hw-04

October 1, 2024

0.1 Lab 4 - part 2: Data Weights and Autocorellation

TU Delft and WUR Q1 2024 Instructor: Theodoros Chatzivasileiadis **Instructor:** Hans Hoogenboom **TA:** Ka Yi Chua **Metropolitan Data 1**

0.2 Exercise 4a: Spatial Autocorrelation and ESDA

USE the Liverpool IMD data we have been using in previous sessions. This will require you to:

- Load up the IMD dataset for Liverpool.
- Create a choropleth of the `imd_score` variable.
- Compute the spatial weights matrix for the LSOAs. Think of one criterium to build it that you think would fit this variable (e.g. contiguity, distance-based, etc.), and apply it.
- Create the standardized version of the IMD scores.
- Calculate the spatial lag of the standardized scores.
- Create the Moran Plot.
- Calculate the value of Moran's I as well as its significance level.
- Perform a LISA analysis and generate a map of the results. What are the main patterns?

[54]:
```
import geopandas as gpd
import osmnx as ox
from pysal.lib import weights
import matplotlib.pyplot as plt
import seaborn as sns
from pysal.explore import esda
from splot.esda import moran_scatterplot, lisa_cluster
import pandas as pd
import numpy as np
```

Load up the IMD dataset for Liverpool.

[55]:
```
'''
Source: lab-04-part-01
Load up whole IMD dataset and save to imd.
'''


imd_shp = 'data/IMD/lab04_imd.shp' #file path
imd = gpd.read_file(imd_shp).set_index("lsoa11cd", drop=False) #read shapefile,␣
↪set unique identifier as lsoa11cd do not drop this column
```

[56]:
```python
'''
Filter for only liverpool.
'''
liverpool_imd = imd[imd['LADnm'] == "Liverpool"] #filter out IMD dataset for␣
↪england for only liverpool (place we are concerned)
liverpool_imd.head() #display dataset.
```

[56]:
```
             lsoa11cd        lsoa11nm       lsoa11nmw     st_areasha  \
lsoa11cd
E01006512  E01006512  Liverpool 031A  Liverpool 031A  283906.863775
E01006513  E01006513  Liverpool 060A  Liverpool 060A  555037.185423
E01006514  E01006514  Liverpool 037A  Liverpool 037A  262030.747956
E01006515  E01006515  Liverpool 037B  Liverpool 037B  366499.754871
E01006518  E01006518  Liverpool 044A  Liverpool 044A  235181.260598

             st_lengths  IMD_Rank  IMD_Decile        LSOA01NM      LADcd  \
lsoa11cd
E01006512  3063.168774     14664           5  Liverpool 031A  E08000012
E01006513  5835.725743     11173           4  Liverpool 060A  E08000012
E01006514  3243.503128      3299           2  Liverpool 037A  E08000012
E01006515  4273.694263      1875           1  Liverpool 037B  E08000012
E01006518  2743.746370       330           1  Liverpool 044A  E08000012

                LADnm  …  IndDec  OutScore  OutRank  OutDec  TotPop  DepChi  \
lsoa11cd               …
E01006512  Liverpool  …       1     0.536     8031       3    2975     255
E01006513  Liverpool  …       1     0.974     4329       2    4418     103
E01006514  Liverpool  …       1     0.550     7888       3    1760      87
E01006515  Liverpool  …       3     0.615     7285       3    1438     176
E01006518  Liverpool  …       2     0.564     7747       3    1732     287

           Pop16_59  Pop60+  WorkPop  \
lsoa11cd
E01006512      2639      81  2649.50
E01006513      4222      93  4257.75
E01006514      1439     234  1538.25
E01006515      1013     249  1014.75
E01006518       971     474   993.00

                                                    geometry
lsoa11cd
E01006512  POLYGON ((335888.000 390042.000, 336203.000 39…
E01006513  POLYGON ((335374.808 390547.998, 335405.406 39…
E01006514  POLYGON ((335650.536 389928.121, 335677.916 38…
E01006515  POLYGON ((335178.000 389642.000, 335222.397 38…
E01006518  POLYGON ((335528.316 389067.946, 335612.896 38…
```

