PUZZLER

Most car headlights have lines across their faces, like those shown here. Without these lines, the headlights either would not function properly or would be much more likely to break from the jarring of the car on a bumpy road. What is the purpose of the lines? *(George Semple)*

chapter 36

Geometric Optics

Chapter Outline

This chapter is concerned with the images that result when spherical waves fall on flat and spherical surfaces. We find that images can be formed either by reflection or by refraction and that mirrors and lenses work because of reflection and refraction. We continue to use the ray approximation and to assume that light travels in straight lines. Both of these steps lead to valid predictions in the field called *geometric optics.* In subsequent chapters, we shall concern ourselves with interference and diffraction effects—the objects of study in the field of *wave optics.*

36.1 IMAGES FORMED BY FLAT MIRRORS

14.6 We begin by considering the simplest possible mirror, the flat mirror. Consider a point source of light placed at O in Figure 36.1, a distance p in front of a flat mirror. The distance p is called the **object distance.** Light rays leave the source and are reflected from the mirror. Upon reflection, the rays continue to diverge (spread apart), but they appear to the viewer to come from a point I behind the mirror. Point I is called the **image** of the object at O. Regardless of the system under study, we always locate images by extending diverging rays back to a point from which they appear to diverge. **Images are located either at the point from which rays of light *actually* diverge or at the point from which they *appear* to diverge.** Because the rays in Figure 36.1 appear to originate at I, which is a distance q behind the mirror, this is the location of the image. The distance q is called the **image distance.**

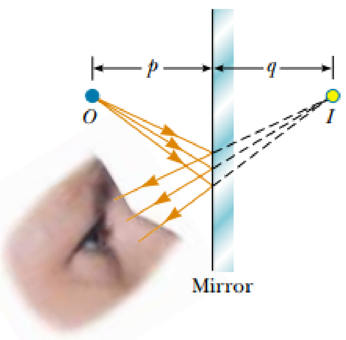


Figure 36.1 An image formed by reflection from a flat mirror. The image point I is located behind the mirror a perpendicular distance q from the mirror (the image distance). Study of Figure 36.2 shows that this image distance has the same magnitude as the object distance p.

Images are classified as real or virtual. **A real image is formed when light rays pass through and diverge from the image point; a virtual image is formed when the light rays do not pass through the image point but appear to diverge from that point.** The image formed by the mirror in Figure 36.1 is virtual. The image of an object seen in a flat mirror is always virtual. Real images can be displayed on a screen (as at a movie), but virtual images cannot be displayed on a screen.

We can use the simple geometric techniques shown in Figure 36.2 to examine the properties of the images formed by flat mirrors. Even though an infinite number of light rays leave each point on the object, we need to follow only two of them to determine where an image is formed. One of those rays starts at P, follows a horizontal path to the mirror, and reflects back on itself. The second ray follows the oblique path PR and reflects as shown, according to the law of reflection. An observer in front of the mirror would trace the two reflected rays back to the point at which they appear to have originated, which is point P' behind the mirror. A continuation of this process for points other than P on the object would result in a virtual image (represented by a yellow arrow) behind the mirror. Because triangles PQR and $P'QR$ are congruent, $PQ = P'Q$. We conclude that **the image formed by an object placed in front of a flat mirror is as far behind the mirror as the object is in front of the mirror.**

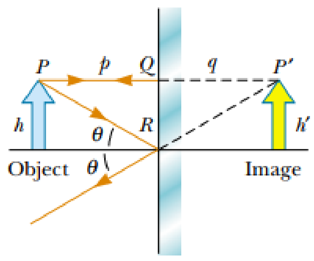


Figure 36.2 A geometric construction that is used to locate the image of an object placed in front of a flat mirror. Because the triangles PQR and $P'QR$ are congruent, $|p| = |q|$ and $h = h'$.

Geometry also reveals that the object height h equals the image height h'. Let us define **lateral magnification** M as follows:

Lateral magnification

$$M \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h} \tag{36.1}$$

Mt. Hood reflected in Trillium Lake. Why is the image inverted and the same size as the mountain?

QuickLab

View yourself in a full-length mirror. Standing close to the mirror, place one piece of tape at the top of the image of your head and another piece at the very bottom of the image of your feet. Now step back a few meters and observe your image. How big is it relative to its original size? How does the distance between the pieces of tape compare with your actual height? You may want to refer to Problem 3.

This is a general definition of the lateral magnification for any type of mirror. For a flat mirror, $M = 1$ because $h' = h$.

Finally, note that a flat mirror produces an image that has an *apparent* left–right reversal. You can see this reversal by standing in front of a mirror and raising your right hand, as shown in Figure 36.3. The image you see raises its left hand. Likewise, your hair appears to be parted on the side opposite your real part, and a mole on your right cheek appears to be on your left cheek.

This reversal is not *actually* a left–right reversal. Imagine, for example, lying on your left side on the floor, with your body parallel to the mirror surface. Now your head is on the left and your feet are on the right. If you shake your feet, the image does not shake its head! If you raise your right hand, however, the image again raises its left hand. Thus, the mirror again appears to produce a left–right reversal but in the up–down direction!

The reversal is actually a *front–back reversal,* caused by the light rays going forward toward the mirror and then reflecting back from it. An interesting exercise is to stand in front of a mirror while holding an overhead transparency in front of you so that you can read the writing on the transparency. You will be able to read the writing on the image of the transparency, also. You may have had a similar experience if you have attached a transparent decal with words on it to the rear window of your car. If the decal can be read from outside the car, you can also read it when looking into your rearview mirror from inside the car.

We conclude that the image that is formed by a flat mirror has the following properties.

- The image is as far behind the mirror as the object is in front of the mirror.
- The image is unmagnified, virtual, and upright. (By *upright* we mean that, if the object arrow points upward as in Figure 36.2, so does the image arrow.)
- The image has front–back reversal.

Figure 36.3 The image in the mirror of a person's right hand is reversed front to back. This makes the right hand appear to be a left hand. Notice that the thumb is on the left side of both real hands and on the left side of the image. That the thumb is not on the right side of the image indicates that there is no left-to-right reversal.

Quick Quiz 36.1

In the overhead view of Figure 36.4, the image of the stone seen by observer 1 is at *C*. Where does observer 2 see the image—at *A*, at *B*, at *C*, at *D*, at *E*, or not at all?

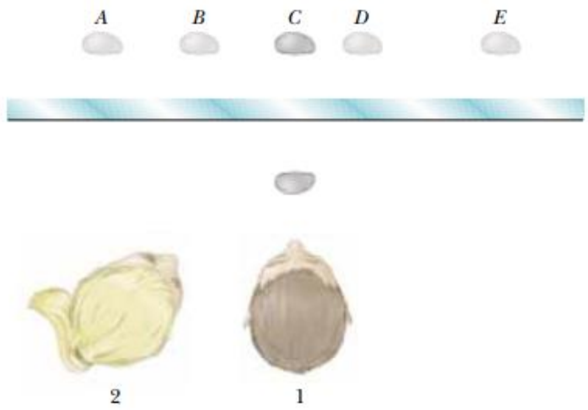


Figure 36.4

CONCEPTUAL EXAMPLE 36.1 Multiple Images Formed by Two Mirrors

Two flat mirrors are at right angles to each other, as illustrated in Figure 36.5, and an object is placed at point *O*. In this situation, multiple images are formed. Locate the positions of these images.

Solution The image of the object is at I_1 in mirror 1 and at I_2 in mirror 2. In addition, a third image is formed at I_3. This third image is the image of I_1 in mirror 2 or, equivalently, the image of I_2 in mirror 1. That is, the image at I_1 (or I_2) serves as the object for I_3. Note that to form this image at I_3, the rays reflect twice after leaving the object at *O*.

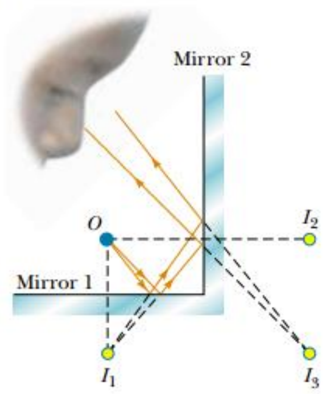


Figure 36.5 When an object is placed in front of two mutually perpendicular mirrors as shown, three images are formed.

CONCEPTUAL EXAMPLE 36.2 The Levitated Professor

The professor in the box shown in Figure 36.6 appears to be balancing himself on a few fingers, with his feet off the floor. He can maintain this position for a long time, and he appears to defy gravity. How was this illusion created?

Solution This is one of many magicians' optical illusions that make use of a mirror. The box in which the professor stands is a cubical frame that contains a flat vertical mirror positioned in a diagonal plane of the frame. The professor straddles the mirror so that one foot, which you see, is in front of the mirror, and one foot, which you cannot see, is behind the mirror. When he raises the foot in front of the mirror, the reflection of that foot also rises, so he appears to float in air.

Figure 36.6 An optical illusion.

CONCEPTUAL EXAMPLE 36.3 The Tilting Rearview Mirror

Most rearview mirrors in cars have a day setting and a night setting. The night setting greatly diminishes the intensity of the image in order that lights from trailing vehicles do not blind the driver. How does such a mirror work?

Solution Figure 36.7 shows a cross-sectional view of a rearview mirror for each setting. The unit consists of a reflective coating on the back of a wedge of glass. In the day setting (Fig. 36.7a), the light from an object behind the car strikes the glass wedge at point 1. Most of the light enters the wedge, refracting as it crosses the front surface, and reflects from the back surface to return to the front surface, where it is refracted again as it re-enters the air as ray B (for *bright*). In addition, a small portion of the light is reflected at the front surface of the glass, as indicated by ray D (for *dim*).

This dim reflected light is responsible for the image that is observed when the mirror is in the night setting (Fig. 36.7b). In this case, the wedge is rotated so that the path followed by the bright light (ray B) does not lead to the eye. Instead, the dim light reflected from the front surface of the wedge travels to the eye, and the brightness of trailing headlights does not become a hazard.

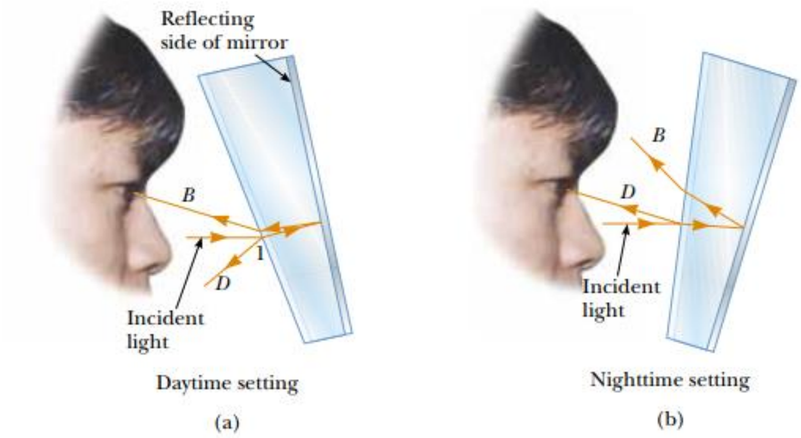


Figure 36.7 Cross-sectional views of a rearview mirror. (a) With the day setting, the silvered back surface of the mirror reflects a bright ray B into the driver's eyes. (b) With the night setting, the glass of the unsilvered front surface of the mirror reflects a dim ray D into the driver's eyes.

36.2 IMAGES FORMED BY SPHERICAL MIRRORS

Concave Mirrors

14.7 A **spherical mirror,** as its name implies, has the shape of a section of a sphere. This type of mirror focuses incoming parallel rays to a point, as demonstrated by the colored light rays in Figure 36.8. Figure 36.9a shows a cross-section of a spherical mirror, with its surface represented by the solid, curved black line. (The blue band represents the structural support for the mirrored surface, such as a curved piece of glass on which the silvered surface is deposited.) Such a mirror, in which light is reflected from the inner, concave surface, is called a **concave mirror.** The mirror has a radius of curvature R, and its center of curvature is point C. Point V is the center of the spherical section, and a line through C and V is called the **principal axis** of the mirror.

Now consider a point source of light placed at point O in Figure 36.9b, where O is any point on the principal axis to the left of C. Two diverging rays that originate at O are shown. After reflecting from the mirror, these rays converge (come together) at the image point I. They then continue to diverge from I as if an object were there. As a result, we have at point I a real image of the light source at O.

We shall consider in this section only rays that diverge from the object and make a small angle with the principal axis. Such rays are called **paraxial rays.** All

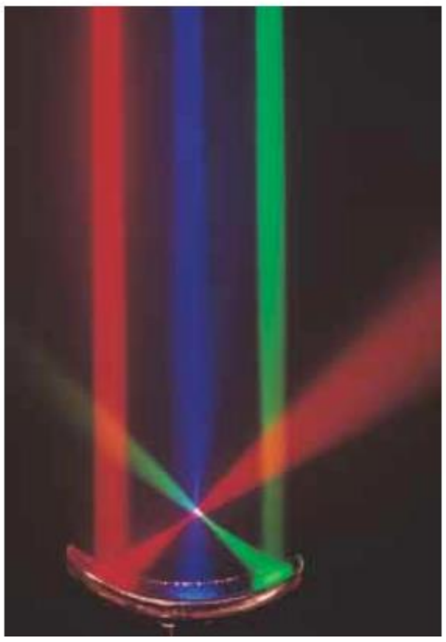


Figure 36.8 Red, blue, and green light rays are reflected by a curved mirror. Note that the point where the three colors meet is white.

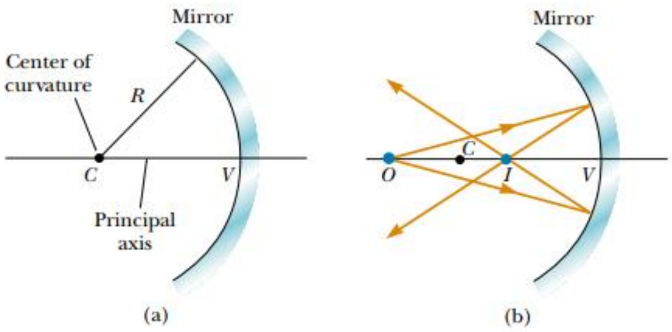


Figure 36.9 (a) A concave mirror of radius R. The center of curvature C is located on the principal axis. (b) A point object placed at O in front of a concave spherical mirror of radius R, where O is any point on the principal axis farther than R from the mirror surface, forms a real image at I. If the rays diverge from O at small angles, they all reflect through the same image point.

such rays reflect through the image point, as shown in Figure 36.9b. Rays that are far from the principal axis, such as those shown in Figure 36.10, converge to other points on the principal axis, producing a blurred image. This effect, which is called **spherical aberration,** is present to some extent for any spherical mirror and is discussed in Section 36.5.

