Chapter 5 Homework

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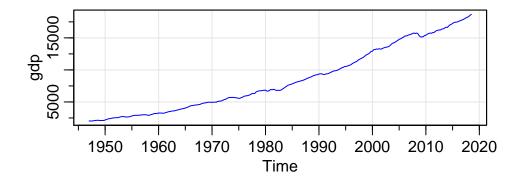
5.2

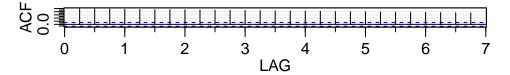
In Example 5.6, we fit an ARIMA model to the quarterly GNP series. Repeat the analysis for the US GDP series in gdp. Discuss all aspects of the fit as specified in the points at the beginning of Section 5.2 from plotting the data to diagnostics and model choice.

To fit an ARIMA model to quarterly data, we follow the following steps: - plot the data - possibly transform the data - identify dependence orders of the model - estimate the parameters - display diagnostics - choose a model

First we graph a tsplot of the gdp data and see what the ACF looks like.

```
layout(1:2, heights = 2:1)
tsplot(gdp, col = 4)
acf1(gdp, main = "")
```

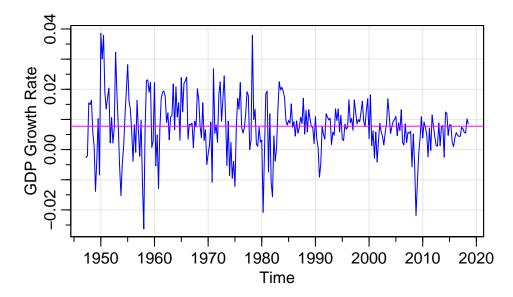




```
## [1] 0.99 0.98 0.97 0.96 0.95 0.94 0.93 0.92 0.91 0.90 0.89 0.88 0.87 0.86 0.85 ## [16] 0.84 0.83 0.82 0.81 0.80 0.79 0.78 0.77 0.76 0.75 0.74 0.73
```

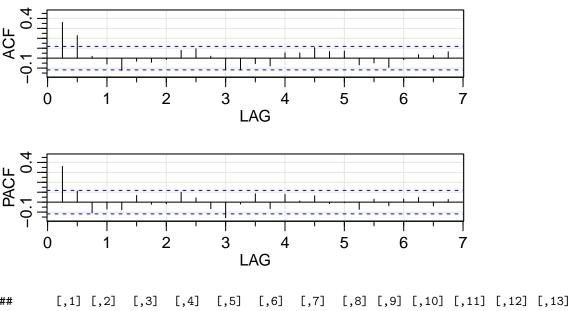
The plot shows a steady incline of gdp data over time, and the trend covers up other potential effects. We might use log to display the data in terms of growth rate rather than actual data.

```
par(mfrow = c(1, 1))
tsplot(diff(log(gdp)), ylab = "GDP Growth Rate", col = 4)
mean_dif = mean(diff(log(gdp)))
abline(h = mean_dif, col = 6)
```



The logged/differenced data is much more stable. Now we can investigate the ACF and PACF to fit the ARIMA model.

```
acf2(diff(log(gdp)), main = "")
```

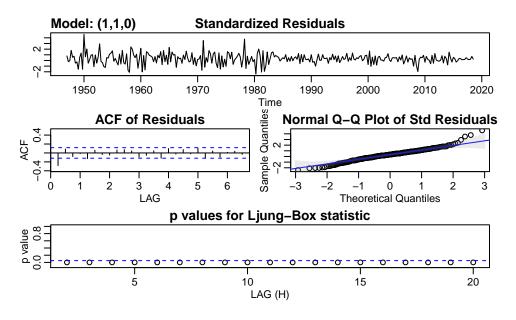


```
## PACF 0.08 -0.06 0.08 0.01 0.06 -0.02 -0.01 -0.07 0.03 -0.04 0.03 0.05 ## [,26] [,27] ## ACF 0.03 0.06 ## PACF -0.04 0.03
```

