DEVELOPMENT OF HIGH-ORDER H(CURL,Ω)-CONFORMING APPROXIMATION SPACES FOR PHOTONIC WAVEGUIDE ANALYSIS

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ABSTRACT

- *Photonics is hard.*
- *Real hard.*
- *People say FEM is crap and pseudospectral methods are the only option.*
- *This is actually not true and I will show you why.*

1. FEM FORMULATION AND THE H(CURL,Ω)-CONFORMING ELEMENTS

The formulation used in this work perform modal analysis on the cross-section of a waveguide, Ω, for a given frequency and it uses $H(curl;\Omega)$ and $H^1(\Omega)$-conforming elements for the transverse and longitudinal components of the electric field, respectively. It is valid for a domain composed of materials presenting at most transverse-anisotropy. From Jin [1]:
Find non-trivial $(\beta^2, e_t, e_z) \in (\mathbb{C} \times [\mathbb{C}]^N \times [\mathbb{C}]^M)$ such that:

$$\begin{aligned}\int_\Omega \Bigg\{ &\sum_j^N e_{tj} \left[\mu_{zz}^{-1} (\nabla_t \times \boldsymbol{\varphi}_j) \cdot (\nabla_t \times \boldsymbol{\varphi}_i)^* - k_0^2 \boldsymbol{\epsilon}_{xy} \boldsymbol{\varphi}_j \cdot \boldsymbol{\varphi}_i^*\right] \\ &+ \beta^2 \sum_l^M e_{zk} \left[\sum_j^N e_{tj} \boldsymbol{\mu}_{xy^S}^{-1} (\nabla_t \varphi_k + \boldsymbol{\varphi}_j) \cdot (\nabla_t \varphi_k + \boldsymbol{\varphi}_i)^* - k_0^2 \epsilon_{zz} \varphi_k \varphi_k^*\right] \Bigg\} \mathrm{d}\Omega = 0, \\ &\forall \boldsymbol{\varphi}_i \in B_{U_h},\ \varphi_k \in B_{V_h},\end{aligned} \tag{1}$$

where B_{U_h} and B_{V_h} denote the FEM basis for the finite-dimensional subspaces of $H(curl;\Omega)$ and $H^1(\Omega)$, respectively.
The $H(curl;\Omega)$-conforming elements are the Nédélec elements of the first kind Nédélec [2] and were constructed in the NeoPZ framework in a hierarchical way.

2. EFFECTS OF GEOMETRICAL REPRESENTATION

When you use high order elements you must be careful with the geometry. Otherwise your results will be crap. Check this out.

