Lecture 12

26.5 Capacitors with Dielectrics

Dielectric – a nonconducting material: glass, rubber, metal oxides

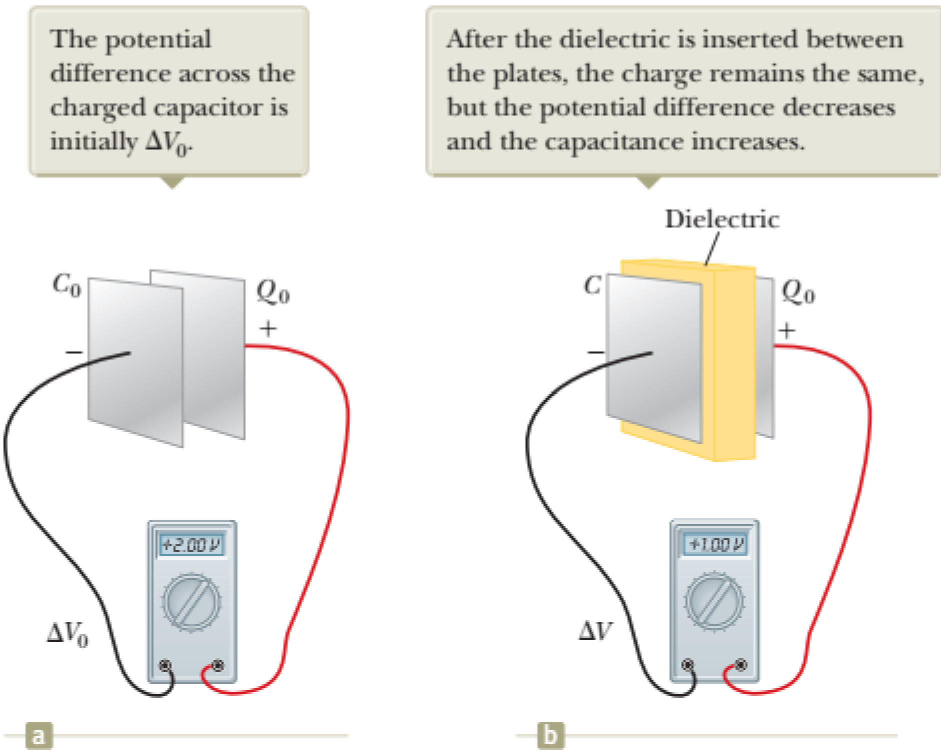

Figure 26.13 A charged capacitor (a) before and (b) after insertion of a dielectric between the plates.

a) The potential difference across the capacitor is $\Delta V_0 = Q_0 / C_0$

we measure the voltage by a voltmeter

b) If a dielectric is now inserted between the plates as in Figure 26.13b, the voltmeter indicates that the voltage between the plates decreases to a value Δ*V.*

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

$\Delta V < \Delta V_0$, so k – a dielectric constant > 1

Charge Q_0 does not change; therefore,

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0 \qquad (26.14)$$

the capacitance *increases* by the factor k when the dielectric completely fills the region between the plates.

Since

$$C_0 = \epsilon_0 A/d \text{ (Eq. 26.3)}$$

$$C = \kappa \frac{\epsilon_0 A}{d} \qquad (26.15)$$

If the electric field inside the capacitor (in the dielectric material) is too high, electrical breakdown can occur

the dielectric strength – maximum electric field

breakdown voltage – maximum voltage for the capacitor

dielectric provides these advantages:

- An increase in capacitance
- An increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the

plates to be close together without touching, thereby decreasing *d* and increasing *C*

