Northeastern University

College of Engineering

Department of Mechanical and Industrial Engineering

ME 2350

CTRUSS™

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Abstract:

This report details the design, construction and testing of a 3D truss that had to be able to support 20 N of weight applied to a hemispherical dome placed atop the structure. The truss had to be built out of only toothpicks and Loctite glue. The team optimized the design using SolidWorks simulations. The final design involved three 2D trusses glued together to create a 3D truss. The truss held a maximum load of 121.1 N, weighed 23 g, and cost $1,000.50. The mechanical advantage of the truss was 5267.44 N/kg and the cost advantage was 0.121 N/$.

Introduction:

The objective of this project was to design and build a 3D truss that could hold a minimum of 20 N applied on the top of a hemispherical dome. The goal was to maximize the load of the truss while minimizing the weight and cost. The truss had to be built using only toothpicks and Loctite glue. The dome had an approximate inner diameter of 12.5 cm, and the truss was placed in a base plate with a 15 cm by 15 cm square opening. The bottom of the hemisphere had to be at least 2.5 cm off the ground, and the truss could only contact the dome within 5 cm of its top. The team assumed that the toothpicks were made of birch, because they found that this was the most common type of wood used in toothpicks. The final design of the truss involved 2D cross sections that were erected vertically and connected using horizontal members. The three cross sections spanned the middle of the square opening, resulting in the truss contacting two sides of the plate for a total of 6 contact points. This design culminated in a rectangle at the top of the truss, creating four corners which touched the dome. The design was optimized for maximum load capacity. There are many truss designs in the public domain; one interesting design is that of a pyramid made of equilateral triangles. The team decided against this design because there would not be 2D cross sections to analyze.

Methodology:

In order to determine their design, the team brainstormed various 2D cross sections of trusses as well as different ways to connect them. SolidWorks simulations were used to determine which design would decrease the maximum stress in each member and thus increase the load capacity. The team's first design is pictured below.

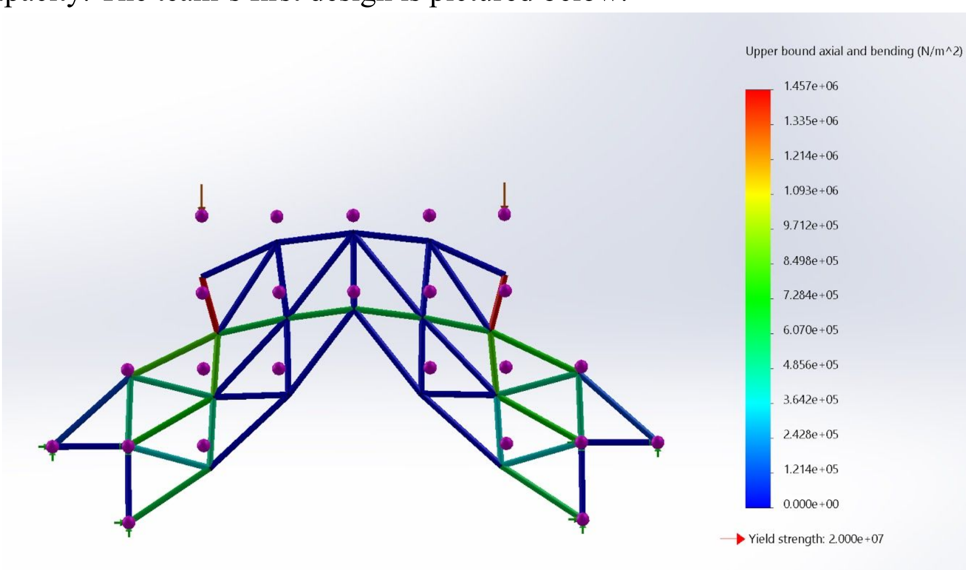


Fig. 1

With this design, the team was going to connect 2 of these trusses in the center of the opening. Thus, the truss design above would have been split into halves, and four of these halves would have spanned from the center of the hole to the edge of the opening. Each contact point with the board has a square opening so that the support could be better taken advantage of. The team decided that this design was not optimal because there was significant force on the members, and the opening in the middle led to weakness. Thus, the team moved on to a design that would more effectively distribute the load.

