Homogeneous Equations with Constant Coefficients

**1**  **English Explanation**

**1.1**  **Definition**

Homogeneous linear differential equations with constant coefficients are a fundamental class of differential equations where the dependent variable and its derivatives appear linearly, and all coefficients are constants, with no non-zero term independent of the dependent variable. These equations take the form:

|  |  |  |  |
| --- | --- | --- | --- |
| *an* | *dny*  *dxn* + *an−*1 | *dn−*1*y*  *dxn−*1 + *· · ·* + *a*1 | *dy*  *dx*+ *a*0*y* = 0 |

where *an, an−*1*, . . . , a*0 are constants, and *an ̸*= 0. The term “homogeneous” indicates that the right-hand side is zero, meaning there is no external forcing function.

**1.2**  **Solution Method**

To solve such equations, we assume a solution of the form *y* = *erx*, where *r* is a constant to be determined. Substituting into the differential equation yields the characteristic equation:   
 *anrn*+ *an−*1*rn−*1+ *· · ·* + *a*1*r* + *a*0 = 0

This is a polynomial equation in *r*. The roots of this characteristic equation determine the form of the general solution:

• **Distinct Real Roots**: If the characteristic equation has *n* distinct real roots *r*1*, r*2*, . . . , rn*, the general solution is:

*y*(*x*) = *c*1*er*1*x*+ *c*2*er*2*x*+ *· · ·* + *cnernx*

where *c*1*, c*2*, . . . , cn* are arbitrary constants determined by initial conditions.

• **Repeated Real Roots**: If a root *ri* has multiplicity *m*, the corresponding solution

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terms are:   
 *erix, xerix, x*2*erix, . . . , xm−*1*erix*

• **Complex Roots**: If the roots include a complex conjugate pair *r* = *α ± iβ*, the solution includes terms of the form:

*eαx*cos(*βx*)*, eαx*sin(*βx*)

**1.3**  **Example**   
Consider the second-order homogeneous equation:

*dx*2 *−*3*dy dx*+ 2*y* = 0 1. **Form the characteristic equation**:

*r*2*−*3*r* + 2 = 0

Factorize:   
 (*r −*1)(*r −*2) = 0 Roots are *r* = 1 and *r* = 2, both real and distinct.

2. **General solution**:   
 *y*(*x*) = *c*1*ex*+ *c*2*e*2*x*

3. **Apply initial conditions** (e.g., *y*(0) = 1, *y′*(0) = 0):• Compute the derivative:

*y′*(*x*) = *c*1*ex*+ 2*c*2*e*2*x*

• At *x* = 0:   
 *y*(0) = *c*1 + *c*2 = 1 *y′*(0) = *c*1 + 2*c*2 = 0

• Solve the system:   
 *c*1 + *c*2 = 1 *c*1 + 2*c*2 = 0

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Subtract the first from the second:

(*c*1 + 2*c*2) *−*(*c*1 + *c*2) = 0 *−*1 =*⇒c*2 = *−*1

Then:   
 *c*1 + (*−*1) = 1 =*⇒c*1 = 2

• Solution:   
*y*(*x*) = 2*ex−e*2*x*

**1.4**  **Interpretation**

The solution *y*(*x*) = 2*ex−e*2*x*describes a function that combines two exponential growth terms. The term *e*2*x*grows faster than *ex*, so as *x* increases, the negative coefficient *−e*2*x* dominates, potentially driving the solution to negative values. The initial conditions ensure the solution satisfies specific starting values, making it unique. This type of equation models systems like mechanical vibrations or electrical circuits without external forces.

