# Assignment 7

Owen Ritzau

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## 5.1

2) $\lambda = 2$ is an eigenvalue of $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ if $A - 2I_2 = 0$

$A - 2I_2 = \begin{bmatrix} 5 & 3 \\ 3 & -3 \end{bmatrix}$

$det(A) = 5(-3) - (3 \cdot 3) \neq 0$, therefore $\lambda$ is not an eigenvalue of $A$

4) $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of $A$ if $\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1+\sqrt{2})+1 \\ -1+\sqrt{2}+4 \end{bmatrix} \approx 4.4142 \cdot \begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$

Therefore, $\begin{bmatrix} -1+\sqrt{2} \\ 1 \end{bmatrix}$ is an eigenvector of A with an eigenvalue of 4.4142

8) $det \begin{bmatrix} -2 & 2 & 2 \\ 3 & -5 & 1 \\ 0 & 1 & -2 \end{bmatrix} = 0$ therefore $\lambda = 3$ is an eigenvalue of $A$.

$\begin{bmatrix} -2 & 2 & 2 & 0 \\ 3 & -5 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Any vector of the form $x_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ where $x_3 \neq 0$ is an eigenvector of $A$.

10) $A - \lambda I_2 = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$

$\begin{bmatrix} 6 & -9 & 0 \\ 4 & -6 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The basis for the eigenspace of $A$ is $\left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix} \right\}$

12) $A - I_2 = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}$

$A - 5I_2 = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$

$\lambda = 1$, basis $= \left\{ \begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix} \right\}$, $\lambda = 5$, basis $= \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$