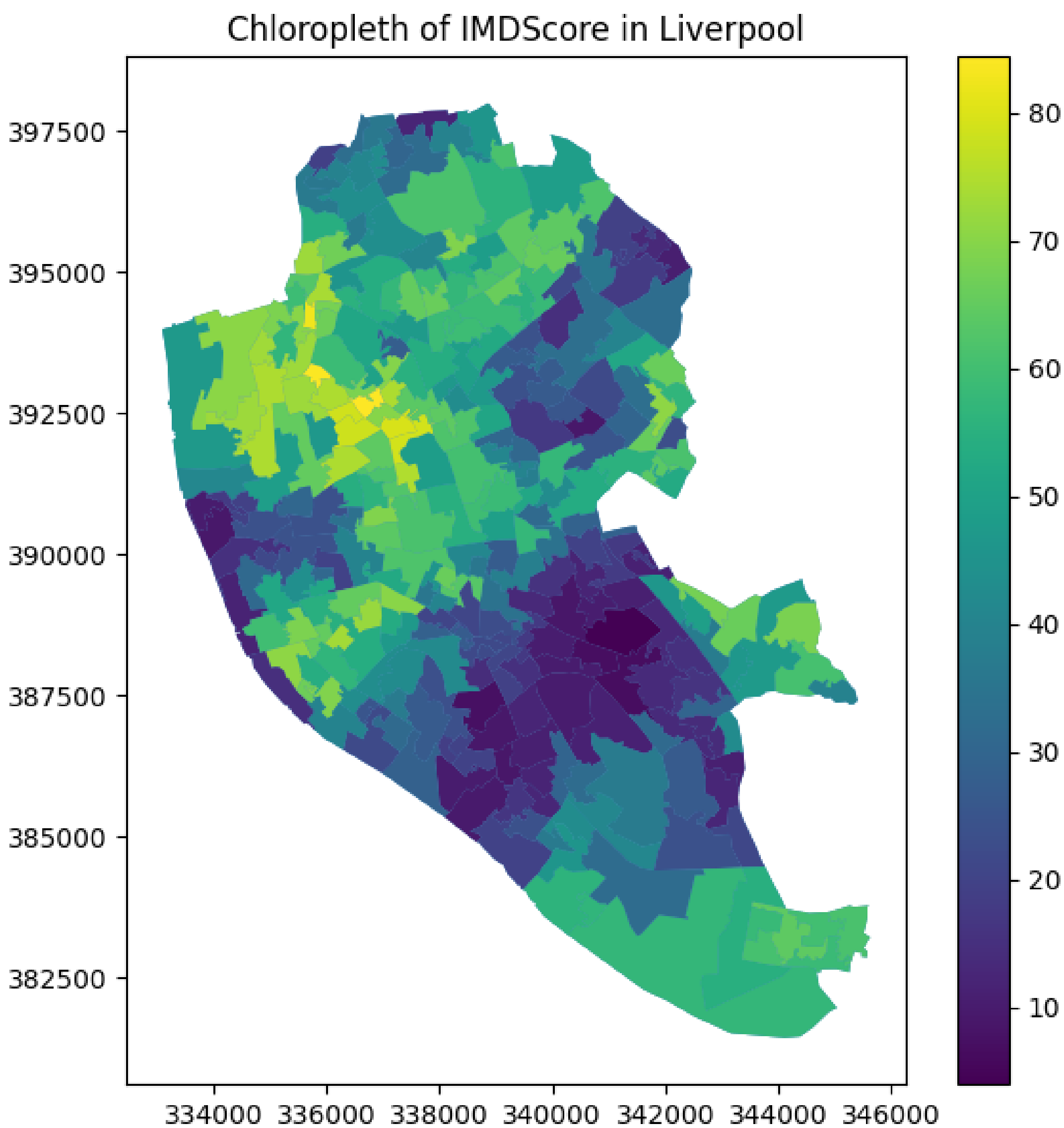
```
[5 rows x 64 columns]
```

Create a choropleth of the `imd_score` variable.

What is chloropleth: https://geographicdata.science/book/notebooks/05_choropleth.html

[57]:
```
'''
Source: lab-04-part-02
Plot base axis for liverpool aread.
Plot chloropleth so that you can see the difference through the city in IMD␣
↪score.
'''
ax = liverpool_imd.plot(figsize=(7, 7)) #from lab-04-part-02
liverpool_imd.plot(column='IMDScore', ax=ax, legend=True) #focused on IMD,␣
↪legend = True shows the colour bar on the side. source: https://geopandas.
↪org/en/stable/docs/reference/api/geopandas.GeoDataFrame.plot.html
plt.title("Chloropleth of IMDScore in Liverpool") #plot title
```

[57]:
```
Text(0.5, 1.0, 'Chloropleth of IMDScore in Liverpool')
```



Compute the spatial weights matrix for the LSOAs. Think of one criterium to build it that you think would fit this variable (e.g. contiguity, distance-based, etc.), and apply it.

[58]:
```
'''
Deprivation (imd score) is often geographically clustered and related to shared␣
↪borders or vertices, therefore will use contiguity.
Source: lab-04-part-01
Make contiguity weights matrix from liverpool only imd dataset.
'''
w_queen = weights.Queen.from_dataframe(liverpool_imd, ids="lsoa11cd") #make␣
↪contiguity from dataframe liverpool_imd
```

```
w_queen
```

[58]:
```
<libpysal.weights.contiguity.Queen at 0x749e39118c40>
```

[59]:
```
w_queen['E01033767'] #test an area to ensure it works (5 neighbours displayed)
```

[59]:
```
{'E01033748': 1.0,
 'E01032505': 1.0,
 'E01006630': 1.0,
 'E01006633': 1.0,
 'E01006632': 1.0}
```

Create the standardized version of the IMD scores.

[60]:
```
'''
Source: lab-04-part-02
'''
liverpool_imd['std_imd'] = (liverpool_imd['IMDScore'] -
 ↪liverpool_imd['IMDScore'].mean()) / liverpool_imd['IMDScore'].std()
 ↪#standardized imd scores
```

```
/mnt/nvme1n1p1/made/data1/env/lib/python3.10/site-
packages/geopandas/geodataframe.py:1528: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  super().__setitem__(key, value)
```

Calculate the spatial lag of the standardized scores.

[61]:
```
'''
Source: lab-04-part-02
'''
w_queen.transform = 'R' #standardise weights matrix
w_queen_score = weights.lag_spatial(w_queen, liverpool_imd['std_imd']) #find
 ↪spatial lag with standarized weights and standardized imd scores
```

