PS1-Structural Macroeconometrics

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1 Data and Sample

The empirical analysis is based on the quarterly U.S. macroeconomic dataset **BCF_US _quarterlydata.xls**, which covers the period from 1954Q3 to 2012Q1. Following the assignment instructions, we restrict the sample to the period from 1960Q1 to 2008Q2. The starting date allows for a short initial burn–in and excludes the early part of the sample where measurement and structural breaks may be more severe, while the ending date ensures that the analysis is not contaminated by the unconventional monetary policy measures adopted by the Federal Reserve during and after the global financial crisis.

We consider the following seven variables:

- $LNRPCNDSVPC$: real per capita personal consumption of non-durables and services, deflated with the GDP deflator, in logs. This will be denoted by CONS_NDS_t.

- $LNRPCDGPC$: real per capita personal consumption of durables, deflated with the GDP deflator, in logs. This will be denoted by CONS_DUR_t.

- $LNRINVPC$: real per capita private fixed investment, deflated with the GDP deflator, in logs, denoted by INV_t.

- $LNRGDPPC$: real per capita GDP, deflated with the GDP deflator, in logs, denoted by Y_t.

- $INFLGDPQ$: quarterly inflation rate based on the GDP deflator, denoted by INFL_t.

- $FFRQ$: quarterly federal funds rate, denoted by FFRATE_t.

- $GS10YRQ$: quarterly 10-year Treasury constant maturity rate, denoted by 10YRRATE_t.

Collecting these variables in a vector, the $M = 7$-dimensional observed process is

$$W_t = \begin{bmatrix} Y_t \\ \text{CONS\_NDS}_t \\ \text{CONS\_DUR}_t \\ \text{INV}_t \\ \text{INFL}_t \\ \text{FFRATE}_t \\ \text{10YRRATE}_t \end{bmatrix}, \qquad t = 1960\text{Q1}, \dots, 2008\text{Q2}.$$

2 Reduced-form VAR specification and OLS estimation

To characterise the joint dynamics of these variables and identify a monetary policy (MP) shock, we estimate a reduced-form vector autoregression of order $p = 4$:

$$W_t = \omega + A_1 W_{t-1} + A_2 W_{t-2} + A_3 W_{t-3} + A_4 W_{t-4} + \varepsilon_t, \tag{1}$$

where ω is a $(M \times 1)$ intercept vector, A_i are $(M \times M)$ coefficient matrices, and ε_t is a $(M \times 1)$ vector of reduced-form innovations with zero mean and covariance matrix

$$\Sigma_u = \mathbb{E}(\varepsilon_t \varepsilon_t').$$

Let T denote the total number of time observations and $T^* = T - p$ the effective sample size after losing the first p observations. We can stack the VAR in matrix form. Define the $T^* \times M$ matrix of dependent variables

$$Y = \begin{bmatrix} W'_{p+1} \\ W'_{p+2} \\ \vdots \\ W'_T \end{bmatrix},$$

and the $T^* \times q$ matrix of regressors

$$X = \begin{bmatrix} 1 & W'_p & W'_{p-1} & \dots & W'_1 \\ 1 & W'_{p+1} & W'_p & \dots & W'_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W'_{T-1} & W'_{T-2} & \dots & W'_{T-p} \end{bmatrix},$$

where $q = 1 + Mp$ is the total number of regressors per equation (intercept plus p lags of all M variables). The coefficient matrices can be stacked as

$$B = \begin{bmatrix} c & A_1 & A_2 & A_3 & A_4 \end{bmatrix}' \in \mathbb{R}^{q \times M},$$

and the reduced-form residuals as

$$\varepsilon = \begin{bmatrix} \varepsilon'_{p+1} \\ \varepsilon'_{p+2} \\ \vdots \\ \varepsilon'_T \end{bmatrix} \in \mathbb{R}^{T^* \times M}.$$

In compact matrix form, the VAR can be written as

$$Y = XB + \varepsilon. \tag{2}$$

The reduced-form VAR is estimated equation-by-equation by ordinary least squares (OLS). In matrix notation, the OLS estimator of B is

$$\hat{B} = (X'X)^{-1}X'Y, \tag{3}$$

and the residuals are given by

$$\hat{\varepsilon} = Y - X\hat{B}.$$

The estimated residual covariance matrix is

$$\hat{\Sigma}_\varepsilon = \frac{1}{T^* - q}\hat{\varepsilon}'\hat{\varepsilon}. \tag{4}$$

