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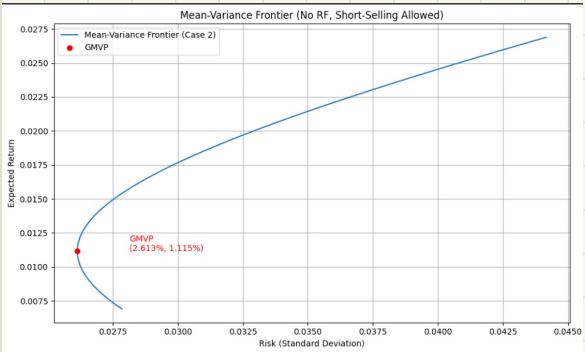
학번 : 20259431

1. Compute the vector of the mean (μ) and covariance matrix (Σ) using all data. Assume that there is no risk-free asset, but short-selling is allowed. (case #2 in our lecture note)

a. Use the parameters to **derive** the mean-variance frontier using the standard deviation for measuring risk. **Plot** the mean-variance frontier. (x: standard deviation, y: expected return) **Indicate** the global minimum portfolio (GMVP) on the plot.



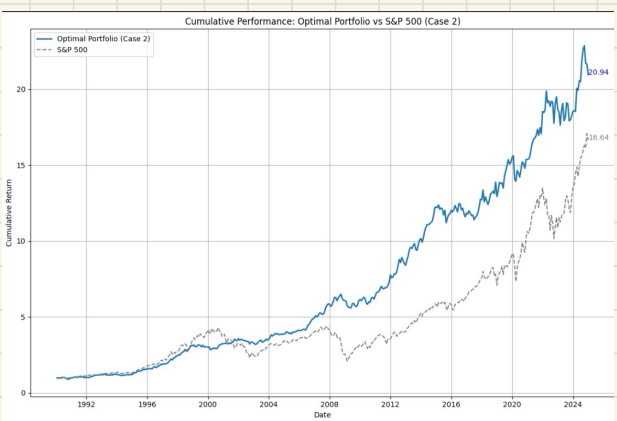

b. Let S&P 500 index be the Benchmark. Then, **derive** the portfolio weight that minimizes portfolio risk while matching the Benchmark expected return. **Report** the portfolio weight in an excel file. And **report** the portfolio weights on 3M CO (22592), Bank of America Corp (59408), Advanced Micro Devices Inc (61241), Apple Inc (14593), and McDonalds Corp (43449).

| permno | comnam | weight |
|---|---|---|
| 22592 | 3M CO | 0.113271627 |
| 59408 | BANK OF AMERICA CORP | -0.033260847 |
| 61241 | ADVANCED MICRO DEVICES INC | -0.019377232 |
| 14593 | APPLE INC | 0.003820170 |
| 43449 | MCDONALDS CORP | 0.022803662 |

c. **Compute** the annualized excess returns, annualized volatility (standard deviation), and annualized Sharpe ratio of the optimal and the S&P 500 portfolios. (Check annualized Sharpe ratio from https://web.stanford.edu/~wfsharpe/ws/wi_perf.htm#:~:text=The%20annualized%20Sharpe%20Ratio%20is,the%20square%20root%20of%2012.)

| | Optimal Portfolio (Case 2) | S&P 500 |
|---|---|---|
| **Excess Return (%)** | 6.529617 | 6.529617 |
| **Volatility (%)** | 9.462050 | 14.814414 |
| **Sharpe Ratio** | 0.690085 | 0.440761 |