Types of Capacitors

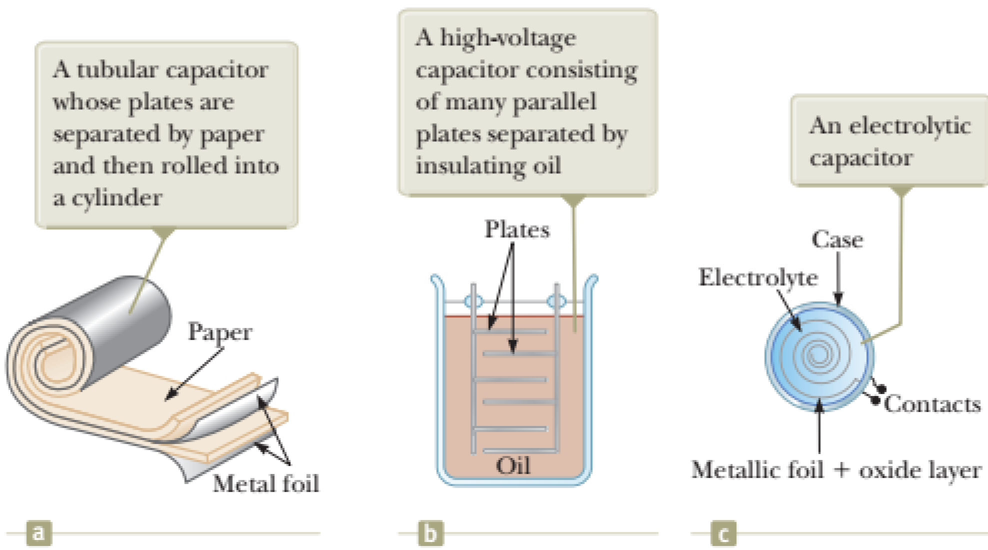

Figure 26.14 Three commercial capacitor designs.

Example 26.5 Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge Q_0. The battery is then removed, and a slab of material that has a dielectric constant k is inserted between the plates. Identify the system as the capacitor and the dielectric. Find the energy stored in the system before and after the dielectric is inserted.

SOLUTION:
find the energy stored in the absence of the dielectric:

$$U_0 = \frac{Q_0^{\ 2}}{2C_0}$$

Find the energy stored in the capacitor after the dielectric is inserted between the plates

$$U = \frac{Q_0^{\ 2}}{2C}$$

Use Equation 26.14 to replace the capacitance C:

$$U = \frac{Q_0^{\ 2}}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Because k <1, the final energy is less than the initial energy. We can account for the decrease in energy of the system by performing an experiment and noting that the dielectric, when inserted, is pulled into the device. To keep the dielectric from accelerating, an external agent must do negative work on the dielectric. Equation 8.2 becomes $\Delta U = W$, where both sides of the equation are negative

26.6 Electric Dipole in an Electric Field

The electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in Figure 26.17.

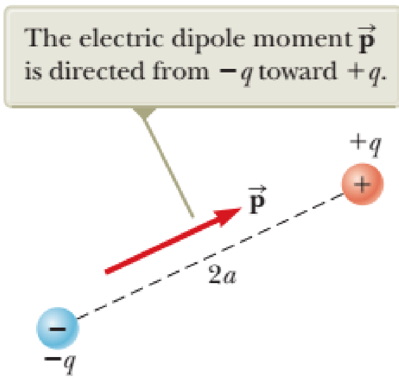

Figure 26.17 An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance of $2a$.

The **electric dipole moment** is defined as the vector **p** directed from -*q* toward +*q* along the line joining the charges and having magnitude

$$p = 2aq \qquad (26.16)$$

The dipole **p** in the electric field **E**:

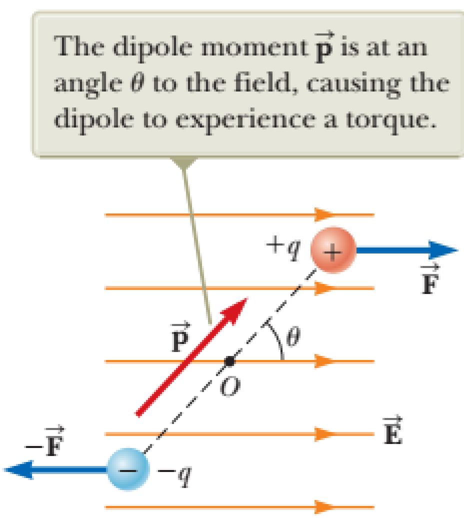

Figure 26.18 An electric dipole in a uniform external electric field.

the magnitude of the net torque about *O* is:

$$\tau = 2Fa \sin \theta$$

Because $F = qE$ and $p = 2aq$, we can express τ as

$$\tau = 2aqE\sin\theta = pE\sin\theta \tag{26.17}$$

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}} \tag{26.18}$$

Potential energy of the dipole in the E field:

Let's determine the potential energy of the system as a function of the dipole's orientation with respect to the field

The work dW required to rotate the dipole through an angle $d\theta$ is $dW = \tau\, d\theta$

since $\tau = pE\sin\theta$

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau\, d\theta = \int_{\theta_i}^{\theta_f} pE\sin\theta\, d\theta = pE\int_{\theta_i}^{\theta_f} \sin\theta\, d\theta$$

$$= pE[-\cos\theta]_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

we choose θ_i (initial angle) equal zero

Hence,

$$U_E = -pE\cos\theta \tag{26.19}$$

Or in the vector form:

$$U_E = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}} \tag{26.20}$$

Molecules are said to be *polarized* when a separation exists between the average position of the negative charges and the average position of the positive charges in the molecule