Compact (stacked) representation

For asymptotic theory and identification it is convenient to also write the VAR in fully stacked form. Let

$$W^* = \text{vec}(Y) \in \mathbb{R}^{T^*M}, \qquad \Pi^* = \text{vec}(B) \in \mathbb{R}^{qM}, \qquad \varepsilon^* = \text{vec}(\epsilon) \in \mathbb{R}^{T^*M},$$

and define the block-diagonal regressor matrix

$$X^* = I_M \otimes X \in \mathbb{R}^{T^*M \times qM},$$

where $\otimes$ denotes the Kronecker product and I_M is the $M \times M$ identity matrix. Then the VAR can be written as a single multivariate regression:

$$W^* = X^*\Pi^* + \varepsilon^*. \tag{5}$$

The OLS estimator of Π^* is

$$\hat{\Pi}^* = (X^{*\prime}X^*)^{-1}X^{*\prime}W^*, \tag{6}$$

and, by construction,

$$\hat{\Pi}^* = \mathrm{vec}(\hat{B}).$$

Thus, estimating the VAR equation-by-equation or via the compact stacked form is algebraically equivalent in this setup, since all equations share the same set of regressors.

2.1 Identification and impulse response analysis

To interpret the dynamic effects of structural monetary policy shocks, we postulate a standard Choleski-SVAR structure. Let ε_t denote the reduced-form innovations and u_t the structural shocks, with

$$\varepsilon_t = Pu_t, \qquad \mathbb{E}(u_t u_t') = I_M, \tag{7}$$

where P is a non-singular $(M \times M)$ impact matrix. Under Choleski identification, P is restricted to be lower triangular. Imposing $PP' = \Sigma_\varepsilon$, a natural choice for $\hat{P}$ is the (lower-triangular) Choleski factor of $\hat{\Sigma}_\varepsilon$. A one-standard-deviation monetary policy shock is then defined as a unit shock in the structural innovation associated with the federal funds rate, i.e. the column of $\hat{P}$ corresponding to FFRATE_t.

To compute impulse response functions (IRFs), we use the companion form representation of the VAR(4). Define the $4M$-dimensional state vector

$$S_t = \begin{bmatrix} W_t \\ W_{t-1} \\ W_{t-2} \\ W_{t-3} \end{bmatrix}.$$

Then the VAR(4) (1) can be written as a first-order vector autoregression:

$$S_t = \mathscr{C} S_{t-1} + P^* u_t, \tag{8}$$

where the companion matrix $\mathscr{C}$ is

$$\mathscr{C} = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ I_M & 0 & 0 & 0 \\ 0 & I_M & 0 & 0 \\ 0 & 0 & I_M & 0 \end{bmatrix} \in \mathbb{R}^{4M \times 4M},$$

and P^* embeds the impact matrix P in the state space,

$$P^* = \begin{bmatrix} P \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{4M \times M}.$$

Let e_j denote the j-th column of the $M \times M$ identity matrix, corresponding to the structural monetary policy shock (the innovation in FFRATE_t). The effect of a one-standard-deviation monetary policy shock at horizon h on the state vector is

$$S_{t+h} - S_{t+h}^{(\text{no shock})} = \mathscr{C}^h P^* e_j.$$

The impulse response of the original variables W_t is obtained by selecting the first M components of this vector. Let $R = [I_M \ 0 \ 0 \ 0]$ be the $(M \times 4M)$ selection matrix that picks out the current variables from S_t. Then the IRF of W_t at horizon h to the monetary policy shock j is

$$\Theta_h^{(j)} = R\mathscr{C}^h P^* e_j, \qquad h = 0, 1, \ldots, H, \tag{9}$$

where in the assignment we focus on $H = 24$ quarters.

In the empirical implementation, these IRFs are computed using the estimated coefficient matrices $\hat{A}_1, \ldots, \hat{A}_4$, the estimated impact matrix $\hat{P}^*$, and the corresponding companion matrix $\hat{\mathscr{C}}$ and impact matrix $\hat{P}^*$. Moreover, to quantify uncertainty around the estimated IRFs, we construct 90% bootstrap confidence intervals using a residual bootstrap scheme based on resampling the reduced-form residuals $\hat{\varepsilon}_t$ and re-estimating the VAR system across bootstrap replications. In our setup, the choice of the number of lags follows the broad consensus in the VAR literature, which commonly adopts four quarterly lags. This choice reflects a balance between information-criteria selection—which typically under-penalize lag length in small and medium-sized systems—and the results of likelihood-ratio test in Gretl, which indicate that at least four lags are required.