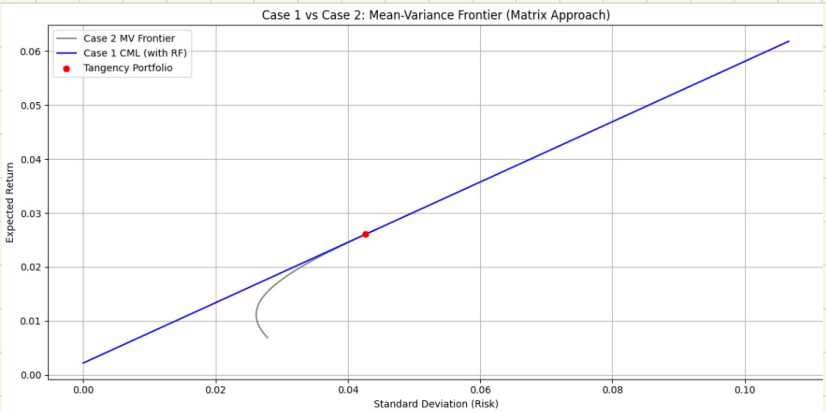
d. **Plot** the accumulated performance of the optimal portfolio and the S&P 500 portfolio. (x: time, y: cumulative return.) **Discuss** the difference in the performance of those two portfolios. (MV case2, Benchmark)




The mean-variance optimized portfolio (case 2) shows superior long-term performance with lower volatility, demonstrating the benefit of quantative portfolio construction under short-selling allowed conditions.

2. Assume that there is a risk-free asset, and short-selling is allowed. (case #1 in our lecture note)

a. **Derive** the mean-variance frontier. **Report** the slope of the mean-variance frontier. Then, **plot** the mean-variance frontier together with the mean-variance frontier in MV case2 in question (1.a).




∴ Slope : 0.559208

b. **Derive** the portfolio weight that minimizes portfolio risk, matching S&P 500 expected return. **Report** the portfolio weight in an excel file. **Report** the portfolio weights on 3M CO (22592), Bank of America Corp (59408), Advanced Micro Devices Inc (61241), Apple Inc (14593), and McDonalds Corp (43449). **What** is the weight of a risk-free asset?

| permno | comnam | weight |
|---|---|---|
| 22592 | 3M CO | 0.009342823 |
| 59408 | BANK OF AMERICA CORP | -0.016050578 |
| 61241 | ADVANCED MICRO DEVICES INC | -0.003344138 |
| 14593 | APPLE INC | 0.019934844 |
| 43449 | MCDONALDS CORP | 0.008284793 |
| RISKFREE | Risk-free asset | 0.771914759 |

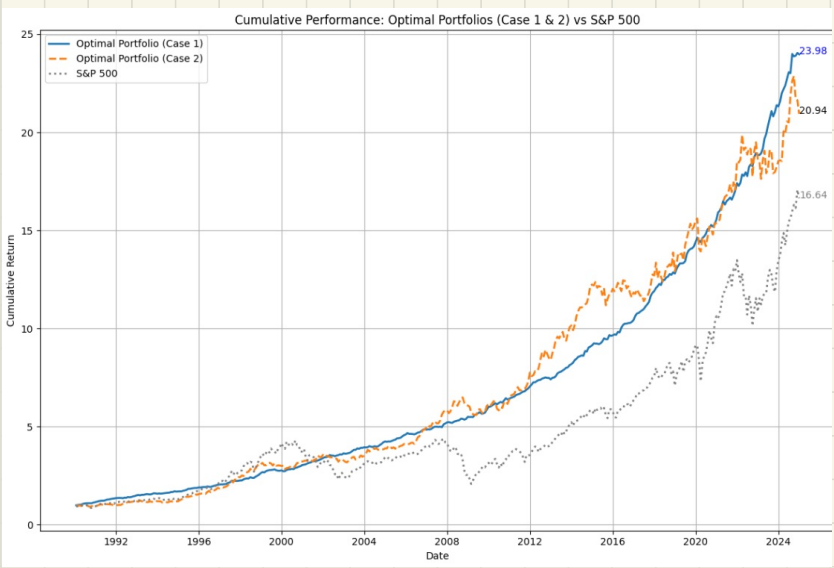
∴ the weight of a risk-free asset : 77%

c. **Compute** the annualized excess returns, annualized volatility, and annualized Sharpe ratio of the optimal and the S&P 500 portfolios. Then, **compare** the results to the results from MV case2 and Benchmark. **How** different are the results?