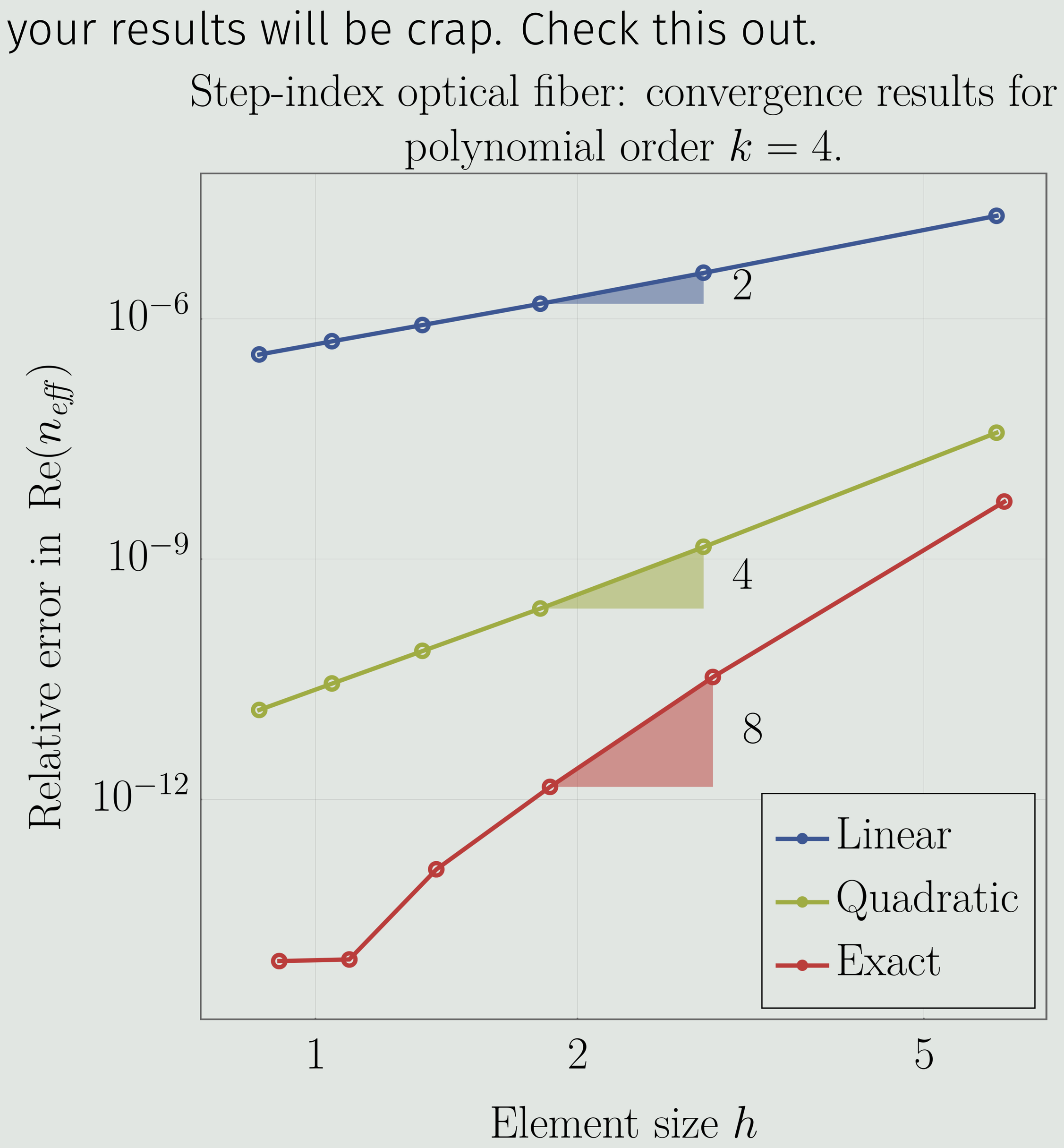

Figure 1: Comparison of convergence rates for the real part of the effective index of a step-index optical fiber. The polynomial order $k = 4$ was used in all three approximations. The geometry was described with elements obtained by linear mapping(blue curve), quadratic mapping(green curve) and exact mapping(red curve).

