Motion in One Dimension

A high-speed car has released a parachute to reduce its speed quickly. The directions of the car's velocity and acceleration are shown by the green $\vec{v}$ and gold $\vec{a}$ arrows. Motion is described using the concepts of velocity and acceleration. In the case shown here, the acceleration $\vec{a}$ is in the opposite direction from the velocity $\vec{v}$, which means the object is slowing down. We examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do. This Chapter and the next deal with kinematics.

Reference Frames and Displacement

A frame of reference is a set of coordinates that can be used to determine the positions and velocities of objects in that frame; different frames of reference move relative to one another. Any measurement of position, distance, or speed must be made concerning a **reference frame**, or **frame of reference**.

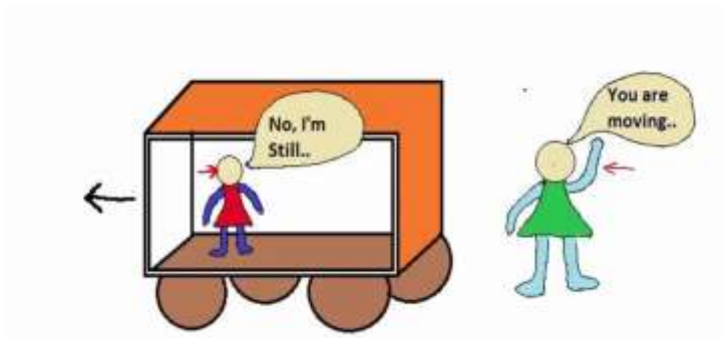


Figure 1.1

The motion of a particle is completely known if the particle's position in space is always known. A particle's **position** is the location of the particle concerning a chosen reference point that we can consider to be the origin of a coordinate system

The **displacement** of a particle is defined as its change in position in some time interval. As the particle moves from an initial position x_i to a final position x_f, its displacement is given by:

$$\Delta x = x_f - x_i$$

It is very important to recognize the difference between displacement and distance traveled. **Distance** is the length of a path followed by a particle. Consider, for example, the basketball players. If a player runs from his own team's basket down the court to the other team's basket and then returns to his basket, the *displacement* of the player during this time interval is zero because he ended up at the same point as he started: $x_f = x_i$, so $\Delta x = 0$. During this time interval, however, he moved through twice the length of the basketball court. Distance is always represented as a positive number, whereas displacement can be either positive or negative. Displacement is an example of a vector quantity. Many other physical quantities, including position, velocity, and acceleration, also are vectors.

Average Velocity

The most obvious aspect of the motion of a moving object is how fast it is moving its speed or velocity The term "speed" refers to how far an object travels in each time interval, regardless of direction.

If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined *as the total distance traveled along its path divided by the time it takes to travel this distance*:

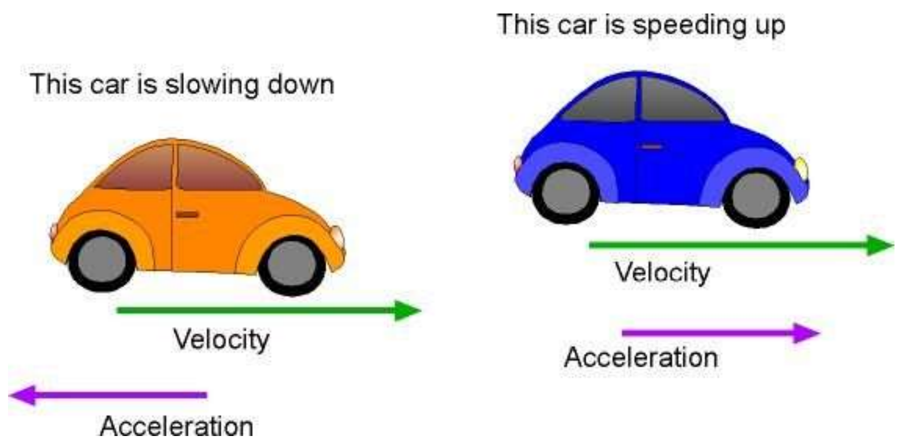


Figure 2.0

The terms “velocity” and “speed” are often used interchangeably in ordinary language. But in physics, we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the magnitude (numerical value) of how fast an object is moving and the direction in which it is moving (Velocity is therefore a vector).

