Assignment 7

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5.1

2) $\lambda = 2$ is an eigenvalue of $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ if $A - 2I_2 = 0$

$A - 2I_2 = \begin{bmatrix} 5 & 3 \\ 3 & -3 \end{bmatrix}$

$det(A) = 5(-3) - (3 \cdot 3) \neq 0$, therefore λ is not an eigenvalue of A

4) $\begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of A if $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1 + \sqrt{2}) + 1 \\ -1 + \sqrt{2} + 4 \end{bmatrix} \approx 4.4142 \cdot \begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix}$

Therefore, $\begin{bmatrix} -1 + \sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of A with an eigenvalue of 4.4142

8) $det \begin{bmatrix} -2 & 2 & 2 \\ 3 & -5 & 1 \\ 0 & 1 & -2 \end{bmatrix} = 0$ therefore $\lambda = 3$ is an eigenvalue of A.

$\begin{bmatrix} -2 & 2 & 2 & 0 \\ 3 & -5 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Any vector of the form $x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ where $x_3 \neq 0$ is an eigenvector of A.

10) $A - \lambda I_2 = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$

$\begin{bmatrix} 6 & -9 & 0 \\ 4 & -6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The basis for the eigenspace of A is $\left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \right\}$

12) $A - I_2 = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}$

$A - 5I_2 = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$

$\lambda = 1$, basis $= \left\{ \begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix} \right\}$, $\lambda = 5$, basis $= \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$

16) basis $= \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$

18) A is in lower triangular form, therefore for λ to be an eigenvalue of A, it must satisfy $(4 - \lambda)(0 - \lambda)(-3 - \lambda) = 0$

Therefore, $\lambda = 4, \lambda = -3$, and $\lambda = 0$ are eigenvalues of A.

22) (a) False, this is the definition of an eigenvector however the vector cannot be the zero vector

(b) False, a matrix can have the same value across its diagonal multiple times, creating multiple linearly independent vectors with identical eigenvalues.

(c) True, a steady state vector $\vec{x}$ is a vector such that $A\vec{x} = x$. Therefore, $\vec{x}$ is an eigenvector with corresponding eigenvalue 1.

(d) False, this is true only for triangular matrices. Because row operations alter eigenvalues, not every matrix has its eigenvalues across its diagonal.

(e) True, the "certain matrix" is $A - \lambda I$

5.2

2)