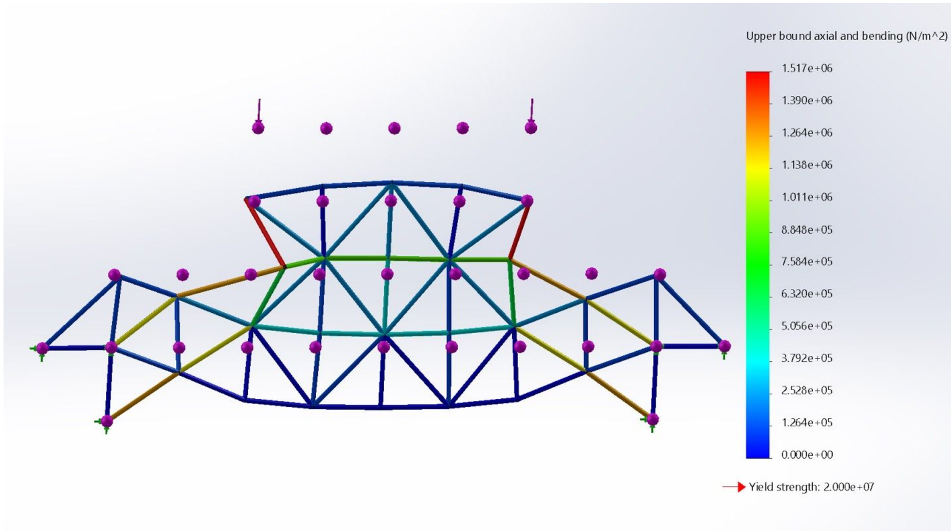


Fig. 2

With this design, the team decided that it would be best to complete several cross sections and connect them horizontally across the length of the square gap. However, the team was still unsatisfied with this design, as its upper bound of axial force and bending moment were still quite high. When reevaluating the design, the team realized that equilateral triangles should be included in the design as they are the strongest type of triangle. Thus, they altered this design to include equilateral triangles. The team's final design is shown below.