There is a second difference between speed and velocity: namely, the **average velocity** is defined in terms of displacement, rather than total distance traveled:

$$\text{average velocity} = {}^{\text{final position - initial position}}/_{\text{time elapsed}}$$

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: A person walked 70 m east and then 30 m west. The total distance traveled was 70 m + 30 m = 100 m, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$${}^{\text{distance}}/_{\text{time elapsed}} = {}^{100\text{m}}/_{70\text{s}} = 1.4 \text{ m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$${}^{\text{displacement}}/_{\text{time elapsed}} = {}^{40\text{m}}/_{70\text{s}} = 0.57 \text{ m/s}.$$

This difference between the speed and the magnitude of the velocity can occur when we calculate average values.

To discuss the one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the *x-axis* at position x_1 in a coordinate system, and at some later time, t_2, suppose it is at position x_2.

The **elapsed time** is $\Delta t = t_2 - t_1$; during this time interval, the displacement of our object is $\Delta x = x_2 - x_1$. Then the average velocity, defined as the displacement divided by the elapsed time, can be written:

$$V = x_2 - x_1 / t_2 - t_1 = \Delta x / \Delta t$$

Example 1.1 Runner's average velocity

A runner’s position as a time function is plotted as moving along the x-axis of a coordinate system. During a 3.00s time interval, the runner’s position changes from x_1 = 50.0 m to x_2 = 30.5 m, as shown in Fig 1.2. What was the runner’s average velocity?

Solution

We want to find the average velocity, which is the displacement divided by the elapsed time.

The displacement is $\Delta x = x_2 - x_1 = 30.5 \text{ m} - 50.0 \text{ m} = -19.5\text{m}$. The elapsed time, or time interval, is $\Delta t = 3.00$ s. The average velocity is

$$v = \Delta x / \Delta t = -19.5\text{m} / 3.00\text{s} = -6.50 \text{ m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the x-axis. Thus, we can say that the runner's average velocity is 6.50 m/s to the left.

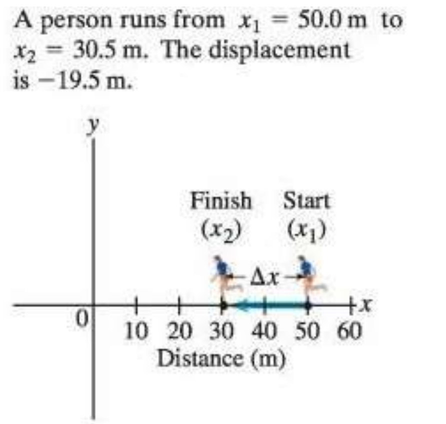


Fig 1.2

Example 1.2 Distance a cyclist travels.

How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

Solution

We want to find the distance traveled, so $\Delta x = v\Delta t$

$\Delta x = v\Delta t$ = (18km/h)(2.5h) = 45km.

Instantaneous Velocity and Speed

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To describe this situation, we need the concept of instantaneous velocity, which is the velocity at any instant in time. (Its magnitude is the number, with units, indicated by a speedometer)

Instantaneous velocity is defined as the rate of change of position for a very small time interval (almost zero). Measured using SI unit m/s. Instantaneous speed is the magnitude of the instantaneous velocity. It has the same value as that of instantaneous velocity but does not have any direction.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

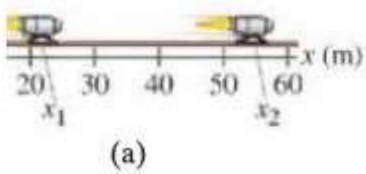
The instantaneous speed is the absolute value (magnitude) of the instantaneous velocity.

If we make a plot of x vs. t for a moving particle the instantaneous velocity is the slope of the tangent to the curve at any point.

