

# Mivne Wet 2

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Our data structure is comprised of the following private members:

1. A hash table for all our customers which is an array of pointers to AVL trees as opposed to the typical array of linked lists. The hash function is  $(\text{customer\_id} \% \text{table\_size})$ .  
Space complexity:  $O(n)$
2. An array of all the records in that month  
Space complexity:  $O(m)$
3. A member\_tree which inherits from the typical AVL tree and overloads rotation operations in order to support member-fee calculations. This, in the worst case, holds all the customers.  
Space complexity:  $O(n)$
4. An int which specifies the number of records that month.  
Space complexity:  $O(1)$
5. An instance of our unionFind class, which holds the “stacks” of our records - all private members of unionFind are proportional to the number of records in that month.  
Space complexity:  $O(m)$

**Total Space Complexity:  $O(n+m)$**

## **RecordsCompany():**

This initializes an empty hash table of shared\_ptrs to customers ( $O(1)$ ), assigns a nullptr as our records array ( $O(1)$ ), initializes an empty tree ( $O(1)$ ), initializes an integer as 0 ( $O(1)$ ), and calls the constructor of our UnionFind class with a size of 1, hence the constructor will do constant time operations proportional to a size of 1:  $O(1)$ . Overall:

**$O(1)$**

## **~RecordsCompany():**

The destructor of our hash table iterates through a table\_size proportional to the number of customers, executing constant time operations with each iteration:  $O(n)$ . The number of nodes throughout all trees in our database is also proportional to the number of customers, and thus deleting these nodes takes  $O(n)$ . All the shared pointers to records are deleted automatically  $O(m)$ , and deleting record\_copies itself is  $O(1)$ . The destructor of Union Find executes a constant number of delete operations:  $O(1)$ . All in all:

**$O(n+m)$**

**NewMonth():**

First, the function assigns the debt and discounts of every customer to zero: this is a constant time operation which is executed on every customer:  $O(n)$ . From there we delete our array of records:  $O(1)$ . We then dynamically allocate an array of size  $m$  to our array of records and for each index, store the necessary information. This is a constant time operation which is executed  $m$  times:  $O(m)$ . The UnionFind constructor also receives a size of  $m$ , and thus executes constant time operations  $m$  times:  $O(m)$ . In total:

**$O(n+m)$**

**AddCustomer():**

As we have seen in the lectures, the insert operation for a typical hash table which is composed of linked lists is  $O(1)$  amortized, even with resizing operations. In this case, we are maintaining the same complexity for our resize operation whilst reducing the complexity of inserting (inserting into a linked list is  $O(n)$  whilst inserting into a tree is  $O(\log n)$ ). Thus, it is trivial that inserting a customer to our linked list is  **$O(1)$  amortized.**

**getPhone():**

Finding a node in a hash table is of the same time complexity as inserting a node, and thus, finding a node, or in this case a customer, is  $O(1)$  amortized based on the same justification provided in addCustomer(). From here, getting the phone number of the customer is a constant time operation. Overall:

**$O(1)$  amortized**

**makeMember():**

Finding the relevant customer tree is a constant time operation as it is simply applying the hash function to the customer's ID. From there, we have to iterate through a tree which, in the worst case, holds all customers in the data structure:  $n$  nodes. As we have seen in the lectures, the complexity of reaching any node in an AVL tree with  $n$  nodes is  $O(\log n)$ . Once we reach the appropriate customer, making him a member is a constant time operation. Therefore:

**$O(\log n)$**

**isMember():**

The justification for the complexity of this function is identical to getPhone(). Just instead of getting their phone number, which is a constant time operation, we are getting their member status, which is also a constant time operation. Overall:

**$O(1)$  amortized**

**buyRecord():**

Based on the same justification as `makeMember()`, finding the relevant customer is  $O(\log n)$ . From there, adding a payment to the customer's monthly debt, if they are a member, is a constant time operation. Accessing the `r_id` index of our record's array is a constant time operation, and adding a purchase to the record is an incrementation of a private data member which is also a constant time operation. All in all:

**$O(\log n)$**

#### **`addPrize():`**

There are two calls to `addPrize_aux`, a function which performs a find operation which iterates through the member tree and finds the relevant node ( $O(\log n)$ ), and at each node, under certain conditions, performs an arithmetic constant time operation ( $O(\log n)$ ). Overall:

**$O(\log n)$**

#### **`getExpenses():`**

Based on the same justification as `makeMember()`, finding the relevant customer is  $O(\log n)$ . From there finding their expenses is simply a constant time summation operation. Overall:

**$O(\log n)$**

#### **`putOnTop():`**

In this function we are executing a “union” operation, similar to the typical union operation which implements both path-compression and rank-optimisation algorithms. As we saw in the lectures, this is of  $O(\log^* m)$  time complexity, where  $m$  is the size within our `UnionFind` class. In addition, we added a constant number of constant time operations throughout the union operation. As such, the total time complexity of `putOnTop` is unaffected by our additions:

**$O(\log^* m)$**

#### **`getPlace():`**

In this function we are executing a “find” operation, which is identical to the find algorithms we have seen in lectures. As we know, this operation is of  $O(\log^* m)$  time complexity:

**$O(\log^* m)$**