Exercise 3

1 Models and Entailment in Propositional Logic

1. For each statement below, determine whether the statement is true or false by building the complete model table.

   (a) $A \wedge \neg B \models A \vee B$

   (b) $A \vee B \models A \wedge \neg B$

   (c) $A \Leftrightarrow B \models A \Rightarrow B$

   (d) $(A \Leftrightarrow B) \Leftrightarrow C \models A \vee \neg B \vee \neg C$

   (e) $(\neg A \wedge B) \wedge (A \Rightarrow B)$ is satisfiable

   (f) $(\neg A \wedge B) \wedge (A \Leftrightarrow B)$ is satisfiable

$\alpha \models \beta$ iff in every model where α is true, β is also true.

- It is **not** raining ($\neg P$)
- It is raining **and** I am indoors ($P \wedge Q$)
- It is raining **or** I am indoors ($P \vee Q$)
- **If** it is raining, **then** I am indoors ($P \rightarrow Q$)
- **If** I am indoors, **then** it is raining ($Q \rightarrow P$)
- I am indoors **if and only if** it is raining ($P \leftrightarrow Q$)

A	B	C	¬ A	¬ B	¬C	A ^ ¬B	¬A ^ B	A v B	A → B	A ↔ B	(A↔B) ↔ C	A v ¬B v ¬C
1	1	1	0	0	0	0	0	1	1	1	1	1
1	1	0	0	0	1	0	0	1	1	1	0	1
1	0	1	0	1	0	1	0	1	0	0	0	1
1	0	0	0	1	1	1	0	1	0	0	0	1
0	1	1	1	0	0	0	1	1	0	0	0	0
0	1	0	1	0	1	0	1	1	0	0	0	1
0	0	1	1	1	0	0	0	0	0	1	1	1
0	0	0	1	1	1	0	0	0	0	1	0	1

a) True
b) False
c) False
d) True

Et uttrykk er "satisfiable" dersom det finnes en kombinasjon som gir uttrykket verdien True.

e) For at uttrykket (¬A ^ B) skal være sant, må A være 0 (A=0), mens B må være 1 (B=1). Det innebærer at uttrykket (A → B) ikke stemmer siden dette uttrykket krever A=1. (Videre refleksjoner: Kan tenke seg at A=0 og B=1 kan fungere)
Not satisfiable

f) For at uttrykket (¬A ^ B) skal være sant, må A være 0 (A=0), mens B må være 1 (B=1). Dette vil være umulig å forenes med uttrykket (A ←→ B) Da dette forutsetter at A=B.
Not satisfiable

2. Consider a logical knowledge base with 100 variables, $A_1, A_2, \ldots, A_{100}$. This will have $Q = 2^{100}$ possible models. For each logical sentence below, give the number of models that satisfy it.

Feel free to express your answer as a fraction of Q (without writing out the whole number 1267650600228229401496703205376 $= 2^{100}$) or to use other symbols to represent large numbers.

Example: The logical sentence A_1 will be satisfied by $\frac{1}{2}Q = \frac{1}{2}2^{100} = 2^{99}$ models.

(a) $A_{31} \wedge \neg A_{76}$

(b) $A_{44} \wedge A_{49} \wedge A_{78}$

(c) $A_{44} \vee A_{49} \vee A_{78}$

(d) $A_{70} \Rightarrow \neg A_{92}$

(e) $(A_7 \Leftrightarrow A_{72}) \wedge (A_{83} \Leftrightarrow A_{84})$

(f) $\neg A_9 \wedge \neg A_{19} \wedge A_{37} \wedge A_{50} \wedge A_{68} \wedge A_{73} \wedge A_{79} \wedge A_{81}$

(a) $1/2 * 1/2 * Q = Q/4 = \mathbf{2^{98}}$

(b) $1/2 * 1/2 * 1/2 = Q/8 = \mathbf{2^{97}}$

(c) $(1/2 + 1/2 * 1/2 + 1/2 * 1/2 * 1/2) * Q = 7/8 * Q = \mathbf{7 * 2^{97}}$

(d) $(1/2 + 1/2 * 1/2) * Q = 3/4\, Q = \mathbf{3 * 2^{98}}$

(e) $((1/2 * 1/2 + 1/2 * 1/2) * (1/2 * 1/2 + 1/2 * 1/2)) = (2/4 * 2/4)Q = Q/4 = \mathbf{2^{98}}$

(f) $1/2 * 1/2 * 1/2 * 1/2 * 1/2 * 1/2 * 1/2 * 1/2 * Q = Q/2^8 = 2^{100-8} = \mathbf{2^{92}}$