| | Case 1 (Optimal) | Case 2 (Optimal) | S&P 500 |
|---|---|---|---|
| **Excess Return (%)** | 6.529617 | 6.529617 | 6.529617 |
| **Volatility (%)** | 3.370308 | 9.462050 | 14.814414 |
| **Sharpe Ratio** | 1.937394 | 0.690085 | 0.440761 |

Although all three portfolios (case 1, case 2, and S&P 500) have the same excess return (6.53%), case 1 achieves the highest Sharpe ratio (1.94) thanks to its lowest volatility (3.30%), due to the inclusion of a risk-free asset. Case 2 has higher volatility (9.46%) resulting in a lower Sharpe ratio (0.69). The S&P 500 performs the worst with the highest volatility and the lowest Sharpe ratio (0.44)

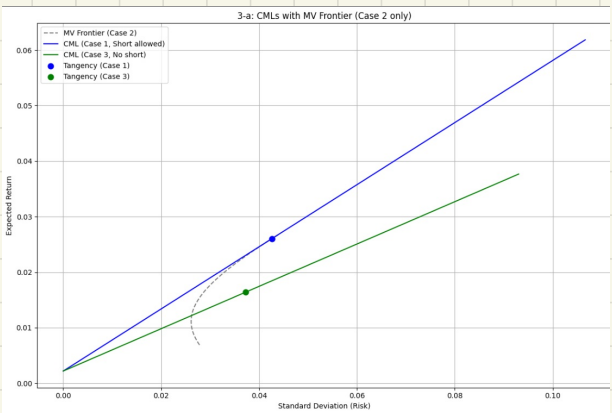
d. **Plot** the accumulated performance of the optimal portfolio with the figure in question (1.d). Then, **discuss** the difference in the performance of those three portfolios. (MV case1, MV case2, and Benchmark)




The case 1 portfolio achieved the highest cumulative return compared to case 2 and the S&P 500. The inclusion of the risk-free asset allowed for better risk-adjusted performance. This result confirms that combining risky assets with a risk-free asset leads to superior long-term returns.

3. Assume that there is a risk-free asset, but short-selling is not allowed. (case #3 in our lecture note)

a. **Derive** the mean-variance frontier. **Report** the slope of the mean-variance frontier. Then, **plot** the mean-variance together with the mean-variance frontier in MV case1 in question (2.a). **Which** one has a higher slope, and **what** is its meaning?




Slope of CML (Case 1) : 0.559208
Slope of CML (Case 3) : 0.381223

Case 1 has a steeper CML than case 3, meaning a higher Sharpe ratio. This is because Case 1 allows short-selling, while Case 3 does not. So, Case 1 gives better risk-adjusted performance.

b. **Derive** the portfolio weight that minimizes portfolio risk, matching S&P 500 expected return. **Report** the portfolio weight in an excel file. **Report** the portfolio weights on 3M CO (22592), Bank of America Corp (59408), Advanced Micro Devices Inc (61241), Apple Inc (14593), and McDonalds Corp (43449). **What** is the weight of a risk-free asset?

| permno | comnam | weight |
|---|---|---|
| 22592 | 3M CO | 0 |
| 59408 | BANK OF AMERICA CORP | 0 |
| 61241 | ADVANCED MICRO DEVICES INC | 0 |
| 14593 | APPLE INC | 0.026603090 |
| 43449 | MCDONALDS CORP | 0 |
| RISKFREE | Risk-free asset | 0.616471065 |

