MA200 Assignment 6

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4.2

16) $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

18) Because A has 4 entries in each column, Col A is a subspace of $\mathbb{R}^4$. A has 4 entries in each row, meaning that a valid vector x such that Ax is defined must have 4 entries, therefore Nul A is a subspace of $\mathbb{R}^4$

20) A has 1 entry in each column, therefore Col A is a subspace of $\mathbb{R}$. A has 5 entries per row, meaning that Nul A is a subspace of $\mathbb{R}^5$

22) $\begin{bmatrix} -6 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ is in Nul A, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is in Col A

24) Col A can be defined as $x_3 \begin{bmatrix} 1 \\ .5 \\ -1 \end{bmatrix}$ $.w = -2 \cdot \begin{bmatrix} 1 \\ .5 \\ -1 \end{bmatrix}$, therefore w is in Col A.

$A \cdot w \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, therefore w is not in Nul A.

26) a. True, a null space has the zero vector and maintains the laws of scalar addition and multiplication, therefore it is a vector space

b. False, the null space is in $\mathbb{R}^n$ but not necessarily $\mathbb{R}^m$

c. False, Col A is $\{b : b = Ax\}$

d. True, the kernel refers specifically to the zero vector, and the set $\{T : T(x) = 0\} = \text{Nul } A$

e. True, the range will maintain addition and multiplication properties and will contain the zero vector

f. True

4.2

4) The set composed of the columns of $\begin{bmatrix} 2 & 1 & -7 \\ -2 & -3 & 5 \\ 1 & 2 & 4 \end{bmatrix}$ is a basis for $\mathbb{R}^3$ as it is both linearly independent and spans $\mathbb{R}^3$

6) The set composed of the columns of $\begin{bmatrix} 1 & -4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$ is not a basis for $\mathbb{R}^3$ as it does not span $\mathbb{R}^3$, however it is linearly independent

8) Because the matrix formed by the vectors of this set has more columns than rows, it is guaranteed that it will not be linearly independent and therefore is not a basis for $\mathbb{R}^3$. This set spans $\mathbb{R}^3$.

10) $\begin{bmatrix} 1 & 0 & -5 & 1 & 4 & 0 \\ -2 & 1 & 6 & -2 & -2 & 0 \\ 0 & 2 & -8 & 1 & 9 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -5 & 0 & 7 & 0 \\ 0 & 1 & -4 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \end{bmatrix}$

$$\begin{bmatrix} x_1 - 5x_3 + 7x_5 = 0 \\ x_2 - 4x_3 + 6x_5 = 0 \\ x_4 - 3x_5 = 0 \end{bmatrix} \to \text{Nul } A = \left\{ \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$$

14) $\text{Nul } A = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ \frac{7}{5} \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -5 \\ -\frac{8}{5} \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, $\text{Col } A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \\ -5 \end{bmatrix} \right\}$

16) The basis for the space spanned by vectors $v_1..v_5 = \mathbb{R}^3$

22) a. False, the set must span H to be a basis for H

b. True, if S spans V then it is guaranteed to have some linearly independent vectors such that it is a basis for V.

c. True, a basis cannot be any larger while maintaining linear independence

d. False, the method will always produce a set

e. False, the pivot columns are not always in the column space of A