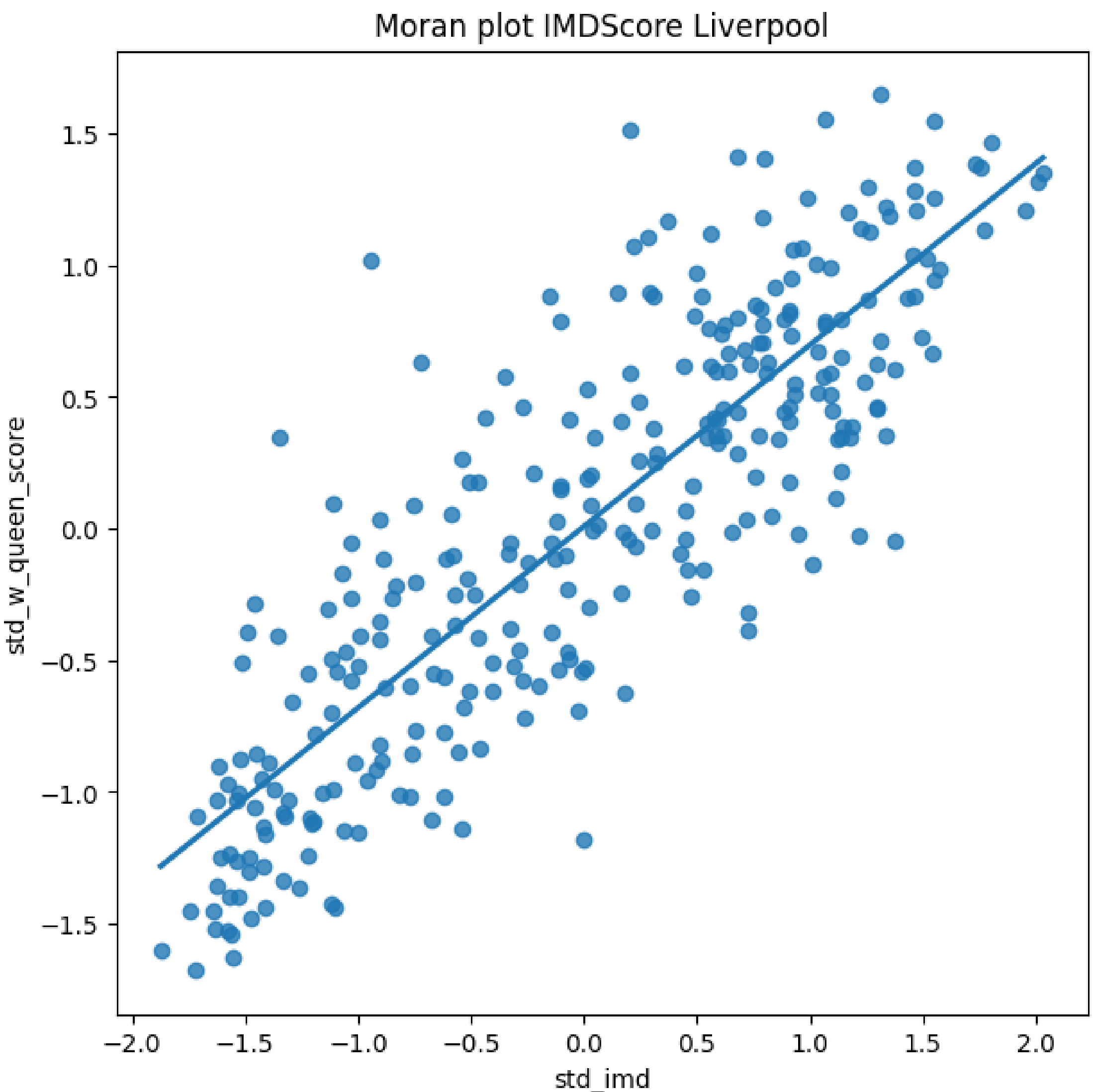
[62]:
```
liverpool_imd['std_w_queen_score'] = w_queen_score #set to new column
```

```
/mnt/nvme1n1p1/made/data1/env/lib/python3.10/site-
packages/geopandas/geodataframe.py:1528: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  super().__setitem__(key, value)
```

Create the Moran Plot.

[63]:
```
'''
Moran plot = attribute on x axis (standardized IMD score) and spatial lag of␣
↪attribute on y axis
Source: lab-04-part-02
'''
f, ax = plt.subplots(1, figsize=(7, 7)) #set up figure
sns.regplot(x='std_imd', y='std_w_queen_score', data=liverpool_imd, ci=None)␣
↪#plot data, set x and y
plt.title("Moran plot IMDScore Liverpool") #make title
plt.show()
```



Calculate the value of Moran's I as well as its significance level.

[64]:
```
'''
Use esda.Moran for moran plot
No need to standardize IMD score before hand.
Source: lab-04-part-02
'''
mi = esda.Moran(liverpool_imd['IMDScore'], w_queen) #prepare moran plot using␣
↪built in function
```