Example 1.3 **Given *x* as a function**

A jet engine moves along an experimental track (which we call the x-axis) as shown in Fig.1.5a We will treat the engine as if it were a particle. Its position as a function of time is given by the equation $x = At^2 + B$, where A = 2.10 m/s2 and B = 2.80 m, and this equation is plotted in Fig.1.5b.

a. Determine the displacement of the engine during the time interval from $t_1 = 3.00$s to $t_2 = 5.00$s
b. Determine the average velocity during this time interval
c. Determine the magnitude of the instantaneous velocity at t = 5.00 s.



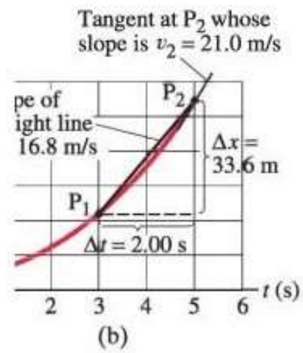


Fig 1.5a

Fig 1.5b

Solution

We substitute values for t_1 and t_2 in the given equation for x to obtain x_1 and x_2. We take the derivative of the given x equation concerning t to find the instantaneous velocity, using the formulas just given.

a. At $t_1 = 3.00$ s, the position (point P_1 in Fig. 1.5b) is $x = At_1^2 + B = (2.10\ m/s^2)\ (5.00s)^2 + 2.80\ m = 21.7\ m$.

   At $t_2 = 5.00$ s, the position (P_2 in Fig. 1.5b) is $x_2 = (2.10\ m/s^2)\ (5.00s)^2 + 2.80\ m = 55.3m$.

   The displacement is thus $x_2 - x_1 = 55.3\ m - 21.7m = 33.6m$

b. The magnitude of the average velocity can then be calculated as
$v = \Delta x / \Delta t = x_2 - x_1 / t_2 - t_1 = 33.6\text{ m} / 2.00\text{ s} = 16.8\text{ m/s}$.

c. The instantaneous velocity at $t = t_2 = 5.00$ s equals the slope of the tangent to the curve at point P_2 shown in Fig. 1.5b. We could measure this slope of the graph to obtain v_2. But we can calculate v more precisely for any time t, using the given formula
$x = At_1^2 + B$,
which is the engine's position x as a function of time t. We take the derivative of x concerning time:
$v = {}^{dx}/_{dt} = {}^{d}/_{dt}(At^2 + B) = 2At$. We are given $A = 2.10\text{ m/s}^2$, so for $t = t_2 = 5.00$ s,
$v_2 = 2At = 2(2.10\text{ m/s}^2)\,(5.00\text{s}) = 21.0\text{ m/s}$

Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how rapidly the velocity of an object is changing.

Average Acceleration

Average acceleration is defined as the change in velocity divided by the time taken to make this change:

average acceleration = change of velocity / time elapsed

In symbols, the average acceleration over a time interval $\Delta t = t_2 - t_1$ during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Because velocity is a vector, acceleration is a vector too. But for one-dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis.

Example 1.4 **Average acceleration.**

A car accelerates along a straight road from rest to 90 km/h in 5.0 s, Fig.2.0 What is the magnitude of its average acceleration?

Solution

Average acceleration is the change in velocity divided by the elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is v_2 = 90 km/h = 90 X 10^3 m/3600 s = 25 m/s.
the average acceleration is