**2**  **Azerbaycan Tercumesi**

**2.1**  **Tanim**

Homojen xetti diferensial tenlikler sabit emsallarla, asili deyisen ve onun toremelerinin xetti sekilde gorunduyu ve butun emsallarin sabit oldugu, asili deyisenden asili olmayan qeyri-sifir uzvun olmadigi tenlikler sinfidir. Bu tenlikler asagidaki formada olur:

|  |  |  |  |
| --- | --- | --- | --- |
| *an* | *dny*  *dxn* + *an−*1 | *dn−*1*y*  *dxn−*1 + *· · ·* + *a*1 | *dy*  *dx*+ *a*0*y* = 0 |

burada *an, an−*1*, . . . , a*0 sabitlerdir ve *an ̸*= 0. “Homojen” termini sag terefinsifir oldugunu gosterir, yani xarici quvve funksiyasi yoxdur.

**2.2**  **Hell Metodu**

Bele tenlikleri hell etmek ucun *y* = *erx*formasnda bir hell oldugunu ferz edirik, burada *r* mueyyen edilmeli sabitdir. Tenliye qoyduqda xarakteristik tenlik elde edilir:

*anrn*+ *an−*1*rn−*1+ *· · ·* + *a*1*r* + *a*0 = 0

Bu, *r*-den asili polinom tenliyidir. Xarakteristik tenliyin kokleri umumi hellin formasini mueyyen edir:

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• **Ferqli Heqiqi Kokler**: Eger xarakteristik tenlikde *n* ferqli heqiqi kok *r*1*, r*2*, . . . , rn*  varsa, umumi hell:   
 *y*(*x*) = *c*1*er*1*x*+ *c*2*er*2*x*+ *· · ·* + *cnernx*   
 burada *c*1*, c*2*, . . . , cn* ilkin sertlerle mueyyen edilen ixtiyari sabitlerdir.

• **Tekrarlanan Heqiqi Kokler**: Eger *ri* koku *m* coxlugu ile tekrarlanirsa, muvafiq hell uzvleri:   
 *erix, xerix, x*2*erix, . . . , xm−*1*erix*

• **Kompleks Kokler**: Eger kokler arasinda *α ± iβ* kompleks konjugat cutu varsa, hell asagidaki formadadir:

*eαx*cos(*βx*)*, eαx*sin(*βx*)

**2.3**  **Numune**   
Ikinci dereceli homojen tenliyi nezarden kecirek:

*dx*2 *−*3*dy dx*+ 2*y* = 0 1. **Xarakteristik tenliyi qururuq**:

*r*2*−*3*r* + 2 = 0

Faktorizasiya:   
 (*r −*1)(*r −*2) = 0 Kokler *r* = 1 ve *r* = 2, her ikisi heqiqi ve ferqlidir.

2. **Umumi hell**:   
 *y*(*x*) = *c*1*ex*+ *c*2*e*2*x*

3. **Ilkin sertleri tetbiq edirik** (meselen, *y*(0) = 1, *y′*(0) = 0):• Toremni hesablayiriq:   
 *y′*(*x*) = *c*1*ex*+ 2*c*2*e*2*x*

• *x* = 0-da:   
 *y*(0) = *c*1 + *c*2 = 1 *y′*(0) = *c*1 + 2*c*2 = 0

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• Sistemi hell edirik:   
 *c*1 + *c*2 = 1

*c*1 + 2*c*2 = 0

Ikinci tenlikden birincini cixiriq:

(*c*1 + 2*c*2) *−*(*c*1 + *c*2) = 0 *−*1 =*⇒c*2 = *−*1

Sonra:   
 *c*1 + (*−*1) = 1 =*⇒c*1 = 2

• Hell:   
*y*(*x*) = 2*ex−e*2*x*

**2.4**  **Serh**

Hell *y*(*x*) = 2*ex−e*2*x*iki eksponensial artim terminini birlesdiren funksiyani tesvir edir. *e*2*x* termini *ex*-den daha suretli artir, buna gore *x* artdiqca menfi emsal *−e*2*x*ustunluk teskil edir ve helli menfi qiymetlere apara biler. Ilkin sertler hellin xususi baslangic qiymetlere uygun olmasini temin edir ve onu unikal edir. Bu tip tenlikler mexaniki vibrasiyalar ve ya elektrik sxemleri kimi xarici quvveler olmadan sistemleri modellestirir.

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