DEVELOPMENT OF HIGH-ORDER $H(curl;\Omega)$-CONFORMING APPROXIMATION SPACES FOR PHOTONIC WAVEGUIDE ANALYSIS

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ABSTRACT

The advances in the fabrication of photonic waveguides in the past twenty years have led the scientific community to seek for numerical methods that could assist in the process of design of such devices. The design photonic waveguides often require relative errors of 10^{-14} on the dispersion parameters. In this context, a hierarchical strategy for constructing $H(curl;\Omega)$-conforming elements is introduced, for application in a Finite Element Method (FEM) scheme for modal analysis of electromagnetic waveguides. The hierarchical $H(curl;\Omega)$-conforming elements are used for the transversal component of the electric field, coupled with scalar $H^1(\Omega)$ elements for its longitudinal component. The Nédélec elements of the first kind were chosen for this work, and the ease of integration with p-adaptivity schemes motivated the hierarchical construction of the FE basis. The scheme is assessed by means of the analysis of well-known waveguides. As a real-world scenario, the modal analysis of a Photonic Crystal Fiber illustrates the accuracy and the generalized eigenvalue problem size when dealing with a design process requiring high precision on the dispersion parameters.

1. FEM FORMULATION

Find non-trivial $(\beta^2, e_t, e_z) \in (\mathbb{C} \times [\mathbb{C}]^N \times [\mathbb{C}]^M)$ such that:

$$\begin{bmatrix} A_{tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} e_t \\ e_z \end{Bmatrix} = -\beta^2 \begin{bmatrix} B_{tt} & B_{tz} \\ B_{zt} & B_{zz} \end{bmatrix} \begin{Bmatrix} e_t \\ e_z \end{Bmatrix}, \tag{1}$$

and the matrices are defined as:

$$[A_{tt}]_{ij} = \int_\Omega \left[\mu_{zz}^{-1} (\nabla_t \times \varphi_i) \cdot (\nabla_t \times \varphi_j) - k_0^2 \epsilon_{xy} \varphi_i \cdot \varphi_j\right] d\Omega, \tag{2}$$

$$[B_{tt}]_{ij} = \int_\Omega \mu_{xy^s}^{-1} \varphi_i \cdot \varphi_j \, d\Omega, \tag{3}$$

$$[B_{tz}]_{ij} = \int_\Omega \mu_{xy^s}^{-1} \varphi_i \cdot \nabla_t \varphi_j \, d\Omega, \tag{4}$$

$$[B_{zt}]_{ij} = \int_\Omega \mu_{xy^s}^{-1} \nabla_t \varphi_i \cdot \varphi_j \, d\Omega, \tag{5}$$

$$[B_{zz}]_{ij} = \int_\Omega \left[\mu_r^{-1} \nabla_t \varphi_i \cdot \nabla_t \varphi_j - k_0^2 \epsilon_{zz} \varphi_i \varphi_j\right] d\Omega. \tag{6}$$