We can use Figure 36.11 to calculate the image distance q from a knowledge of the object distance p and radius of curvature R. By convention, these distances are measured from point V. Figure 36.11 shows two rays leaving the tip of the object. One of these rays passes through the center of curvature C of the mirror, hitting the mirror perpendicular to the mirror surface and reflecting back on itself. The second ray strikes the mirror at its center (point V) and reflects as shown, obeying the law of reflection. The image of the tip of the arrow is located at the point where these two rays intersect. From the gold right triangle in Figure 36.11, we see that $\tan\theta = h/p$, and from the blue right triangle we see that $\tan\theta = -h'/q$. The negative sign is introduced because the image is inverted, so h' is taken to be negative. Thus, from Equation 36.1 and these results, we find that the magnification of the mirror is

$$M = \frac{h'}{h} = -\frac{q}{p} \tag{36.2}$$

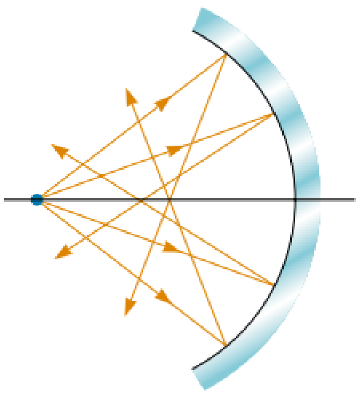


Figure 36.10 Rays diverging from the object at large angles from the principal axis reflect from a spherical concave mirror to intersect the principal axis at different points, resulting in a blurred image. This condition is called *spherical aberration.*

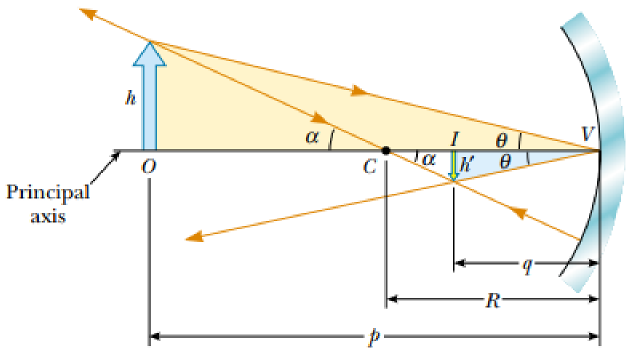


Figure 36.11 The image formed by a spherical concave mirror when the object O lies outside the center of curvature C.

We also note from the two triangles in Figure 36.11 that have α as one angle that

$$\tan \alpha = \frac{h}{p - R} \qquad \text{and} \qquad \tan \alpha = -\frac{h'}{R - q}$$

from which we find that

$$\frac{h'}{h} = -\frac{R - q}{p - R} \tag{36.3}$$

If we compare Equations 36.2 and 36.3, we see that

$$\frac{R - q}{p - R} = \frac{q}{p}$$

Simple algebra reduces this to

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \tag{36.4}$$

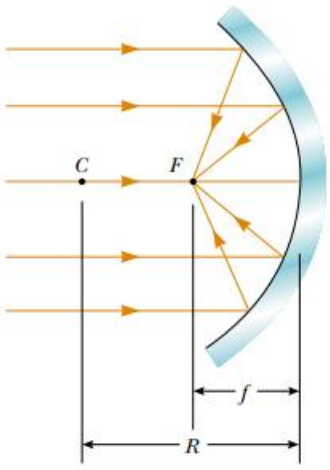
Mirror equation in terms of R

This expression is called the **mirror equation.** It is applicable only to paraxial rays.

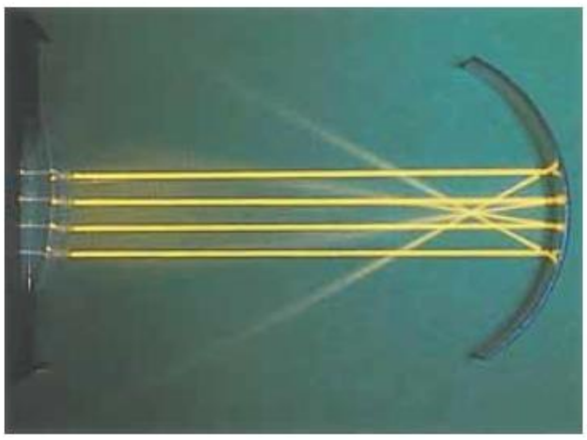
If the object is very far from the mirror—that is, if p is so much greater than R that p can be said to approach infinity—then $1/p \approx 0$, and we see from Equation 36.4 that $q \approx R/2$. That is, when the object is very far from the mirror, the image point is halfway between the center of curvature and the center point on the mirror, as shown in Figure 36.12a. The incoming rays from the object are essentially parallel in this figure because the source is assumed to be very far from the mirror. We call the image point in this special case the **focal point** F and the image distance the **focal length** f, where

$$f = \frac{R}{2} \tag{36.5}$$

Focal length



(a)



(b)

Figure 36.12 (a) Light rays from a distant object ($p \approx \infty$) reflect from a concave mirror through the focal point F. In this case, the image distance $q \approx R/2 = f$, where f is the focal length of the mirror. (b) Reflection of parallel rays from a concave mirror.

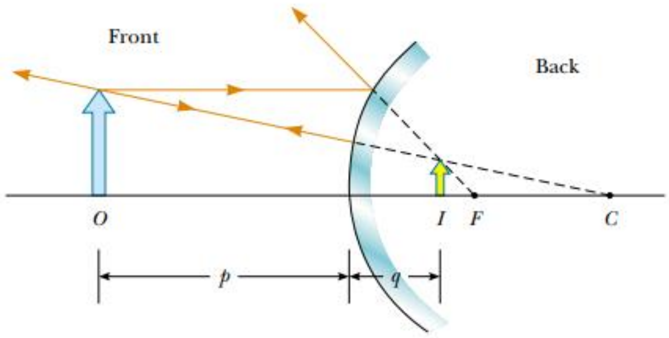


Figure 36.13 Formation of an image by a spherical convex mirror. The image formed by the real object is virtual and upright.

Focal length is a parameter particular to a given mirror and therefore can be used to compare one mirror with another. The mirror equation can be expressed in terms of the focal length:

Mirror equation in terms of f

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{36.6}$$

Notice that the focal length of a mirror depends only on the curvature of the mirror and not on the material from which the mirror is made. This is because the formation of the image results from rays reflected from the surface of the material. We shall find in Section 36.4 that the situation is different for lenses; in that case the light actually passes through the material.

Convex Mirrors

Figure 36.13 shows the formation of an image by a **convex mirror**—that is, one silvered so that light is reflected from the outer, convex surface. This is sometimes called a **diverging mirror** because the rays from any point on an object diverge after reflection as though they were coming from some point behind the mirror. The image in Figure 36.13 is virtual because the reflected rays only appear to originate at the image point, as indicated by the dashed lines. Furthermore, the image is always upright and smaller than the object. This type of mirror is often used in stores to foil shoplifters. A single mirror can be used to survey a large field of view because it forms a smaller image of the interior of the store.

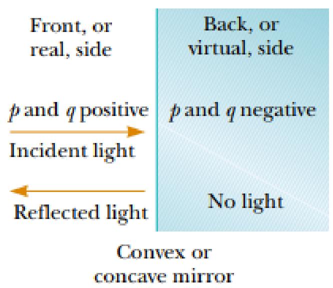


Figure 36.14 Signs of p and q for convex and concave mirrors.

We do not derive any equations for convex spherical mirrors because we can use Equations 36.2, 36.4, and 36.6 for either concave or convex mirrors if we adhere to the following procedure. Let us refer to the region in which light rays move toward the mirror as the *front side* of the mirror, and the other side as the *back side*. For example, in Figures 36.10 and 36.12, the side to the left of the mirrors is the front side, and the side to the right of the mirrors is the back side. Figure 36.14 states the sign conventions for object and image distances, and Table 36.1 summarizes the sign conventions for all quantities.

TABLE 36.1 Sign Conventions for Mirrors

p is **positive** if object is in **front** of mirror (real object).
p is **negative** if object is in **back** of mirror (virtual object).
q is **positive** if image is in **front** of mirror (real image).
q is **negative** if image is in **back** of mirror (virtual image).
Both f and R are **positive** if center of curvature is in **front** of mirror (concave mirror).
Both f and R are **negative** if center of curvature is in **back** of mirror (convex mirror).
If M is **positive,** image is **upright.**
If M is **negative,** image is **inverted.**

Reflection of parallel lines from a convex cylindrical mirror. The image is virtual, upright, and reduced in size.

Ray Diagrams for Mirrors

The positions and sizes of images formed by mirrors can be conveniently determined with *ray diagrams.* These graphical constructions reveal the nature of the image and can be used to check results calculated from the mirror and magnification equations. To draw a ray diagram, we need to know the position of the object and the locations of the mirror's focal point and center of curvature. We then draw three rays to locate the image, as shown by the examples in Figure 36.15. These rays all start from the same object point and are drawn as follows. We may choose any point on the object; here, we choose the top of the object for simplicity:

- Ray 1 is drawn from the top of the object parallel to the principal axis and is reflected through the focal point F.
- Ray 2 is drawn from the top of the object through the focal point and is reflected parallel to the principal axis.
- Ray 3 is drawn from the top of the object through the center of curvature C and is reflected back on itself.

The intersection of any two of these rays locates the image. The third ray serves as a check of the construction. The image point obtained in this fashion must always agree with the value of q calculated from the mirror equation.

With concave mirrors, note what happens as the object is moved closer to the mirror. The real, inverted image in Figure 36.15a moves to the left as the object approaches the focal point. When the object is at the focal point, the image is infinitely far to the left. However, when the object lies between the focal point and the mirror surface, as shown in Figure 36.15b, the image is virtual, upright, and enlarged. This latter situation applies in the use of a shaving mirror or a makeup mirror. Your face is closer to the mirror than the focal point, and you see an upright, enlarged image of your face.

In a convex mirror (see Fig. 36.15c), the image of an object is always virtual, upright, and reduced in size. In this case, as the object distance increases, the virtual image decreases in size and approaches the focal point as p approaches infinity. You should construct other diagrams to verify how image position varies with object position.

QuickLab

Compare the images formed of your face when you look first at the front side and then at the back side of a shiny soup spoon. Why do the two images look so different from each other?

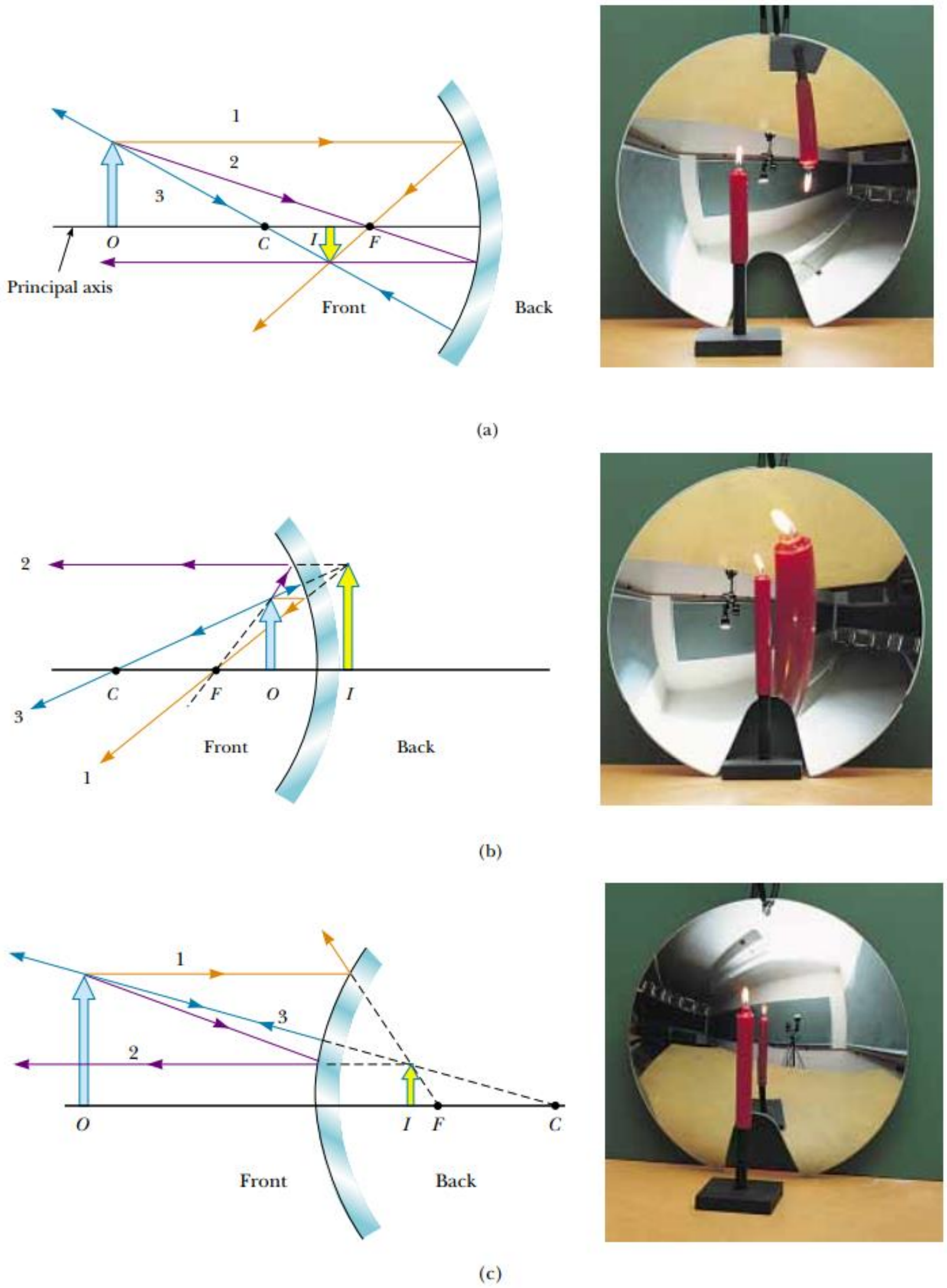


Figure 36.15 Ray diagrams for spherical mirrors, along with corresponding photographs of the images of candles. (a) When the object is located so that the center of curvature lies between the object and a concave mirror surface, the image is real, inverted, and reduced in size. (b) When the object is located between the focal point and a concave mirror surface, the image is virtual, upright, and enlarged. (c) When the object is in front of a convex mirror, the image is virtual, upright, and reduced in size.

EXAMPLE 36.4 The Image from a Mirror

Assume that a certain spherical mirror has a focal length of $+10.0$ cm. Locate and describe the image for object distances of (a) 25.0 cm, (b) 10.0 cm, and (c) 5.00 cm.

