Task report

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Analitical calculation

The code plot of the $P_F, P_{SF}, P_{SF}^{new}$ for ε, t, m, n was written in python, according the analytic formulas:

$$P_F = 1 - \sum_{j=0}^{t} \binom{n}{j} \varepsilon^j \left(1-\varepsilon\right)^{n-j}$$

$$P_{SF} = 1 - \left[\left(1-p_f\right)^m\right]$$

$$P_{SF}^{new} = 1 - \left[\left(1-p_f\right)^m + m \times p_f \times \left(1-p_f\right)^m\right]$$

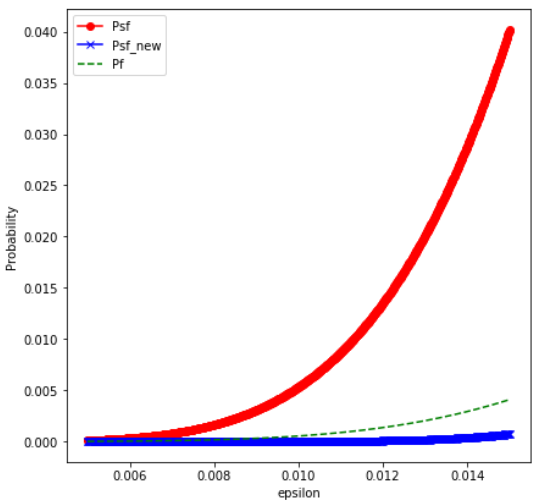
A little bit on the code:

The code was written in consideration of modularity and as compact as possible without losing readability, it's possible to add another functions as needed to the code and change all the drawing with almost no additional coding.

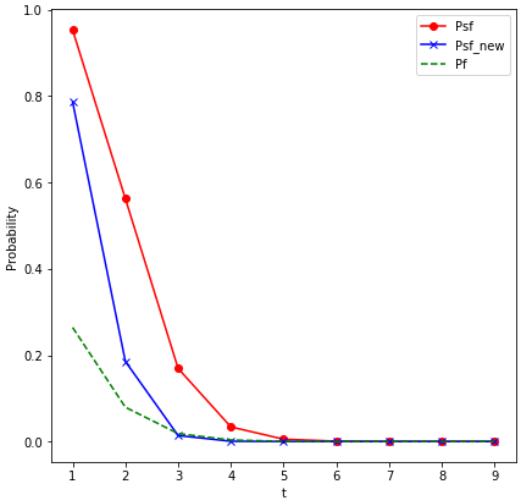
The advantages mentioned above came in a cost of ordering constraint, i.e. the order of parameters inserted to the function needed to be the same.

The graphs:

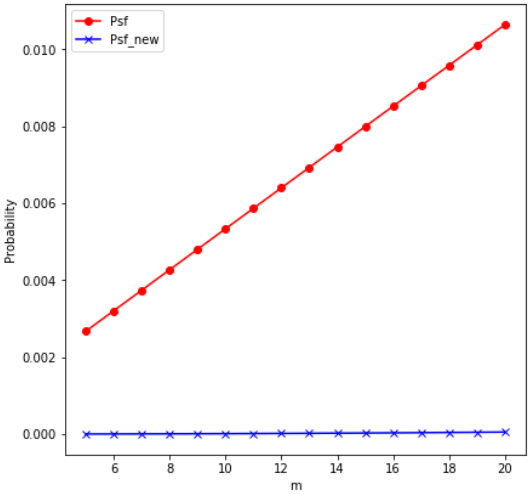
The graphs with the default values of: $\varepsilon = 10^{-2}, t = 5, m = 9, n = 100$.