Etc

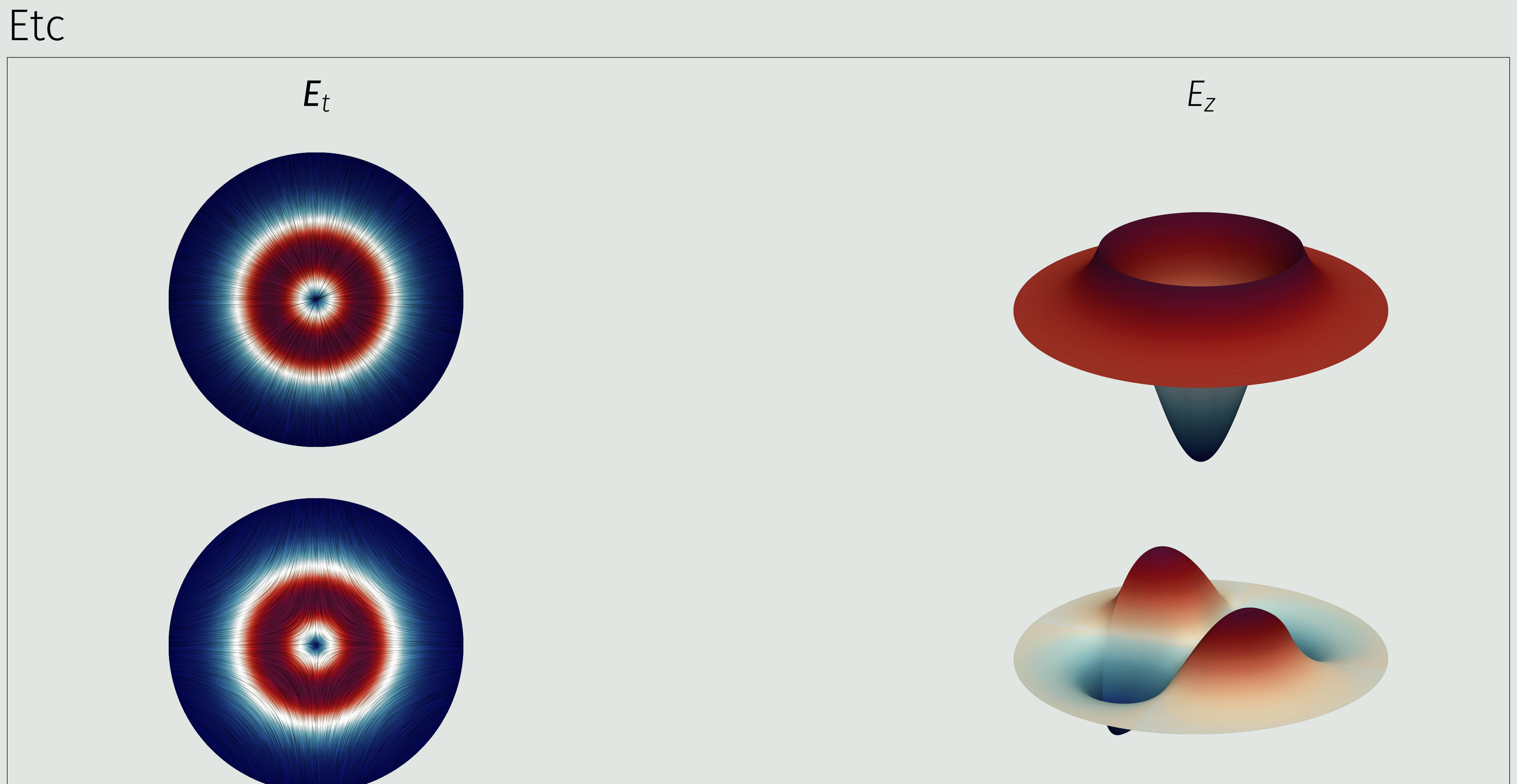

Figure 2: These are really cool plots.

3. HP-ADAPTIVITY CAPABILITIES

The developed basis functions, due to the hierarchical construction, can be integrated in the *hp*-algorithms of the `NeoPZ` framework[3].
Figure 3 shows a *hp*-adaptive mesh, in which the refinements were performed upon observation of the solution obtained with a fine mesh.