[65]:
```
mi.I #get moran I score
```

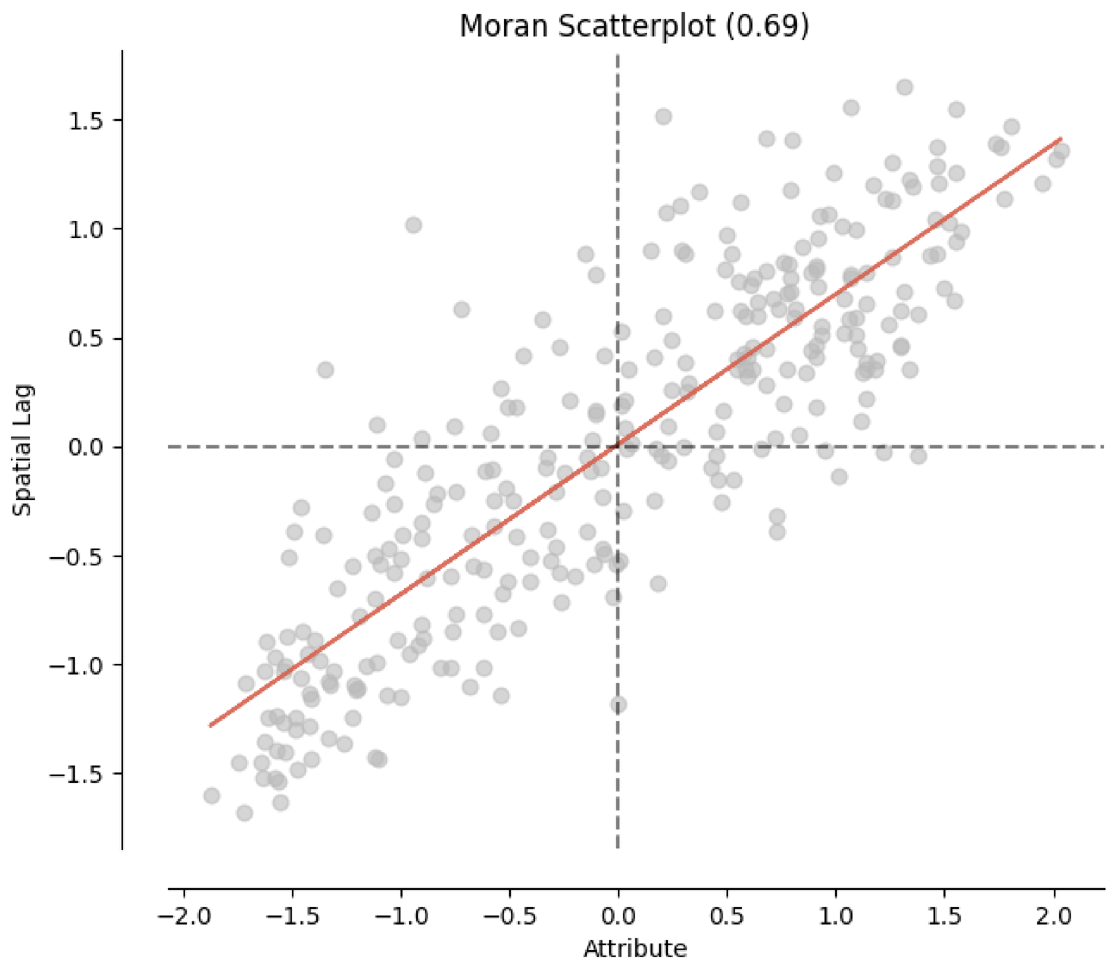
[65]: 0.6896035100024694

[66]:
```
mi.p_sim #get significance level
```

[66]: 0.001

[67]:
```
moran_scatterplot(mi) #plot Moran to check previous plot (identical to manual␣
↪plot made above)
```

[67]:
```
(<Figure size 700x700 with 1 Axes>,
 <Axes: title={'center': 'Moran Scatterplot (0.69)'}, xlabel='Attribute',
ylabel='Spatial Lag'>)
```



Perform a LISA analysis and generate a map of the results. What are the main patterns?

[68]:
```
'''
Use esda.Moran_local for LISA analysis
Source: lab-04-part-02
'''
lisa = esda.Moran_Local(liverpool_imd['IMDScore'], w_queen) #prepares LISA␣
↪analysis
```

[69]:
```
#Source: lab-04-part-02

liverpool_imd['significant'] = lisa.p_sim < 0.05 # Break observations into␣
↪significant or not
liverpool_imd['quadrant'] = lisa.q # Store the quadrant they belong to
```

/mnt/nvme1n1p1/made/data1/env/lib/python3.10/site-

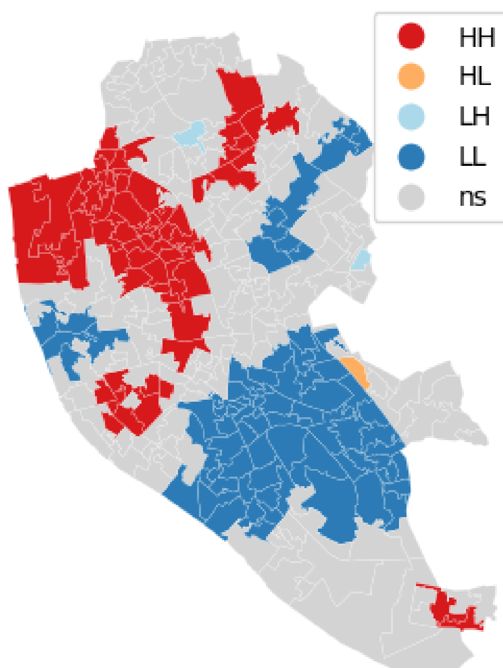
```
packages/geopandas/geodataframe.py:1528: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  super().__setitem__(key, value)
/mnt/nvme1n1p1/made/data1/env/lib/python3.10/site-
packages/geopandas/geodataframe.py:1528: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  super().__setitem__(key, value)
```

[70]:
```
lisa_cluster(lisa, liverpool_imd) # Plot LISA analysis to show spatial␣
↪relevance of IMD score.
```

[70]: (<Figure size 640x480 with 1 Axes>, <Axes: >)



What are main patterns?

In the northwest, there is a noticeable cluster of deprived areas, where both each region and its neighbors have high IMD scores, highlighting the spatial significance of this metric. Conversely, in

South-Central Liverpool, the area appears wealthier and less deprived, with surrounding neighborhoods following a similar trend. Additionally, there are smaller clusters showing a mix of spatially more deprived and less deprived areas.

There are a few exceptions: two neighborhoods with low IMD scores are surrounded by highly deprived areas with high IMD scores, and one neighborhood with a high IMD score is surrounded by wealthier areas with low IMD scores

For this part I would like you to experiment with Data from Amsterdam. However this will require you to find the dataset yourself!