Solution Because the focal length is positive, we know that this is a concave mirror (see Table 36.1). (a) This situation is analogous to that in Figure 36.15a; hence, we expect the image to be real and closer to the mirror than the object. According to the figure, it should also be inverted and reduced in size. We find the image distance by using the Equation 36.6 form of the mirror equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{25.0\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$q=\boxed{16.7\text{ cm}}$$

The magnification is given by Equation 36.2:

$$M=-\frac{q}{p}=-\frac{16.7\text{ cm}}{25.0\text{ cm}}=-0.668$$

The fact that the absolute value of M is less than unity tells us that the image is smaller than the object, and the negative sign for M tells us that the image is inverted. Because q is positive, the image is located on the front side of the mirror and is real. Thus, we see that our predictions were correct.

(b) When the object distance is 10.0 cm, the object is located at the focal point. Now we find that

$$\frac{1}{10.0\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$q=\boxed{\infty}$$

which means that rays originating from an object positioned at the focal point of a mirror are reflected so that the image is formed at an infinite distance from the mirror; that is, the rays travel parallel to one another after reflection. This is the situation in a flashlight, where the bulb filament is placed at the focal point of a reflector, producing a parallel beam of light.

(c) When the object is at $p = 5.00$ cm, it lies between the focal point and the mirror surface, as shown in Figure 36.15b. Thus, we expect a magnified, virtual, upright image. In this case, the mirror equation gives

$$\frac{1}{5.00\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$q=\boxed{-10.0\text{ cm}}$$

The image is virtual because it is located behind the mirror, as expected. The magnification is

$$M=-\frac{q}{p}=-\left(\frac{-10.0\text{ cm}}{5.00\text{ cm}}\right)=2.00$$

The image is twice as large as the object, and the positive sign for M indicates that the image is upright (see Fig. 36.15b).

Exercise At what object distance is the magnification -1.00?

Answer 20.0 cm.

EXAMPLE 36.5 The Image from a Convex Mirror

A woman who is 1.5 m tall is located 3.0 m from an antishoplifting mirror, as shown in Figure 36.16. The focal length of the mirror is -0.25 m. Find (a) the position of her image and (b) the magnification.

Solution (a) This situation is depicted in Figure 36.15c. We should expect to find an upright, reduced, virtual image. To find the image position, we use Equation 36.6:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{-0.25\text{ m}}$$

$$\frac{1}{q}=\frac{1}{-0.25\text{ m}}-\frac{1}{3.0\text{ m}}$$

$$q=\boxed{-0.23\text{ m}}$$

Figure 36.16 Convex mirrors, often used for security in department stores, provide wide-angle viewing.

The negative value of q indicates that her image is virtual, or behind the mirror, as shown in Figure 36.15c.

(b) The magnification is

$$M = -\frac{q}{p} = -\left(\frac{-0.23 \text{ m}}{3.0 \text{ m}}\right) = \boxed{0.077}$$

The image is much smaller than the woman, and it is upright because M is positive.

Exercise Find the height of the image.

Answer 0.12 m.

36.3 IMAGES FORMED BY REFRACTION

In this section we describe how images are formed when light rays are refracted at the boundary between two transparent materials. Consider two transparent media having indices of refraction n_1 and n_2, where the boundary between the two media is a spherical surface of radius R (Fig. 36.17). We assume that the object at O is in the medium for which the index of refraction is n_1, where $n_1 < n_2$. Let us consider the paraxial rays leaving O. As we shall see, all such rays are refracted at the spherical surface and focus at a single point I, the image point.

Figure 36.18 shows a single ray leaving point O and focusing at point I. Snell's law of refraction applied to this refracted ray gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Because θ_1 and θ_2 are assumed to be small, we can use the small-angle approximation $\sin \theta \approx \theta$ (angles in radians) and say that

$$n_1\theta_1 = n_2\theta_2$$

Now we use the fact that an exterior angle of any triangle equals the sum of the two opposite interior angles. Applying this rule to triangles OPC and PIC in Figure 36.18 gives

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

If we combine all three expressions and eliminate θ_1 and θ_2, we find that

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta \tag{36.7}$$

Looking at Figure 36.18, we see three right triangles that have a common vertical leg of length d. For paraxial rays (unlike the relatively large-angle ray shown in Fig.

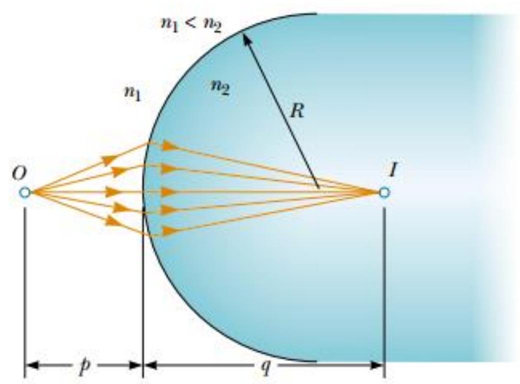


Figure 36.17 An image formed by refraction at a spherical surface. Rays making small angles with the principal axis diverge from a point object at O and are refracted through the image point I.

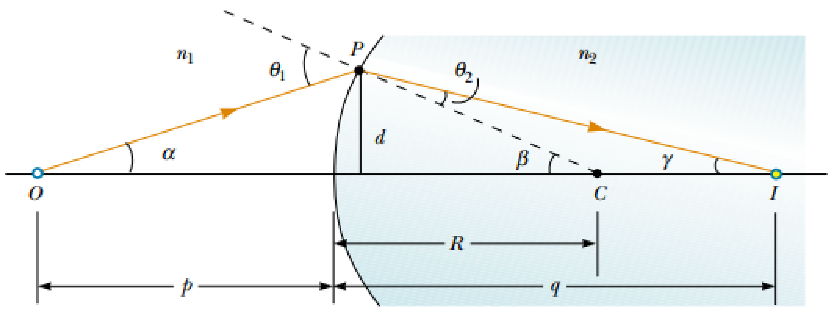


Figure 36.18 Geometry used to derive Equation 36.8.

36.18), the horizontal legs of these triangles are approximately p for the triangle containing angle α, R for the triangle containing angle β, and q for the triangle containing angle γ. In the small-angle approximation, $\tan\theta \approx \theta$, so we can write the approximate relationships from these triangles as follows:

$$\tan\alpha \approx \alpha \approx \frac{d}{p} \qquad \tan\beta \approx \beta \approx \frac{d}{R} \qquad \tan\gamma \approx \gamma \approx \frac{d}{q}$$

We substitute these expressions into Equation 36.7 and divide through by d to get

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad \text{(36.8)}$$

For a fixed object distance p, the image distance q is independent of the angle that the ray makes with the axis. This result tells us that all paraxial rays focus at the same point I.

As with mirrors, we must use a sign convention if we are to apply this equation to a variety of cases. We define the side of the surface in which light rays originate as the front side. The other side is called the back side. Real images are formed by refraction in back of the surface, in contrast with mirrors, where real images are formed in front of the reflecting surface. Because of the difference in location of real images, the refraction sign conventions for q and R are opposite the reflection sign conventions. For example, q and R are both positive in Figure 36.18. The sign conventions for spherical refracting surfaces are summarized in Table 36.2.

We derived Equation 36.8 from an assumption that $n_1 < n_2$. This assumption is not necessary, however. Equation 36.8 is valid regardless of which index of refraction is greater.

TABLE 36.2 Sign Conventions for Refracting Surfaces

p is **positive** if object is in **front** of surface (real object). p is **negative** if object is in **back** of surface (virtual object).
q is **positive** if image is in **back** of surface (real image). q is **negative** if image is in **front** of surface (virtual image).
R is **positive** if center of curvature is in **back** of convex surface. R is **negative** if center of curvature is in **front** of concave surface.

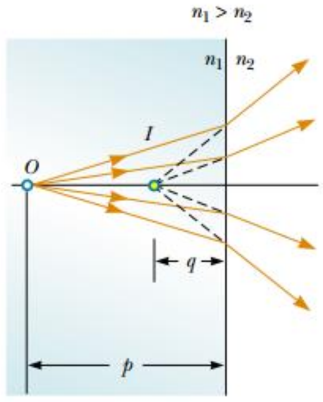


Figure 36.19 The image formed by a flat refracting surface is virtual and on the same side of the surface as the object. All rays are assumed to be paraxial.

Flat Refracting Surfaces

If a refracting surface is flat, then R is infinite and Equation 36.8 reduces to

$$\frac{n_1}{p} = -\frac{n_2}{q}$$

$$q = -\frac{n_2}{n_1}p \tag{36.9}$$

From this expression we see that the sign of q is opposite that of p. Thus, according to Table 36.2, **the image formed by a flat refracting surface is on the same side of the surface as the object.** This is illustrated in Figure 36.19 for the situation in which the object is in the medium of index n_1 and n_1 is greater than n_2. In this case, a virtual image is formed between the object and the surface. If n_1 is less than n_2, the rays in the back side diverge from each other at lesser angles than those in Figure 36.19. As a result, the virtual image is formed to the left of the object.

CONCEPTUAL EXAMPLE 36.6 Let's Go Scuba Diving!

It is well known that objects viewed under water with the naked eye appear blurred and out of focus. However, a scuba diver using a mask has a clear view of underwater objects. (a) Explain how this works, using the facts that the indices of refraction of the cornea, water, and air are 1.376, 1.333, and 1.000 29, respectively.

Solution Because the cornea and water have almost identical indices of refraction, very little refraction occurs when a person under water views objects with the naked eye. In this case, light rays from an object focus behind the retina, resulting in a blurred image. When a mask is used, the air space between the eye and the mask surface provides the normal amount of refraction at the eye–air interface, and the light from the object is focused on the retina.

(b) If a lens prescription is ground into the glass of a mask, should the curved surface be on the inside of the mask, the outside, or both?

Solution If a lens prescription is ground into the glass of the mask so that the wearer can see without eyeglasses, only the inside surface is curved. In this way the prescription is accurate whether the mask is used under water or in air. If the curvature were on the outer surface, the refraction at the outer surface of the glass would change depending on whether air or water were present on the outside of the mask.

EXAMPLE 36.7 Gaze into the Crystal Ball

A dandelion seed ball 4.0 cm in diameter is embedded in the center of a spherical plastic paperweight having a diameter of 6.0 cm (Fig. 36.20a). The index of refraction of the plastic is $n_1 = 1.50$. Find the position of the image of the near edge of the seed ball.

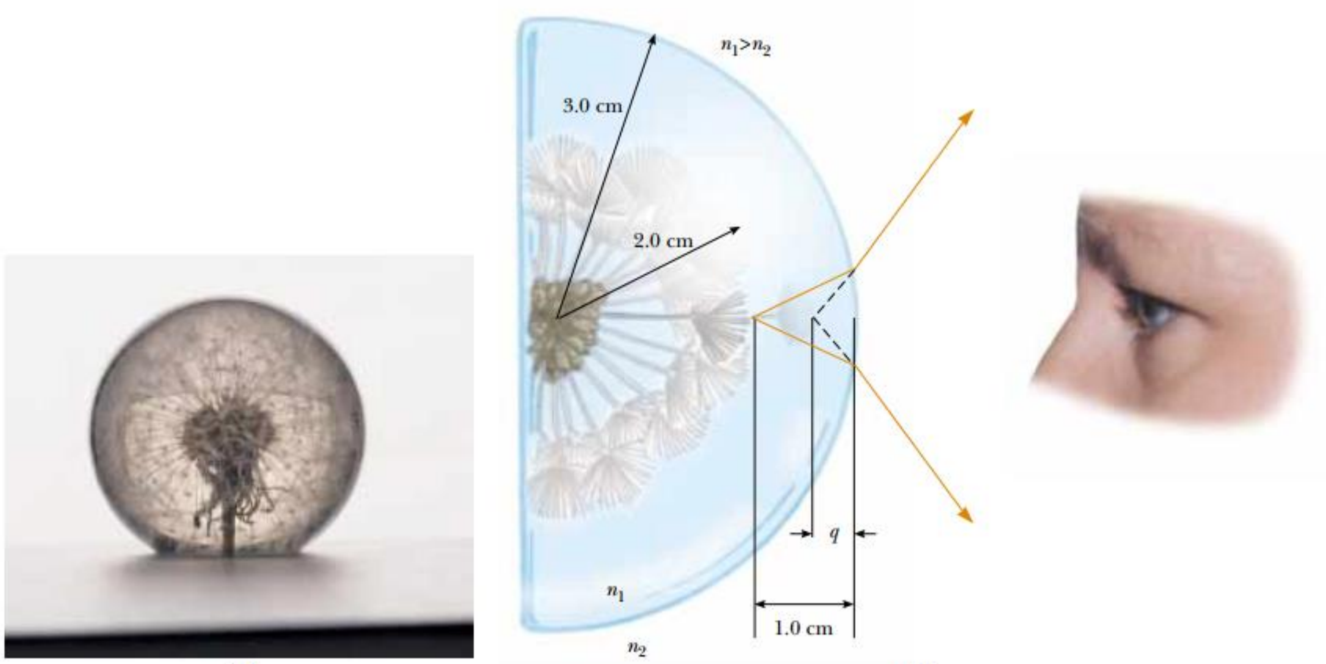
Solution Because $n_1 > n_2$, where $n_2 = 1.00$ is the index of refraction for air, the rays originating from the seed ball are refracted away from the normal at the surface and diverge outward, as shown in Figure 36.20b. Hence, the image is formed inside the paperweight and is virtual. From the given dimensions, we know that the near edge of the seed ball is 1.0 cm beneath the surface of the paperweight. Applying Equation 36.8 and noting from Table 36.2 that R is negative, we obtain

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1.50}{1.0 \text{ cm}} + \frac{1}{q} = \frac{1.00 - 1.50}{-3.0 \text{ cm}}$$

$$q = \boxed{-0.75 \text{ cm}}$$

The negative sign for q indicates that the image is in front of the surface—in other words, in the same medium as the object, as shown in Figure 36.20b. Being in the same medium as the object, the image must be virtual (see Table 36.2). The surface of the seed ball appears to be closer to the paperweight surface than it actually is.



(a) (b)

Figure 36.20 (a) An object embedded in a plastic sphere forms a virtual image between the surface of the object and the sphere surface. All rays are assumed paraxial. Because the object is inside the sphere, the front of the refracting surface is the *interior* of the sphere. (b) Rays from the surface of the object form an image that is still inside the plastic sphere but closer to the plastic surface.

EXAMPLE 36.8 The One That Got Away

A small fish is swimming at a depth d below the surface of a pond (Fig. 36.21). What is the apparent depth of the fish, as viewed from directly overhead?

Solution Because the refracting surface is flat, R is infinite. Hence, we can use Equation 36.9 to determine the location of the image with $p = d$. Using the indices of refraction given in Figure 36.21, we obtain

$$q = -\frac{n_2}{n_1} p = -\frac{1.00}{1.33} d = \boxed{-0.752d}$$

Because q is negative, the image is virtual, as indicated by the dashed lines in Figure 36.21. The apparent depth is three-fourths the actual depth.