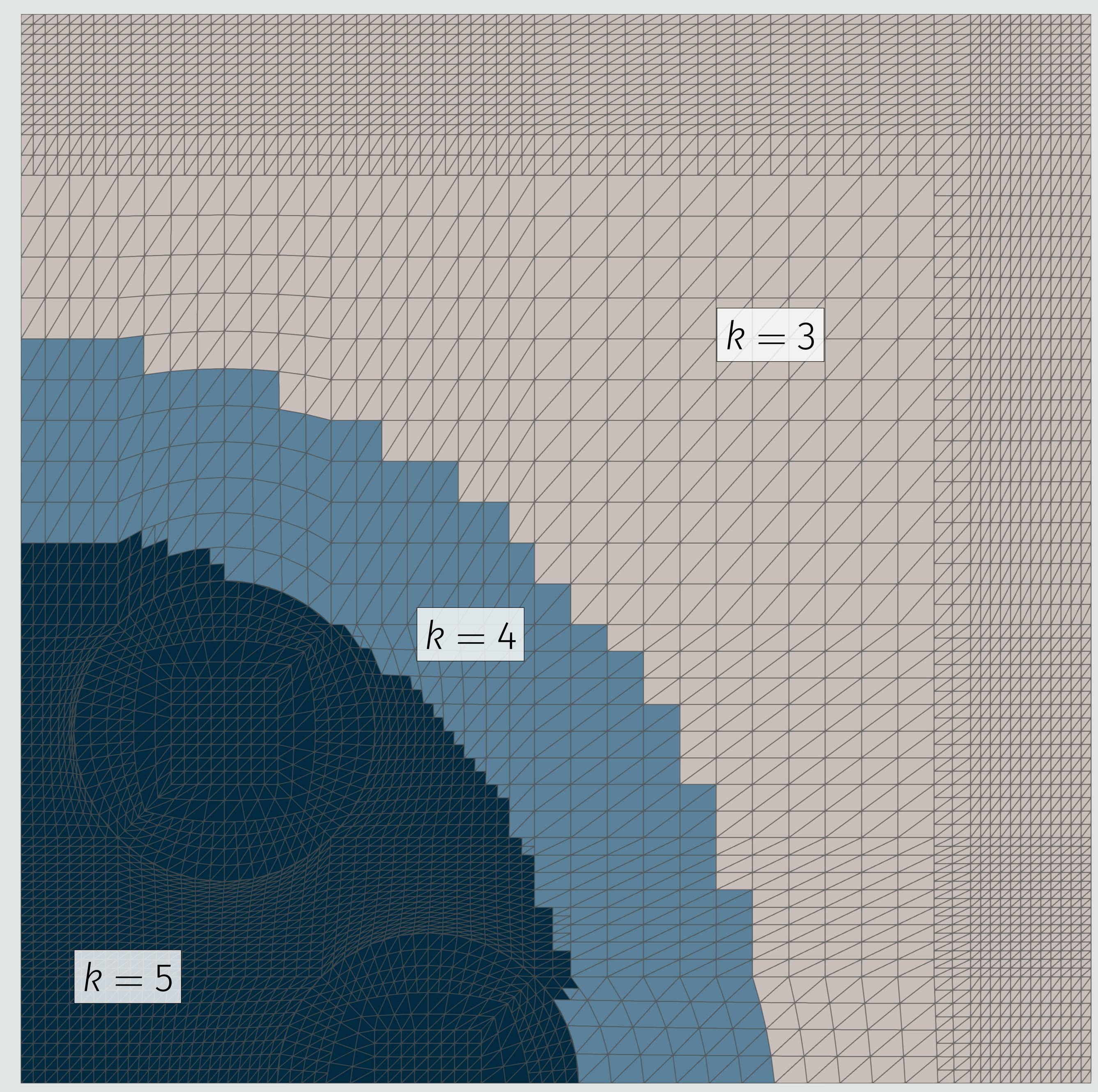

Figure 3: The Finite Element mesh of the microstructured fiber with six air-holes. The waveguide has a hole diameter of $d = 5\,\mu m$ and a hole pitch of $\Lambda = 6.75\,\mu m$. The computational domain is a square with side $l = W + d_{pml}$, with $W = 15.75\,\mu m$ and $d_{pml} = 2\,\mu m$. The elements have varying polynomial order from $k = 3$ to $k = 5$, and the mesh was refined on the region near the holes.