0.3 Question 1: Building a Contiguity-Based Weight Matrix

Use PySAL to create a contiguity-based weight matrix (W) for a given spatial dataset of polygons. Write a function create_contiguity_weights that:

```
Reads in a shapefile of polygons.
Constructs a Queen contiguity-based spatial weights matrix.
Returns the weight matrix.
```

0.4 Question 2: Calculating Moran's I

Write a function calculate_morans_i that:

```
Takes a spatial weight matrix W and an attribute array y.
Calculates Moran's I for the given attribute array.
Returns the Moran's I value and its p-value.
```

0.5 Question 3: Standardizing Spatial Weight Matrices

Create a function standardize_weights that:

```
Takes a spatial weight matrix W.
Standardizes it so that the weights of each row sum to one.
Returns the standardized weight matrix.
```

0.6 Question 4: Creating Distance-Based Weights

Write a function create_distance_weights that:

```
Takes a set of point coordinates and a threshold distance.
Constructs a distance-based spatial weight matrix where all points within the threshold distanc
Returns the weight matrix.
```

0.7 Question 5: Visualizing a Moran Plot

Using PySAL, write a function plot_moran that:

```
Takes a spatial weights matrix W and an attribute array y.
Creates and displays a Moran plot for the attribute.
Saves the Moran plot as an image file.
```

Question 0: Prepare dataset

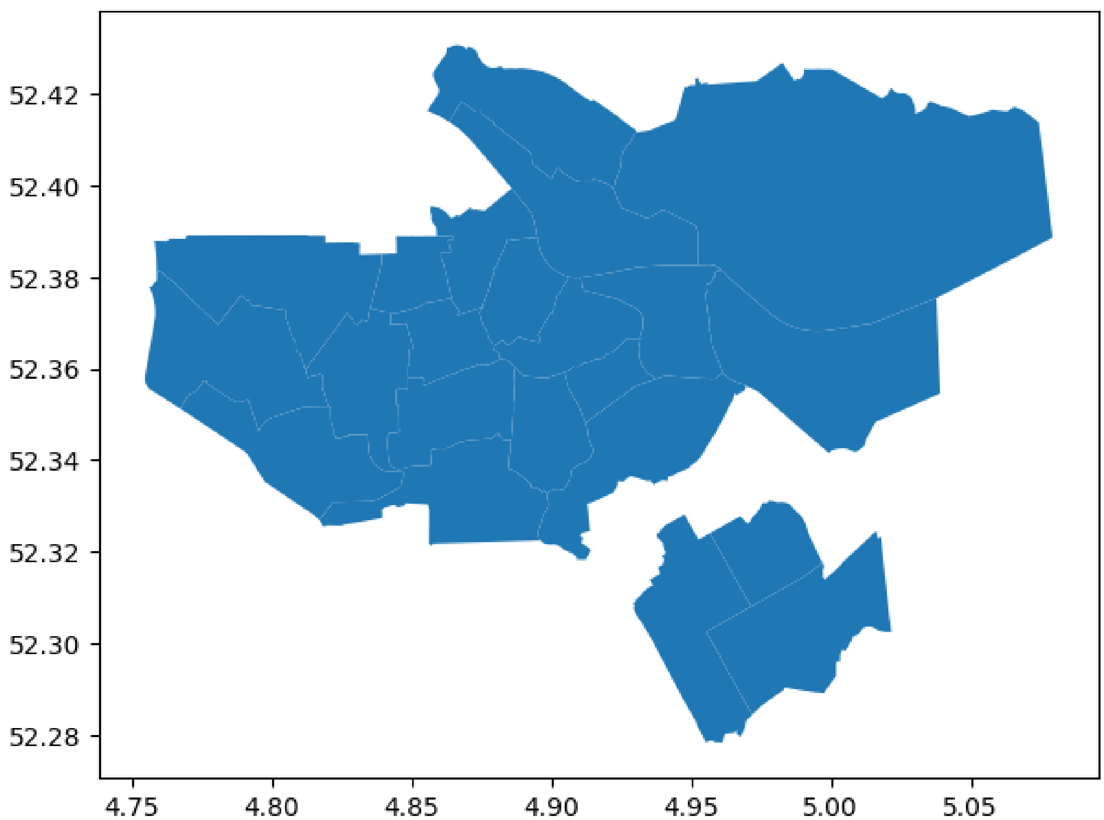
[71]:
```
'''
No dataset found with shapefile + useful metric for amsterdam.
Found a shapefile and separate csv for amsterdam airbnb scraped data.
Therefore must combine to have usefull attributes we can analysis spatially.
Data source: https://www.kaggle.com/code/erikbruin/
 ↪airbnb-the-amsterdam-story-with-interactive-maps
'''

ams_shp = 'data/mygeodata/neighbourhoods-polygon.shp' #shapefile path
ams = gpd.read_file(ams_shp) #read shapefile
ams = ams.rename(columns={'neighbourh': 'neighbourhood'}) #make sure columns␣
 ↪are labelled well
ams = ams.drop(columns=['neighbou_1']) #drop useless column
ams.head() #display shapefile dataframe
```

[71]:

	neighbourhood	geometry
0	Bijlmer-Oost	POLYGON Z ((4.99167 52.32444 43.06929, 4.99176…
1	Noord-Oost	POLYGON Z ((5.07916 52.38865 42.95663, 5.06710…
2	Noord-West	POLYGON Z ((4.93072 52.41161 42.91539, 4.93051…
3	Oud-Noord	POLYGON Z ((4.95242 52.38983 42.95411, 4.95242…
4	IJburg - Zeeburgereiland	POLYGON Z ((5.03906 52.35458 43.01664, 5.02022…

[72]:
```
ax = ams.plot(figsize=(7, 7)) #plot base to ensure it works
```



[73]:
```python
'''
Load and display airbnb data that is going to be combined with shape data
'''
airbnb_listings=pd.read_csv('data/mygeodata/listings.csv') #load csv file with␣
↪pandas to later insert into shapefile
airbnb_listings.head() #display csv of airbnb scraped data
```