Polar molecules and **nonpolar** molecules induced polarization and permanent polarization

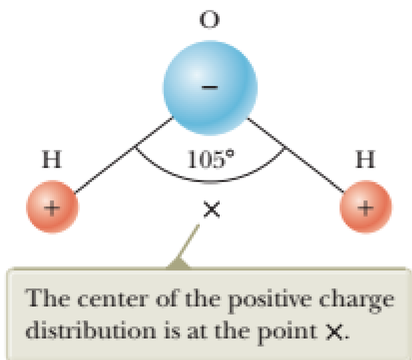

Figure 26.19 The water molecule, H_2O, has a permanent polarization resulting from its nonlinear geometry.

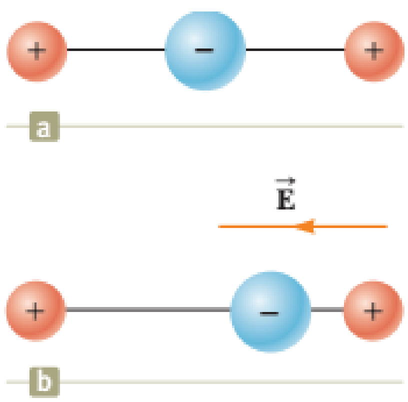

Figure 26.20 (a) A linear symmetric molecule has no permanent polarization. (b) An external electric field induces a polarization in the molecule.

Example 26.6 The H_2O Molecule

The water (H2O) molecule has an electric dipole moment of $6.3 * 10^{-30}$ C m. A sample contains 10^{21} water molecules, with the dipole moments all oriented in the direction of an electric field of magnitude $2.5 * 10^5$ N/C. How much work is required to rotate the dipoles from this orientation ($\theta = 0^0$) to one in which all the moments are perpendicular to the field ($\theta = 90^0$)?

SOLUTION:

at $\theta = 0$ the dipole in the electric field has the minimum potential energy we need to apply torque and do some work to increase the potential energy. Work should be equal to the change of the potential energy:

$$(1) \quad \Delta U_E = W$$

$$W = U_{90°} - U_{0°} = (-NpE\cos 90°) - (-NpE\cos 0°)$$

$$= NpE = (10^{21})(6.3 \times 10^{-30}\ \mathrm{C \cdot m})(2.5 \times 10^{5}\ \mathrm{N/C})$$

$$= \boxed{1.6 \times 10^{-3}\ \mathrm{J}}$$

26.7 An Atomic Description of Dielectrics

When we introduce a dielectric into a capacitor the potential difference between plates decreases k times:

$$\Delta V = V_0 / k$$

Reason : E decreases k times:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{E}}_0}{\kappa} \qquad (26.21)$$