$a = v_2 - v_1 / t_2 - t_1$ = 25 m/s -0 m/s / 5.0 s = 5.0 m/s^2

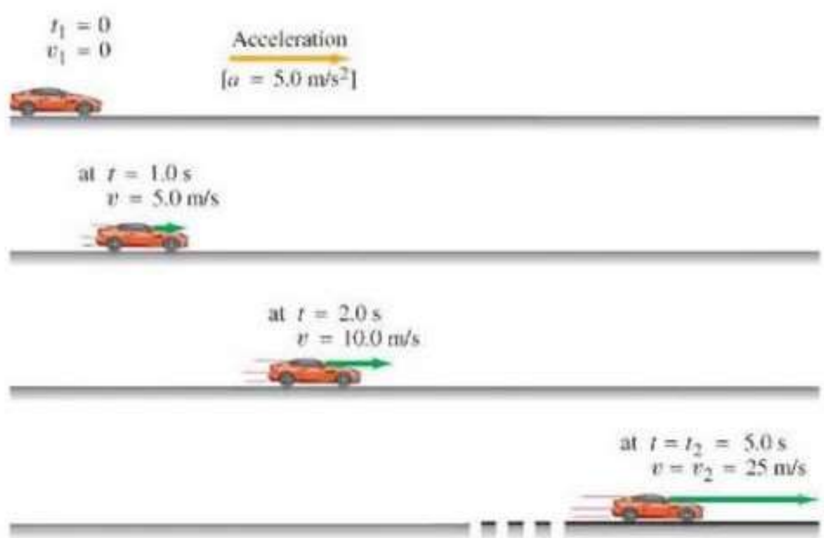


Fig. 2.1

Example 1.5 **Car slowing down.**

An automobile is moving to the right along a straight highway, which we choose to be the positive x-axis (Fig. 2.1). Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is v_1 = 15.0 m/s, and it takes 5.0 s to slow down to v_2 = 5.0 m/s, what was the car's average acceleration?

Solution

$t_1 = 0$ and $t_2 = 5.0$ s. $\Delta t = t_2 - t_1$

a = 5.0 m/s - 15.0 m/s / 5.0 s = -2.0 m/s

The negative sign appears because the final velocity is less than the initial velocity. In this case, the direction of the acceleration is to the left (in the negative x-direction) even though the velocity is always pointing to the right. We say that the acceleration is 2.0 m/s^2 to the left.

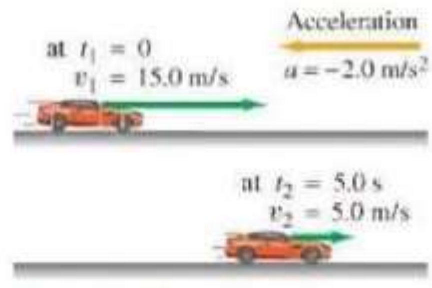


Fig. 2.2

Instantaneous acceleration is defined as the ratio of change in velocity during a given time interval such that the time interval goes to zero.

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

Motion at Constant Acceleration

We now examine the situation when the magnitude of the acceleration is constant, and the motion is in a straight line. In this case, the instantaneous and average accelerations are equal. We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate x, v, a, and t when a is constant, allowing us to determine any one of these variables if we know the others.

To simplify our notation, let us take the initial time in any discussion to be zero, and we call it t_0: $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at t0.) We can then let $t_2 = t$ be the elapsed time. The initial position(x_1) and the initial velocity (v_1) of an object will now be represented by x_0 and v_0 since they represent x and v at t = 0. At time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time interval $t - t_0$ will be

$$v = {}^{\Delta x}/_{\Delta t} = {}^{x - x_0}/_{t - t0} = {}^{x - x_0}/_{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is

$$a = {}^{v - v_0}/_{t}$$

A common problem is to determine the velocity of an object after any elapsed time t when we are given the object's constant acceleration. We can solve such problems by solving for v in the last equation to obtain:

$$v = v_0 + at$$

If an object starts from rest ($v_0 = 0$) and accelerates at 4.0 m/s^2, after an elapsed time t = 6.0 s its velocity will be v = at = (4.0 m/s^2) (6.0 s) = 24 m/s. Next, let us see how to calculate the position x of an object after a time t when it

undergoes constant acceleration. The definition of average velocity is $v = (x - x_0)/t$, which we can rewrite as

$x = x_0 + Vt$

Because the velocity increases at a uniform rate, the average velocity, v, will be midway between the initial and final velocities:

$$v = {}^{v}{}_{0} + v/_{2}$$

We combine the last two equations and find

$$x = x_0 + vt = x_0 + ({}^{v}{}_{0} + v/_{2})t = x_0 + ({}^{v}{}_{0} + v_0 + at/_{2})t$$

or

$$x = x_0 + v_0 t + {}^{1}/_{2}at^2$$

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

Example 1.6 **Runaway Design.**

You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h) and can accelerate at 2.00 m/s2.
(a) If the runway is 150 m long, can this airplane reach the required speed for takeoff? (*b*) If not, what minimum length must the runway have?