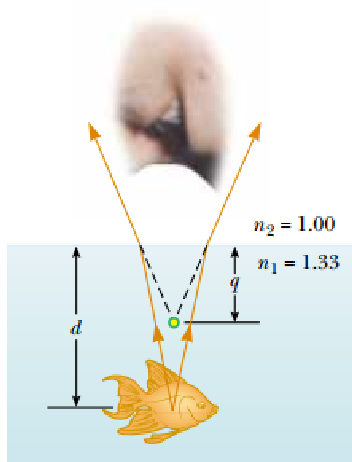


Figure 36.21 The apparent depth q of the fish is less than the true depth d. All rays are assumed to be paraxial.

36.4 THIN LENSES

14.8 Lenses are commonly used to form images by refraction in optical instruments, such as cameras, telescopes, and microscopes. We can use what we just learned about images formed by refracting surfaces to help us locate the image formed by a lens. We recognize that light passing through a lens experiences refraction at two surfaces. The development we shall follow is based on the notion that **the image formed by one refracting surface serves as the object for the second surface.** We shall analyze a thick lens first and then let the thickness of the lens be approximately zero.

Consider a lens having an index of refraction n and two spherical surfaces with radii of curvature R_1 and R_2, as in Figure 36.22. (Note that R_1 is the radius of curvature of the lens surface that the light leaving the object reaches first and that R_2 is the radius of curvature of the other surface of the lens.) An object is placed at point O at a distance p_1 in front of surface 1. If the object were far from surface 1, the light rays from the object that struck the surface would be almost parallel to each other. The refraction at the surface would focus these rays, forming a real image to the right of surface 1 in Figure 36.22 (as in Fig. 36.17). If the object is placed close to surface 1, as shown in Figure 36.22, the rays diverging from the object and striking the surface cover a wide range of angles and are not parallel to each other. In this case, the refraction at the surface is not sufficient to cause the rays to converge on the right side of the surface. They still diverge, although they are closer to parallel than they were before they struck the surface. This results in a virtual image of the object at I_1 to the left of the surface, as shown in Figure 36.22. This image is then used as the object for surface 2, which results in a real image I_2 to the right of the lens.

Let us begin with the virtual image formed by surface 1. Using Equation 36.8 and assuming that $n_1 = 1$ because the lens is surrounded by air, we find that the image I_1 formed by surface 1 satisfies the equation

$$(1) \qquad \frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}$$

where q_1 is a negative number because it represents a virtual image formed on the front side of surface 1.

Now we apply Equation 36.8 to surface 2, taking $n_1 = n$ and $n_2 = 1$. (We make this switch in index because the light rays from I_1 approaching surface 2 are *in the material of the lens,* and this material has index n. We could also imagine removing the object at O, filling all of the space to the left of surface 1 with the mate-

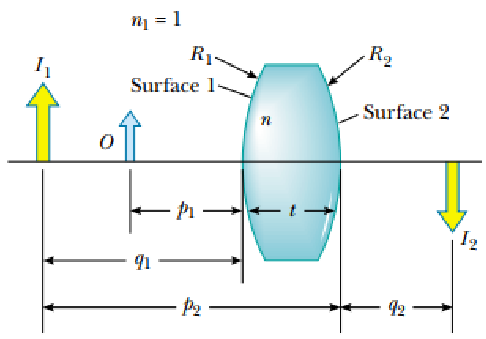


Figure 36.22 To locate the image formed by a lens, we use the virtual image at I_1 formed by surface 1 as the object for the image formed by surface 2. The final image is real and is located at I_2.

rial of the lens, and placing the object at I_1; the light rays approaching surface 2 would be the same as in the actual situation in Fig. 36.22.) Taking p_2 as the object distance for surface 2 and q_2 as the image distance gives

$$(2) \qquad \frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

We now introduce mathematically the fact that the image formed by the first surface acts as the object for the second surface. We do this by noting from Figure 36.22 that p_2 is the sum of q_1 and t and by setting $p_2 = -q_1 + t$, where t is the thickness of the lens. (Remember that q_1 is a negative number and that p_2 must be positive by our sign convention—thus, we must introduce a negative sign for q_1.) For a *thin* lens (for which the thickness is small compared to the radii of curvature), we can neglect t. In this approximation, we see that $p_2 = -q_1$. Hence, Equation (2) becomes

$$(3) \qquad -\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

Adding Equations (1) and (3), we find that

$$(4) \qquad \frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a thin lens, we can omit the subscripts on p_1 and q_2 in Equation (4) and call the object distance p and the image distance q, as in Figure 36.23. Hence, we can write Equation (4) in the form

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \textbf{(36.10)}$$

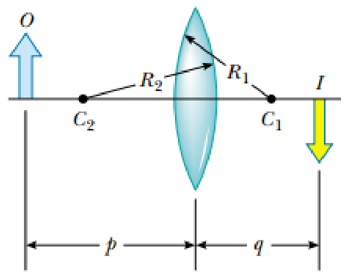


Figure 36.23 Simplified geometry for a thin lens.

This expression relates the image distance q of the image formed by a thin lens to the object distance p and to the thin-lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is much less than R_1 and R_2.

The **focal length** f of a thin lens is the image distance that corresponds to an infinite object distance, just as with mirrors. Letting p approach ∞ and q approach f in Equation 36.10, we see that the inverse of the focal length for a thin lens is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \textbf{(36.11)}$$

Lens makers' equation

This relationship is called the **lens makers' equation** because it can be used to determine the values of R_1 and R_2 that are needed for a given index of refraction and a desired focal length f. Conversely, if the index of refraction and the radii of curvature of a lens are given, this equation enables a calculation of the focal length. If the lens is immersed in something other than air, this same equation can be used, with n interpreted as the *ratio* of the index of refraction of the lens material to that of the surrounding fluid.

Quick Quiz 36.2

What is the focal length of a pane of window glass?

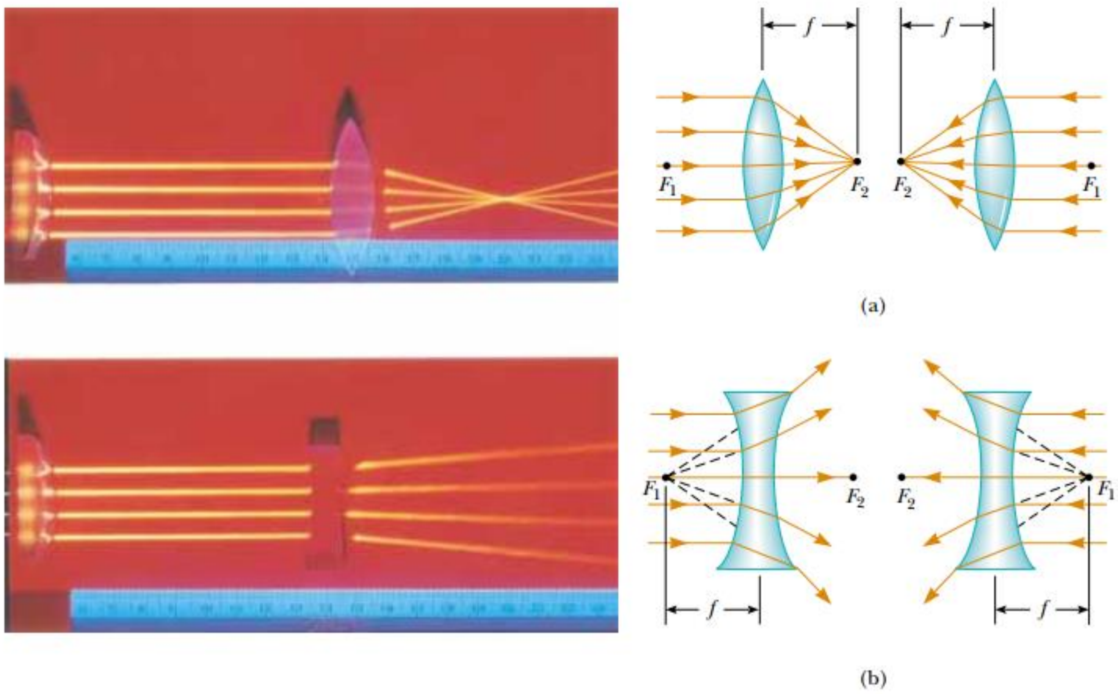


Figure 36.24 (*Left*) Effects of a converging lens (top) and a diverging lens (bottom) on parallel rays. (*Right*) The object and image focal points of (a) a converging lens and (b) a diverging lens.

Using Equation 36.11, we can write Equation 36.10 in a form identical to Equation 36.6 for mirrors:

Thin-lens equation

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \tag{36.12}$$

This equation, called the **thin-lens equation,** can be used to relate the image distance and object distance for a thin lens.

Because light can travel in either direction through a lens, each lens has two focal points, one for light rays passing through in one direction and one for rays passing through in the other direction. This is illustrated in Figure 36.24 for a biconvex lens (two convex surfaces, resulting in a converging lens) and a biconcave lens (two concave surfaces, resulting in a diverging lens). Focal point F_1 is sometimes called the *object focal point,* and F_2 is called the *image focal point.*

Figure 36.25 is useful for obtaining the signs of p and q, and Table 36.3 gives the sign conventions for thin lenses. Note that these sign conventions are the same as those for refracting surfaces (see Table 36.2). Applying these rules to a biconvex lens, we see that when $p > f$, the quantities p, q, and R_1 are positive, and R_2 is negative. Therefore, p, q, and f are all positive when a converging lens forms a real image of an object. For a biconcave lens, p and R_2 are positive and q and R_1 are negative, with the result that f is negative.

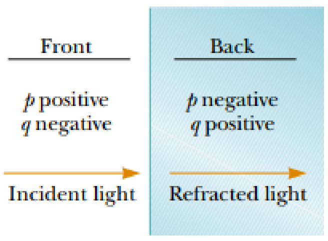


Figure 36.25 A diagram for obtaining the signs of p and q for a thin lens. (This diagram also applies to a refracting surface.)

Various lens shapes are shown in Figure 36.26. Note that a converging lens is thicker at the center than at the edge, whereas a diverging lens is thinner at the center than at the edge.

TABLE 36.3 Sign Conventions for Thin Lenses

p is **positive** if object is in **front** of lens (real object).
p is **negative** if object is in **back** of lens (virtual object).

q is **positive** if image is in **back** of lens (real image).
q is **negative** if image is in **front** of lens (virtual image).

R_1 and R_2 are **positive** if center of curvature is in **back** of lens.
R_1 and R_2 are **negative** if center of curvature is in **front** of lens.

f is **positive** if the lens is **converging.**
f is **negative** if the lens is **diverging.**

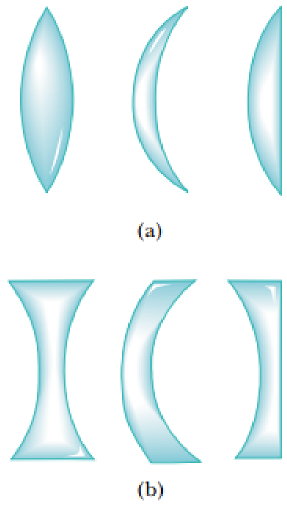


Figure 36.26 Various lens shapes. (a) Biconvex, convex–concave, and plano–convex. These are all converging lenses; they have a positive focal length and are thickest at the middle. (b) Biconcave, convex–concave, and plano–concave. These are all diverging lenses; they have a negative focal length and are thickest at the edges.

Magnification of Images

Consider a thin lens through which light rays from an object pass. As with mirrors (Eq. 36.2), the lateral magnification of the lens is defined as the ratio of the image height h' to the object height h:

$$M = \frac{h'}{h} = -\frac{q}{p}$$

From this expression, it follows that when M is positive, the image is upright and on the same side of the lens as the object. When M is negative, the image is inverted and on the side of the lens opposite the object.

Ray Diagrams for Thin Lenses

Ray diagrams are convenient for locating the images formed by thin lenses or systems of lenses. They also help clarify our sign conventions. Figure 36.27 shows such diagrams for three single-lens situations. To locate the image of a converg-

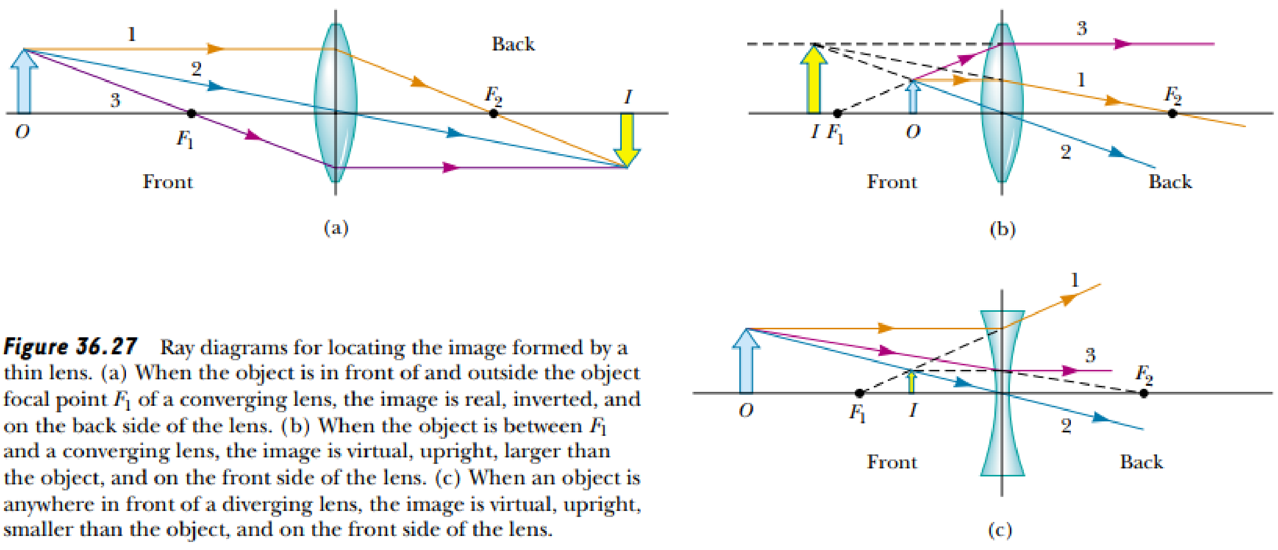


Figure 36.27 Ray diagrams for locating the image formed by a thin lens. (a) When the object is in front of and outside the object focal point F_1 of a converging lens, the image is real, inverted, and on the back side of the lens. (b) When the object is between F_1 and a converging lens, the image is virtual, upright, larger than the object, and on the front side of the lens. (c) When an object is anywhere in front of a diverging lens, the image is virtual, upright, smaller than the object, and on the front side of the lens.

ing lens (Fig. 36.27a and b), the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn through that focal point on the front side of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.