2. EFFECTS OF GEOMETRICAL REPRESENTATION

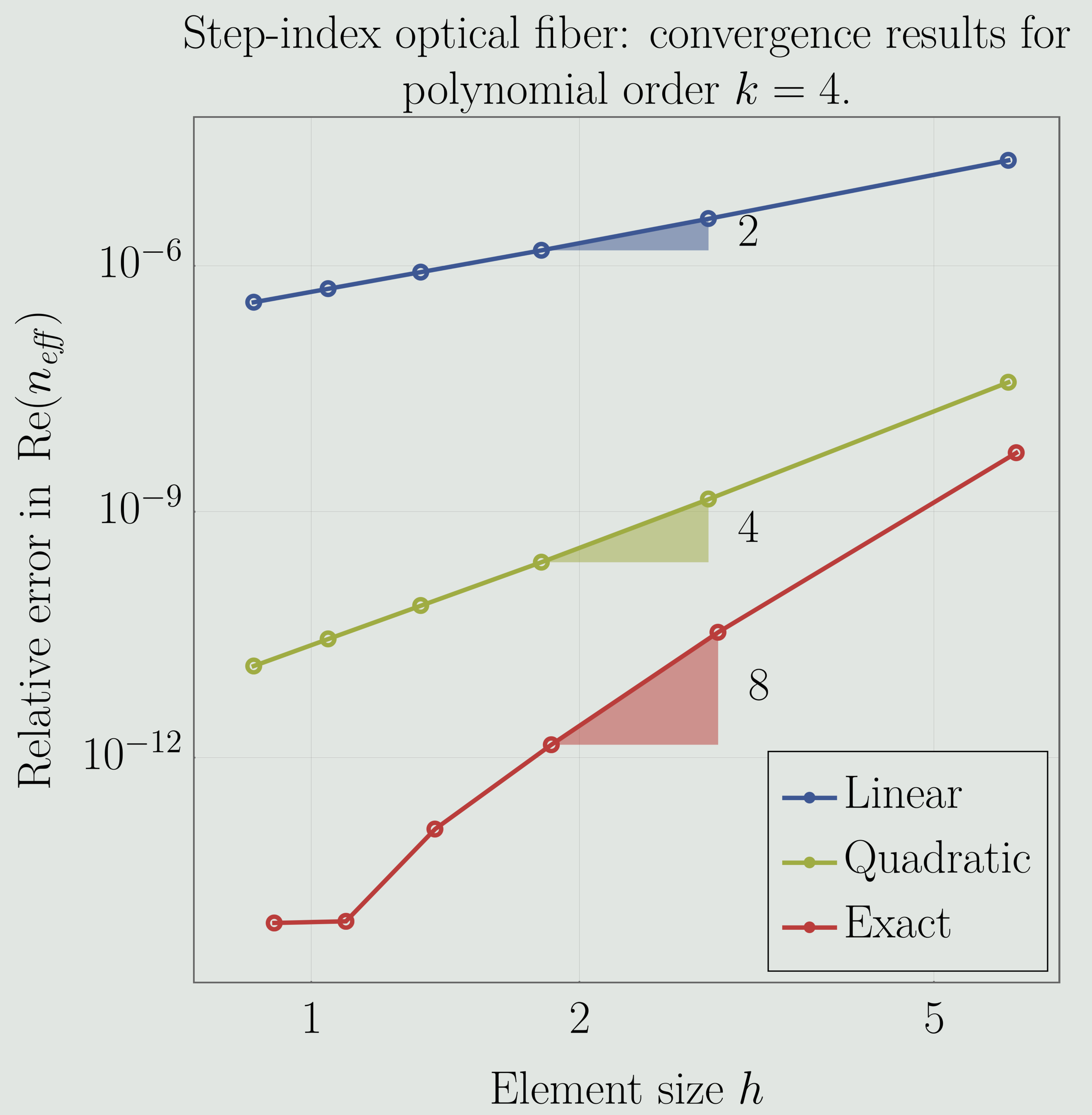

Figure 1: BlaBlabla

On a last note regarding implementation choices, there is the matter of *scaling*. The electromagnetic quantities present in a computation of the generalized eigenvalue problem obtained in Equation (1) vary widely in magnitude. The k_0 factor, in the context of photonics, can easily reach orders of magnitude around 10^6, while the cross-sectional area of waveguides such as optical fibers can be as small as $10^{-14}m^2$. In order to reduce the floating-point errors, Equation (1) was scaled by the factor k_0, so the resulting eigenvalue of Equation (1) is now $-n_{eff}^2$, given that the *effective refractive index* is calculated by $n_{eff} = \beta/k_0$.