2 Resolution in Propositional Logic

1. Convert each of the following sentences to Conjunctive Normal Form (CNF).
   (a) $\neg A \vee (B \wedge C)$
   (b) $\neg(A \Rightarrow B) \wedge \neg(C \Rightarrow D)$
   (c) $\neg(A \Rightarrow B) \vee \neg(C \Rightarrow D)$
   (d) $(A \Rightarrow B) \Leftrightarrow C$
2. Consider the following Knowledge Base (KB):
   - $(X \wedge \neg Y) \Rightarrow \neg B$
   - $\neg X \Rightarrow C$
   - $B \wedge \neg Y$
   - $A \Rightarrow \neg C$

   Use resolution to show that $KB \models \neg A$
3. Do exercise 7.17 / 6.18 from the textboox ("Consider the following sentence…"), but with the following the sentence instead of the one in the textbook:

   $((Food \vee Drinks) \Rightarrow Party) \Rightarrow (\neg Party \Rightarrow \neg Food)$

CNF: Logical statements without ^, ∃, ∀, →

1.
   a. $\neg A$ v (B^C)
      CNF:
      ¬A v B
      ¬A v C
   b. $\neg(A \rightarrow B)$^ $\neg(C \rightarrow D)$
      CNF:
      $\neg(\neg A \vee B) \wedge \neg(\neg C \vee D)$
      $(A \vee \neg B) \wedge (C \vee \neg D)$
   c. $\neg(A \rightarrow B)$v $\neg(C \rightarrow D)$
      CNF:
      $\neg(\neg A \vee B)$ v $\neg(\neg C \vee D)$
      (A v $\neg(\neg C \vee D)$) v (¬B v $\neg(\neg C \vee D)$)
      (A v C) v (A v $\neg D$)v (¬B v C) v (¬B v ¬D)
   d. $(A \rightarrow B) \leftrightarrow$ C
      CNF:
      $(\neg A \vee B)$ v $\neg C \wedge C$ v $\neg(\neg A \vee B)$
      $(\neg A \vee B \vee \neg C) \wedge (A \vee \neg B \vee C)$
2.

3 Representations in First-Order Logic

1. Consider a first-order logical knowledge base that describes worlds containing movies, actors, directors and characters. The vocabulary contains the following symbols:

   - PlayedInMovie(a,m): predicate. Actor/person a played in the movie m
   - PlayedCharacter(a,c): predicate. Actor/person a played character c
   - CharacterInMovie(c,m): predicate. Character c is in the movie m.
   - Directed(p,m): person p directed movie m.
   - Constants related to the name of the movie, person or character with obvious meaning (to simplify you may use the surname or abbreviation).

   Express the following statements in First-Order Logic:

   (a) The character "Batman" was played by Christian Bale, George Clooney and Val Kilmer.

   (b) Heath Ledger and Christian Bale did not play the same characters.

   (c) Christian Bale played in all "Batman" movies directed by Christopher Nolan (*tip*: note that in this case Batman is a character of the movie, not the name of the movie).

   (d) "The Joker" and "Batman" are characters that appear together in some movies.

   (e) Kevin Costner directed and starred in the same movie.

   (f) George Clooney and Tarantino never played in the same movie and Tarantino never directed a film that George Clooney played.

   (g) Uma Thurman played a character in *some* movies directed by Tarantino.

Symbols: ∃:There exist an x such that…*or* for some x…

(a) PlayedCharacter(Bale,Batman) ^ PlayedCharacter(Clooney,Batman) ^ PlayedCharacter(Kilmer, Batman)
(b) $\neg\exists c$ PlayedCharacter(Ledger,c)=PlayedCharacter(Bale,c)
(c) $\forall m\ Directed(Nolan, m) \rightarrow PlayedCharacter(Bale, Batman)$
(d) $\neg\forall m\ CharacterInMovie(Batman, m)$^CharacterInMovie(Joker, m)
(e) $\exists m\ Directed(Costner, m)$ ^ PlayedInMovie(Costner, m)
(f) $\neg\exists m\ PlayedInMovie(Clooney, m)$ ^ PlayedInMovie(Tarantino, m) v $Directed(Tarantino, m)$ ^ PlayedInMovie(Clooney, m)
(g) $\exists m\ Directed(Tarantino, m)$ ^ PlayedCharacter(Thurman, m)

2. Arithmetic assertions can be written using FOL. Use the predicates ($<, \leq, \neq, =$), usual arithmetic operations as function symbol (+,-,x,/), biconditionals to create new predicates, and integer number constants to express the following statements in FOL:

   (a) An integer number x is divisible by y if there is some integer z less than x such that $x = z \times y$ (in other words, define the predicate $Divisible(x, y)$).

   (b) A number is even if and only if it is divisible by 2 (define the predicate $Even(x)$).

   (c) The result of summing an even number with 1 is an odd number (define the predicate $Odd(x)$).

   (d) A prime number is divisible only by itself.

(a) Devisible(x,y)= $\frac{x}{y}\ |\exists z\ z < x$ ^ $x = z \times y$
(b) Even(x) ↔ xmod2=0
(c) Odd(x) ↔ xmod2=1
(d) Prime(x) ↔ $\neg\exists y\ Divisible(x, y)\ |y \neq x$