To locate the image of a diverging lens (Fig. 36.27c), the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray emerges such that it appears to have passed through the focal point on the front side of the lens. (This apparent direction is indicated by the dashed line in Fig. 36.27c.)
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn toward the focal point on the back side of the lens and emerges from the lens parallel to the optic axis.

Quick Quiz 36.3

In Figure 36.27a, the blue object arrow is replaced by one that is much taller than the lens. How many rays from the object will strike the lens?

For the converging lens in Figure 36.27a, where the object is to the left of the object focal point ($p > f_1$), the image is real and inverted. When the object is between the object focal point and the lens ($p < f_1$), as shown in Figure 36.27b, the image is virtual and upright. For a diverging lens (see Fig. 36.27c), the image is always virtual and upright, regardless of where the object is placed. These geometric constructions are reasonably accurate only if the distance between the rays and the principal axis is much less than the radii of the lens surfaces.

It is important to realize that refraction occurs only at the surfaces of the lens. A certain lens design takes advantage of this fact to produce the *Fresnel lens,* a powerful lens without great thickness. Because only the surface curvature is important in the refracting qualities of the lens, material in the middle of a Fresnel lens is removed, as shown in Figure 36.28. Because the edges of the curved segments cause some distortion, Fresnel lenses are usually used only in situations in which image quality is less important than reduction of weight.

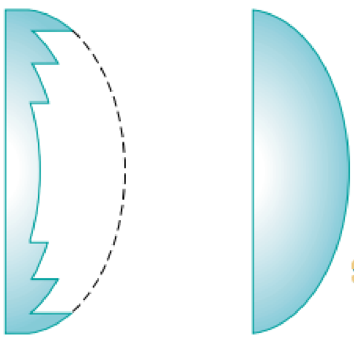


Figure 36.28 The Fresnel lens on the left has the same focal length as the thick lens on the right but is made of much less glass.

The lines that are visible across the faces of most automobile headlights are the edges of these curved segments. A headlight requires a short-focal-length lens to collimate light from the nearby filament into a parallel beam. If it were not for the Fresnel design, the glass would be very thick in the center and quite heavy. The weight of the glass would probably cause the thin edge where the lens is supported to break when subjected to the shocks and vibrations that are typical of travel on rough roads.

Quick Quiz 36.4

If you cover the top half of a lens, which of the following happens to the appearance of the image of an object? (a) The bottom half disappears; (b) the top half disappears; (c) the entire image is visible but has half the intensity; (d) no change occurs; (e) the entire image disappears.

EXAMPLE 36.9 An Image Formed by a Diverging Lens

A diverging lens has a focal length of -20.0 cm. An object 2.00 cm tall is placed 30.0 cm in front of the lens. Locate the image.

Solution Using the thin-lens equation (Eq. 36.12) with $p = 30.0$ cm and $f = -20.0$ cm, we obtain

$$\frac{1}{30.0\text{ cm}} + \frac{1}{q} = \frac{1}{-20.0\text{ cm}}$$

$$q = \boxed{-12.0\text{ cm}}$$

The negative sign tells us that the image is in front of the lens and virtual, as indicated in Figure 36.27c.

Exercise Determine both the magnification and the height of the image.

Answer $M = 0.400$; $h' = 0.800$ cm.

EXAMPLE 36.10 An Image Formed by a Converging Lens

A converging lens of focal length 10.0 cm forms an image of each of three objects placed (a) 30.0 cm, (b) 10.0 cm, and (c) 5.00 cm in front of the lens. In each case, find the image distance and describe the image.

Solution (a) The thin-lens equation can be used again:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30.0\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$

$$q = \boxed{15.0\text{ cm}}$$

The positive sign indicates that the image is in back of the lens and real. The magnification is

$$M = -\frac{q}{p} = -\frac{15.0\text{ cm}}{30.0\text{ cm}} = \boxed{-0.500}$$

The image is reduced in size by one half, and the negative sign for M means that the image is inverted. The situation is like that pictured in Figure 36.27a.

(b) No calculation is necessary for this case because we know that, when the object is placed at the focal point, the image is formed at infinity. We can readily verify this by substituting $p = 10.0$ cm into the thin-lens equation.

(c) We now move inside the focal point, to an object distance of 5.00 cm:

$$\frac{1}{5.00\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$

$$q = \boxed{-10.0\text{ cm}}$$

$$M = -\frac{q}{p} = -\left(\frac{-10.0\text{ cm}}{5.00\text{ cm}}\right) = \boxed{2.00}$$

The negative image distance indicates that the image is in front of the lens and virtual. The image is enlarged, and the positive sign for M tells us that the image is upright, as shown in Figure 36.27b.

EXAMPLE 36.11 A Lens Under Water

A converging glass lens ($n = 1.52$) has a focal length of 40.0 cm in air. Find its focal length when it is immersed in water, which has an index of refraction of 1.33.

Solution We can use the lens makers' equation (Eq. 36.11) in both cases, noting that R_1 and R_2 remain the same in air and water:

$$\frac{1}{f_{\text{air}}} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_{\text{water}}} = (n'-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n' is the ratio of the index of refraction of glass to that of water: $n' = 1.52/1.33 = 1.14$. Dividing the first equation by the second gives

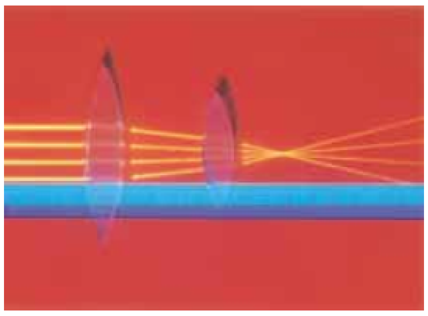
$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{n-1}{n'-1} = \frac{1.52-1}{1.14-1} = 3.71$$

Because $f_{\text{air}} = 40.0$ cm, we find that

$$f_{\text{water}} = 3.71 f_{\text{air}} = 3.71(40.0 \text{ cm}) = \boxed{148 \text{ cm}}$$

The focal length of any glass lens is increased by a factor $(n-1)/(n'-1)$ when the lens is immersed in water.

Combination of Thin Lenses



Light from a distant object brought into focus by two converging lenses.

If two thin lenses are used to form an image, the system can be treated in the following manner. First, the image formed by the first lens is located as if the second lens were not present. Then a ray diagram is drawn for the second lens, with the image formed by the first lens now serving as the object for the second lens. The second image formed is the final image of the system. One configuration is particularly straightforward; that is, if the image formed by the first lens lies on the back side of the second lens, then that image is treated as a **virtual object** for the second lens (that is, p is negative). The same procedure can be extended to a system of three or more lenses. The overall magnification of a system of thin lenses equals the product of the magnifications of the separate lenses.

Let us consider the special case of a system of two lenses in contact. Suppose two thin lenses of focal lengths f_1 and f_2 are placed in contact with each other. If p is the object distance for the combination, application of the thin-lens equation (Eq. 36.12) to the first lens gives

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_1}$$

where q_1 is the image distance for the first lens. Treating this image as the object for the second lens, we see that the object distance for the second lens must be $-q_1$ (negative because the object is virtual). Therefore, for the second lens,

$$\frac{1}{-q_1} + \frac{1}{q} = \frac{1}{f_2}$$

where q is the final image distance from the second lens. Adding these equations eliminates q_1 and gives

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2}$$

Focal length of two thin lenses in contact

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \textbf{(36.13)}$$

Because the two thin lenses are touching, q is also the distance of the final image from the first lens. Therefore, **two thin lenses in contact with each other are equivalent to a single thin lens having a focal length given by Equation 36.13.**

Example 36.12 Where Is the Final Image?

Even when the conditions just described do not apply, the lens equations yield image position and magnification. For example, two thin converging lenses of focal lengths $f_1 = 10.0$ cm and $f_2 = 20.0$ cm are separated by 20.0 cm, as illustrated in Figure 36.29. An object is placed 15.0 cm to the left of lens 1. Find the position of the final image and the magnification of the system.

Solution First we locate the image formed by lens 1 while ignoring lens 2:

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{15.0\text{ cm}} + \frac{1}{q_1} = \frac{1}{10.0\text{ cm}}$$

$$q_1 = 30.0\text{ cm}$$

where q_1 is measured from lens 1. A positive value for q_1 means that this first image is in back of lens 1.

Because q_1 is greater than the separation between the two lenses, this image formed by lens 1 lies 10.0 cm to the right of lens 2. We take this as the object distance for the second lens, so $p_2 = -10.0$ cm, where distances are now measured from lens 2:

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{-10.0\text{ cm}} + \frac{1}{q_2} = \frac{1}{20.0\text{ cm}}$$

$$q_2 = \boxed{6.67\text{ cm}}$$

The final image lies 6.67 cm to the right of lens 2.

The individual magnifications of the images are

$$M_1 = -\frac{q_1}{p_1} = -\frac{30.0\text{ cm}}{15.0\text{ cm}} = -2.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{6.67\text{ cm}}{-10.0\text{ cm}} = 0.667$$

The total magnification M is equal to the product $M_1M_2 = (-2.00)(0.667) = \boxed{-1.33.}$ The final image is real because q_2 is positive. The image is also inverted and enlarged.

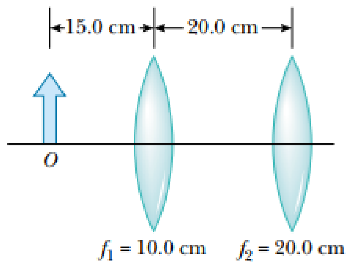


Figure 36.29 A combination of two converging lenses.

Conceptual Example 36.13 Watch Your *p*'s and *q*'s!

Use a spreadsheet or a similar tool to create two graphs of image distance as a function of object distance—one for a lens for which the focal length is 10 cm and one for a lens for which the focal length is -10 cm.

Solution The graphs are shown in Figure 36.30. In each graph a gap occurs where $p = f$, which we shall discuss. Note the similarity in the shapes—a result of the fact that image and object distances for both lenses are related according to the same equation—the thin-lens equation.

The curve in the upper right portion of the $f = +10$ cm graph corresponds to an object on the *front* side of a lens, which we have drawn as the left side of the lens in our previous diagrams. When the object is at positive infinity, a real image forms at the focal point on the back side (the positive side) of the lens, $q = f$. (The incoming rays are parallel in this case.) As the object gets closer to the lens, the image moves farther from the lens, corresponding to the upward path of the curve. This continues until the object is located at the focal point on the near side of the lens. At this point, the rays leaving the lens are parallel, making the image infinitely far away. This is described in the graph by the asymptotic approach of the curve to the line $p = f = 10$ cm.

As the object moves inside the focal point, the image becomes virtual and located near $q = -\infty$. We are now following the curve in the lower left portion of Figure 36.30a. As the object moves closer to the lens, the virtual image also moves closer to the lens. As $p \rightarrow 0$, the image distance q also approaches 0. Now imagine that we bring the object to the back side of the lens, where $p < 0$. The object is now a virtual object, so it must have been formed by some other lens. For all locations of the virtual object, the image distance is positive and less than the focal length. The final image is real, and its position approaches the focal point as p gets more and more negative.

The $f = -10$ cm graph shows that a distant real object forms an image at the focal point on the front side of the lens. As the object approaches the lens, the image remains virtual and moves closer to the lens. But as we continue toward the left end of the p axis, the object becomes virtual. As the position of this virtual object approaches the focal point, the image recedes toward infinity. As we pass the focal point, the image shifts from a location at positive infinity to one at negative infinity. Finally, as the virtual object continues moving away from the lens, the image is virtual, starts moving in from negative infinity, and approaches the focal point.

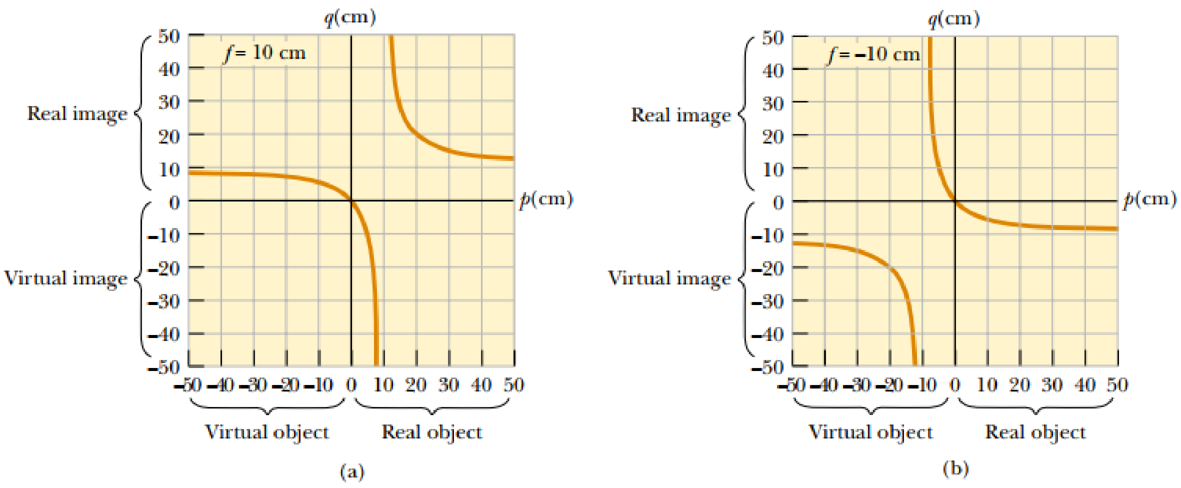


Figure 36.30 (a) Image position as a function of object position for a lens having a focal length of +10 cm. (b) Image position as a function of object position for a lens having a focal length of −10 cm.

Optional Section

36.5 LENS ABERRATIONS

One problem with lenses is imperfect images. The theory of mirrors and lenses that we have been using assumes that rays make small angles with the principal axis and that the lenses are thin. In this simple model, all rays leaving a point source focus at a single point, producing a sharp image. Clearly, this is not always true. When the approximations used in this theory do not hold, imperfect images are formed.