[73]:
```
      id                                              name  host_id  \
0   2818            Quiet Garden View Room & Super Fast WiFi     3159
1   3209                  Quiet apt near center, great view     3806
2  20168          100%Centre-Studio 1 Private Floor/Bathroom    59484
3  25428                 Lovely apt in City Centre (Jordaan)    56142
4  27886  Romantic, stylish B&B houseboat in canal district    97647

  host_name  neighbourhood_group                           neighbourhood  \
0    Daniel                  NaN  Oostelijk Havengebied - Indische Buurt
1   Maartje                  NaN                              Westerpark
2      Alex                  NaN                            Centrum-Oost
3      Joan                  NaN                            Centrum-West
4      Flip                  NaN                            Centrum-West

    latitude  longitude        room_type  price  minimum_nights  \
0  52.365755   4.941419     Private room     59               3
1  52.390225   4.873924  Entire home/apt    160               4
2  52.365087   4.893541  Entire home/apt     80               1
3  52.373114   4.883668  Entire home/apt    125              14
4  52.386727   4.892078     Private room    150               2

   number_of_reviews last_review  reviews_per_month  \
0                248  2018-11-28               2.10
1                 42  2018-08-29               1.03
2                233  2018-11-30               2.18
3                  1  2018-01-21               0.09
4                171  2018-11-25               2.03

   calculated_host_listings_count  availability_365
0                               1                44
1                               1                47
2                               2               198
3                               2               141
4                               1               199
```

[74]:
```python
'''
Only take relevant data.
Number of listings per neighbourhood.
```

```
And average price of listings per neighbourhood.
'''
grouped_df = airbnb_listings.groupby('neighbourhood').agg(count=('price',␣
↪'size'), average_price=('price', 'mean')).reset_index() #per neighbourhood␣
↪make count of all listings, make mean of all the prices
grouped_df.head() #display refined dataset
```

[74]:

	neighbourhood	count	average_price
0	Bijlmer-Centrum	111	91.216216
1	Bijlmer-Oost	96	101.604167
2	Bos en Lommer	1145	116.996507
3	Buitenveldert - Zuidas	262	153.087786
4	Centrum-Oost	1730	189.376301

[75]:
```
'''
Combine datasets
'''
ams = ams.merge(grouped_df, on='neighbourhood', how='left') #use geopandas to␣
↪merge attribute data with shapedata source: https://geopandas.org/en/stable/
↪docs/user_guide/mergingdata.html
ams.head() #display merged datasets
```

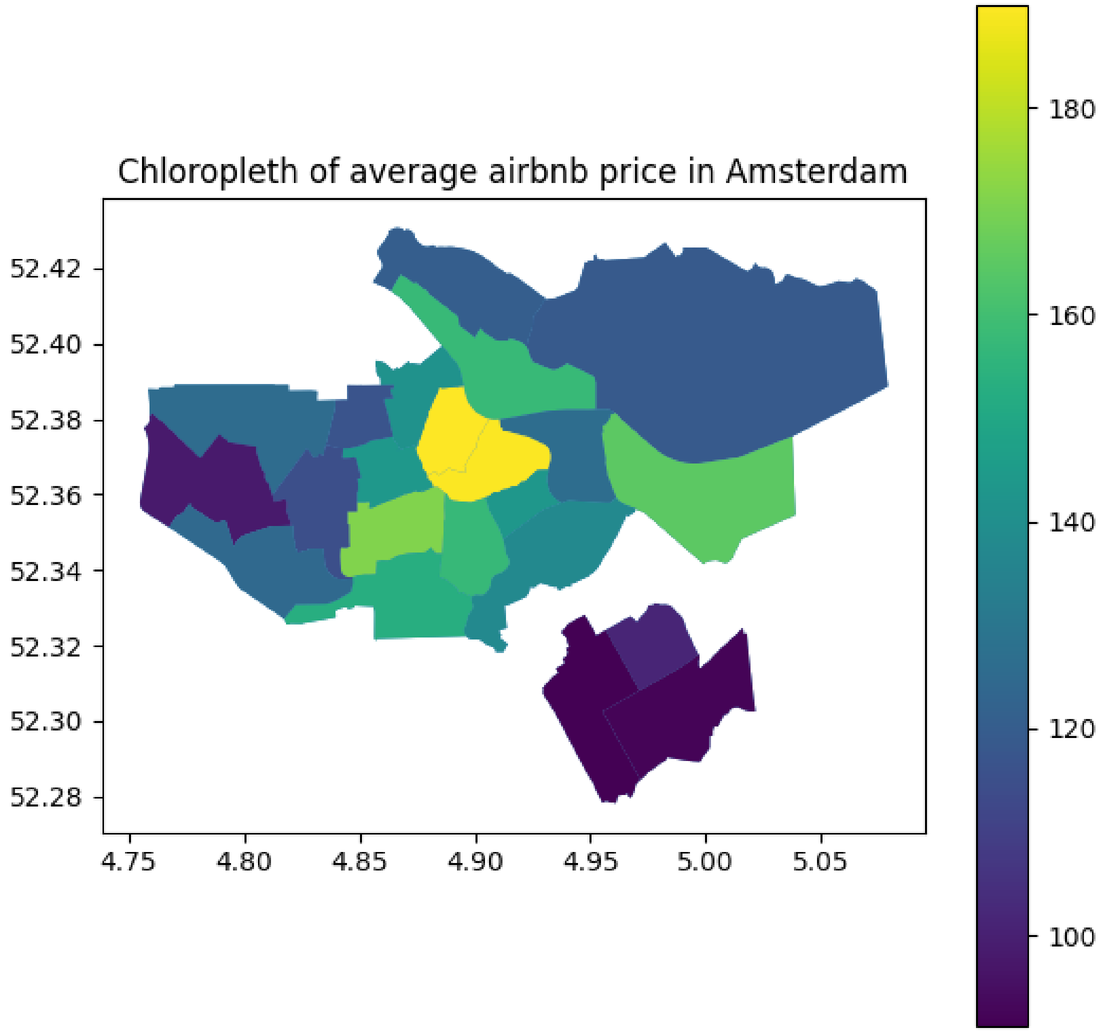
[75]:

```
                neighbourhood  \
0                Bijlmer-Oost
1                  Noord-Oost
2                  Noord-West
3                   Oud-Noord
4  IJburg - Zeeburgereiland

                                           geometry  count  average_price
0  POLYGON Z ((4.99167 52.32444 43.06929, 4.99176...     96     101.604167
1  POLYGON Z ((5.07916 52.38865 42.95663, 5.06710...    257     118.638132
2  POLYGON Z ((4.93072 52.41161 42.91539, 4.93051...    320     120.171875
3  POLYGON Z ((4.95242 52.38983 42.95411, 4.95242...    571     157.684764
4  POLYGON Z ((5.03906 52.35458 43.01664, 5.02022...    452     165.157080
```