3 Variables ordering and interpretation

Following the standard Choleski-SVAR identification procedure, we address the restrictions in the variance-covariance matrix Σ_ε imposing $\frac{m(m+1)}{2}$ free parameters. In particular the ordering follows the usual assumption that only few variables responds simultaneously to structural shocks. The variables are ordered from the least contemporaneously responsive (first) to the most responsive (last), so that each structural shock affects only the disturbances of the variables below it on impact. In our specification, real activity variables are placed first and are therefore assumed to react only to their own structural disturbances on impact. Inflation, appearing after the real block, is allowed to respond contemporaneously to shocks in output and spending. The short-term federal funds rate reacts instantaneously to macroeconomic and price shocks, reflecting the high-frequency adjustment of monetary policy to incoming information. Finally, long-term interest rates, as the fastest-moving financial variables, are placed last and are allowed to adjust contemporaneously to all structural shocks in the system, including monetary policy.

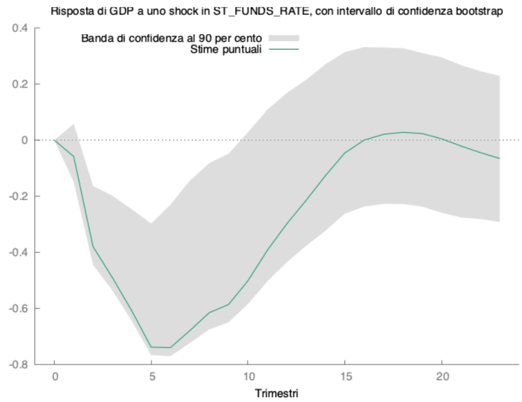
$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Mt} \end{bmatrix} = \begin{bmatrix} p_{11} & 0 & \cdots & 0 \\ p_{21} & p_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_{M1} & p_{M2} & \cdots & p_{MM} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{Mt} \end{bmatrix}. \tag{10}$$

Under the Cholesky identification, the variables are ordered from the least contemporaneously responsive (first) to the most responsive (last), so that each structural shock affects only the variables -the disturbances- below it on impact.

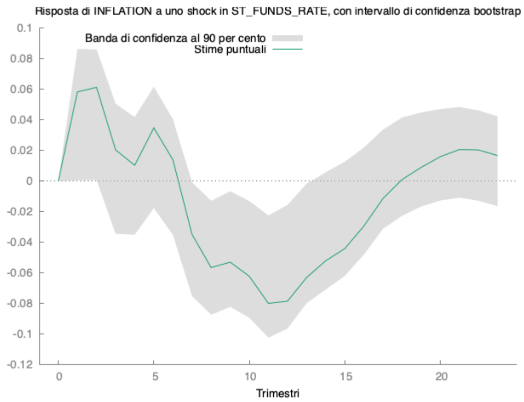
The results are overall consistent with economic theory (Figure 2) : real variables react negatively to a monetary tightening surprise, with similar persistence across output, consumption and investment. However, the magnitude of the response of investment and durable consumption-two components well known for their high interest-rate sensitivity-is much larger, with investment reaching a trough of approximately -2% around 18 months after the shock. A notable exception concerns the response of inflation, which displays a short-run increase following the monetary policy tightening. This "price puzzle" is a well-documented empirical regularity (Sims, 1992; Christiano, Eichenbaum and Evans, 1996) and typically arises when the VAR omits variables that the central bank observes and that contain information about future inflation, such as commodity prices. In such cases, the identified policy shock is contaminated by non-policy disturbances that signal rising expected inflation to the central bank, generating an apparent increase in prices immediately after the policy tightening. In line with this literature, the presence of a price puzzle in our results likely reflects this omitted-variable mechanism rather than a genuine expansionary effect of contractionary monetary policy. However, this price puzzle in our setting is quite weak in magnitude and significative only in the first 9 months after the monetary surprise.