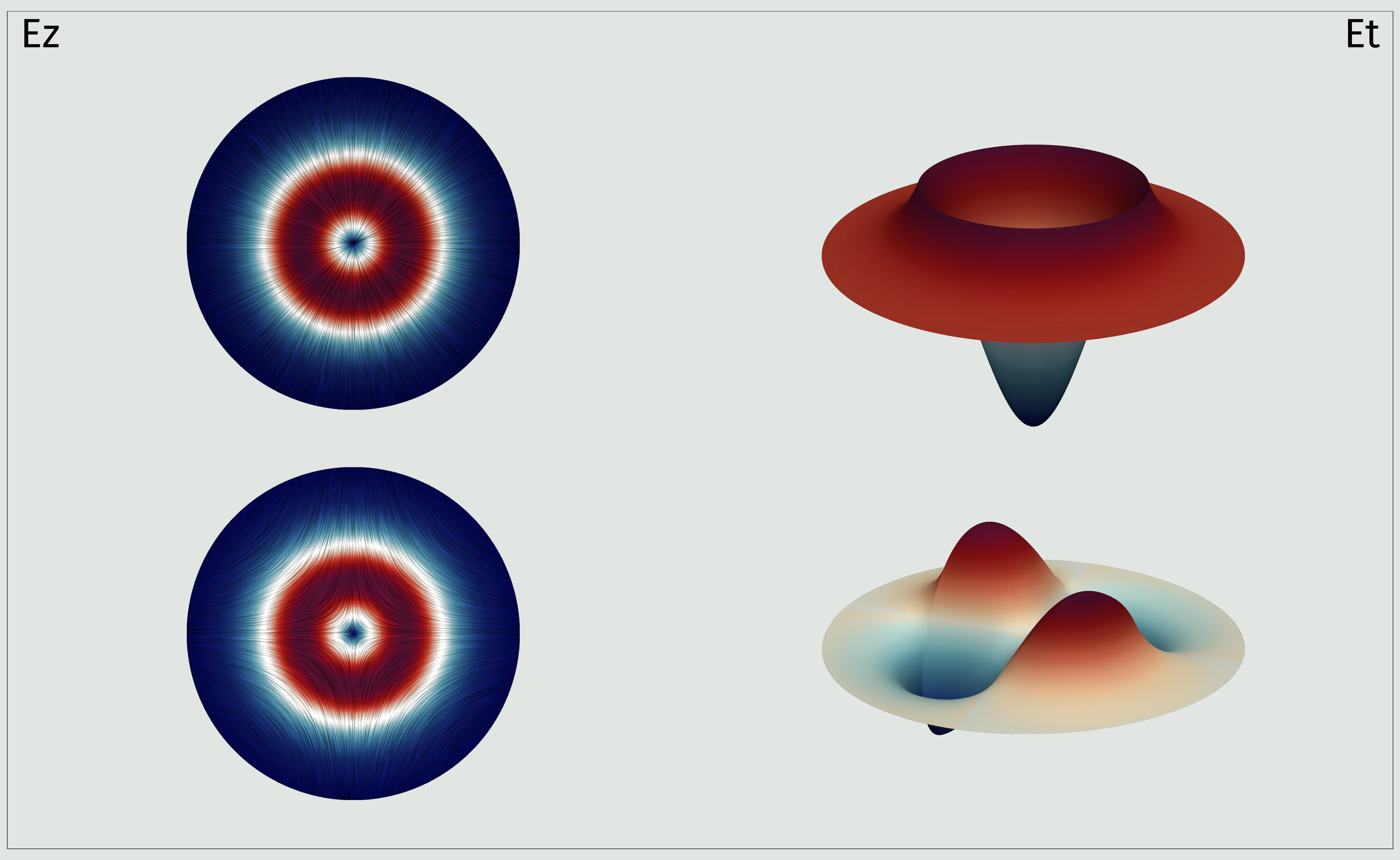

Figure 2: BlaBlabla