A precise analysis of image formation requires tracing each ray, using Snell's law at each refracting surface and the law of reflection at each reflecting surface. This procedure shows that the rays from a point object do not focus at a single point, with the result that the image is blurred. The departures of actual (imperfect) images from the ideal predicted by theory are called **aberrations.**

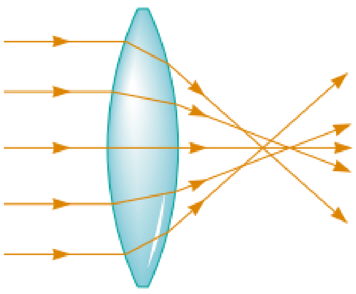
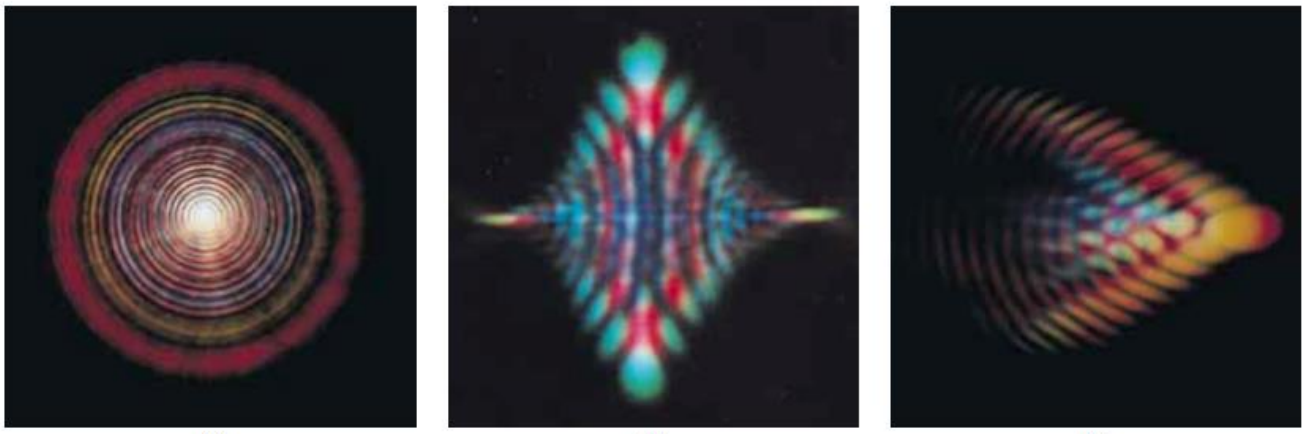


Figure 36.31 Spherical aberration caused by a converging lens. Does a diverging lens cause spherical aberration?

Spherical Aberrations

Spherical aberrations occur because the focal points of rays far from the principal axis of a spherical lens (or mirror) are different from the focal points of rays of the same wavelength passing near the axis. Figure 36.31 illustrates spherical aberration for parallel rays passing through a converging lens. Rays passing through points near the center of the lens are imaged farther from the lens than rays passing through points near the edges.

Many cameras have an adjustable aperture to control light intensity and reduce spherical aberration. (An aperture is an opening that controls the amount of light passing through the lens.) Sharper images are produced as the aperture size is reduced because with a small aperture only the central portion of the lens is exposed to the light; as a result, a greater percentage of the rays are paraxial. At the



(a) (b) (c)

Lens aberrations. (a) *Spherical aberration* occurs when light passing through the lens at different distances from the principal axis is focused at different points. (b) *Astigmatism* occurs for objects not located on the principal axis of the lens. (c) *Coma* occurs as light passing through the lens far from the principal axis and light passing near the center of the lens focus at different parts of the focal plane.

same time, however, less light passes through the lens. To compensate for this lower light intensity, a longer exposure time is used.

In the case of mirrors used for very distant objects, spherical aberration can be minimized through the use of a parabolic reflecting surface rather than a spherical surface. Parabolic surfaces are not used often, however, because those with high-quality optics are very expensive to make. Parallel light rays incident on a parabolic surface focus at a common point, regardless of their distance from the principal axis. Parabolic reflecting surfaces are used in many astronomical telescopes to enhance image quality.

Chromatic Aberrations

The fact that different wavelengths of light refracted by a lens focus at different points gives rise to chromatic aberrations. In Chapter 35 we described how the index of refraction of a material varies with wavelength. For instance, when white light passes through a lens, violet rays are refracted more than red rays (Fig. 36.32). From this we see that the focal length is greater for red light than for violet light. Other wavelengths (not shown in Fig. 36.32) have focal points intermediate between those of red and violet.

Chromatic aberration for a diverging lens also results in a shorter focal length for violet light than for red light, but on the front side of the lens. Chromatic aberration can be greatly reduced by combining a converging lens made of one type of glass and a diverging lens made of another type of glass.

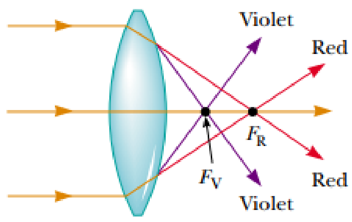


Figure 36.32 Chromatic aberration caused by a converging lens. Rays of different wavelengths focus at different points.

Optional Section

36.6 THE CAMERA

The photographic **camera** is a simple optical instrument whose essential features are shown in Figure 36.33. It consists of a light-tight box, a converging lens that produces a real image, and a film behind the lens to receive the image. One focuses the camera by varying the distance between lens and film. This is accom-

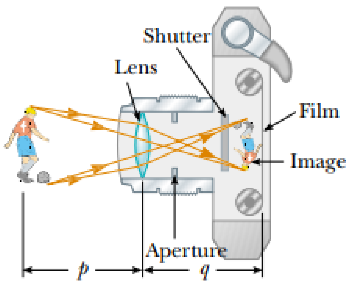


Figure 36.33 Cross-sectional view of a simple camera. Note that in reality, $p \gg q$.

plished with an adjustable bellows in older-style cameras and with some other mechanical arrangement in modern cameras. For proper focusing—which is necessary for the formation of sharp images—the lens-to-film distance depends on the object distance as well as on the focal length of the lens.

The shutter, positioned behind the lens, is a mechanical device that is opened for selected time intervals, called *exposure times.* One can photograph moving objects by using short exposure times, or photograph dark scenes (with low light levels) by using long exposure times. If this adjustment were not available, it would be impossible to take stop-action photographs. For example, a rapidly moving vehicle could move enough in the time that the shutter was open to produce a blurred image. Another major cause of blurred images is the movement of the camera while the shutter is open. To prevent such movement, either short exposure times or a tripod should be used, even for stationary objects. Typical shutter speeds (that is, exposure times) are 1/30, 1/60, 1/125, and 1/250 s. For handheld cameras, the use of slower speeds can result in blurred images (due to movement), but the use of faster speeds reduces the gathered light intensity. In practice, stationary objects are normally shot with an intermediate shutter speed of 1/60 s.

More expensive cameras have an aperture of adjustable diameter to further control the intensity of the light reaching the film. As noted earlier, when an aperture of small diameter is used, only light from the central portion of the lens reaches the film; in this way spherical aberration is reduced.

The intensity I of the light reaching the film is proportional to the area of the lens. Because this area is proportional to the square of the diameter D, we conclude that I is also proportional to D^2. Light intensity is a measure of the rate at which energy is received by the film per unit area of the image. Because the area of the image is proportional to q^2, and $q \approx f$ (when $p \gg f$, so p can be approximated as infinite), we conclude that the intensity is also proportional to $1/f^2$, and thus $I \propto D^2/f^2$. The brightness of the image formed on the film depends on the light intensity, so we see that the image brightness depends on both the focal length and the diameter of the lens.

The ratio f/D is called the ***f*-number** of a lens:

$$f\text{-number} \equiv \frac{f}{D} \tag{36.14}$$

Hence, the intensity of light incident on the film can be expressed as

$$I \propto \frac{1}{(f/D)^2} \propto \frac{1}{(f\text{-number})^2} \tag{36.15}$$

The *f*-number is often given as a description of the lens "speed." The lower the *f*-number, the wider the aperture and the higher the rate at which energy from the light exposes the film—thus, a lens with a low *f*-number is a "fast" lens. The conventional notation for an *f*-number is "$f/$" followed by the actual number. For example, "$f/4$" means an *f*-number of 4—it *does not* mean to divide f by 4! Extremely fast lenses, which have *f*-numbers as low as approximately $f/1.2$, are expensive because it is very difficult to keep aberrations acceptably small with light rays passing through a large area of the lens. Camera lens systems (that is, combinations of lenses with adjustable apertures) are often marked with multiple *f*-numbers, usually $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, and $f/16$. Any one of these settings can be selected by adjusting the aperture, which changes the value of D. Increasing the setting from one *f*-number to the next higher value (for example, from $f/2.8$ to $f/4$) decreases the area of the aperture by a factor of two. The lowest *f*-number setting on a camera lens corresponds to a wide-open aperture and the use of the maximum possible lens area.

Simple cameras usually have a fixed focal length and a fixed aperture size, with an *f*-number of about $f/11$. This high value for the *f*-number allows for a large **depth of field,** meaning that objects at a wide range of distances from the lens form reasonably sharp images on the film. In other words, the camera does not have to be focused.

EXAMPLE 36.14 Finding the Correct Exposure Time

The lens of a certain 35-mm camera (where 35 mm is the width of the film strip) has a focal length of 55 mm and a speed (an *f*-number) of $f/1.8$. The correct exposure time for this speed under certain conditions is known to be (1/500) s. (a) Determine the diameter of the lens.

Solution From Equation 36.14, we find that

$$D = \frac{f}{f\text{-number}} = \frac{55 \text{ mm}}{1.8} = \boxed{31 \text{ mm}}$$

(b) Calculate the correct exposure time if the *f*-number is changed to $f/4$ under the same lighting conditions.

Solution The total light energy hitting the film is proportional to the product of the intensity and the exposure time. If I is the light intensity reaching the film, then in a time t the energy per unit area received by the film is proportional to It. Comparing the two situations, we require that $I_1t_1 = I_2t_2$, where t_1 is the correct exposure time for $f/1.8$ and t_2 is the correct exposure time for $f/4$. Using this result together with Equation 36.15, we find that

$$\frac{t_1}{(f_1\text{-number})^2} = \frac{t_2}{(f_2\text{-number})^2}$$

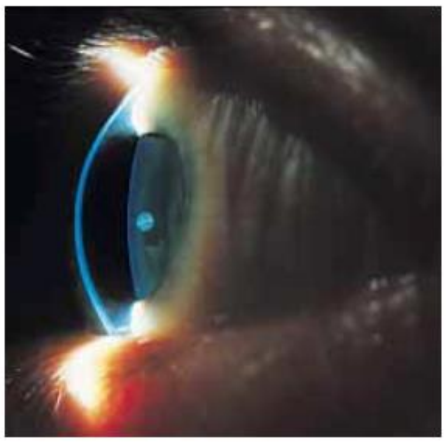
$$t_2 = \left(\frac{f_2\text{-number}}{f_1\text{-number}}\right)^2 t_1$$

$$= \left(\frac{4}{1.8}\right)^2\left(\frac{1}{500}\text{ s}\right) \approx \boxed{\frac{1}{100}\text{ s}}$$

As the aperture size is reduced, exposure time must increase.

Optional Section

36.7 THE EYE



Close-up photograph of the cornea of the human eye.

Like a camera, a normal eye focuses light and produces a sharp image. However, the mechanisms by which the eye controls the amount of light admitted and adjusts to produce correctly focused images are far more complex, intricate, and effective than those in even the most sophisticated camera. In all respects, the eye is a physiological wonder.

Figure 36.34 shows the essential parts of the human eye. Light entering the eye passes through a transparent structure called the *cornea,* behind which are a clear liquid (the *aqueous humor*), a variable aperture (the *pupil,* which is an opening in the *iris*), and the *crystalline lens.* Most of the refraction occurs at the outer surface of the eye, where the cornea is covered with a film of tears. Relatively little refraction occurs in the crystalline lens because the aqueous humor in contact with the lens has an average index of refraction close to that of the lens. The iris, which is the colored portion of the eye, is a muscular diaphragm that controls pupil size. The iris regulates the amount of light entering the eye by dilating the pupil in low-light conditions and contracting the pupil in high-light conditions. The *f*-number range of the eye is from about $f/2.8$ to $f/16$.

The cornea–lens system focuses light onto the back surface of the eye, the *retina,* which consists of millions of sensitive receptors called *rods* and *cones.* When stimulated by light, these receptors send impulses via the optic nerve to the brain,

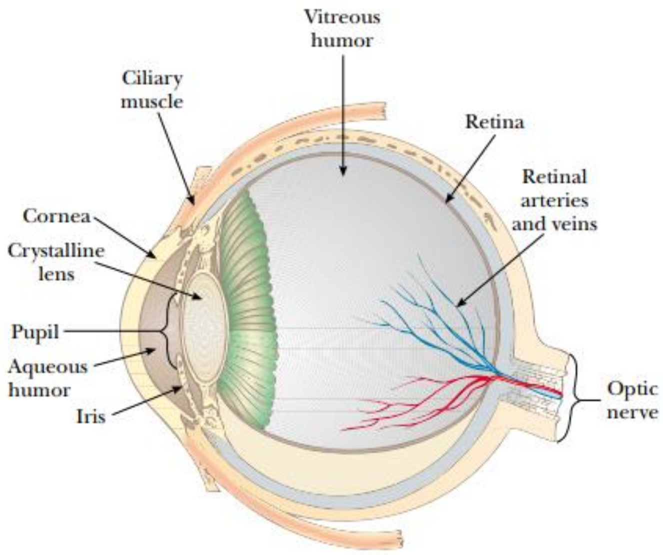


Figure 36.34 Essential parts of the eye.

where an image is perceived. By this process, a distinct image of an object is observed when the image falls on the retina.

The eye focuses on an object by varying the shape of the pliable crystalline lens through an amazing process called **accommodation.** An important component of accommodation is the *ciliary muscle,* which is situated in a circle around the rim of the lens. Thin filaments, called *zonules,* run from this muscle to the edge of the lens. When the eye is focused on a distant object, the ciliary muscle is relaxed, tightening the zonules that attach the muscle to the edge of the lens. The force of the zonules causes the lens to flatten, increasing its focal length. For an object distance of infinity, the focal length of the eye is equal to the fixed distance between lens and retina, about 1.7 cm. The eye focuses on nearby objects by tensing the ciliary muscle, which relaxes the zonules. This action allows the lens to bulge a bit, and its focal length decreases, resulting in the image being focused on the retina. All these lens adjustments take place so swiftly that we are not even aware of the change. In this respect, even the finest electronic camera is a toy compared with the eye.

QuickLab

Move this book toward your face until the letters just begin to blur. The distance from the book to your eyes is your near point.

Accommodation is limited in that objects that are very close to the eye produce blurred images. The **near point** is the closest distance for which the lens can accommodate to focus light on the retina. This distance usually increases with age and has an average value of 25 cm. Typically, at age 10 the near point of the eye is about 18 cm. It increases to about 25 cm at age 20, to 50 cm at age 40, and to 500 cm or greater at age 60. The **far point** of the eye represents the greatest distance for which the lens of the relaxed eye can focus light on the retina. A person with normal vision can see very distant objects, such as the Moon, and thus has a far point near infinity.