Solution

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\text{ m}$	
$a = 2.00\text{ m/s}^2$	

$V_2 = v_1 + 2a(x - x_0)$

$= 0 + 2(2.00\text{ m/s}^2)(150\text{m}) = 600\text{m}^2/\text{s}^2$

$V_2 = \sqrt{600\text{ m}^2/\text{s}^2} = 24.5\text{ m/s}.$

This runway length is *not* sufficient.

(*b*) Now we want to find the minimum length of the runway, $x - x_0$, given
$v = 27.8$ m/s and $a = 2.00\text{ m/s}^2$.

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\,\mathrm{m/s})^2 - 0}{2(2.00\,\mathrm{m/s^2})} = 193\,\mathrm{m}.$$

A 200-m runway is more appropriate for this plane.

Example 1.7 Two moving objects: Police and Speeder.

A car speeding at 150 km/h passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at a constant speed, estimate how long it takes the police car to overtake the speeder.
Then estimate the police car's speed at that moment and decide if the assumptions were reasonable.

Solution

$$x_S = v_{0S}t + \tfrac{1}{2}a_S t^2 = (150\,\mathrm{km/h})t = (42\,\mathrm{m/s})t$$
$$x_P = v_{0P}t + \tfrac{1}{2}a_P t^2 = \tfrac{1}{2}(5.6\,\mathrm{m/s^2})t^2,$$

where we have set $v_{0P} = 0$ and $a_S = 0$ (speeder assumed to move at constant speed). We want the time when the cars meet, so we set $x_S = x_P$ and solve for t:

$$(42\,\mathrm{m/s})t = (2.8\,\mathrm{m/s^2})t^2.$$

The solutions are

$$t = 0 \quad \text{and} \quad t = \frac{42\,\mathrm{m/s}}{2.8\,\mathrm{m/s^2}} = 15\,\mathrm{s}.$$

The first solution corresponds to the instant the speeder passed the police car. The second solution tells us when the police car catches up to the speeder, 15 s later. This is our answer, but is it reasonable? The police car's speed at $t = 15\,\mathrm{s}$ is

$$v_P = v_{0P} + a_P t = 0 + (5.6\,\mathrm{m/s^2})(15\,\mathrm{s}) = 84\,\mathrm{m/s}$$

or 300 km/h ($\approx$ 190 mi/h). Not reasonable, and highly dangerous.

Freely Falling Objects

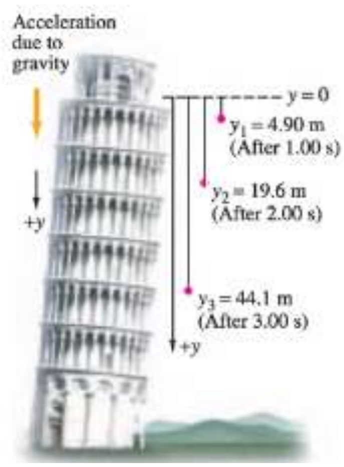
In Newtonian physics, **free fall** is any motion of a body where gravity is the only force acting upon it. In the context of general relativity, where gravitation is reduced to a space-time curvature, a body in free fall has no force acting on it. An object in the technical sense of the term "free fall" may not necessarily be falling in the usual sense of the term. An object moving upwards might not normally be falling, but if it is subject to only the force of gravity, it is said to be in free fall. The Moon is thus in free fall around the Earth, though its orbital speed keeps it in a very far orbit from the Earth's surface.

In a roughly uniform gravitational field, in the absence of any other forces, gravitation acts on each part of the body roughly equally. When there is no normal force exerted between a body (e.g., an astronaut in orbit) and its surrounding objects, it will result in the sensation of weightlessness, a condition that also occurs when the gravitational field is weak (such as when far away from any source of gravity). The term "free fall" is often used more loosely than in the strict sense defined above. Thus, falling through an atmosphere without a deployed parachute, or lifting device, is also often referred to as *free fall*. The aerodynamic drag forces in such situations prevent them from producing full weightlessness, and thus a skydiver's "free fall" after reaching terminal velocity produces the sensation of the body's weight being supported on a cushion of air.