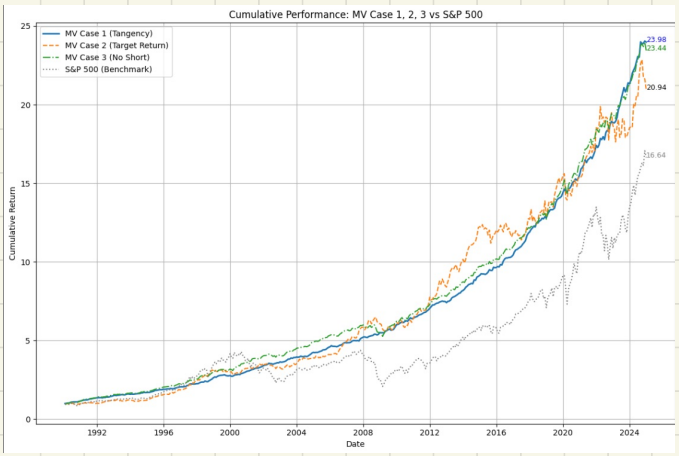
∴ the weight of a risk-free asset : 62%

c. **Compute** the annualized excess returns, annualized volatility, and annualized Sharpe ratio of the optimal and the S&P 500 portfolios. Then, **compare** the results to MV case1, MV case2, and Benchmark. **How** different are the results?

| | MV Case 1 | MV Case 2 | MV Case 3 | S&P 500 |
|---|---|---|---|---|
| **Excess Return (%)** | 6.529617 | 6.529617 | 6.529617 | 6.529617 |
| **Volatility (%)** | 3.370308 | 9.462050 | 4.944451 | 14.814414 |
| **Sharpe Ratio** | 1.937394 | 0.690085 | 1.320595 | 0.440761 |

All portfolios have the same excess return (6.53%). Case 1 shows the best risk-adjusted performance (Sharpe ratio 1.94) due to lowest volatility. Case 3 improves over Case 2 by including a risk-free asset, which lowers volatility and increases the Sharpe ratio (1.32 vs 0.69). The S&P too has the worst Sharpe ratio (0.44) due to the highest level of volatility.

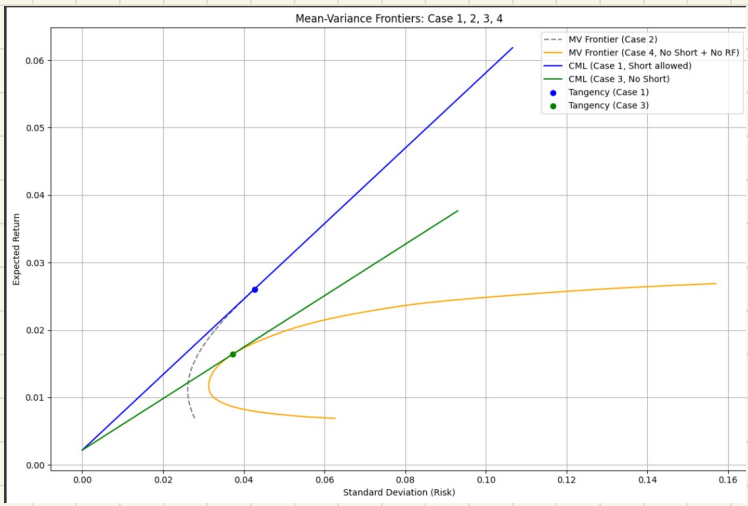
d. **Plot** the accumulated performance of the optimal portfolio with the figure in question (2.d). Then, **discuss** the difference in the performance of those four portfolios. (MV case1, MV case2, MV case3, Benchmark)




Case 1 shows the highest cummulative return, followed closely by case3, then Case 2, and lastly the S&P too. The inclusion of a risk-free asset (in case 1 and Case 3) improves performance compared to Case 2. Overall, optimized portfolios outperform the benchmark, and fewer constraints lead to better results.

4. Assume that there is no risk-free asset, and short-selling is not allowed. (case #4 in our lecture note)

a. **Derive** the mean-variance frontier. Then, **plot** the mean-variance frontier together with the mean-variance frontiers from MV case1, MV case2, MV case3, Benchmark in questions (1.a), (2.a), and (3.a). **Explain** the differences in the plotted frontiers. **Do** the efficient frontiers expand with lesser constraints?