[76]:
```
'''
Plot chlorpleth average airbnb price in amsterdam to check data.
'''
ax = ams.plot(figsize=(7, 7)) #set up base axis to plot chlorpleth
ams.plot(column='average_price', ax=ax, legend=True) #plot to display differing␣
↪average airbnb price source: https://geopandas.org/en/stable/docs/reference/
↪api/geopandas.GeoDataFrame.plot.html
plt.title("Chloropleth of average airbnb price in Amsterdam") #make title
```

[76]: Text(0.5, 1.0, 'Chloropleth of average airbnb price in Amsterdam')



[77]:
```
'''
Use to save shapefile+attributes dataframe
'''
#ams.to_file('airbnb-amsterdam.shp')
```

Question 1: Building a Contiguity-Based Weight Matrix

[78]:
```
ams.head() #display ready dataframe
```

[78]:
```
              neighbourhood  \
0              Bijlmer-Oost
1                Noord-Oost
2                Noord-West
3                 Oud-Noord
4  IJburg - Zeeburgereiland
```

```
                                            geometry  count  average_price
0  POLYGON Z ((4.99167 52.32444 43.06929, 4.99176…     96     101.604167
1  POLYGON Z ((5.07916 52.38865 42.95663, 5.06710…    257     118.638132
2  POLYGON Z ((4.93072 52.41161 42.91539, 4.93051…    320     120.171875
3  POLYGON Z ((4.95242 52.38983 42.95411, 4.95242…    571     157.684764
4  POLYGON Z ((5.03906 52.35458 43.01664, 5.02022…    452     165.157080
```

[80]:
```python
def create_contiguity_weights(dataframe): #define function, input geopandas␣
↪dataframe
    '''
    Create and return contiguity matrix for dataframe (no set index)
    Source: lab-04-part-01
    '''
    w_queen = weights.Queen.from_dataframe(dataframe) #create contiguity weights
    return(w_queen) #return

#test function
w_queen = create_contiguity_weights(ams) #create weights for ams airbnb␣
↪dataframe
w_queen[1] #display neighbours of Noord-oost
```

```
/tmp/ipykernel_16495/4180433704.py:2: FutureWarning: `use_index` defaults to
False but will default to True in future. Set True/False directly to control
this behavior and silence this warning
  w_queen = weights.Queen.from_dataframe(dataframe)
/mnt/nvme1n1p1/made/data1/env/lib/python3.10/site-
packages/libpysal/weights/contiguity.py:347: UserWarning: The weights matrix is
not fully connected:
 There are 2 disconnected components.
  W.__init__(self, neighbors, ids=ids, **kw)
```

[80]:
```
{2: 1.0, 3: 1.0, 4: 1.0, 6: 1.0}
```

Question 2: Calculating Moran's I

[31]:
```python
def calculate_morans_i(W,y): #define function input weights matrix and␣
↪attribute list
    '''
    Create and return contiguity matrix for dataframe (no set index)
    Source: lab-04-part-02
    '''
    mi = esda.Moran(y, W) #use package esda.Moran to prepare Moran analysis
    return mi.I, mi.p_sim #return Moran I score and significance level

#test function
```

```python
i,p_sim = calculate_morans_i(w_queen, ams['average_price']) #find Moran I and
↪significance with contiguity matrix and average price of airbnb
print(i,p_sim) #print found data
```

```
0.546869482158763 0.001
```

Question 3: Standardizing Spatial Weight Matrices

[32]:
```python
def standardize_weights(W): #define function, input weights matrix
    '''
    Standardize weights for all rows of neighbourhours (example: 1,1 -> 0.5,0.5)
    Source: lab-04-part-01
    '''
    W.transform = 'R' #standardize
    return W #return standardized matrix


#test function
w_queen = standardize_weights(w_queen) #standardized contiguity matrix for ams
↪airbnb dataset
w_queen[1] #print neighbourhours (previously seen as 1 1 1 1)
```

[32]:
```
{2: 0.25, 3: 0.25, 4: 0.25, 6: 0.25}
```

Question 4: Creating Distance-Based Weights

[42]:
```python
'''
Check crs as inportant when calculating distance of neighbourhoods
Currently in long-lat degrees
Source: lab-04-part-01
'''
ams.crs
```

[42]:
```
<Geographic 2D CRS: EPSG:4326>
Name: WGS 84
Axis Info [ellipsoidal]:
- Lat[north]: Geodetic latitude (degree)
- Lon[east]: Geodetic longitude (degree)
Area of Use:
- name: World.
- bounds: (-180.0, -90.0, 180.0, 90.0)
Datum: World Geodetic System 1984 ensemble
- Ellipsoid: WGS 84
- Prime Meridian: Greenwich
```

[46]:
```python
'''
Must change crs to metres in order to create distance weights with coordinate
↪points.
```

```
Source: lab-04-part-01
'''
ams=ox.project_gdf(ams) #convert to metres
ams.crs #display to ensure successful
```