We can see that the ACF cuts off around lag 2 and the PACF cuts off around 2, which suggests an ARMA(1, 2) process after differencing, or ARIMA(1, 1, 2) on the logged data log(gdp). We now can fit the model and analyze the diagnostics. First we should try AR(1) and MA(2) alone to check out the diagnostics and compare to the ARIMA model proposed.

```
sarima(log(gdp), 1, 1, 0, no.constant = TRUE)
```

```
## initial value -4.409274
## iter
          2 value -4.653694
## iter
          3 value -4.653694
## iter
          4 value -4.653694
          4 value -4.653694
## iter
## final
         value -4.653694
## converged
## initial
            value -4.654504
          2 value -4.654507
## iter
## iter
          3 value -4.654508
## iter
          3 value -4.654508
## iter
          3 value -4.654508
## final value -4.654508
## converged
```



```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
## Q), period = S), include.mean = !no.constant, transform.pars = trans, fixed = fixed,
## optim.control = list(trace = trc, REPORT = 1, reltol = tol))
```

```
##
## Coefficients:
##
           ar1
##
        0.6203
## s.e. 0.0461
##
## sigma^2 estimated as 9.045e-05: log likelihood = 925.37, aic = -1846.75
## $degrees_of_freedom
## [1] 285
##
## $ttable
      Estimate
                   SE t.value p.value
## ar1 0.6203 0.0461 13.4416
##
## $AIC
## [1] -6.457153
##
## $AICc
## [1] -6.457103
##
## $BIC
## [1] -6.431586
sarima(log(gdp), 0, 1, 2, no.constant = TRUE)
## initial value -4.410941
## iter 2 value -4.571030
## iter 3 value -4.612413
## iter 4 value -4.623939
## iter 5 value -4.625520
## iter 6 value -4.625527
## iter 7 value -4.625527
## iter 7 value -4.625527
## iter 7 value -4.625527
## final value -4.625527
## converged
## initial value -4.625172
```

iter 2 value -4.625173
iter 2 value -4.625173
iter 2 value -4.625173
final value -4.625173

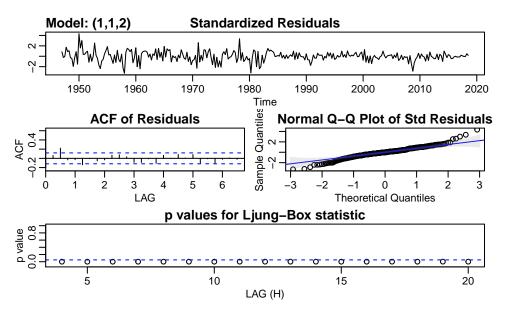
converged

```
Model: (0,1,2)
                           Standardized Residuals
                                                                      2020
        1950
                          1970
                                           1990
                                                    2000
                                                             2010
                 1960
                                   1980
                                     Time
                                     Quantiles
            ACF of Residuals
                                         Normal Q-Q Plot of Std Residuals
  0.4
                                     Sample
             2
                  3
                           5
                                6
                                                    -1
                                                         0
                                                                   2
                                                                        3
                                          -3
                                               -2
                                                  Theoretical Quantiles
                  LAG
                       p values for Ljung-Box statistic
                                                   15
                                10
                                                                      20
                                    LAG (H)
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, q))
##
       Q), period = S), include.mean = !no.constant, transform.pars = trans, fixed = fixed,
       optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
##
##
   Coefficients:
##
             ma1
                     ma2
##
         0.4560 0.3341
## s.e. 0.0607 0.0473
##
## sigma^2 estimated as 9.596e-05: log likelihood = 916.98, aic = -1827.97
##
## $degrees_of_freedom
## [1] 284
##
## $ttable
##
       Estimate
                     SE t.value p.value
## ma1
         0.4560 0.0607 7.5096
         0.3341 0.0473 7.0653
   ma2
                                        0
##
## $AIC
## [1] -6.39149
##
## $AICc
## [1] -6.391342
##
## $BIC
## [1] -6.353141
sarima(log(gdp), 1, 1, 2, no.constant = TRUE)
```