Recall that the light leaving the mirror in Figure 36.8 becomes white where it comes together but then diverges into separate colors again. Because nothing but air exists at the point where the rays cross (and hence nothing exists to cause the colors to separate again), seeing white light as a result of a combination of colors must be a visual illusion. In fact, this is the case. Only three types of color-sensitive

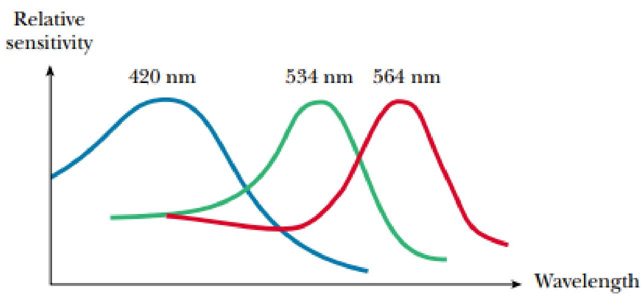


Figure 36.35 Approximate color sensitivity of the three types of cones in the retina.

cells are present in the retina; they are called red, green, and blue cones because of the peaks of the color ranges to which they respond (Fig. 36.35). If the red and green cones are stimulated simultaneously (as would be the case if yellow light were shining on them), the brain interprets what we see as yellow. If all three types of cones are stimulated by the separate colors red, blue, and green, as in Figure 36.8, we see white. If all three types of cones are stimulated by light that contains *all* colors, such as sunlight, we again see white light.

QuickLab

Pour a pile of salt or sugar into your palm. Compare its white appearance with the transparency of a single grain.

Color televisions take advantage of this visual illusion by having only red, green, and blue dots on the screen. With specific combinations of brightness in these three primary colors, our eyes can be made to see any color in the rainbow. Thus, the yellow lemon you see in a television commercial is not really yellow, it is red and green! The paper on which this page is printed is made of tiny, matted, translucent fibers that scatter light in all directions; the resultant mixture of colors appears white to the eye. Snow, clouds, and white hair are not really white. In fact, there is no such thing as a white pigment. The appearance of these things is a consequence of the scattering of light containing all colors, which we interpret as white.

Conditions of the Eye

When the eye suffers a mismatch between the focusing range of the lens–cornea system and the length of the eye, with the result that light rays reach the retina before they converge to form an image, as shown in Figure 36.36a, the condition is known as **farsightedness** (or *hyperopia*). A farsighted person can usually see faraway objects clearly but not nearby objects. Although the near point of a normal eye is approximately 25 cm, the near point of a farsighted person is much farther away. The eye of a farsighted person tries to focus by accommodation—that is, by shortening its focal length. This works for distant objects, but because the focal length of the farsighted eye is greater than normal, the light from nearby objects cannot be brought to a sharp focus before it reaches the retina, and it thus causes a blurred image. The refracting power in the cornea and lens is insufficient to focus the light from all but distant objects satisfactorily. The condition can be corrected by placing a converging lens in front of the eye, as shown in Figure 36.36b. The lens refracts the incoming rays more toward the principal axis before entering the eye, allowing them to converge and focus on the retina.

A person with **nearsightedness** (or *myopia*), another mismatch condition, can focus on nearby objects but not on faraway objects. In the case of *axial myopia,* the nearsightedness is caused by the lens being too far from the retina. In *refractive my-*

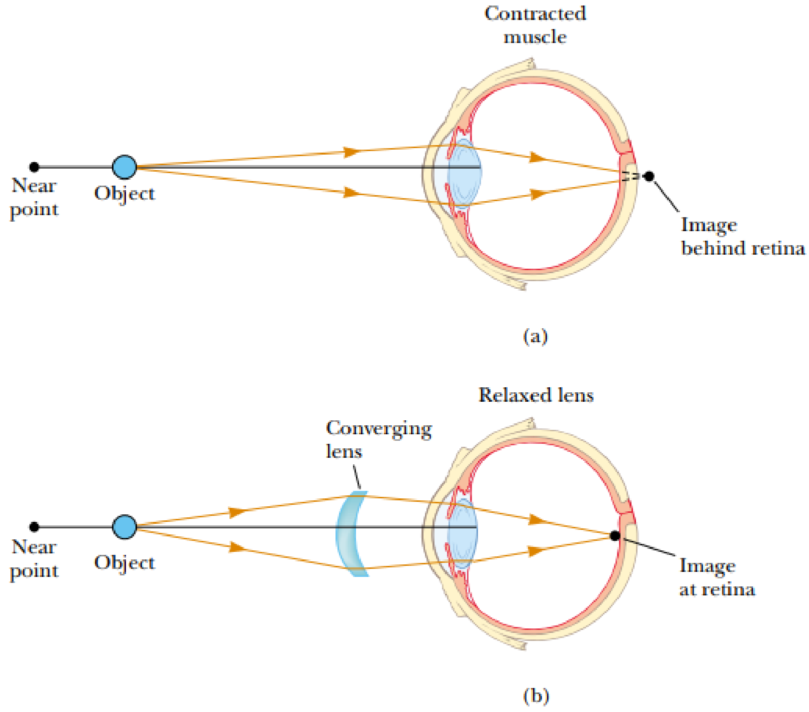


Figure 36.36 (a) When a farsighted eye looks at an object located between the near point and the eye, the image point is behind the retina, resulting in blurred vision. The eye muscle contracts to try to bring the object into focus. (b) Farsightedness is corrected with a converging lens.

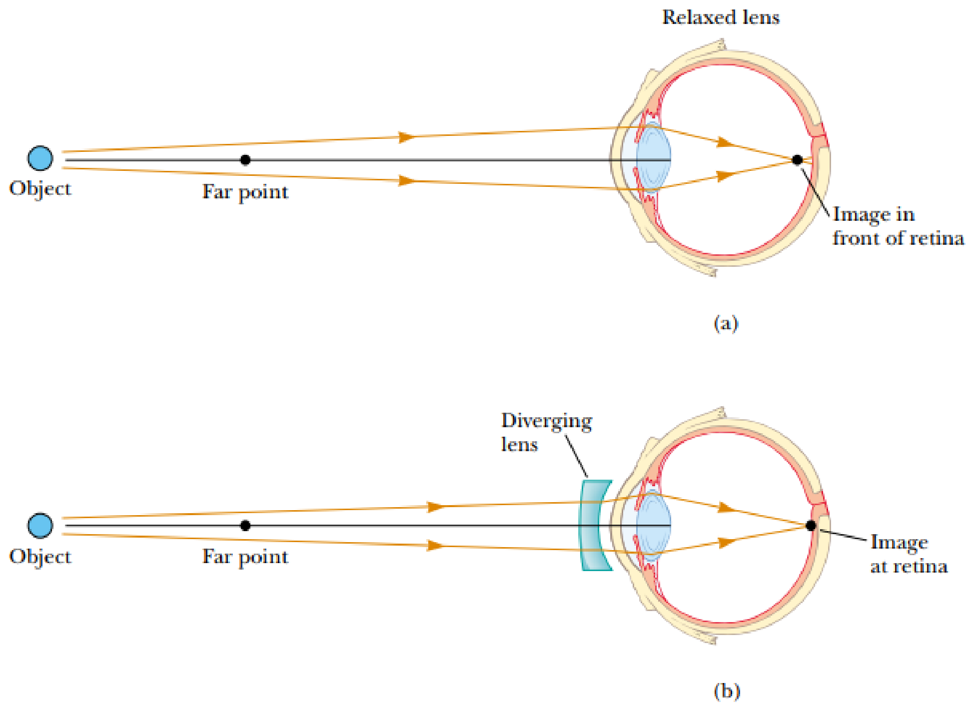


Figure 36.37 (a) When a nearsighted eye looks at an object that lies beyond the eye's far point, the image is formed in front of the retina, resulting in blurred vision. (b) Nearsightedness is corrected with a diverging lens.

opia, the lens–cornea system is too powerful for the length of the eye. The far point of the nearsighted eye is not infinity and may be less than 1 m. The maximum focal length of the nearsighted eye is insufficient to produce a sharp image on the retina, and rays from a distant object converge to a focus in front of the retina. They then continue past that point, diverging before they finally reach the retina and causing blurred vision (Fig. 36.37a). Nearsightedness can be corrected with a diverging lens, as shown in Figure 36.37b. The lens refracts the rays away from the principal axis before they enter the eye, allowing them to focus on the retina.

Quick Quiz 36.5

Which glasses in Figure 36.38 correct nearsightedness and which correct farsightedness?

(a) (b)

Figure 36.38

Beginning in middle age, most people lose some of their accommodation ability as the ciliary muscle weakens and the lens hardens. Unlike farsightedness, which is a mismatch between focusing power and eye length, **presbyopia** (literally, "old-age vision") is due to a reduction in accommodation ability. The cornea and lens do not have sufficient focusing power to bring nearby objects into focus on the retina. The symptoms are the same as those of farsightedness, and the condition can be corrected with converging lenses.

In the eye defect known as **astigmatism,** light from a point source produces a line image on the retina. This condition arises when either the cornea or the lens or both are not perfectly symmetric. Astigmatism can be corrected with lenses that have different curvatures in two mutually perpendicular directions.

Optometrists and ophthalmologists usually prescribe lenses[1] measured in **diopters:**

The **power** P of a lens in diopters equals the inverse of the focal length in meters: $P = 1/f$.

For example, a converging lens of focal length $+20$ cm has a power of $+5.0$ diopters, and a diverging lens of focal length -40 cm has a power of -2.5 diopters.

EXAMPLE 36.15 A Case of Nearsightedness

A particular nearsighted person is unable to see objects clearly when they are beyond 2.5 m away (the far point of this particular eye). What should the focal length be in a lens prescribed to correct this problem?

Solution The purpose of the lens in this instance is to "move" an object from infinity to a distance where it can be seen clearly. This is accomplished by having the lens produce an image at the far point. From the thin-lens equation, we have

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} + \frac{1}{-2.5\ \text{m}} = \frac{1}{f}$$

$$f = \boxed{-2.5\ \text{m}}$$

Why did we use a negative sign for the image distance? As you should have suspected, the lens must be a diverging lens (one with a negative focal length) to correct nearsightedness.

Exercise What is the power of this lens?

Answer -0.40 diopter.

Optional Section

36.8 THE SIMPLE MAGNIFIER

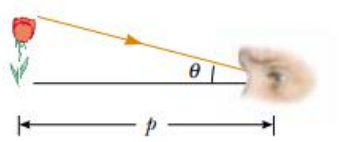


Figure 36.39 The size of the image formed on the retina depends on the angle θ subtended at the eye.

The simple magnifier consists of a single converging lens. As the name implies, this device increases the apparent size of an object.

Suppose an object is viewed at some distance p from the eye, as illustrated in Figure 36.39. The size of the image formed at the retina depends on the angle θ subtended by the object at the eye. As the object moves closer to the eye, θ increases and a larger image is observed. However, an average normal eye cannot focus on an object closer than about 25 cm, the near point (Fig. 36.40a). Therefore, θ is maximum at the near point.

To further increase the apparent angular size of an object, a converging lens can be placed in front of the eye as in Figure 36.40b, with the object located at point O, just inside the focal point of the lens. At this location, the lens forms a virtual, upright, enlarged image. We define **angular magnification** m as the ratio of the angle subtended by an object with a lens in use (angle θ in Fig. 36.40b) to the angle subtended by the object placed at the near point with no lens in use (angle

[1] The word *lens* comes from *lentil,* the name of an Italian legume. (You may have eaten lentil soup.) Early eyeglasses were called "glass lentils" because the biconvex shape of their lenses resembled the shape of a lentil. The first lenses for farsightedness and presbyopia appeared around 1280; concave eyeglasses for correcting nearsightedness did not appear for more than 100 years after that.

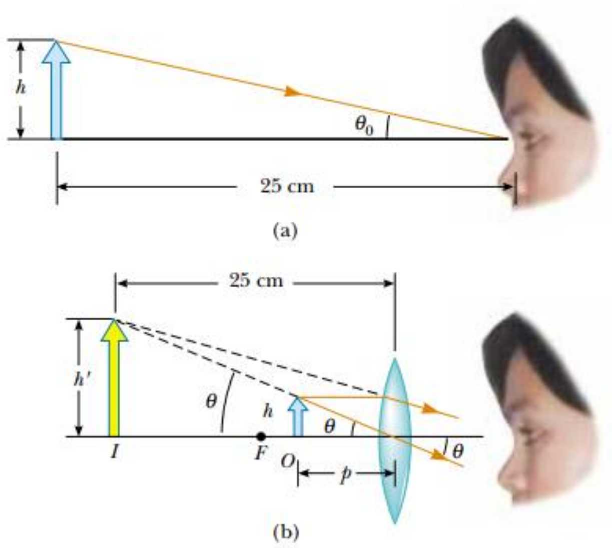


Figure 36.40 (a) An object placed at the near point of the eye ($p = 25$ cm) subtends an angle $\theta_0 \approx h/25$ at the eye. (b) An object placed near the focal point of a converging lens produces a magnified image that subtends an angle $\theta \approx h'/25$ at the eye.

θ_0 in Fig. 36.40a):

Angular magnification with the object at the near point

$$m \equiv \frac{\theta}{\theta_0} \tag{36.16}$$

The angular magnification is a maximum when the image is at the near point of the eye—that is, when $q = -25$ cm. The object distance corresponding to this image distance can be calculated from the thin-lens equation:

$$\frac{1}{p} + \frac{1}{-25\text{ cm}} = \frac{1}{f}$$

$$p = \frac{25f}{25 + f}$$

where f is the focal length of the magnifier in centimeters. If we make the small-angle approximations

$$\tan\theta_0 \approx \theta_0 \approx \frac{h}{25} \quad \text{and} \quad \tan\theta \approx \theta \approx \frac{h}{p} \tag{36.17}$$

Equation 36.16 becomes

$$m_{\text{max}} = \frac{\theta}{\theta_0} = \frac{h/p}{h/25} = \frac{25}{p} = \frac{25}{25f/(25 + f)}$$

$$m_{\text{max}} = 1 + \frac{25\text{ cm}}{f} \tag{36.18}$$

Although the eye can focus on an image formed anywhere between the near point and infinity, it is most relaxed when the image is at infinity. For the image formed by the magnifying lens to appear at infinity, the object has to be at the focal point of the lens. In this case, Equations 36.17 become

$$\theta_0 \approx \frac{h}{25} \quad \text{and} \quad \theta \approx \frac{h}{f}$$

and the magnification is

$$m_{\min} = \frac{\theta}{\theta_0} = \frac{25 \text{ cm}}{f} \quad \textbf{(36.19)}$$

With a single lens, it is possible to obtain angular magnifications up to about 4 without serious aberrations. Magnifications up to about 20 can be achieved by using one or two additional lenses to correct for aberrations.

EXAMPLE 36.16 Maximum Magnification of a Lens

What is the maximum magnification that is possible with a lens having a focal length of 10 cm, and what is the magnification of this lens when the eye is relaxed?