3. HP-ADAPTIVITY CAPABILITIES

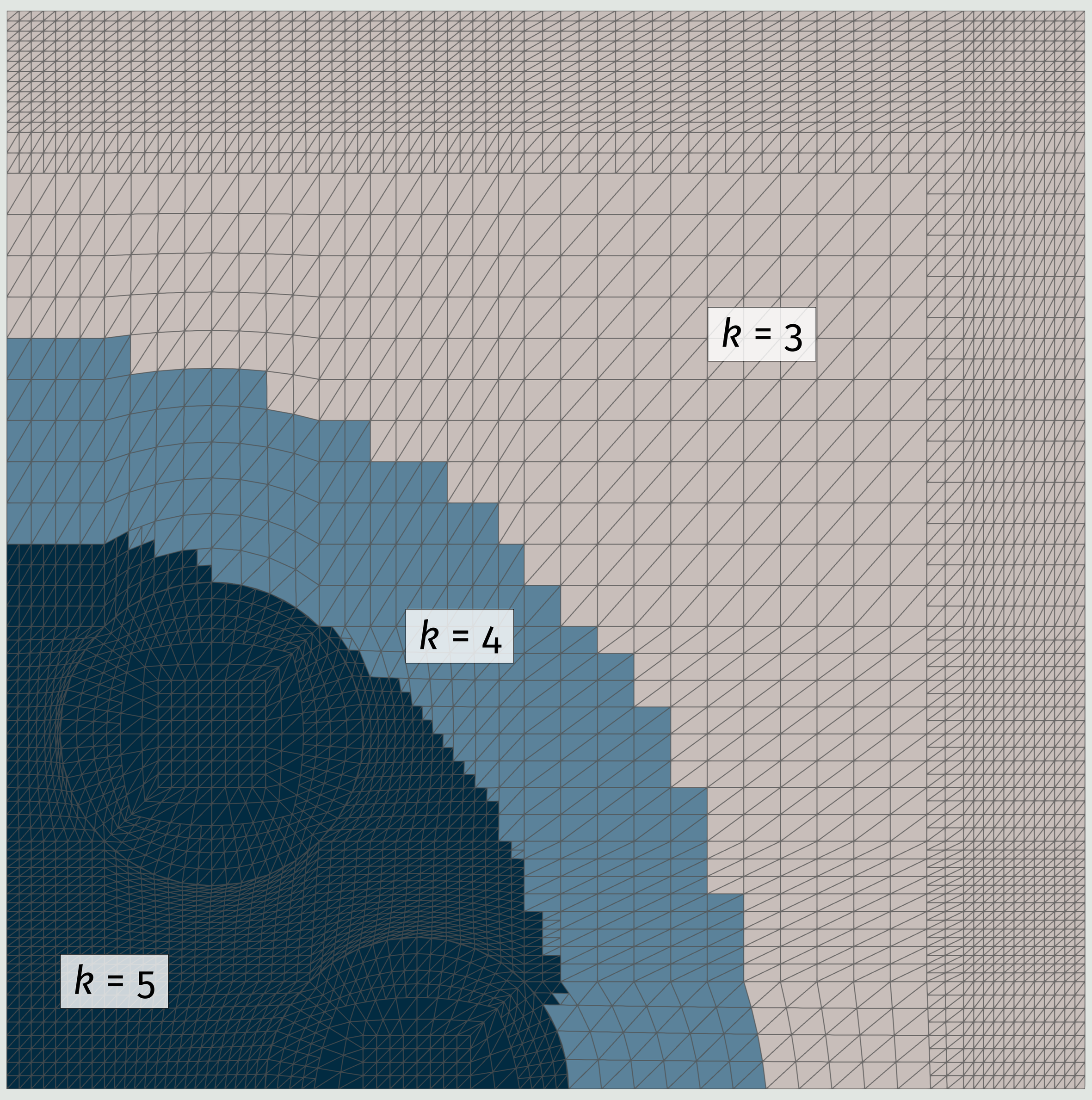