initial value -4.409274

```
## iter
        2 value -4.575603
## iter
        3 value -4.657000
        4 value -4.662588
## iter
## iter
        5 value -4.670962
## iter
         6 value -4.680038
## iter
         7 value -4.685092
         8 value -4.686879
## iter
        9 value -4.687059
## iter
## iter 10 value -4.687137
## iter
        11 value -4.687328
## iter
        12 value -4.687746
## iter
        13 value -4.688211
## iter
       14 value -4.688439
## iter 15 value -4.692273
## iter 16 value -4.693780
## iter 17 value -4.697493
## iter 18 value -4.697681
        19 value -4.697708
## iter 20 value -4.697730
## iter 21 value -4.697732
## iter 22 value -4.697732
## iter 22 value -4.697732
## final value -4.697732
## converged
## initial value -4.705641
## iter
        2 value -4.709167
## iter
        3 value -4.709318
        4 value -4.709768
## iter
## iter
        5 value -4.710408
## iter
        6 value -4.710814
        7 value -4.710946
## iter
## iter
         8 value -4.713375
## iter
         9 value -4.713640
       10 value -4.714262
## iter
## iter
        11 value -4.714305
## iter
       12 value -4.714345
## iter 13 value -4.714545
## iter 14 value -4.714875
## iter 15 value -4.715446
## iter 16 value -4.715451
       17 value -4.715794
## iter
## iter 18 value -4.715800
## iter 19 value -4.715811
## iter 20 value -4.715842
## iter 21 value -4.715904
        22 value -4.716015
## iter
## iter 23 value -4.716034
## iter
       24 value -4.716098
## iter 25 value -4.716102
## iter 26 value -4.716103
## iter 27 value -4.716115
## iter 28 value -4.716134
## iter 29 value -4.716173
## iter 30 value -4.716175
```

```
## iter 31 value -4.716195
## iter
        32 value -4.716200
## iter
        33 value -4.716200
         34 value -4.716210
## iter
## iter
         35 value -4.716219
## iter
        36 value -4.716231
## iter
        37 value -4.716233
         38 value -4.716234
## iter
## iter
         39 value -4.716234
        39 value -4.716234
## iter
## iter 39 value -4.716234
## final value -4.716234
## converged
```



```
## $fit
##
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##
       Q), period = S), include.mean = !no.constant, transform.pars = trans, fixed = fixed,
       optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
##
##
  Coefficients:
##
            ar1
                     ma1
##
         0.9997
                 -0.7205
                          -0.2591
## s.e. 0.0006
                  0.0479
                           0.0472
##
## sigma^2 estimated as 7.931e-05: log likelihood = 943.03, aic = -1878.05
## $degrees_of_freedom
## [1] 283
##
## $ttable
##
       Estimate
                    SE
                         t.value p.value
## ar1 0.9997 0.0006 1760.8080
```

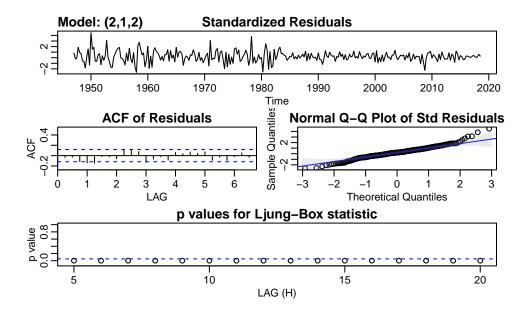
```
-0.7205 0.0479
                         -15.0370
        -0.2591 0.0472
                                         0
## ma2
                          -5.4867
##
## $AIC
##
  [1] -6.56662
##
## $AICc
## [1] -6.566322
##
## $BIC
## [1] -6.515487
```

We see that the ACF of residuals is essentially all 0, the q-q plot shows normal diagnostics, and the p-values are where we want them. We might try a higher order to see if it is even better, and if not, use AIC or BIC to choose the best.

```
sarima(log(gdp), 2, 1, 2, no.constant = TRUE)
```

```
## initial
            value -4.407567
## iter
          2 value -4.611974
## iter
          3 value -4.674509
## iter
          4 value -4.681211
## iter
          5 value -4.683491
## iter
          6 value -4.687128
          7 value -4.687295
## iter
## iter
          8 value -4.687315
## iter
          9 value -4.687327
         10 value -4.687396
## iter
## iter
         11 value -4.687423
## iter
         12 value -4.687441
## iter
         13 value -4.687467
         14 value -4.687513
## iter
## iter
         15 value -4.687587
## iter
         16 value -4.687692
## iter
         17 value -4.687872
         18 value -4.688091
## iter
## iter
         19 value -4.688336
## iter
         20 value -4.688425
## iter
         21 value -4.688551
         22 value -4.688559
## iter
## iter
         23 value -4.688682
## iter
         24 value -4.688710
## iter
         25 value -4.688719
         26 value -4.688725
## iter
         27 value -4.688744
## iter
## iter
         28 value -4.688791
## iter
         29 value -4.688806
## iter
         30 value -4.688816
## iter
         31 value -4.688839
## iter
         32 value -4.688862
         33 value -4.688872
## iter
## iter
         34 value -4.688885
## iter 35 value -4.688931
```

```
## iter 36 value -4.688974
## iter
        37 value -4.689027
         38 value -4.689080
         39 value -4.689206
## iter
## iter
         40 value -4.689322
## iter
        41 value -4.689476
## iter
         42 value -4.689613
         43 value -4.689656
## iter
## iter
         44 value -4.689698
## iter
        45 value -4.689742
## iter
        46 value -4.689785
## iter
         47 value -4.689789
        48 value -4.689791
  iter
         49 value -4.689794
  iter
## iter
         50 value -4.689795
## iter
        51 value -4.689796
## iter
        51 value -4.689796
## iter 51 value -4.689796
## final value -4.689796
## converged
## initial value -4.692166
## iter
          2 value -4.692169
          3 value -4.692183
## iter
## iter
          4 value -4.692184
## iter
          5 value -4.692184
## iter
          6 value -4.692185
## iter
          6 value -4.692185
## iter
          6 value -4.692185
## final value -4.692185
## converged
```