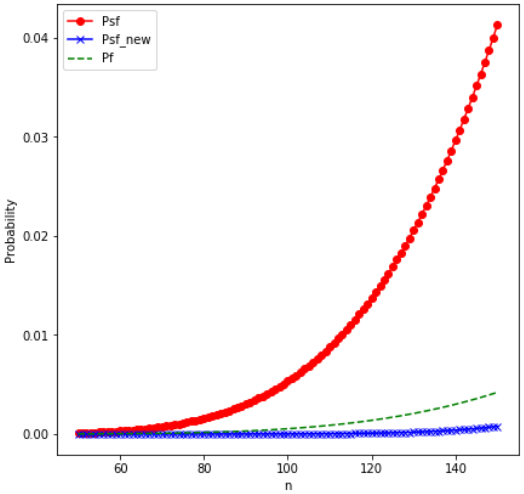
(a)



(b)



(c)



(d)

Figure (1): (a) Plot of the $P_F, P_{SF}, P_{SF}^{new}$ for different epsilon. Threshold, frames, bits are default.

(b) Plot of the $P_F, P_{SF}, P_{SF}^{new}$ for different threshold .Epsilon, frames, bits are default.

(c) Plot of the $P_F, P_{SF}, P_{SF}^{new}$ for different frames. Threshold, bits, epsilon are default.

(d) Plot of the $P_F, P_{SF}, P_{SF}^{new}$ for different bits. Threshold, frames, epsilon are default.

Because the blue line (P_{SF}) seem to be flat because it in another scale,
Hereinafter draw only of this line in smaller scale:

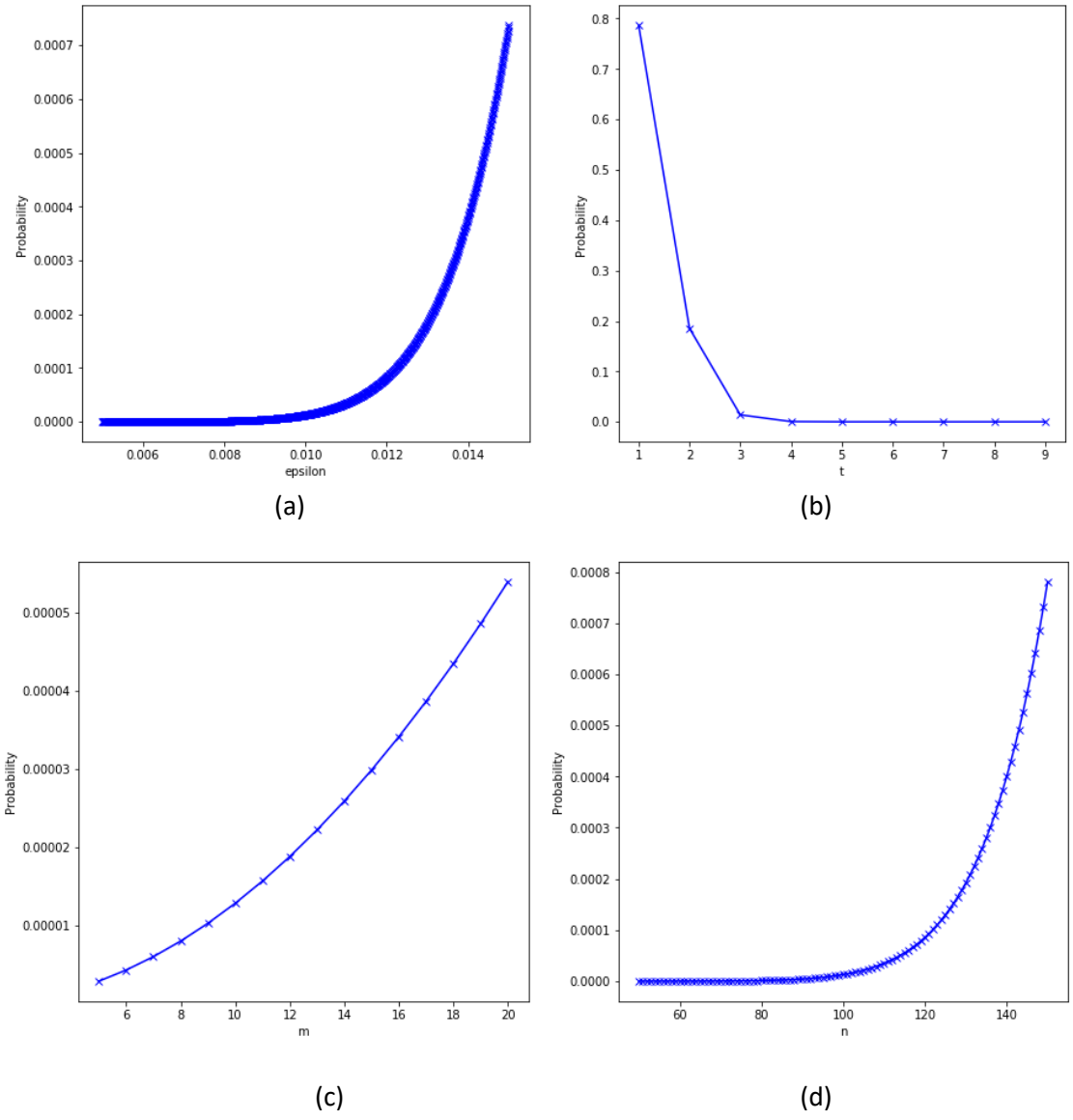


Figure (2): (a) Plot of the P_{SF}^{new} for different epsilon. Threshold, frames, bits are default.

(b) Plot of the P_{SF}^{new} for different threshold .Epsilon, frames, bits are default.

(c) Plot of the P_{SF}^{new} for different frames. Threshold, bits, epsilon are default.

(d) Plot of the P_{SF}^{new} for different bits. Threshold, frames, epsilon are default.

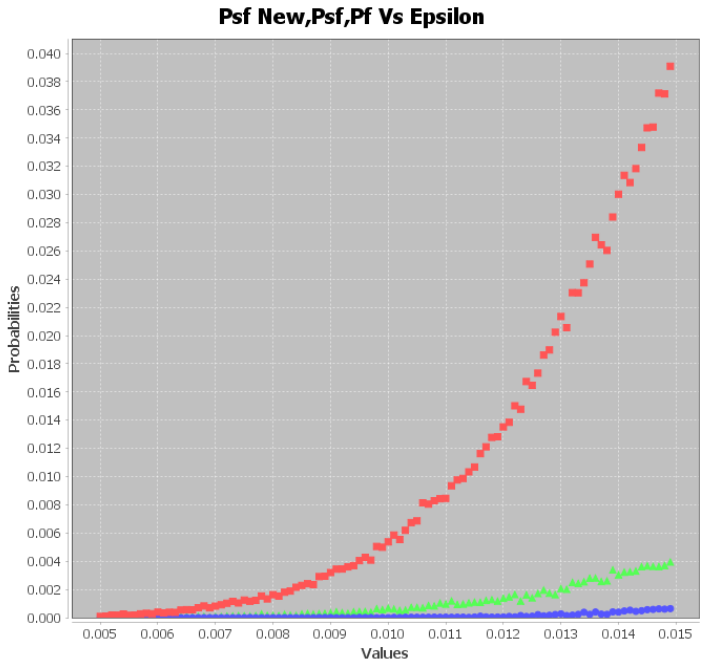
Monte Carlo simulation

Simulation of $P_F, P_{SF}, P_{SF}^{new}$ for different ε, t, m, n was written in java.