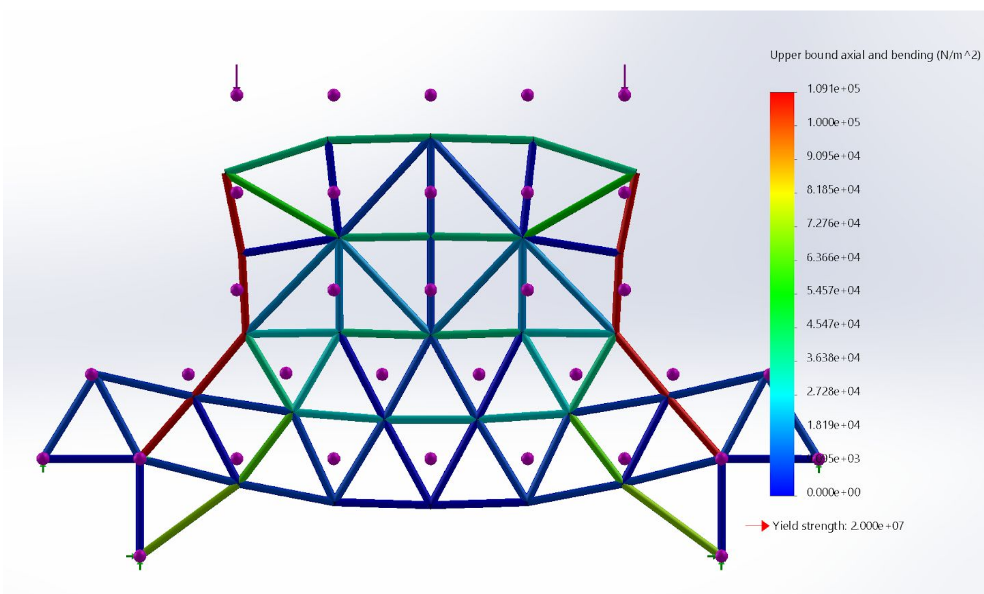


Fig. 3

This design went through many slight changes until the team landed on the one above. For example, it was determined that the diagonal members on the left and right sides of the top row should point inwards in order to counteract the force applied by the bowl. This 2D cross section could sustain significantly more weight than other previous designs because it distributed the weight better, which is partially due to the equilateral triangles. The free body diagram is shown below.

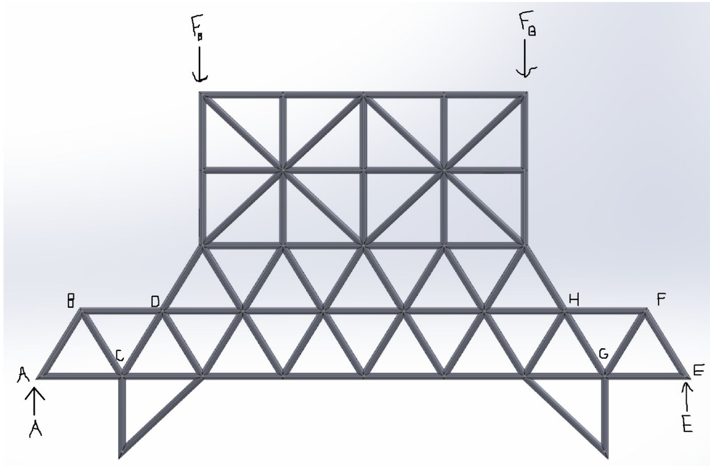


Fig. 4

After the team finalized this design, they then had to determine how they would connect the 2D cross sections with horizontal members to make a 3D truss. The length of the horizontal members was determined to be approximately 3 cm so that the truss would contact the bowl 4.5 cm from the top. They placed the horizontal members between joints at which the stress was the largest, and which would be important points to sustain the structure of the truss. For example, there were horizontal members connected to each joint that were red in the picture above because it indicated that they experienced the most stress. Each of the 2D trusses were built by making templates on cardboard with holes where members should connect, taping down the toothpicks, and gluing the joints. An example of this setup is shown below. The team made 6 cross sections in total so that they could build a test truss.

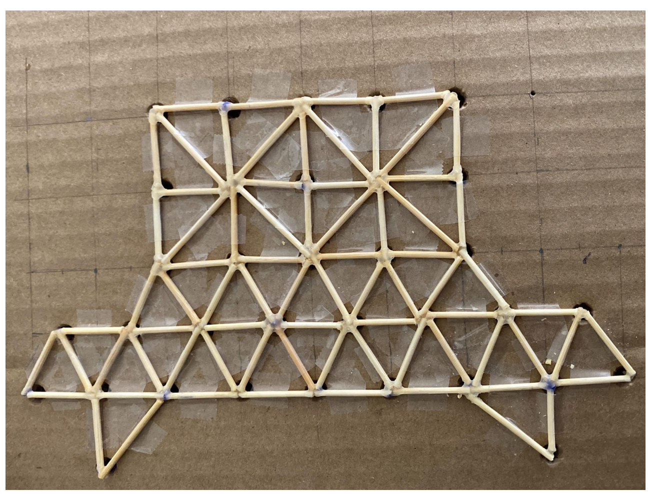


Fig. 5

The test truss was constructed so that the team could evaluate the results of loading and improve the final design. It was tested to failure, as shown below.