\$fit ## ## Call:

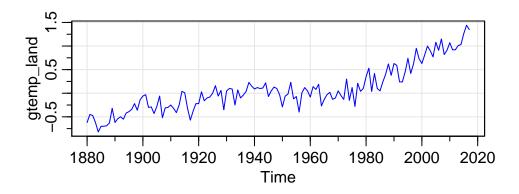
```
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
##
       Q), period = S), include.mean = !no.constant, transform.pars = trans, fixed = fixed,
##
       optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
##
  Coefficients:
##
             ar1
                     ar2
                             ma1
                                      ma2
         -0.1292
                  0.8286
                          0.5741
                                  -0.3684
##
## s.e.
         0.0637
                  0.0656 0.1048
                                   0.1053
##
## sigma^2 estimated as 8.381e-05: log likelihood = 936.15, aic = -1862.3
## $degrees_of_freedom
## [1] 282
##
## $ttable
##
       Estimate
                    SE t.value p.value
## ar1 -0.1292 0.0637 -2.0272 0.0436
         0.8286 0.0656 12.6402
         0.5741 0.1048 5.4757
                                0.0000
## ma1
## ma2
       -0.3684 0.1053 -3.4969 0.0005
##
## $AIC
## [1] -6.511527
##
## $AICc
## [1] -6.51103
##
## $BIC
## [1] -6.447611
```

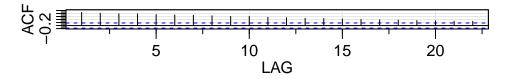
There is not significant change, and the AIC and BIC point towards using the ARIMA(1, 1, 2) model to fit the logged gdp data.

5.4

Fit an ARIMA(p, d, q) model to gtemp_land, the land-based global temperature data, performing all of the necessary diagnostics; include a model choice analysis. After deciding on an appropriate model, forecast (with limits) the next 10 years. Comment.

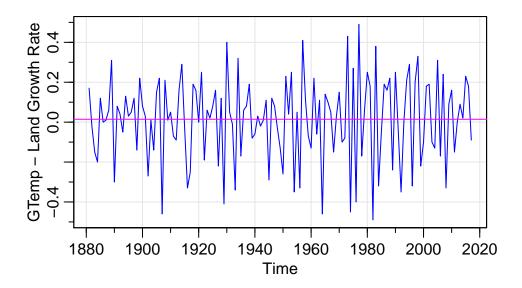
```
layout(1:2, heights = 2:1)
tsplot(gtemp_land, col = 4)
acf1(gtemp_land, main = "")
```



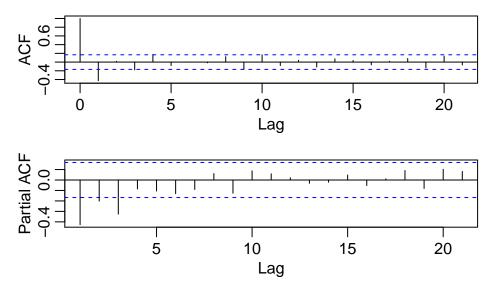


[1] 0.87 0.82 0.78 0.77 0.73 0.70 0.68 0.65 0.61 0.61 0.55 0.52 0.48 0.47 0.44 ## [16] 0.40 0.38 0.37 0.33 0.32 0.29 0.28

```
par(mfrow = c(1, 1))
tsplot(diff(gtemp_land), ylab = "GTemp - Land Growth Rate", col = 4)
mean_dif2 = mean(diff(gtemp_land))
abline(h = mean_dif2, col = 6)
```