Solution The maximum magnification occurs when the image is located at the near point of the eye. Under these circumstances, Equation 36.18 gives

$$m_{\max} = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{10 \text{ cm}} = \boxed{3.5}$$

When the eye is relaxed, the image is at infinity. In this case, we use Equation 36.19:

$$m_{\min} = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{10 \text{ cm}} = \boxed{2.5}$$

Optional Section

36.9 THE COMPOUND MICROSCOPE

A simple magnifier provides only limited assistance in inspecting minute details of an object. Greater magnification can be achieved by combining two lenses in a device called a **compound microscope,** a schematic diagram of which is shown in Figure 36.41a. It consists of one lens, the *objective,* that has a very short focal length

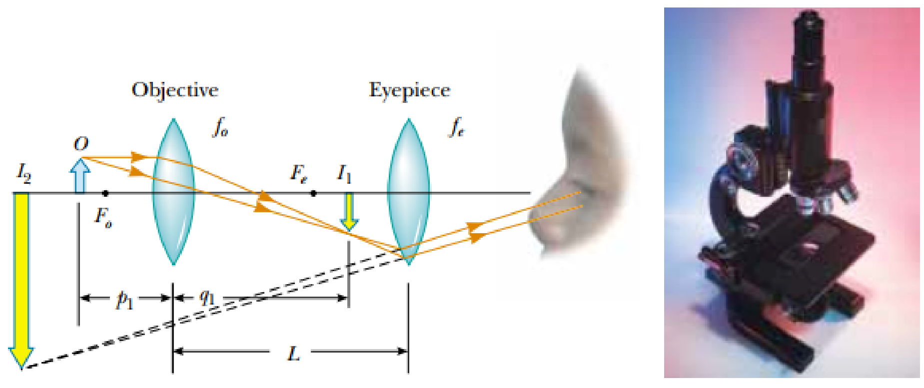


Figure 36.41 (a) Diagram of a compound microscope, which consists of an objective lens and an eyepiece lens. (b) A compound microscope. The three-objective turret allows the user to choose from several powers of magnification. Combinations of eyepieces with different focal lengths and different objectives can produce a wide range of magnifications.

$f_o < 1$ cm and a second lens, the *eyepiece,* that has a focal length f_e of a few centimeters. The two lenses are separated by a distance L that is much greater than either f_o or f_e. The object, which is placed just outside the focal point of the objective, forms a real, inverted image at I_1, and this image is located at or close to the focal point of the eyepiece. The eyepiece, which serves as a simple magnifier, produces at I_2 a virtual, inverted image of I_1. The lateral magnification M_1 of the first image is $-q_1/p_1$. Note from Figure 36.41a that q_1 is approximately equal to L and that the object is very close to the focal point of the objective: $p_1 \approx f_o$. Thus, the lateral magnification by the objective is

$$M_1 \approx -\frac{L}{f_o}$$

The angular magnification by the eyepiece for an object (corresponding to the image at I_1) placed at the focal point of the eyepiece is, from Equation 36.19,

$$m_e = \frac{25 \text{ cm}}{f_e}$$

The overall magnification of the compound microscope is defined as the product of the lateral and angular magnifications:

$$M = M_1 m_e = -\frac{L}{f_o}\left(\frac{25 \text{ cm}}{f_e}\right) \qquad \textbf{(36.20)}$$

The negative sign indicates that the image is inverted.

The microscope has extended human vision to the point where we can view previously unknown details of incredibly small objects. The capabilities of this instrument have steadily increased with improved techniques for precision grinding of lenses. An often-asked question about microscopes is: "If one were extremely patient and careful, would it be possible to construct a microscope that would enable the human eye to see an atom?" The answer is no, as long as light is used to illuminate the object. The reason is that, for an object under an optical microscope (one that uses visible light) to be seen, the object must be at least as large as a wavelength of light. Because the diameter of any atom is many times smaller than the wavelengths of visible light, the mysteries of the atom must be probed using other types of "microscopes."

The ability to use other types of waves to "see" objects also depends on wavelength. We can illustrate this with water waves in a bathtub. Suppose you vibrate your hand in the water until waves having a wavelength of about 15 cm are moving along the surface. If you hold a small object, such as a toothpick, so that it lies in the path of the waves, it does not appreciably disturb the waves; they continue along their path "oblivious" to it. Now suppose you hold a larger object, such as a toy sailboat, in the path of the 15-cm waves. In this case, the waves are considerably disturbed by the object. Because the toothpick was smaller than the wavelength of the waves, the waves did not "see" it (the intensity of the scattered waves was low). Because it is about the same size as the wavelength of the waves, however, the boat creates a disturbance. In other words, the object acts as the source of scattered waves that appear to come from it.

Light waves behave in this same general way. The ability of an optical microscope to view an object depends on the size of the object relative to the wavelength of the light used to observe it. Hence, we can never observe atoms with an optical microscope[2] because their dimensions are small (≈ 0.1 nm) relative to the wavelength of the light (≈ 500 nm).

Optional Section

36.10 THE TELESCOPE

14.1 & 14.9

Two fundamentally different types of **telescopes** exist; both are designed to aid in viewing distant objects, such as the planets in our Solar System. The **refracting telescope** uses a combination of lenses to form an image, and the **reflecting telescope** uses a curved mirror and a lens.

The lens combination shown in Figure 36.42a is that of a refracting telescope. Like the compound microscope, this telescope has an objective and an eyepiece. The two lenses are arranged so that the objective forms a real, inverted image of the distant object very near the focal point of the eyepiece. Because the object is essentially at infinity, this point at which I_1 forms is the focal point of the objective. Hence, the two lenses are separated by a distance $f_o + f_e$, which corresponds to the length of the telescope tube. The eyepiece then forms, at I_2, an enlarged, inverted image of the image at I_1.

The angular magnification of the telescope is given by θ/θ_o, where θ_o is the angle subtended by the object at the objective and θ is the angle subtended by the final image at the viewer's eye. Consider Figure 36.42a, in which the object is a very great distance to the left of the figure. The angle θ_o (to the *left* of the objective) subtended by the object at the objective is the same as the angle (to the *right* of the objective) subtended by the first image at the objective. Thus,

$$\tan \theta_o \approx \theta_o \approx -\frac{h'}{f_o}$$

where the negative sign indicates that the image is inverted.

The angle θ subtended by the final image at the eye is the same as the angle that a ray coming from the tip of I_1 and traveling parallel to the principal axis makes with the principal axis after it passes through the lens. Thus,

$$\tan \theta \approx \theta \approx \frac{h'}{f_e}$$

We have not used a negative sign in this equation because the final image is not inverted; the object creating this final image I_2 is I_1, and both it and I_2 point in the same direction. To see why the adjacent side of the triangle containing angle θ is f_e and not $2f_e$, note that we must use only the bent length of the refracted ray. Hence, the angular magnification of the telescope can be expressed as

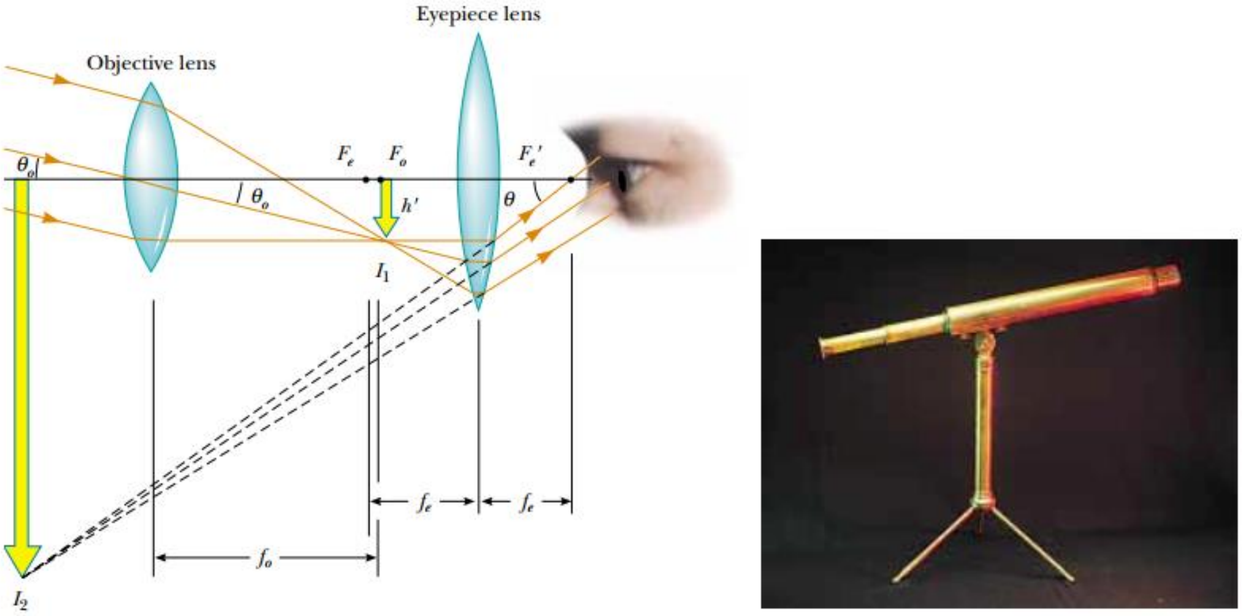
$$m = \frac{\theta}{\theta_o} = \frac{h'/f_e}{-h'/f_o} = -\frac{f_o}{f_e} \qquad \textbf{(36.21)}$$

and we see that the angular magnification of a telescope equals the ratio of the objective focal length to the eyepiece focal length. The negative sign indicates that the image is inverted.

Quick Quiz 36.6

Why isn't the lateral magnification given by Equation 36.1 a useful concept for telescopes?

[2] Single-molecule near-field optic studies are routinely performed with visible light having wavelengths of about 500 nm. The technique uses very small apertures to produce images having resolution as small as 10 nm.



(a) (b)

Figure 36.42 (a) Lens arrangement in a refracting telescope, with the object at infinity. (b) A refracting telescope.

When we look through a telescope at such relatively nearby objects as the Moon and the planets, magnification is important. However, stars are so far away that they always appear as small points of light no matter how great the magnification. A large research telescope that is used to study very distant objects must have a great diameter to gather as much light as possible. It is difficult and expensive to manufacture large lenses for refracting telescopes. Another difficulty with large lenses is that their weight leads to sagging, which is an additional source of aberration. These problems can be partially overcome by replacing the objective with a concave mirror, which results in a reflecting telescope. Because light is reflected from the mirror and does not pass through a lens, the mirror can have rigid supports on the back side. Such supports eliminate the problem of sagging.

Figure 36.43 shows the design for a typical reflecting telescope. Incoming light rays pass down the barrel of the telescope and are reflected by a parabolic mirror at the base. These rays converge toward point A in the figure, where an image would be formed. However, before this image is formed, a small, flat mirror M reflects the light toward an opening in the side of the tube that passes into an eyepiece. This particular design is said to have a Newtonian focus because Newton developed it. Note that in the reflecting telescope the light never passes through glass (except through the small eyepiece). As a result, problems associated with chromatic aberration are virtually eliminated.

The largest reflecting telescopes in the world are at the Keck Observatory on Mauna Kea, Hawaii. The site includes two telescopes with diameters of 10 m, each containing 36 hexagonally shaped, computer-controlled mirrors that work together to form a large reflecting surface. In contrast, the largest refracting telescope in the world, at the Yerkes Observatory in Williams Bay, Wisconsin, has a diameter of only 1 m.

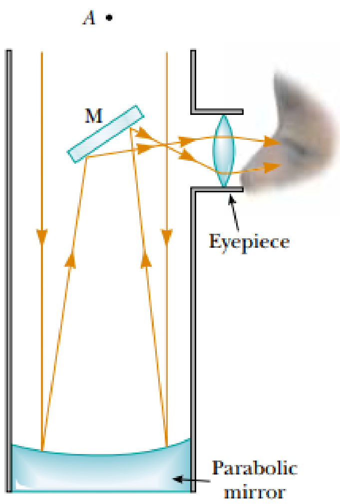


Figure 36.43 A Newtonian-focus reflecting telescope.

web

For more information on the Keck telescopes, visit **http://www2.keck.hawaii.edu:3636/**

SUMMARY

The **lateral magnification** M of a mirror or lens is defined as the ratio of the image height h' to the object height h:

$$M = \frac{h'}{h} \tag{36.1}$$

In the paraxial ray approximation, the object distance p and image distance q for a spherical mirror of radius R are related by the **mirror equation:**

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{f} \qquad \textbf{(36.4, 36.6)}$$

where $f = R/2$ is the **focal length** of the mirror.

An image can be formed by refraction from a spherical surface of radius R. The object and image distances for refraction from such a surface are related by

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \tag{36.8}$$

where the light is incident in the medium for which the index of refraction is n_1 and is refracted in the medium for which the index of refraction is n_2.

The inverse of the **focal length** f of a thin lens surrounded by air is given by the **lens makers' equation:**

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{36.11}$$

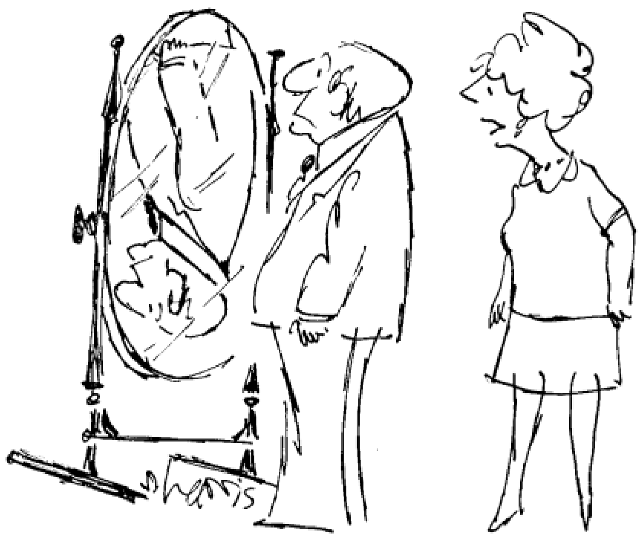
Converging lenses have positive focal lengths, and **diverging lenses** have negative focal lengths.

For a thin lens, and in the paraxial ray approximation, the object and image distances are related by the **thin-lens equation:**

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{36.12}$$

QUESTIONS

1. What is wrong with the caption of the cartoon shown in Figure Q36.1?
2. Using a simple ray diagram, such as the one shown in Figure 36.2, show that a flat mirror whose top is at eye level need not be as long as you are for you to see your entire body in it.
3. Consider a concave spherical mirror with a real object. Is the image always inverted? Is the image always real? Give conditions for your answers.
4. Repeat the preceding question for a convex spherical mirror.
5. Why does a clear stream of water, such as a creek, always appear to be shallower than it actually is? By how much is its depth apparently reduced?
6. Consider the image formed by a thin converging lens. Under what conditions is the image (a) inverted, (b) upright, (c) real, (d) virtual, (e) larger than the object, and (f) smaller than the object?
7. Repeat Question 6 for a thin diverging lens.
8. Use the lens makers' equation to verify the sign of the focal length of each of the lenses in Figure 36.26.



"Most mirrors reverse left and right. This one reverses top and bottom."

Figure Q36.1