Fig. 6

The team took a slow motion video of the truss collapsing and evaluated the weakest points of the truss. This resulted in adding 4 more horizontal members in the structure to reinforce the weakest section at the top. Then, the team constructed the final design. The largest difficulty with the project was putting it together in 3D, because some joins were hard to access with precision. The team cut a 15 cm by 15 cm hole in a pizza box so that the truss could be constructed while oriented correctly. A picture of the construction of the truss is shown below.

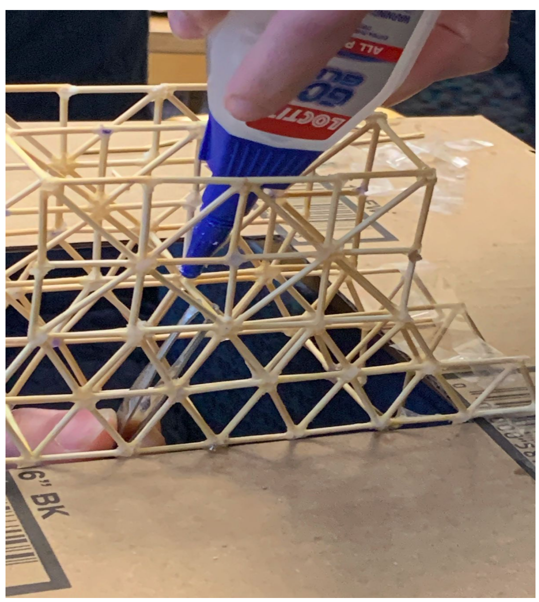


Fig.7

<u>Analysis:</u>

The final truss design was too complicated to solve using the method of joints and the method of sections; however, it was possible to solve for some of the beams by analysing one of the 2D trusses that make up the 3D truss. The 2D cross section can be seen in figure 8. The lengths of the members were scaled up to account for the units of the simulated applied force.

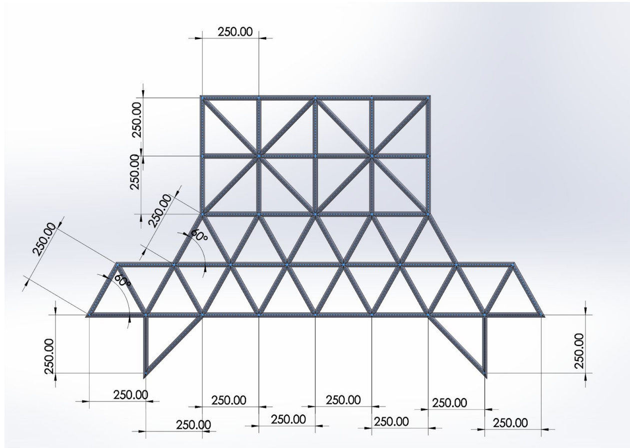


Fig. 8

Assuming that the bowl places an equal amount of force at each of the truss' four corners that contact it, that the force from the bowl is completely vertical, and that the support acts only on the joints, the support reactions can be calculated using the free body diagram in figure 4.

In figure 4, F_B is equal to ¼ of the force put on the bowl, and A and E are the support reactions. Horizontal support reactions can be ignored because there is no external horizontal force. The equations used to solve for the support reactions are shown below.

$$\Sigma F_y = A + E - 2F_B = 0$$

Eq. 1

$$\Sigma M_A =- F_B(400) - F_B(1600) + E(1600) = 0$$

Eq. 2

Solving for A and E, $A = E =- F_B$.

The truss and the forces placed on it are symmetrical, which means AB=FE, BC=GF, CD=GH, ect., and so solving half of the truss gives the solution for the other half. In addition, all of the triangles on the bottom half of the truss are equilateral, so all of the angles on the bottom half of the truss are 60°. Using this information, the method of joints can be used to solve for AB, AC, and BD. The equations used to solve for joint A can be seen below.

$$\Sigma F_y = A + ABsin60 = 0$$

Eq. 2

$$\Sigma F_x = AC + ABcos60 = 0$$

Eq. 3

By solving these equations it can be determined that $AB = \frac{F_B}{0.866}$ and $AC = 0.577F_B$. Using these numbers it is possible to solve for joint B using the equations shown below.

$$\Sigma F_y = ABsin60 - BCsin60 = 0$$

Eq. 4

$$\Sigma F_x = ABcos60 - BCcos60 + BD = 0$$

Eq. 5

Solving these equations, $BC = AB = \frac{F_B}{0.866}$, and $BD = 0$. After solving for joints A and B, the truss becomes too complicated to continue to solve using taught methods.