Figure 3: Hello

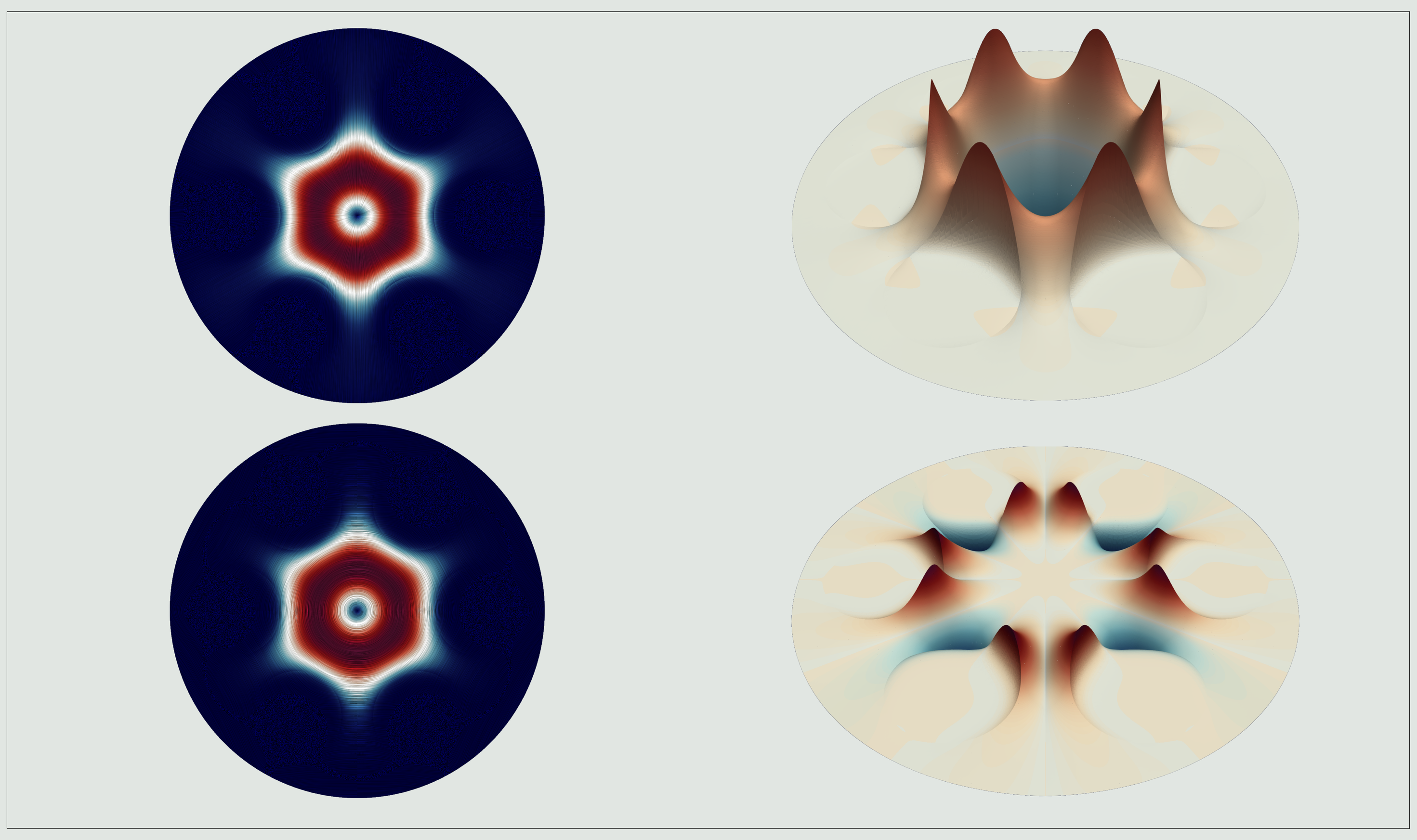

Figure 4: BlaBlabla