Assignment 7

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5.1

2) $\lambda = 2$ is an eigenvalue of $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ if $A - 2I_2 = 0$

$A - 2I_2 = \begin{bmatrix} 5 & 3 \\ 3 & -3 \end{bmatrix}$

$det(A) = 5(-3) - (3 \cdot 3) \neq 0$, therefore λ is not an eigenvalue of A

4) $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of A if $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1+\sqrt{2})+1 \\ -1+\sqrt{2}+4 \end{bmatrix} \approx 4.4142 \cdot \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$

Therefore, $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of A with an eigenvalue of 4.4142

8) $det \begin{bmatrix} -2 & 2 & 2 \\ 3 & -5 & 1 \\ 0 & 1 & -2 \end{bmatrix} = 0$ therefore $\lambda = 3$ is an eigenvalue of A.

$\begin{bmatrix} -2 & 2 & 2 & 0 \\ 3 & -5 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Any vector of the form $x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ where $x_3 \neq 0$ is an eigenvector of A.

10) $A - \lambda I_2 = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$

$\begin{bmatrix} 6 & -9 & 0 \\ 4 & -6 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The basis for the eigenspace of A is $\left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \right\}$

12) $A - I_2 = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}$

$A - 5I_2 = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$

$\lambda = 1$, basis $= \left\{ \begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix} \right\}$, $\lambda = 5$, basis $= \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$

16) basis $= \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$

18) A is in lower triangular form, therefore for λ to be an eigenvalue of A, it must satisfy $(4 - \lambda)(0 - \lambda)(-3 - \lambda) = 0$

Therefore, $\lambda = 4, \lambda = -3$, and $\lambda = 0$ are eigenvalues of A.

22) (a) False, this is the definition of an eigenvector however the vector cannot be the zero vector

(b) False, a matrix can have the same value across its diagonal multiple times, creating multiple linearly independent vectors with identical eigenvalues.

(c) True, a steady state vector $\vec{x}$ is a vector such that $A\vec{x} = \vec{x}$. Therefore, $\vec{x}$ is an eigenvector with corresponding eigenvalue 1.

(d) False, this is true only for triangular matrices. Because row operations alter eigenvalues, not every matrix has its eigenvalues across its diagonal.

(e) True, the "certain matrix" is $A - \lambda I$

5.2

2) $(5 - \lambda)(5 - \lambda) - (3)(3) = \lambda^2 - 10\lambda + 16$

The eigenvalues of A are $\lambda = 8, 2$

6) $(3 - \lambda)(8 - \lambda) - (4)(-4) = \lambda^2 - 11\lambda + 40$

A has no real eigenvalues

8) $(7 - \lambda)(3 - \lambda) - (2)(-2) = \lambda^2 - 10\lambda + 25$

The eigenvalue of A is $\lambda = 5$

10) $det \begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} = 1(6 - \lambda^2) - 2(\lambda^2 - 9) = -3\lambda^2 + 24$

$-3(\lambda^2 - 8) = 0$

12) $det(A) = 2((-1 - \lambda)(4 - \lambda)) = 2(\lambda^2 - 3\lambda - 4)$

16) $det \begin{bmatrix} 5-\lambda & 0 & 0 & 0 \\ 8 & -4-\lambda & 0 & 0 \\ 0 & 7 & 1-\lambda & 0 \\ 1 & -5 & 2 & 1-\lambda \end{bmatrix} = (5-\lambda) \cdot \begin{bmatrix} -4-\lambda & 0 & 0 \\ 7 & 1-\lambda & 0 \\ -5 & 2 & 1-\lambda \end{bmatrix}$

$= (5-\lambda)(-4-\lambda)(1-\lambda)(1-\lambda)$

I now realize the original matrix was triangular and I did not have to expand. That's 5 minutes I'll never get back.

Eigenvalues: $\lambda = 5, -4, 1, 1$

22) (a) False, the volume is equal to $|det(A)|$

(b) False, $det(A^T) = det(A)$

(c) True, identical to the rule given in example 3 of this section

(d) False, row operations can (and typically do) change the eigenvalues of a matrix