive x direction? Explain.
2. Two charged particles are projected into a region where a magnetic field is directed perpendicular to their velocities. If the charges are deflected in opposite directions, what can be said about them?
3. If a charged particle moves in a straight line through some region of space, can one say that the magnetic field in that region is zero?
4. Suppose an electron is chasing a proton up this page when suddenly a magnetic field directed perpendicular into the page is turned on. What happens to the particles?
5. How can the motion of a moving charged particle be used to distinguish between a magnetic field and an electric field? Give a specific example to justify your argument.
6. List several similarities and differences between electric and magnetic forces.
7. Justify the following statement: "It is impossible for a constant (in other words, a time-independent) magnetic field to alter the speed of a charged particle."
8. In view of the preceding statement, what is the role of a magnetic field in a cyclotron?
9. A current-carrying conductor experiences no magnetic force when placed in a certain manner in a uniform magnetic field. Explain.
10. Is it possible to orient a current loop in a uniform magnetic field such that the loop does not tend to rotate? Explain.
11. How can a current loop be used to determine the presence of a magnetic field in a given region of space?
12. What is the net force acting on a compass needle in a uniform magnetic field?
13. What type of magnetic field is required to exert a resultant force on a magnetic dipole? What is the direction of the resultant force?
14. A proton moving horizontally enters a region where a uniform magnetic field is directed perpendicular to the proton's velocity, as shown in Figure Q29.14. Describe the subsequent motion of the proton. How would an electron behave under the same circumstances?
15. In a magnetic bottle, what causes the direction of the velocity of the confined charged particles to reverse? (*Hint:* Find the direction of the magnetic force acting on the particles in a region where the field lines converge.)
16. In the cyclotron, why do particles of different velocities take the same amount of time to complete one half-circle trip around one dee?

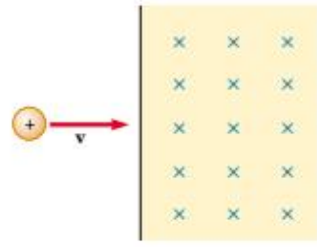


Figure Q29.14

17. The *bubble chamber* is a device used for observing tracks of particles that pass through the chamber, which is immersed in a magnetic field. If some of the tracks are spirals and others are straight lines, what can you say about the particles?
18. Can a constant magnetic field set into motion an electron initially at rest? Explain your answer.
19. You are designing a magnetic probe that uses the Hall effect to measure magnetic fields. Assume that you are restricted to using a given material and that you have already made the probe as thin as possible. What, if anything, can be done to increase the Hall voltage produced for a given magnetic field?
20. The electron beam shown in Figure Q29.20 is projected to the right. The beam deflects downward in the presence of a magnetic field produced by a pair of current-carrying coils. (a) What is the direction of the magnetic field? (b) What would happen to the beam if the current in the coils were reversed?

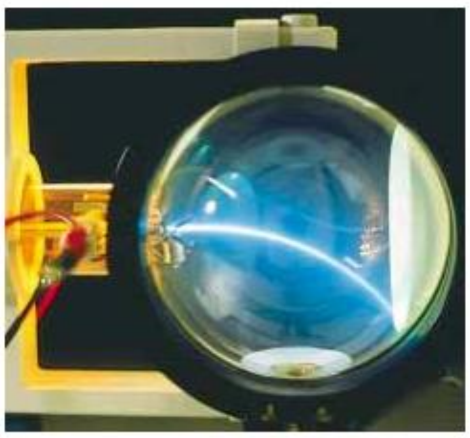


Figure Q29.20