Assignment 8

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5.3

4) $A^k = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} -3(2^k) & 4(3(2^k)) - 3(4^k) \\ -2^k & 4(2^k) - 3(1^k) \end{bmatrix}$

6) Eigenvalues: $\lambda = 5, 4$ Basis: $\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$

10) $P = \begin{bmatrix} 1 & -\frac{3}{4} \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$

12) $P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

14) $P = \begin{bmatrix} 0 & -2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

16) Not diagonalizable

22) (a) False, for a matrix to be diagonalizable it must have linearly independent eigenvectors

(b) False, eigenvalues do not need to be unique

(c) True, this formula can be derived from the $A = PDP^{-1}$ formula for diagonals by multiplying both sides by P

(d) False, invertability does not affect diagonalizability

22) A is not diagonalizable, as the dimensions of its eigenspaces need to sum to 3 but only sum to 2