The Case 1 frontier (blue CML) is the highest and steepest, showing the best risk-return trade-off. Case 3 (green CML) is lower due to the no short-selling constraint. Case 2 (gray curve) lies below both. Case 4 (orange) is the lowest, as it includes neither a risk-free asset nor short-selling.

∴ As constraints increase, the efficient frontier shrinks. The efficient frontier expand with fewer constraints.

b. **Derive** the portfolio weight that minimizes portfolio risk, matching S&P 500 expected return. Report the portfolio weight in an excel file. **Report** the portfolio weights on 3M CO (22592), Bank of America Corp (59408), Advanced Micro Devices Inc (61241), Apple Inc (14593), and McDonalds Corp (43449).

| permno | comnam | weight |
|---|---|---|
| 22592 | 3M CO | 0.043442185 |
| 59408 | BANK OF AMERICA CORP | 0 |
| 61241 | ADVANCED MICRO DEVICES INC | 1.06646E-17 |
| 14593 | APPLE INC | 0 |
| 43449 | MCDONALDS CORP | 0 |

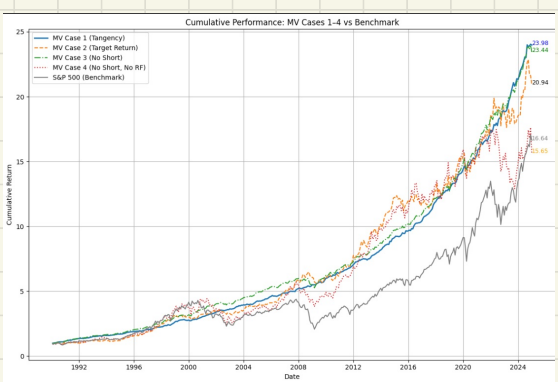
c. **Compute** the annualized excess returns, annualized volatility, and annualized Sharpe ratio of the optimal and the S&P 500 portfolios. Then, **compare** the results to MV case1, MV case2, MV case3, and Benchmark. **How** different are the results?

| | MV Case 1 | MV Case 2 | MV Case 3 | MV Case 4 | S&P 500 |
|---|---|---|---|---|---|
| **Excess Return (%)** | 6.529617 | 6.529617 | 6.529617 | 6.529617 | 6.529617 |
| **Volatility (%)** | 3.370308 | 9.462050 | 4.944451 | 16.104665 | 14.814414 |
| **Sharpe Ratio** | 1.937394 | 0.690085 | 1.320595 | 0.405449 | 0.440761 |

All portfolios have the same excess return (6.53%), but their risk and Sharpe ratios vary. Case 1 has the lowest volatility and the highest Sharpe ratio (1.94) due to the inclusion of a risk-free asset and short-selling. Case 3 performs better than Case 2 because it includes a risk-free asset. Case 4 has the highest volatility and the lowest Sharpe ratio (0.41) due to both constraints. S&P 500 performs slightly better than Case 4 but worse than all optimized portfolios with fewer constraints.

⇒ More constraints lead to lower risk-adjusted performance.

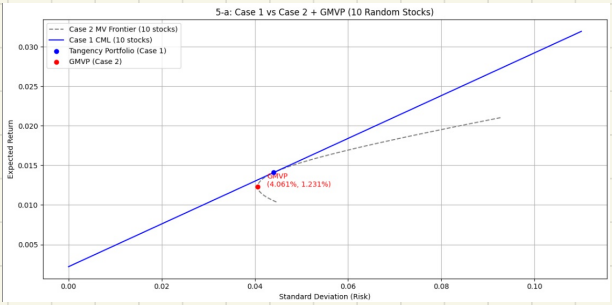
d. **Plot** the accumulated performance of the optimal portfolio with the figure in question (3.d). Then, **discuss** the difference in the performance of those five portfolios. (MV case1, MV case2, MV case3, MV case4, Benchmark)




Case 1 shows the highest cumulative return (23.98), followed by Case 3 (23.44), Case 2 (20.94), and Case 4 (15.65). The S&P 500 benchmark is the lowest (16.64), except slightly above Case 4. Portfolios with fewer constraints (Case 1, Case 3) perform better, confirming that optimization with a risk-free asset and flexible allocation enhances long-term performance.