The member that experienced the most compression, and thus that will break first, was found to be the 2 vertical members on the outermost sides of the top row from the SolidWorks simulation. This makes sense, because they were less reinforced with other 3D members. When testing our prototype truss, this member broke first.

Given the final design and results from testing, the team was able to evaluate several attributes of the design. Mechanical advantage is a measure of the force amplification of a structure. A higher mechanical advantage means that the truss is lighter and more efficient in supporting the applied load. Though a stronger truss could be created by using many toothpicks, this would not be feasible in the real world. Thus, there must be a balance between the weight of the truss and the load capacity. The equation used for the calculation of mechanical advantage of our truss is shown below.

$$Mechanical\ Advantage = \frac{Load\ Capacity}{Weight\ of\ Truss} = \frac{121.1\ N}{0.023\ kg} = 5267.44\ N/kg$$

Eq. 6

Cost advantage is another important attribute that was examined. Cost advantage analyzes the performance of the truss in relation to its cost. Once again, it is important that there is a balance between the materials used and the effectiveness of the design. The equation used to calculate the cost of the truss is shown below, where n is the number of joints and l_i is the length of individual beams. A lower cost advantage means that the product is more marketable.

$$Cost = \$1 \times \sum_i l_i + \$3 \times n$$

Eq. 7

The table below was used to calculate the total length of all members. It was found to be 694.5 cm.

Table 1: Beam lengths

Beam Length (cm)	*Quantity*	*Total Length (cm)*
2.5	195	487.5
3.5	30	105
3	34	102

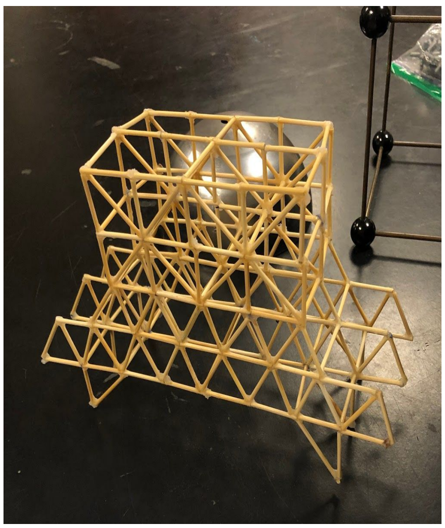
There were 102 joints in the truss. Thus, the cost was found to be $\$1 \times 694.5 + \$3 \times 102 = \$1000.50$. The cost advantage is calculated below.

$$Cost\ Advantage = \frac{Load\ Capacity}{cost} = \frac{121.1\ N}{\$1000.50} = 0.121\ N/\$ \quad .$$

Eq. 8

Measurements:

The measurements for testing the truss were taken with an Instron universal testing machine. The truss was placed on an acrylic base plate, with an edge length of 15cm, and the metal dome was mounted on top of the truss upside down. The Instron testing machine then gradually added weight to the truss until failure occurred. The figures below show the truss before, during and after testing.