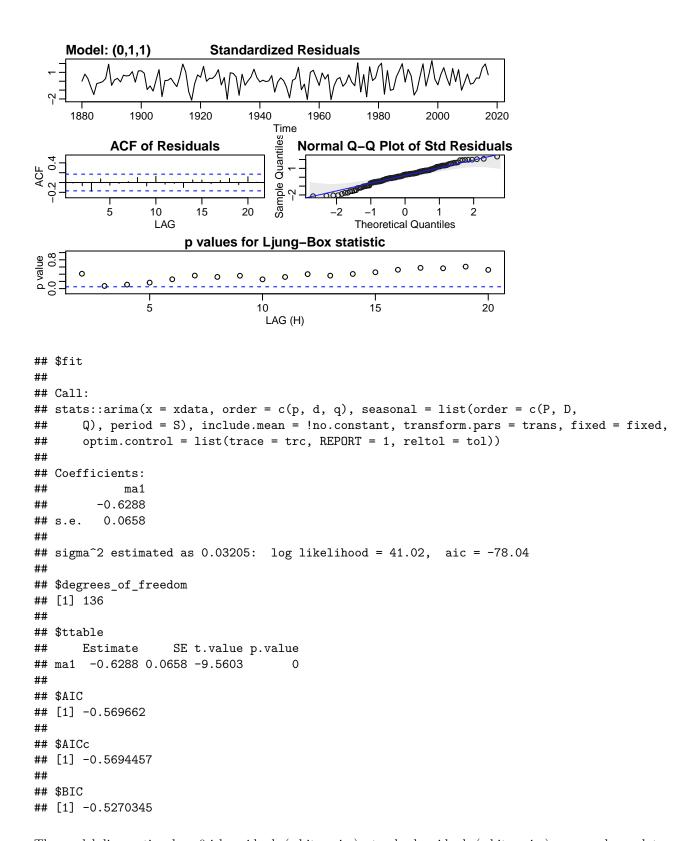
```
par(mfrow = c(2, 1))
acf(diff(gtemp_land))
pacf(diff(gtemp_land))
```



Repeating the steps mentioned in 5.2, the graph of the gtemp_land data shows a trend prime for differencing. We cannot/need not take the log this time, and we can see from the differenced data that it is much more stable. The ACF cuts off at about lag 1, and the PACF trails off, leading us to fit an MA(1) model on the differenced data, or ARIMA(0, 1, 1).

```
sarima(gtemp_land, 0, 1, 1, no.constant = TRUE)
```

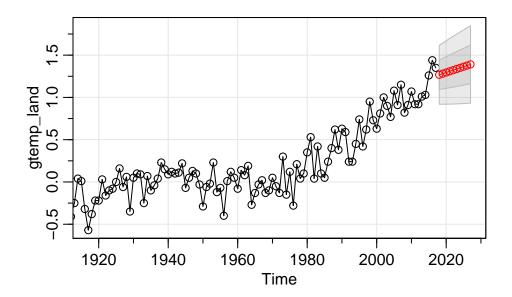
```
value -1.566843
## initial
## iter
          2 value -1.694707
  iter
          3 value -1.707823
          4 value -1.714248
  iter
##
   iter
          5 value -1.719347
          6 value -1.719752
##
   iter
## iter
          7 value -1.719812
          8 value -1.719812
## iter
## iter
          9 value -1.719812
## iter
          9 value -1.719812
          9 value -1.719812
## iter
          value -1.719812
## final
## converged
## initial
            value -1.718348
## iter
          2 value -1.718358
## iter
          3 value -1.718368
## iter
          3 value -1.718368
          3 value -1.718368
## iter
## final
          value -1.718368
## converged
```



The model diagnostics show 0-ish residuals (white noise), standard residuals (white noise), a normal q-q plot (minimal divergence from the normal line), and acceptable p-values.

Now we can forecast to the next 10 years.

```
par(mfrow = c(1, 1))
sarima.for(gtemp_land, 10, 0, 1, 1)
```



```
## $pred
## Time Series:
## Start = 2018
## End = 2027
## Frequency = 1
## [1] 1.267809 1.281445 1.295081 1.308716 1.322352 1.335987 1.349623 1.363258
## [9] 1.376894 1.390529
##
## $se
## Time Series:
## Start = 2018
## End = 2027
## Frequency = 1
## [1] 0.1741466 0.1810522 0.1877038 0.1941277 0.2003458 0.2063765 0.2122360
## [8] 0.2179380 0.2234945 0.2289163
```

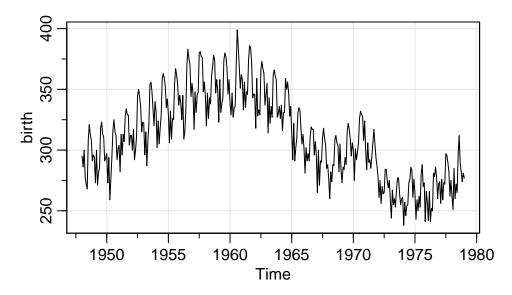
The forecast from the ARIMA(0, 1, 1) model shows a steady upward trend over the next 10 years, consistent with the trajectory of the past 30 years or so.