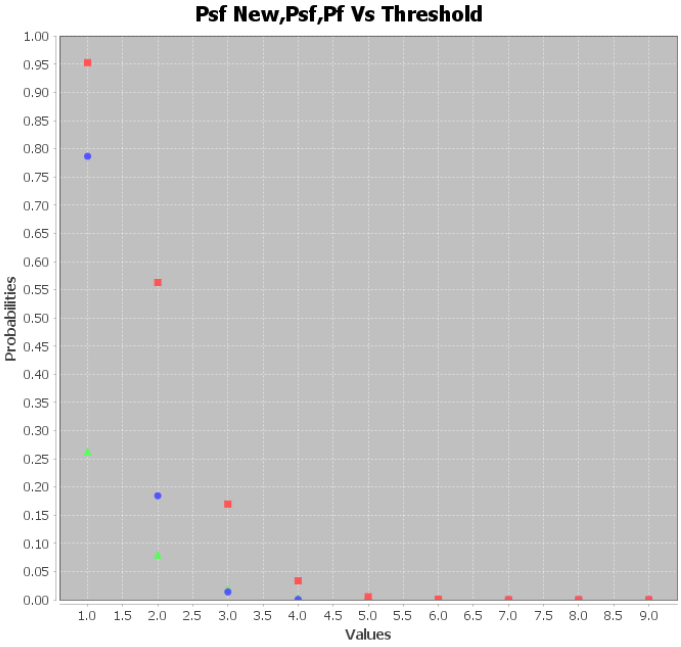
The number of repetitions for each sample was 10^6 .

Because the blue line(P_{SF}) seem to be flat because it in another scale,

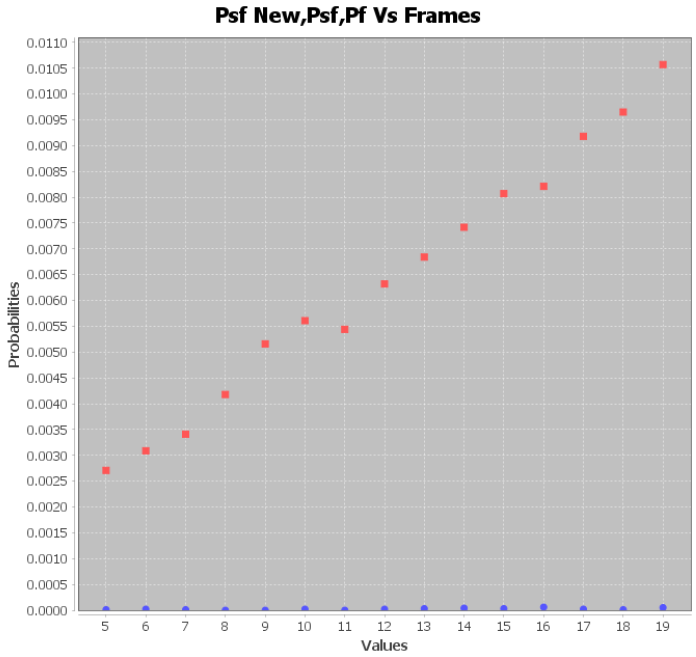
After each graph, a new graph was draw with only this line in smaller scale.



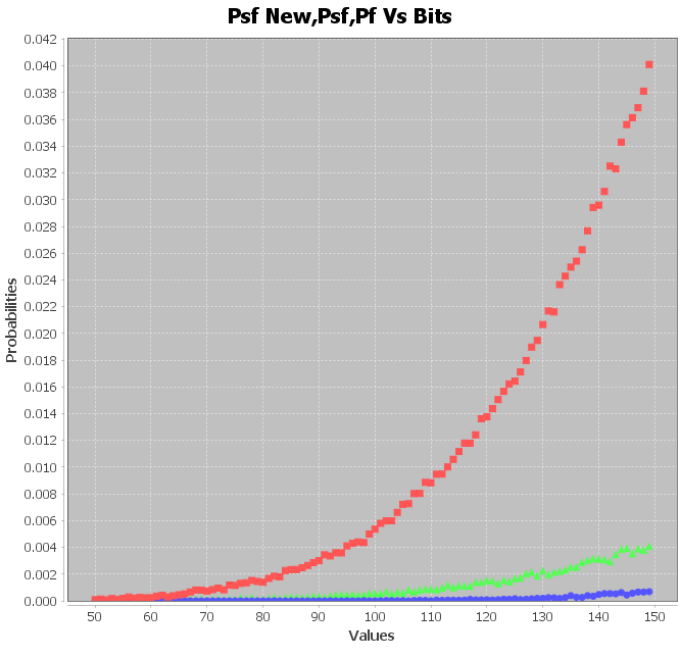
(a)



(b)