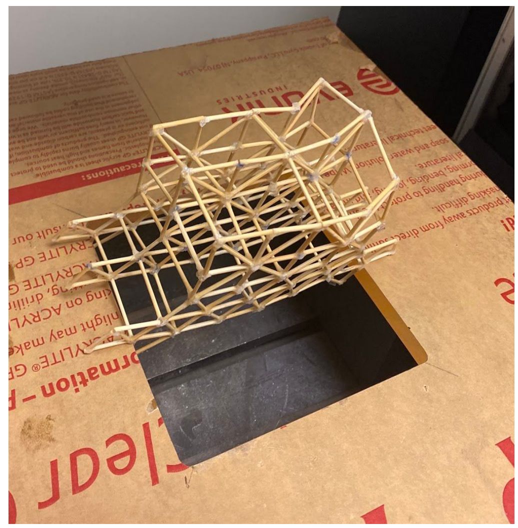


Fig. 9: Truss before testing **Fig. 10: Truss during testing** **Fig. 11: Truss after testing**

Furthermore, a video of the truss's performance can be found at the following link: https://www.youtube.com/watch?v=laA_z-K_BvI&feature=youtu.be. The truss was able to hold a maximum of 121.1 before failing. This maximum load for the truss greatly exceeded the minimum required load of 20 N, fulfilling the design objective. The first members to fail were the 3-dimensional members added to support the vertical members on the outermost sides of the top row. These vertical members were predicted to have the most stress in the software analysis, which is why the extra support was added. Therefore, the results of the testing confirmed the predicted location of failure and the benefit of these added supports. It was also observed that the truss was pushed forward as more weight was added. It is believed that this may have been due to the final truss not fitting as snuggly on the bowl as intended in the design. Also, slight variations between the two dimensional cross sections created during the build processes may have factored into the design's failure. Both errors could have caused a more uneven distribution of the load than expected, pushing the bowl forward. One of the downward facing triangular supports also snapped when fitting the truss in the hole, but this most likely did not make a significant difference. A graph of the load during testing plotted against the time is shown below.

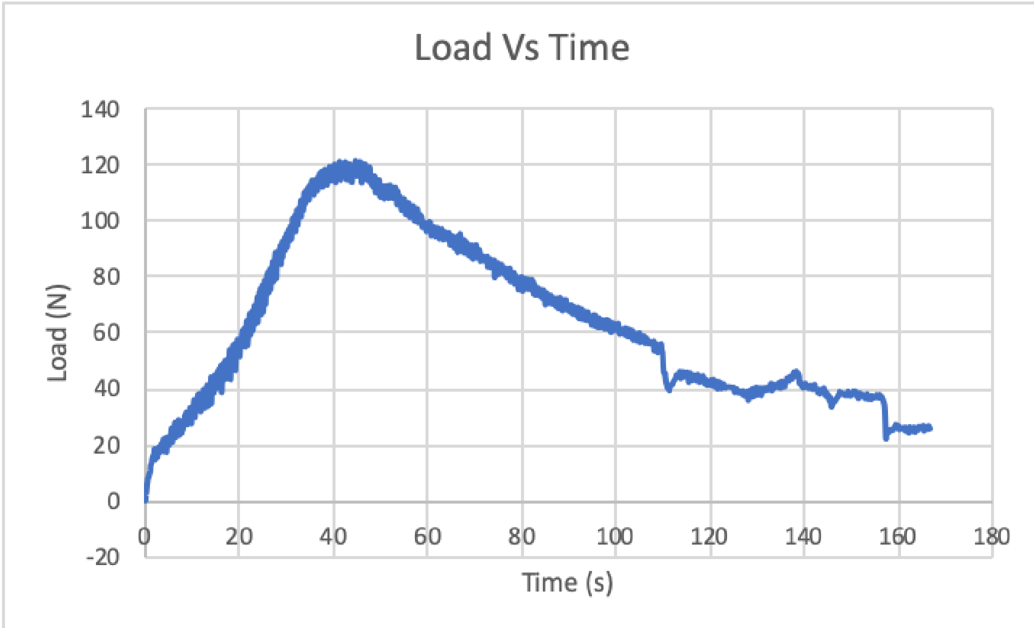


Fig. 12

The mechanical advantage for the design was $5267.44\ N/kg$, which meant the truss held a significant amount of weight for its small mass. Furthermore, the cost advantage was $0.121\ N/\$$, which meant the truss was cost efficient in its design and held a lot of weight for the amount of members and joints used.