5.11

Fit a seasonal ARIMA model of your choice to the U.S. Live Birth Series, birth. Use the estimated model to forecast the next 12 months.

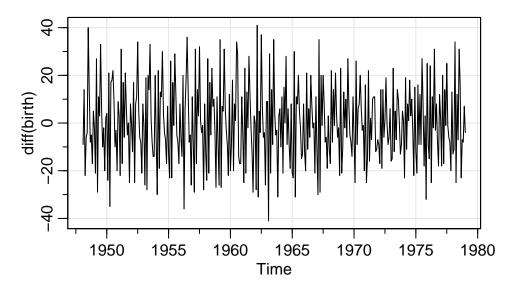
First, plot the data.

```
tsplot(birth)
```

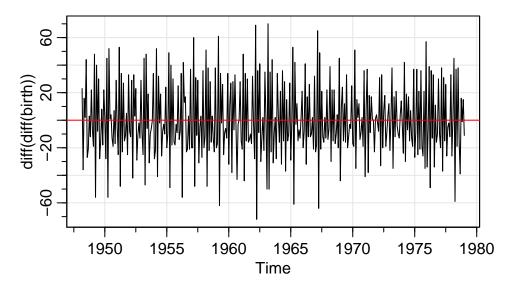


We need to transform the data before fitting a model. The first difference doesn't quite get rid of the trend, but the second one does much better.

tsplot(diff(birth))

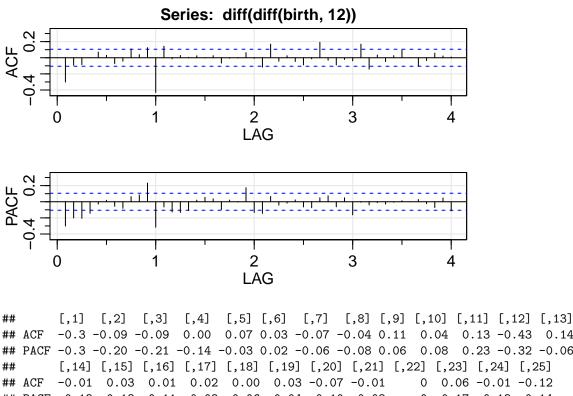


```
tsplot(diff(diff(birth)))
abline(h = mean(diff(diff(birth))), col = "red")
```



Now we can fit the model. First we investigate the ACF, PACF of the differenced data.

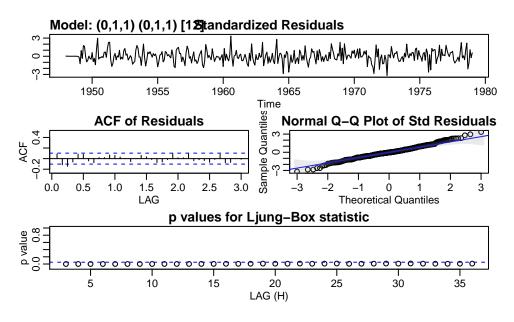
acf2(diff(diff(birth, 12)))



We can see that the ACF repeats at lags 1, 2, 3, etc. and the PACF tails off, indicating s=12 and MA(1), for seasonal component. The ACF, without the seasonal component, seems to cut off at lag 1 indicating p=1 and PACF tails off, which is also an MA(1) for the non-seasonal component. We can model SARIMA(0, 1, 1)x(0, 1, 1) with s=12.

```
sarima(birth, 0, 1, 1, 0, 1, 1, 12)
```

```
## initial
            value 2.219164
## iter
          2 value 2.013310
## iter
          3 value 1.988107
          4 value 1.980026
## iter
## iter
          5 value 1.967594
## iter
          6 value 1.965384
## iter
          7 value 1.965049
##
  iter
          8 value 1.964993
## iter
          9 value 1.964992
## iter
          9 value 1.964992
          9 value 1.964992
## iter
## final
          value 1.964992
## converged
## initial
            value 1.951264
## iter
          2 value 1.945867
          3 value 1.945729
##
  iter
          4 value 1.945723
##
  iter
          5 value 1.945723
## iter
          5 value 1.945723
## iter
          5 value 1.945723
## iter
## final value 1.945723
## converged
```



