MA200 Assignment 6

Owen Ritzau

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3.1

2) $\begin{bmatrix} 0 & 4 & 1 \\ 5 & -3 & 0 \\ 2 & 3 & 1 \end{bmatrix} \rightarrow 0 \cdot (-3 \cdot 1 - 3 \cdot 0) - 4 \cdot (5 \cdot 1 - 2 \cdot 0) + 1 \cdot (5 \cdot 3 - 2 \cdot -3)$

$= 0 - 20 + 21 = 1$

6) $\begin{bmatrix} 5 & -2 & 2 \\ 0 & 3 & -3 \\ 2 & -4 & 7 \end{bmatrix} \rightarrow 5 \cdot (3 \cdot 7 - (-4 \cdot -3)) + 2 \cdot (0 \cdot 7 - 2 \cdot -3) + 2 \cdot (0 \cdot -4 - 2 \cdot 3)$

$= 5 \cdot (21 - 12) + 2 \cdot (0 + 6) + 2 \cdot (0 - 6) = 45 + 12 - 12 = 45$

10) $A = \begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 3 & 0 \\ 1 & -2 & 5 & 2 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{bmatrix}$

$det(A) = 0 \cdot A_{11} + 0 \cdot A_{21} + 3 \cdot \begin{bmatrix} 1 & -2 & 2 \\ 2 & -4 & 5 \\ 2 & 0 & 5 \end{bmatrix} + 0 \cdot A_{41}$

$3 \cdot \begin{bmatrix} 1 & -2 & 2 \\ 2 & -4 & 5 \\ 2 & 0 & 5 \end{bmatrix} = 3 \cdot (2(-10 + 8) - 0(5 - 4) + 5(-4 + 4)) = 3 \cdot -4 = -12$

12) $A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 7 & -2 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 3 & -8 & 4 & -3 \end{bmatrix}$

$det(A) = 3 \cdot \begin{bmatrix} -2 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{bmatrix} + 0 \cdot A_{21} + 0 \cdot A_{31} + 0 \cdot A_{41}$

$= 3 \cdot (-2 \cdot (-9 - 0) - 0 + 0) = 54$

3.2

8) $det(A) = 1 \cdot \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & -10 \end{bmatrix}$

$= 1 \cdot \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & -1 & -1 & -10 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & -10 \end{bmatrix} = 1 \cdot 1 \cdot 1 \cdot -10 = -10$

12) $A = \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 11 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix} \to \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 3 & 0 & -2 & 0 \\ 4 & 2 & 4 & 3 \end{bmatrix}$

$det(A) = 3 \cdot \begin{bmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 3 & 0 & -2 \end{bmatrix} = 3 \cdot 2 = 6$

14) $A = \begin{bmatrix} 1 & 5 & 4 & 1 \\ 0 & -2 & -4 & 0 \\ 3 & 5 & 4 & 1 \\ -6 & 5 & 5 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 5 & 4 & 1 \\ 0 & -2 & -4 & 0 \\ 2 & 0 & 0 & 0 \\ -6 & 5 & 5 & 0 \end{bmatrix}$

$det(A) = 1 \cdot \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & 0 \\ -6 & 5 & 5 \end{bmatrix} = 1 \cdot 2 \cdot \begin{bmatrix} -2 & -4 \\ 5 & 5 \end{bmatrix} = 20$

24) $A = \begin{bmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 2 & 7 & -2 \end{bmatrix} \to \begin{bmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 6 & 0 & -5 \end{bmatrix}$

$det(A) = 7 \cdot 0 = 0$, therefore the set of vectors composed of the columns of A is not linearly independent.

26) $A = \begin{bmatrix} 3 & 2 & -2 & 0 \\ 5 & -6 & -1 & 0 \\ -6 & 0 & 3 & 0 \\ 4 & 7 & 0 & -2 \end{bmatrix}$

$det(A) = -2 \cdot \begin{bmatrix} 3 & 2 & -2 \\ 5 & -6 & -1 \\ -6 & 0 & 3 \end{bmatrix} = 2 \cdot -3 \neq 0$, therefore the set of vectors composed of the columns of A is linearly independent.

4.1

6) Let W be the set of all polynomials of form $a + t^2$. If $w_1 = 1 + t^2$ and $w_2 = 2 + t^2$, then $w_1 + w_2 = 3 + 2t^2$, which is not of the form $a + t^2$.

Therefore, W does not have closure under vector addition and is not a subspace

8) This is a subspace, $w_1 + w_2 = w_3$ such that $w_3(0) = 0$, $0 \cdot 0 = 0$, and $0 + 0 = 0$

12) $W = s \cdot \begin{bmatrix}1\\1\\2\end{bmatrix} + t \cdot \begin{bmatrix}3\\-1\\-1\\4\end{bmatrix}$, $W = \text{span}\left\{\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\-1\\4\end{bmatrix}\right\}$

14) If W is in the subspace spanned by $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$, then W can be expressed as a linear combination of those vectors.
$\begin{bmatrix}1&2&4&8\\0&1&2&4\\-1&3&6&7\end{bmatrix} \rightarrow \begin{bmatrix}1&0&0&0\\0&1&2&4\\0&0&0&1\end{bmatrix}$. The system is inconsistent, so W is in the span$\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$.

16) $-a + 1 = 0$. $a - 6b = 0$, therefore $a = 1$ and $b = \frac{1}{6}$. $2 \cdot \frac{1}{6} - 2 \neq 0$, therefore the zero vector doesn't exist in W and W is not a vector space.

4.2

2) $A \cdot w = \begin{bmatrix}5&21&19\\13&23&2\\8&14&1\end{bmatrix} \cdot \begin{bmatrix}5\\-3\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$, therefore w is in Nul A.

4) $\begin{bmatrix}1&-6&4&0&0\\0&0&2&0&0\end{bmatrix} \rightarrow \begin{bmatrix}1&-6&4&0&0\\0&0&1&0&0\end{bmatrix}$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}6x_2 - 4x_3\\x_2\\0\\x_4\end{bmatrix} = x_2 \cdot \begin{bmatrix}6\\1\\0\\0\end{bmatrix} + x_4 \cdot \begin{bmatrix}-4\\0\\0\\1\end{bmatrix}$$

The vectors that span Nul A are $\begin{bmatrix}6\\1\\0\\0\end{bmatrix}$ and $\begin{bmatrix}-4\\0\\0\\1\end{bmatrix}$

6) $x_1 = -5x_2 + 4x_3 + 3x_4 - x_5$

$x_2 = 2x_3 - x_4$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = x_3 \cdot \begin{bmatrix}4\\2\\1\\0\\0\end{bmatrix} + x_4 \cdot \begin{bmatrix}3\\-1\\0\\1\\0\end{bmatrix} + x_5 \cdot \begin{bmatrix}-1\\0\\0\\0\\1\end{bmatrix}$$

The vectors that span Nul A are $\begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

8) If W is a vector space, the zero vector $\begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ must exist inside W.

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ does not satisfy the constraint that $5r - 1 = s + 2t$, as $-1 \neq 0$.

Therefore, W is not a vector space.

12) If W is a vector space, the zero vector $\begin{bmatrix} b - 5d \\ 2b \\ 2d + 1 \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ must exist inside W. Both d and $2d + 1$ cannot be equal to zero simultaneously, therefore the zero vector cannot exist inside W and W is not a vector space.