5. Let's check how portfolio optimization performance depends on the number of stocks in a portfolio and the correlation of returns among stocks.

a. Randomly **choose** 10 stocks. **Plot** the mean-variance frontiers for Case 1 (with the MV frontier in the question (2.a)) and Case 2 (with the MV frontier in the question (1.a)). **Compare** annualized Sharpe ratio to the answers in (2.c) and (1.c).

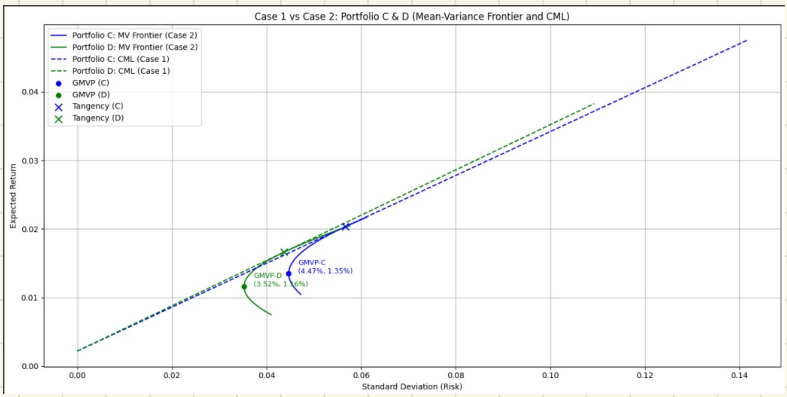



| | Case 1 (Optimal) | Case 2 (Optimal) | S&P 500 | Random 10 Stocks (Case 1) | Random 10 Stocks (Case 2) |
|---|---|---|---|---|---|
| **Excess Return (%)** | 6.529617 | 6.529617 | 6.529617 | 6.529617 | 6.529617 |
| **Volatility (%)** | 3.370308 | 9.462050 | 14.814414 | 6.979780 | 20.906622 |
| **Sharpe Ratio** | 1.937394 | 0.690085 | 0.440761 | 0.935505 | 0.312323 |

performance: Case 1 > Case 1 with random 10 stocks > Case 2 > Case 2 with random 10 stocks

More stocks and access to the risk-free asset provide better diversification and higher risk-adjusted returns.

b. Let's construct a portfolio C with only stocks in the Money sector (19 stocks with flag_sector=1 in the provided data). We also construct a portfolio D with 17 stocks in diverse industries (17 stocks with flag_sector=0 in the provided data). **First, plot** the mean-variance frontiers of C and D portfolios in the case of MV case 2. **Next, plot** the mean-variance frontiers of C and D portfolios in the case of MV case 1. **Whose** Sharpe ratio is higher? **Why**?




| | Portfolio C (Case 1) | Portfolio C (Case 2) | Portfolio D (Case 1) | Portfolio D (Case 2) | S&P 500 |
|---|---|---|---|---|---|
| **Excess Return (%)** | 6.529617 | 6.529617 | 6.529617 | 6.529617 | 6.529617 |
| **Volatility (%)** | 5.890327 | 18.597500 | 5.710355 | 14.073323 | 14.814414 |
| **Sharpe Ratio** | 1.108532 | 0.351102 | 1.143469 | 0.463971 | 0.440761 |

In both case 1 and Case 2, Portfolio D has a higher Sharpe ratio than portfolio C. Because it achieves better diversification and lower volatility, resulting in a higher risk-adjusted return.