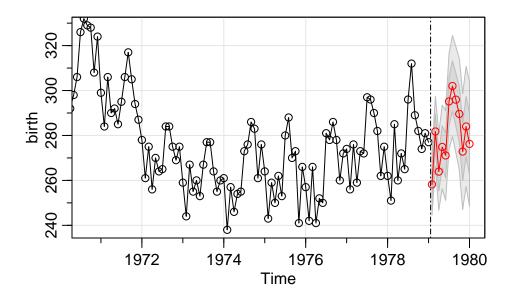
```
## $fit
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
```

```
##
       Q), period = S), include.mean = !no.constant, transform.pars = trans, fixed = fixed,
##
       optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
##
  Coefficients:
##
             ma1
                     sma1
         -0.4734
                  -0.7861
##
          0.0598
                   0.0451
## s.e.
##
## sigma^2 estimated as 47.4: log likelihood = -1211.28, aic = 2428.56
##
## $degrees_of_freedom
  [1] 358
##
##
## $ttable
##
        Estimate
                     SE
                        t.value p.value
## ma1
         -0.4734 0.0598
                         -7.9097
        -0.7861 0.0451 -17.4227
                                        0
##
  sma1
##
## $AIC
## [1] 6.545975
##
## $AICc
## [1] 6.546062
## $BIC
## [1] 6.577399
```

After testing a few different SARIMA models to the birth data, I landed on ARIMA(0, 1, 1)X(0, 1, 1)s = 12, where the diagnostics most agree with our requirements for a well-fit model. The forecasted model is below:

```
sarima.for(birth, 12, 0, 1, 1, 0, 1, 1, 12)
```

```
## $pred
##
              Jan
                       Feb
                                                             Jun
                                                                       Jul
                                Mar
                                          Apr
                                                    May
                                                                                Aug
## 1979
                  258.2171 281.7558 263.9016 274.8702 271.1040 295.1588 302.0120
## 1980 276.2490
##
             Sep
                       Oct
                                Nov
                                          Dec
## 1979 295.9419 289.5935 272.8265 283.9909
## 1980
##
## $se
##
               Jan
                         Feb
                                    Mar
                                              Apr
                                                         May
                                                                    Jun
                                                                              Jul
## 1979
                    6.884911
                              7.781346
                                         8.584677
                                                    9.319014 9.999569 10.636668
## 1980 13.857229
                                              Nov
                         Sep
                                    Oct
              Aug
## 1979 11.237707 11.808192 12.352358 12.873542 13.374432
## 1980
abline(v = 1979.05, lty = 6)
```

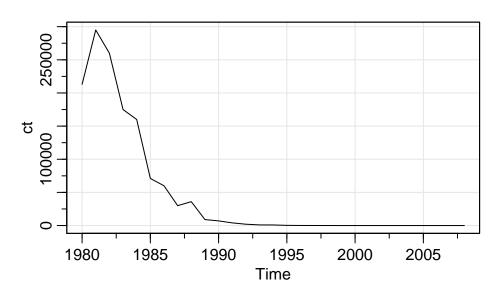


5.14

One of the remarkable technological developments in the computer industry has been the ability to store information densely on a hard drive. In addition, the cost of storage has steadily declined causing problems of too much data as opposed to big data. The data set for this assignment is cpg, which consists of the median annual retail price per GB of hard drives, say ct, taken from a sample of manufacturers from 1980 to 2008.

(a) Plot ct and describe what you see.



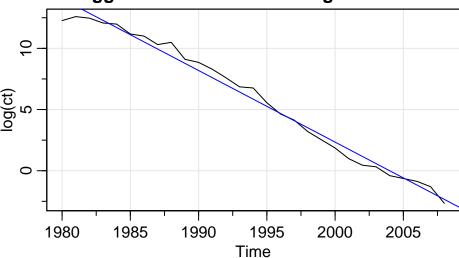


The cpg dataset is high to begin with in the 1980s and sharply decreases until 1990, where it smooths out around 0 for the rest of the time interval.

(b) Argue that the curve ct versus t behaves like $ct = ae^bt$ by fitting a linear regression of log ct on t and then plotting the fitted line to compare it to the logged data. Comment.

```
t = seq(1980, 2008, 1)
data = cbind(ct, t)
model = lm(log(ct) \sim t)
summary(model)
##
## Call:
## lm(formula = log(ct) ~ t)
## Residuals:
##
       Min
                  1Q
                       Median
                                    30
                                        1.13129
  -1.77156 -0.39840 0.04726 0.42186
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1172.49431
                            27.57793
                                       42.52
                 -0.58508
                             0.01383
                                      -42.30
                                               <2e-16 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6231 on 27 degrees of freedom
## Multiple R-squared: 0.9851, Adjusted R-squared: 0.9846
## F-statistic: 1790 on 1 and 27 DF, p-value: < 2.2e-16
tsplot(log(ct), main = "Logged ct data with linear regression line")
abline(model, col = 4)
```

Logged ct data with linear regression line

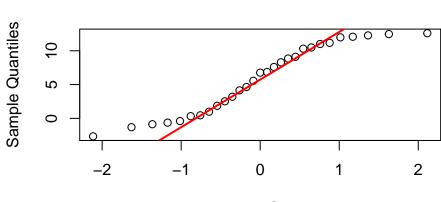


The logged data looks smoother, rather than a sharp decrease it steadily decreases over the time interval of the data. The fitted linear model of logged ct onto t (year) has promising diagnostics for modeling the logged ct data. We now have a model of $\log(ct) = Bt$, or $ct = e^Bt$.

