Time value of money

Real-Estate Investments - Tutorial 1

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Time value of money

Value of an initial deposit at the beginning (PV) and end of a period (FV) can be expressed by :

$$FV = PV * (1 + i)^T$$

The relationship between PV and FV depends on compunding interstest rates. Example :

- PV = 100,000
- i = 0.1

After 1 year: $100000 * (1 + 0.1)^1 = 110,000$

After 2 years: $100000 * (1 + 0.1)^2 = 121,000$

After 10 years: $100000 * (1 + 0.1)^{10} = 259,374.2$

Time value of money - Compounding intervals

To adjust for different compunding interavls m we apply:

$$FV = PV * (1 + \frac{i}{m})^{T*m}$$

or:

$$PV = \frac{FV}{(1 + \frac{i}{m})^{T*m}}$$

Examples of future value of 100,000 Euro after 1 year, compunded:

$Quarterly : 100000 * (1 + \frac{0.1}{4})^4 = 110,381.3$

$Monthly : 100000 * (1 + \frac{0.1}{12})^{12} = 110,471.3$

$Daily : 100000 * (1 + \frac{0.1}{365})^{365} = 110,515.6$

Annuity

Often we are interested in the value of a series of payments, defined as *annuity*.

The basic annuity formula refers to the payment of an amount P over T periods , while the interest rate is equal to i.

$$FV = P(1+i)^{T-1} + P(1+i)^{T-2} + \ldots + P(1+i) + P$$

Example:

- $P = 1{,}000$
- $i = 5\%$
- $T = 10$ years

$1000 * (1.05)^9 + \ldots + 1000 * (1.05) + 1000 = 12,577.89$

Present value of annuity

The *present value* of annuity payments is particularly relevant for investment decision. It is equal to:

$$L = \frac{P}{(1+i)} + \frac{P}{(1+i)^2} + \ldots + \frac{P}{(1+i)^T}$$

or:

$$L = \frac{P}{i}(1 - \frac{1}{(1+i)^T})$$

Exercises

Ex. 1

Jim makes a deposit of 12,000 Euro in a bank account. The deposit is to earn interest compounded annually at the rate of 6% for 7 years.

1. How much will Jim have on deposit at the end of 7 years?
2. Assuming the deposit earned a 9% rate of interest compounded quarterly, how much would he have at the end of 7 years?
3. In comparing (1) and (2), what are the respective effective annual yields?

Ex. 1 Answers

1. $12,000 * (1 + 0.06)^7 = 18,043.56$
2. $12,000 * (1 + \frac{0.09}{4})^{7*4} = 22,374.54$
3. $(Q1) : (1 + \frac{18043-12000}{12000})^{\frac{1}{7}} - 1 = 6\%$
   $(Q2) : (1 + \frac{22374-12000}{12000})^{\frac{1}{7}} - 1 = 9.3\%$

Ex. 2

An investor can make an investment in a real estate development and receive an expected cash return of 45,000 Euro at the end of 6 years. She believes that a 9% annual return compounded quarterly is a reasonable return on this investment.

1. How much should she pay for it today?

An investor is considering an investment that will pay 2,150 Euro at the end of each year for the next 10 years. He expects to earn a return of 12% on his investment, compounded annually.

2. How much should he pay today for the investment?
3. How much should he pay if the investment returns are received at the beginning of each year?

Ex. 2 Answers

1.

$$PV = \frac{45{,}000}{(1+\frac{0.09}{4})^{6*4}} = 26,381.1$$

2.

$$PV = \frac{2150}{1.12} + \frac{2150}{1.12^2} + \ldots + \frac{2150}{1.12^{10}} =$$

$$\frac{2150}{0.12}[1 - \frac{1}{(1+0.12)^{10}}] = 12,147.98$$

3.

$$PV = 2150 + \frac{2150}{1.12} + \frac{2150}{1.12^2} + \ldots + \frac{2150}{1.12^9} =$$

$$\frac{2150}{0.12}[1 - \frac{1}{(1+0.12)^{9}}] = 13,605.74$$

Ex. 3

You want to buy a house with price €250,000. Since you didn't save, the house has to be completely financed. The bank offers you an annuity mortgage with an interest rate of 6% and a term of 25 years.

1. What is the annual amount you have to pay on interest and amortization per year? (*Hint:* Use the annuity formula to recover the annual payments)

After 5 years the interest rate has decreased to 4%, and you consider refinancing the loan.

2. What is the outstanding amount of the loan after 5 years?
3. What will be the annual payment if it is transferred to an annuity mortgage loan with an interest rate of 4% and a term of 20 years?

Ex .3 Answers

1. Invert the annuity formula : $p = \frac{L*i}{1-\frac{1}{(1+i)^T}}$

Using the formula derived above: $\frac{250000*0.06}{1-\frac{1}{1.06}^{25}} = 19,556.68$

2. The outstanding amount is the present value of an annuity that pays 19,556 over a term of $25-5=20$ years, while the interest rate is 6%: $\frac{19,556.68}{0.06}(1-\frac{1}{(1+0.06)^{20}}) = 224,313.6$
3. The annual payment of the new loan is:
$\frac{224,313.6*0.04}{1-\frac{1}{1.04}^{20}} = 16,505.39$