3.1 Pre and Post 1984

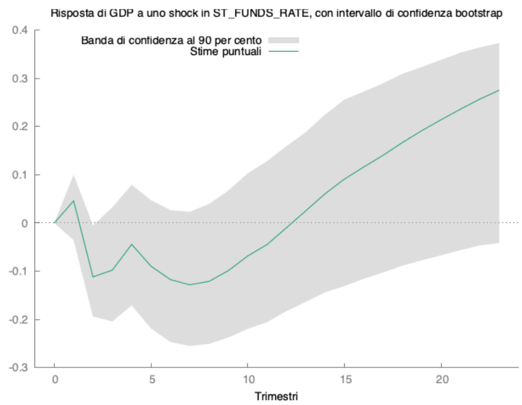
To explore whether the transmission of monetary policy has changed over time, we repeat the same Cholesky-SVAR analysis separately over two subsamples: 1960Q1–1983Q4 and 1984Q1–2008Q2 (Figure 1). The break date corresponds to the beginning of the so-called "Great Moderation", a period often associated with structural changes in the conduct and effectiveness of monetary policy. The results display a clear shift in the propagation mechanism of monetary policy shocks. In the first subsample, a contractionary monetary policy shock generates a sharp and statistically significant decline in real activity: output, consumption (both non-durable and durable), and investment all fall markedly, with deeper and more persistent troughs than in the full-sample estimates. Conversely, in the post-1984 subsample the responses of real variables are considerably milder. Output and consumption still decrease after a tightening, but the effects are smaller in magnitude and often not statistically significant at 90% confidence levels, while investment displays a markedly reduced sensitivity compared to the earlier period. Overall, the evidence suggests that monetary policy shocks had substantially stronger real effects before 1984, consistent with the view that the monetary transmission mechanism weakened during the Great Moderation. Regarding inflation, both subsamples display a mild "price puzzle": the response of inflation is slightly positive on impact, but in neither subsample is this increase statistically significant, reinforcing the interpretation that this pattern reflects noise or omitted-variable bias rather than a genuine expansionary effect of monetary tightening.