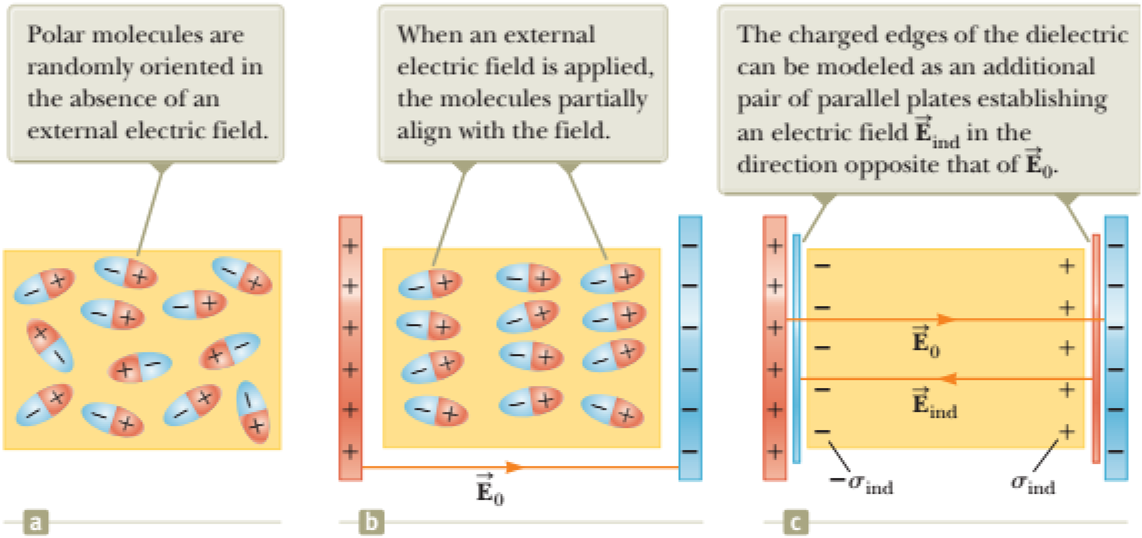

Figure 26.21 (a) Polar molecules in a dielectric. (b) An electric field is applied to the dielectric. (c) Details of the electric field inside the dielectric.

1.polar dielectrics

Polar molecules initially are randomly oriented (in the absence of E)

When E_0 is applied, dipoles partially align with the field

2. nonpolar dielectrics

molecules get polarized by the applied field (induced polarization)

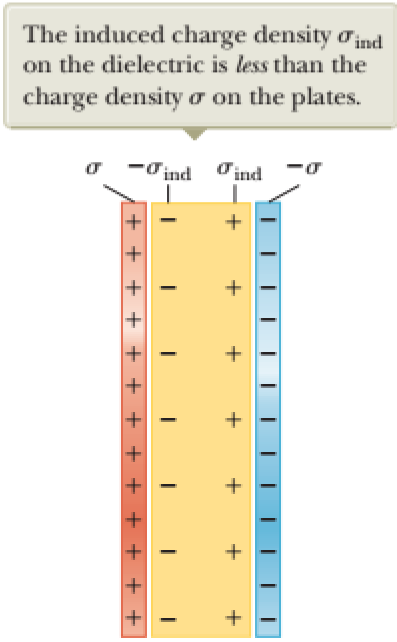

Figure 26.22 Induced charge on a dielectric placed between the plates of a charged capacitor.

Consider a capacitor with dielectric (Fig 26.22)

charged plates create a uniform E-field and polarize the dielectric

The net effect: positive surface charge density σ_{ind} on the right side of the dielectric slab, and negative surface charge density $-\sigma_{ind}$ on the left side

induced charges form the $\vec{E}_{ind}$ in the direction opposite to $\vec{E}_0$

Hence, the net electric field $\vec{E}$ has a magnitude

$$E = E_0 - E_{ind} \tag{26.22}$$

$E_0 = \sigma/\varepsilon_0$

$E_{ind} = \sigma_{ind}/\varepsilon_0$

$E = E_0/k = \sigma/k\varepsilon_0$

Hence, $\sigma/k\varepsilon_0 = \sigma/\varepsilon_0 - \sigma_{ind}/\varepsilon_0$

$$\sigma_{\text{ind}} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma \qquad (26.23)$$

If k > 1: dielectric: the charge density σ_{ind} is less than σ (charge density on the plates)

k = 1? no dielectric. Hence no induced charge

For a conductor: E =0 (no electric field inside conductor)

Hence from (26.22) : $\sigma_{ind} = \sigma$ (metal surfaces have the same charge density as the plates)

That is, the surface charge induced on the conductor is equal in magnitude but opposite in sign to that on the plates, resulting in a net electric field of zero in the conductor (see Fig. 24.16)

Example 26.7 **Effect of a Metallic Slab**

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates.

(A) Find the capacitance of the device.

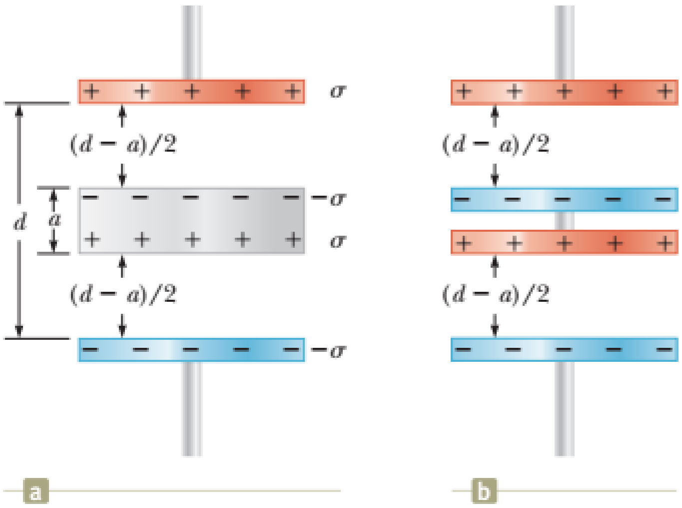

Figure 26.23 (Example 26.7) (a) A parallel-plate capacitor of plate separation d partially filled with a metallic slab of thickness a. (b) The equivalent circuit of the device in (a) consists of two capacitors in series, each having a plate separation $(d - a)/2$.

we can model the edges of the slab as conducting planes and the bulk of the slab as a wire. As a result, the capacitor in Figure 26.23a is equivalent to two capacitors in series, each having a plate separation $(d\ 2\ a)/2$ as shown in Figure 26.23b.