(c) Inspect the residuals of the linear regression fit and comment.

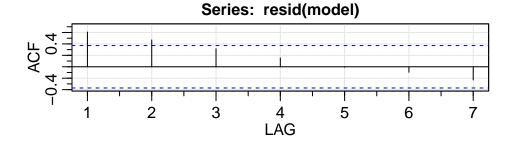
```
qqnorm(log(ct), pch = 1)
qqline(log(ct), col = "red", lwd = 2)
```

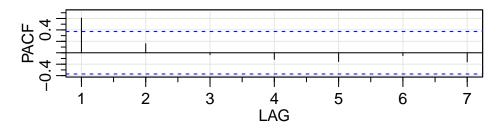
Normal Q-Q Plot



Theoretical Quantiles

acf2(resid(model))





```
## [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## ACF 0.61 0.47 0.32 0.15 -0.01 -0.10 -0.23
## PACF 0.61 0.16 -0.03 -0.11 -0.15 -0.05 -0.16
```

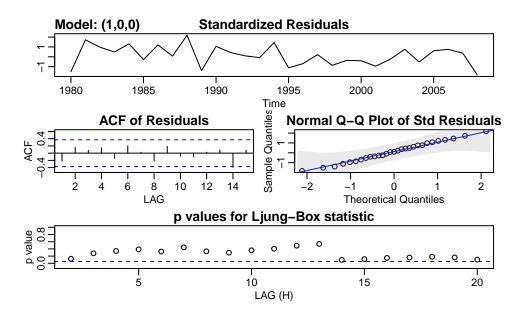
The trend present in the residuals shows that they are likely correlated. A good model would have the residuals mostly flatly on the qqline, but here we see that they trend somewhat cyclically around the line. The ACF and PACF of the residuals also show that they are not white.

(d) Fit the regression again, but now using the fact that the errors are autocorrelated. Comment.

Since the PACF lagged at 1 and the ACF tailed off, we can try an AR(1) model.

sarima(log(ct), 1, 0, 0, xreg = t)

```
## initial value -0.669056
## iter
          2 value -0.999488
          3 value -1.088763
## iter
## iter
          4 value -1.102248
## iter
          5 value -1.128914
          6 value -1.131945
## iter
## iter
          7 value -1.132479
## iter
          8 value -1.132525
          9 value -1.132540
## iter
         10 value -1.132543
## iter
         11 value -1.132545
## iter
         12 value -1.132545
## iter
        12 value -1.132545
## iter
## iter 12 value -1.132545
## final value -1.132545
## converged
## initial value -0.701381
## iter
          2 value -0.882862
          3 value -0.886699
## iter
          4 value -0.888651
## iter
## iter
          5 value -0.888966
          6 value -0.889035
## iter
          7 value -0.889043
## iter
## iter
          8 value -0.889045
## iter
          9 value -0.889045
## iter
        10 value -0.889045
## iter 10 value -0.889045
## iter 10 value -0.889045
## final value -0.889045
## converged
```



\$fit

```
##
## Call:
## stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
       Q), period = S), xreg = xreg, transform.pars = trans, fixed = fixed, optim.control = list(trace
##
       REPORT = 1, reltol = tol))
##
##
## Coefficients:
##
            ar1 intercept
##
         0.8297 1113.0105 -0.5554
## s.e. 0.1190
                  73.5665
                             0.0368
## sigma^2 estimated as 0.1623: log likelihood = -15.37, aic = 38.73
## $degrees_of_freedom
## [1] 26
##
## $ttable
##
              Estimate
                            SE t.value p.value
               0.8297 0.1190
                                6.9741
                                              0
## ar1
## intercept 1113.0105 73.5665 15.1293
                                              0
## xreg
              -0.5554 0.0368 -15.0716
                                              0
##
## $AIC
## [1] 1.335649
##
## $AICc
## [1] 1.368752
## $BIC
## [1] 1.524241
```

Now, the residuals are fairly white and we are happy with the model.