Example 1.8 Free falling from a tower.

Suppose that a ball is dropped ($v_0 = 0$) from a tower 70.0 m high. How far will it have fallen after a time $t_1 = 1.00$ s, $t_2 = 2.00$ s, and $t_3 = 3.00$ s? Ignore air resistance.

Solution



We set $t = t_1 = 1.00$ s

$y_1 = v_0 t_1 + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 = \frac{1}{2}(9.80\ m/s^2)(1.00s)^2 = 4.90m.$

The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00$ s. Similarly, after 2.00 s ($= t_2$), the ball's position is

$y = \frac{1}{2}at^2 = \frac{1}{2}(9.80\ m/s^2)(2.00\ s)^2 = 19.6$ m.

Finally, after 3.00 s ($= t_3$), the ball's position is

$y_3 = \frac{1}{2}at^2 = \frac{1}{2}(9.80\ m/s^2)(3.00\ s)^2 = 44.1m.$

Example 1.9 A ball is thrown upward.

A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate *(a)* how high it goes, and (*b*) how long the ball is in the air before it comes back to the hand. Ignore air resistance.

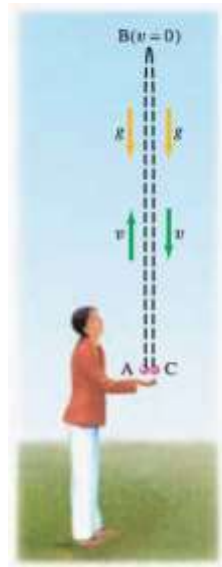


Figure 2 - 30

Solution

(a) We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2-30) we have $y_0 = 0$, $v_0 = 15.0$ m/s, and $a = -9.80$ m/s2. At time t (maximum height), $v = 0$, $a = -9.80$ m/s2, and we wish to find y.

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 11.5\,\mathrm{m}.$$

The ball reaches a height of 11.5 m above the hand.

(b) Now we need to choose a different time interval to calculate how long the the ball is in the air before it returns to the hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 2-30) in one step. We can do this because y represents position or displacement, and not the total distance traveled.

Thus, at both points A and C, y = 0. We use Eq. 2-12b with a = -9.80 m/s2 and find

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$
$$0 = 0 + (15.0\,\mathrm{m/s})t + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)t^2.$$

This equation is readily factored (we factor out one t):

$$\left(15.0\,\mathrm{m/s} - 4.90\,\mathrm{m/s^2}\,t\right)t = 0.$$

There are two solutions:

$$t = 0 \quad \text{and} \quad t = \frac{15.0\,\mathrm{m/s}}{4.90\,\mathrm{m/s^2}} = 3.06\,\mathrm{s}.$$

The first solution $(t = 0)$ corresponds to the initial point (A) in Fig. 2–30, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06$ s, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

NOTE We have ignored air resistance, which could be significant, so our result is only an approximation to a real, practical situation.

Summary

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame.**

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the elapsed time or time interval, *At,* the period over which we choose to make our observations. An object's **average velocity** over a particular time interval Δ*t* is its displacement *Δx* during that time interval, divided by *Δt:*

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

The **instantaneous velocity,** whose magnitude is the same as the *instantaneous speed* is defined as the average velocity taken over an infinitesimally short time interval *(Δt* —» 0):

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt},$$

where *dx/dt* is the derivative of *x* concerning *t.* On a graph of position vs. time, the slope is equal to the instantaneous velocity.

Acceleration is the change of velocity per unit of time. An object's average acceleration over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t},$$

Instantaneous acceleration is the average acceleration taken over an infinitesimally short time interval:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

If an object moves in a straight line with constant acceleration, the velocity v and position x are related to the acceleration a, the elapsed time t, the initial position x_0, and the initial velocity v_0

$$v = v_0 + at, \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0), \quad \bar{v} = \frac{v + v_0}{2}.$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward acceleration due to gravity, whose magnitude is $g = 9.80\ m/s^2$ if air resistance can be ignored.