(c)



(d)

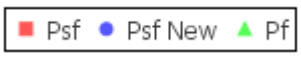
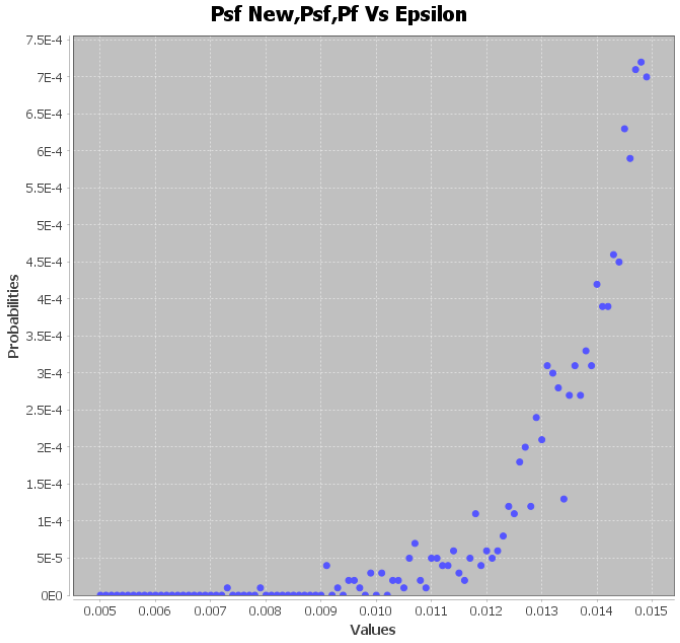


Figure (3): (a) Simulation of the $P_F, P_{SF}, P_{SF}^{new}$ *for different epsilon. Threshold, frames, bits are default.*

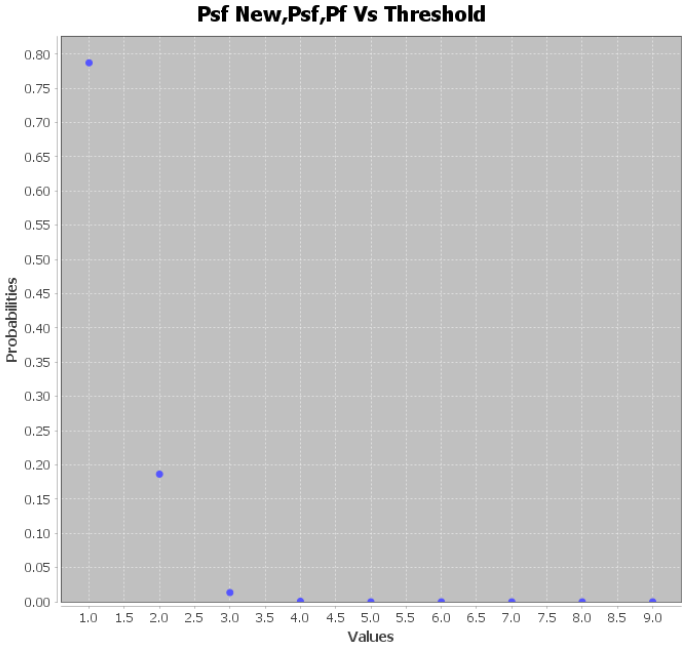
(b) Simulation of the $P_F, P_{SF}, P_{SF}^{new}$ *for different threshold .Epsilon, frames, bits are default.*

(c) Simulation of the $P_F, P_{SF}, P_{SF}^{new}$ *for different frames. Threshold, bits, epsilon are default.*