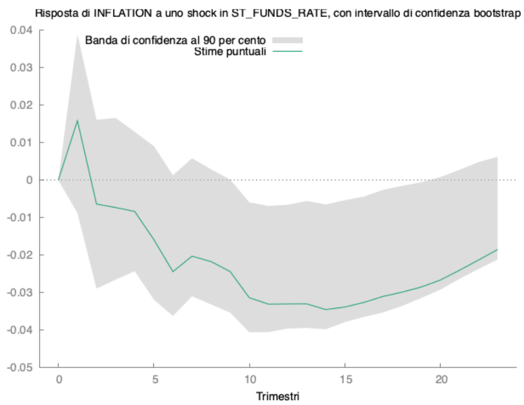
(a) Output response, 1960Q1–1983Q4



(b) Inflation response, 1960Q1–1983Q4

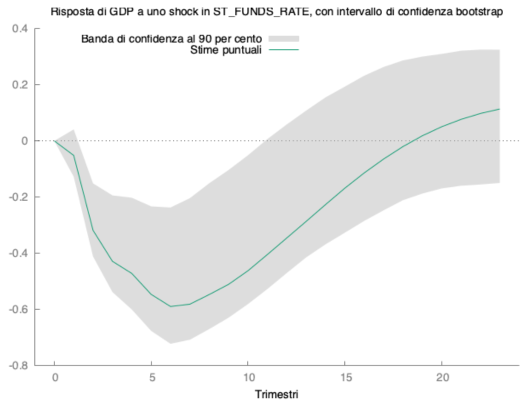


(c) Output response, 1984Q1–2008Q2

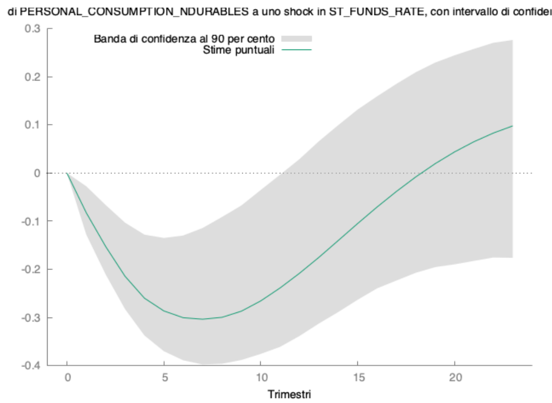


(d) Inflation response, 1984Q1–2008Q2

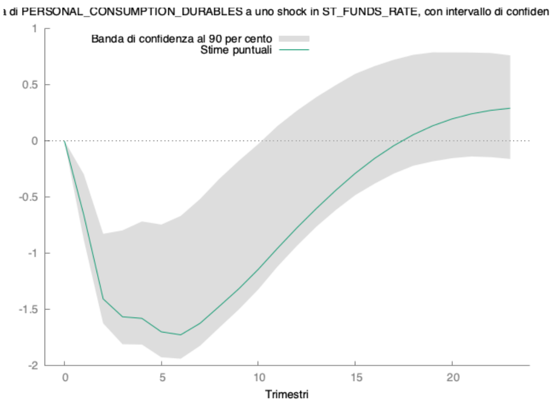
Figure 1: Impulse responses of output and inflation to a monetary policy shock across sub-samples.



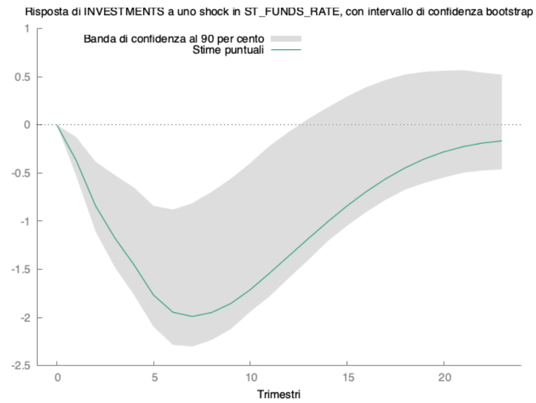
(a) Y_t



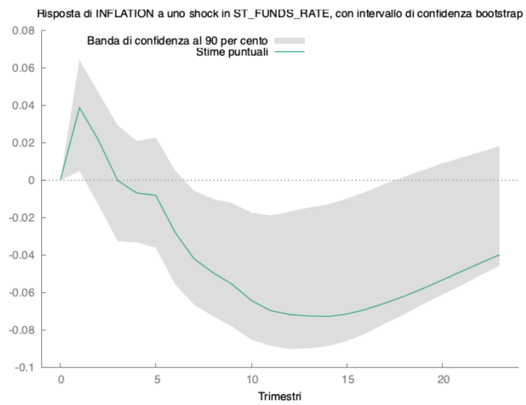
(b) CONS_NDS_t



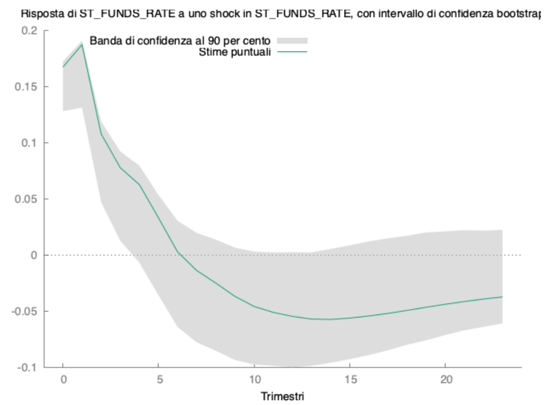
(c) CONS_DUR_t



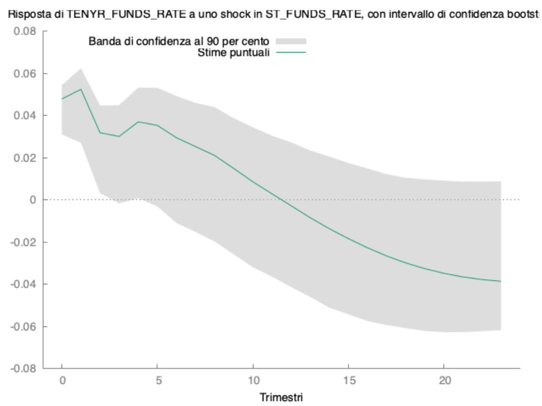
(d) INV_t



(e) INFL_t



(f) FFRATE_t



(g) 10YRRATE_t

Figure 2: Impulse responses to a tightening monetary policy shock over the full sample.