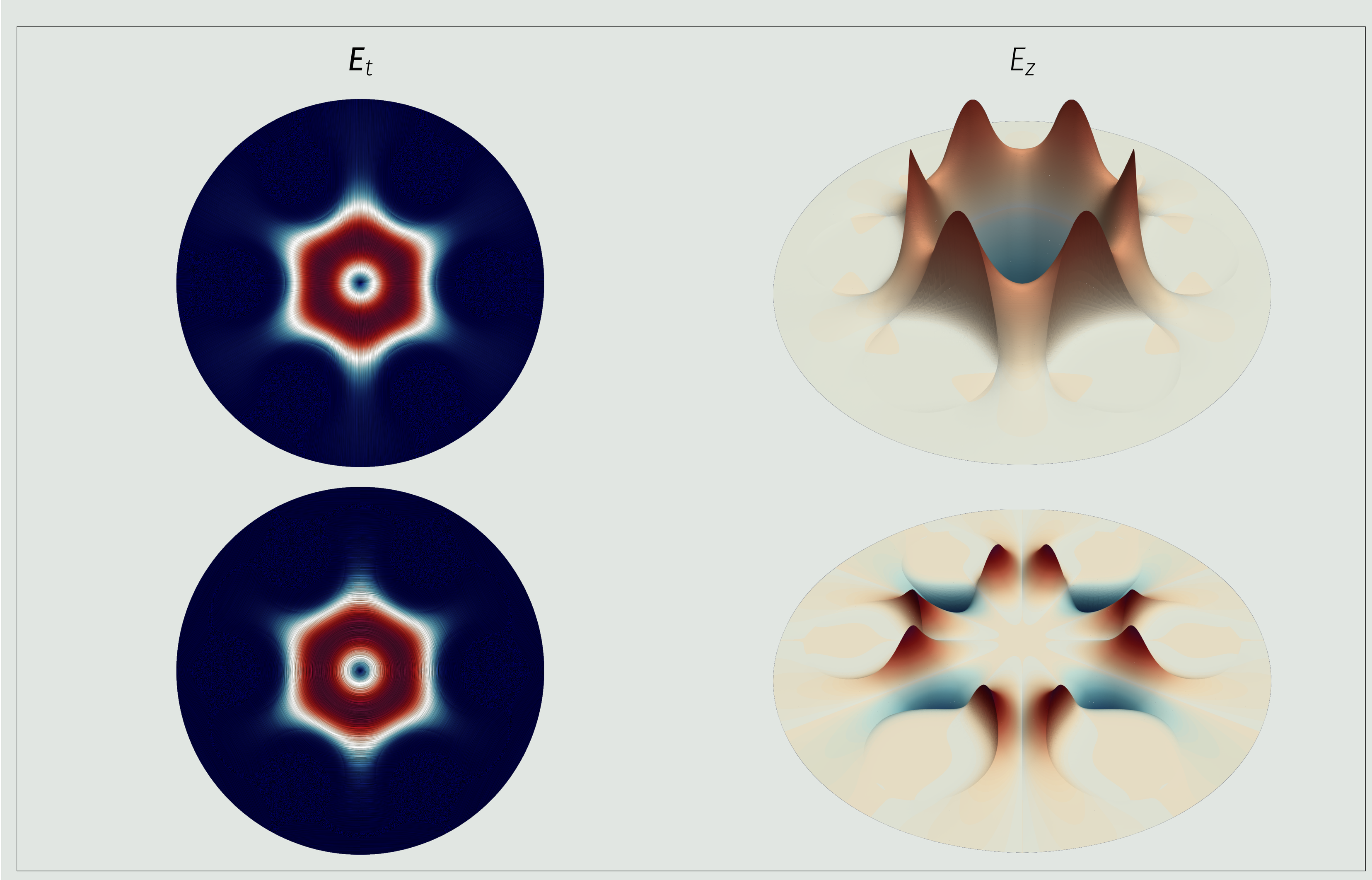

Figure 4: These are also really cool plots.

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