Conclusion:

The goal of this project was to construct a truss out of toothpicks and glue that supported a minimum of 20 N applied to a hemispherical dome. The final design was able to hold 121.1 N, thus holding over 100 N above the minimum weight. The truss weighed 23 g and cost $1,000.50, which contributed to a mechanical advantage of 5267.44 N/kg and a cost advantage of 0.121 N/$. While building the truss, the team had trouble making all 2D cross sections the exact same. The team learned that the members should have been aligned in the same manner for all trusses, such as placing all horizontal members first. One of the vertical members at a support also broke off when placing the truss in the hole. The team realized they should have ensured that the length of the sections of the truss that went in the hole were less than 15 cm. In order to improve the design, the team could have included more horizontal members between the 2D cross sections. Specifically, adding four more members on the topmost level connecting the trusses would have made it stronger, as this is where the majority of deformation occurred.

References:

Bedford, A., et al. *Engineering Mechanics*. Pearson Learning Solutions, 2011.

“Toothpick and Technology.” *The Toothpick - Technology & Culture*, userwww.sfsu.edu/art511_c/tele07/spmasterf/finalSite/technology.html.

Appendix:

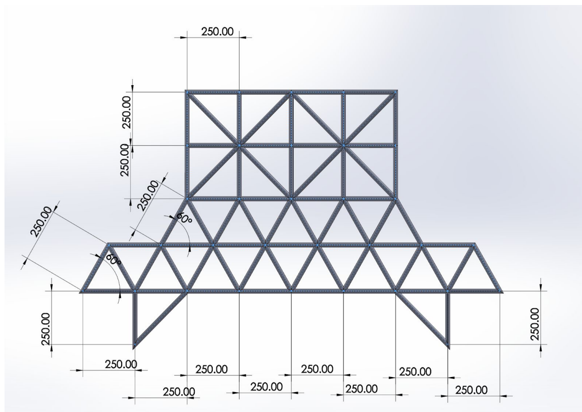


Fig. 13- Solidworks drawing of final truss design

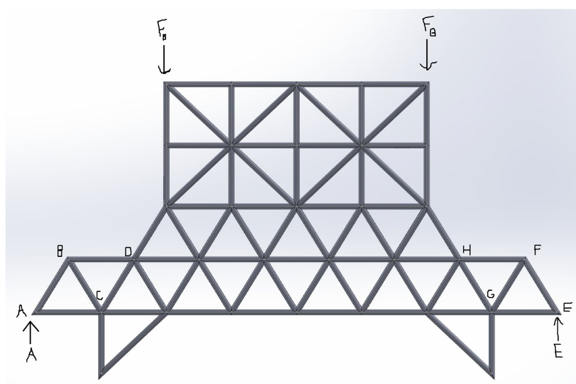


Fig 14- Force diagram of truss

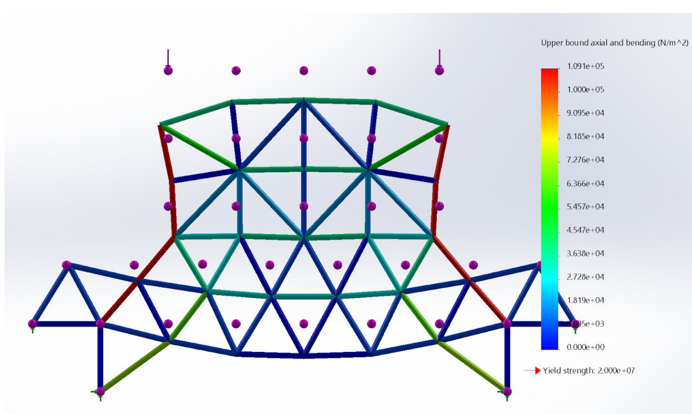


Fig 15- Solidworks stress analysis of final truss design