[46]:
```
<Projected CRS: EPSG:32631>
Name: WGS 84 / UTM zone 31N
Axis Info [cartesian]:
- E[east]: Easting (metre)
- N[north]: Northing (metre)
Area of Use:
- name: Between 0°E and 6°E, northern hemisphere between equator and 84°N,
onshore and offshore. Algeria. Andorra. Belgium. Benin. Burkina Faso. Denmark -
North Sea. France. Germany - North Sea. Ghana. Luxembourg. Mali. Netherlands.
Niger. Nigeria. Norway. Spain. Togo. United Kingdom (UK) - North Sea.
- bounds: (0.0, 0.0, 6.0, 84.0)
Coordinate Operation:
- name: UTM zone 31N
- method: Transverse Mercator
Datum: World Geodetic System 1984 ensemble
- Ellipsoid: WGS 84
- Prime Meridian: Greenwich
```

[52]:
```
'''
Two functions:
Create distance weights, which takes an array of coordinates and a threshod in␣
↪metres.
Get centre points, assumes dataframe axis in metres and gets the centre points␣
↪of each area in dataframe
Source: lab-04-part-01
'''

def create_distance_weights(pts, threshold):#input is threshold (m) and␣
↪coordinates
    w_distkm = weights.DistanceBand.from_array(pts, threshold) #use from_array␣
↪as input is not matrix but coordinates
    return w_distkm #return matrix of for certain threshold neighbours

# test
def get_centre_points(dataframe): #define function input dataframe (axis in␣
↪metres)
    #Source: lab-04-part-01
    cents = dataframe.centroid #gets centre points for dataframe
    pts = np.array([(pt.x, pt.y) for pt in cents]) #splits them into a list of␣
↪tuples for x,y of each centre point
    return pts #returns list of coordinates
```

```
#test function
pts=get_centre_points(ams) #get points for amsterdam dataframe
w_dist5km=create_distance_weights(pts, 5000) #get neighbourhood matrix for each␣
↪in threshold of 5km
w_dist5km[1] #display 5km neighbour from centre of Noord-oost
```

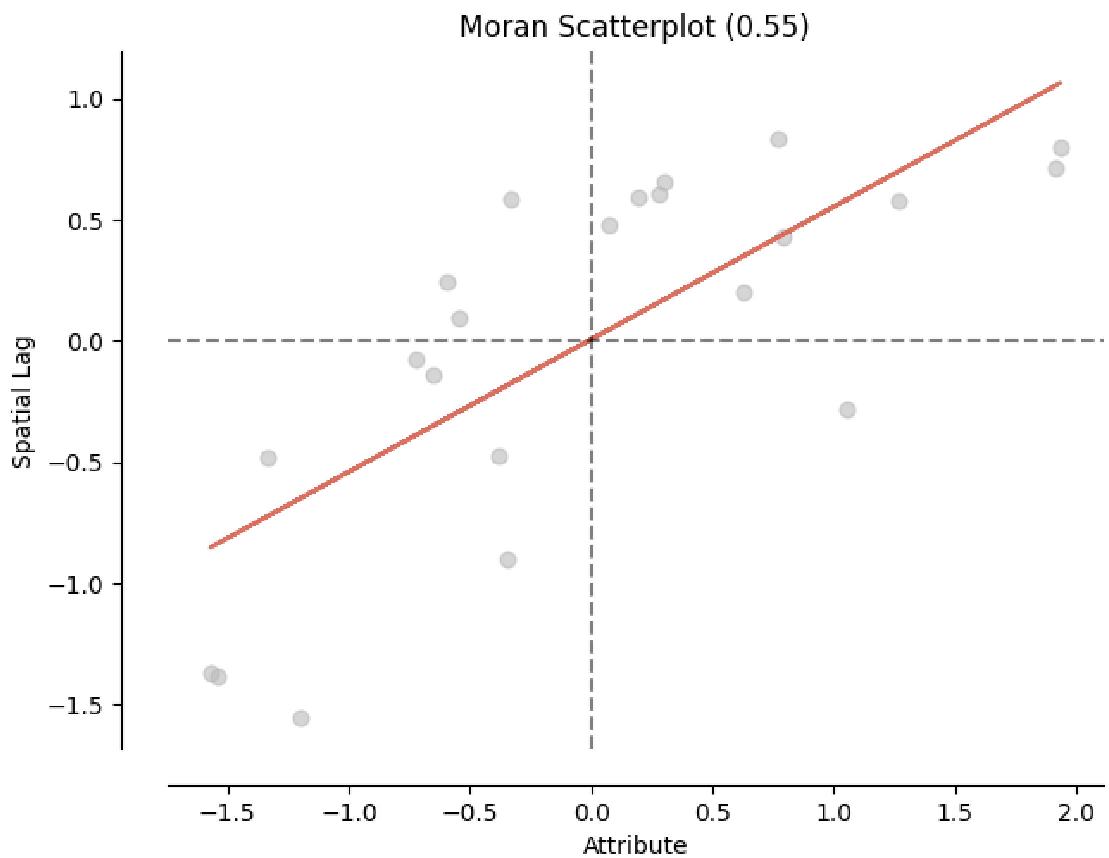
[52]: {4: 1.0}

Question 5: Visualizing a Moran Plot

[53]:
```
'''
Use esda.Moran to plot attribute against spatial lag
Save to hardcoded file moranplot.png
Source: lab-04-part-02
'''

def plot_moran(W,y):#define function input weights neighbour matrix and list of␣
↪attribute
    mi = esda.Moran(y, W) #prepare moran
    moran_scatterplot(mi) #plot moran plot
    plt.savefig("moranplot.png") #save moran plot

plot_moran(w_queen, ams['average_price']) #test with amsterdam contiguity␣
↪matrix on attribute average airbnb price in amsterdam
```


Moran Scatterplot (0.55)
Spatial Lag
Attribute
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0