For a capacitor with the area A and separation d:

$$C = \varepsilon_0 A/d$$

For two capacitors in series,

$1/C = 1/C_1 + 1/C_2$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}} + \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}}$$

$$C = \boxed{\frac{\epsilon_0 A}{d-a}}$$

Example 26.8 A Partially Filled Capacitor

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness fd is inserted between the plates (Fig. 26.24a), where f is a fraction between 0 and 1?

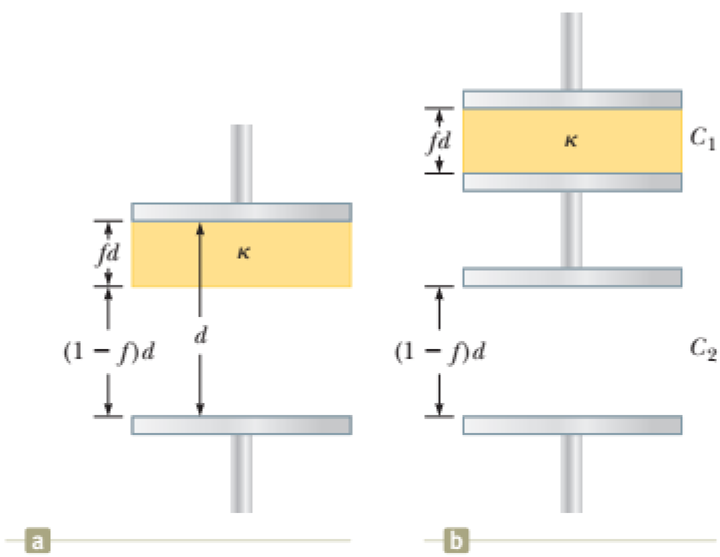

Figure 26.24 (Example 26.8) (a) A parallel-plate capacitor of plate separation d partially filled with a dielectric of thickness fd. (b) The equivalent circuit of the capacitor consists of two capacitors connected in series.

Summary

A **capacitor** consists of two conductors carrying charges of equal magnitude and opposite sign. The **capacitance** C of any capacitor is the ratio of the charge Q on either conductor to the potential difference ΔV between them:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference. The SI unit of capacitance is coulombs per volt, or the **farad** (F): 1 F = 1 C/V.

The **electric dipole moment** $\vec{\mathbf{p}}$ of an electric dipole has a magnitude

$$p \equiv 2aq \tag{26.16}$$

where $2a$ is the distance between the charges q and $-q$. The direction of the electric dipole moment vector is from the negative charge toward the positive charge.

Concepts and Principles Capacitors in parallel combination

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \tag{26.8}$$

Capacitors in series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{26.10}$$

Energy stored in a charged capacitor:

$$U_E = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C\,(\Delta V)^2 \qquad \textbf{(26.11)}$$

Capacitance of a capacitor filled with a dielectric material

$$C = \kappa C_0 \qquad \textbf{(26.14)}$$

k – dielectric constant, C_0 capacitance without dielectric

The torque acting on an electric dipole in a uniform electric field $\vec{\mathbf{E}}$ is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}} \qquad \textbf{(26.18)}$$

The potential energy of the system of an electric dipole in a uniform external electric field $\vec{\mathbf{E}}$ is

$$U_E = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}} \qquad \textbf{(26.20)}$$

Problems:

1. A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities (a) increase, (b) decrease, or (c) stay the same? **(i)** C **(ii)** Q **(iii)** ΔV **(iv)** the energy stored in the capacitor

2. By what factor is the capacitance of a metal sphere multiplied if its volume is tripled? (a) 3 (b) $3^{1/3}$ (c) 1 (d) $3^{-1/3}$ (e) $\frac{1}{3}$

2. Assume you want to increase the maximum operating voltage of a parallel-plate capacitor. Describe how you can do that with a fixed plate separation.