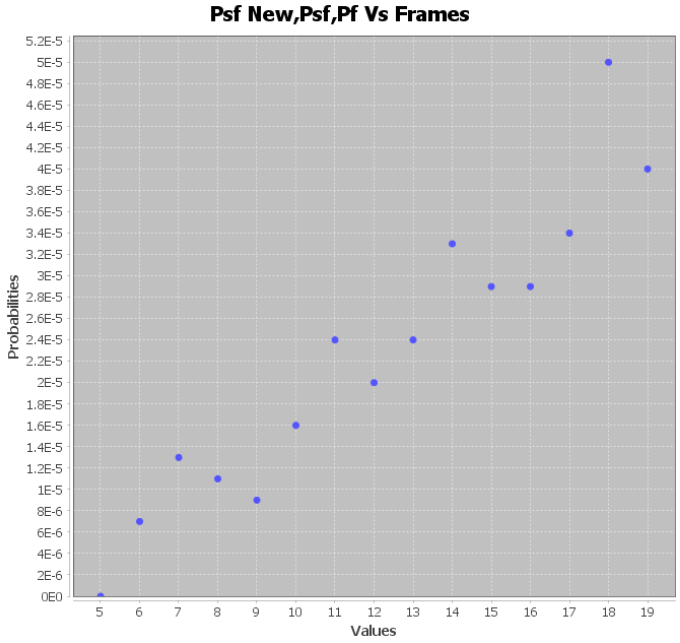
(d) Simulation of the $P_F, P_{SF}, P_{SF}^{new}$ *for different bits. Threshold, frames, epsilon are default.*



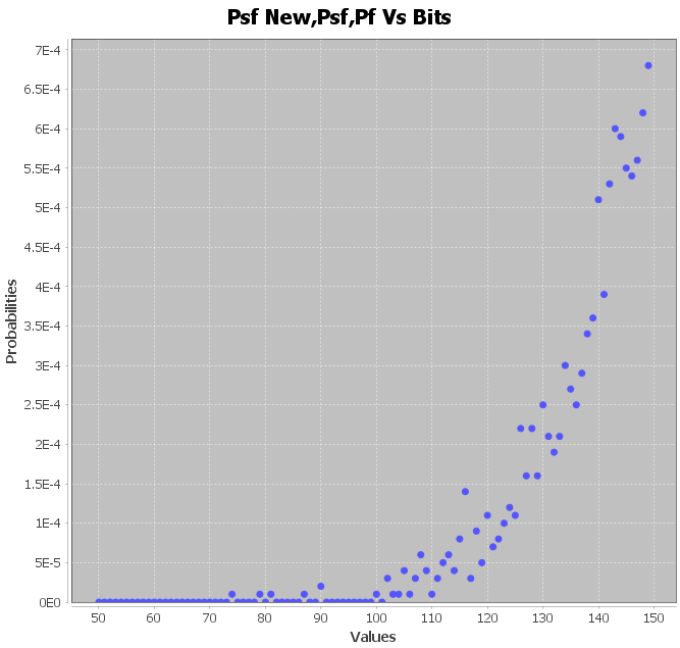
(a)



(b)



(c)



(d)

Figure (4): (a) Simulation of the P_{SF}^{new} *for different epsilon. Threshold, frames, bits are default.*

(b) Simulation of the P_{SF}^{new} *for different threshold .Epsilon, frames, bits are default.*

(c) Simulation of the P_{SF}^{new} *for different frames. Threshold, bits, epsilon are default.*

(d) Simulation of the P_{SF}^{new} *for different bits. Threshold, frames, epsilon are default.*

Conclusions

- In section (a), it is clear that the probability of errors are as follows:

  $$P_{SF} > P_{SF}^{new}$$

  So improvement has been made in the new super frame.
  Furthermore:

  $$P_{SF} > P_F$$

  The error of the super frame is as expected greater than the error of the single frame, because it compose of multiple frames.
  In total :

  $$P_{SF}\left(\sim \mathrm{m}\times P_F\right) > P_F > P_{SF}^{new}\left(\sim \mathrm{m}^2\times P_F^2\right)$$

- A clear agreement between the numeric simulation and the theoretical calculation is obtained.
- Even though the typical scale of $\varepsilon = 10^{-3}$, the value of $\varepsilon = 10^{-2}$ were choose because, the number of repetitions in the simulation should be at least one magnitude of order bigger than the one that been used(10^{-6}), it was time consuming.

* The code of the simulation and the analytic calculation is attached.