ESEMPIO ZAINO 07/11/2023

$$PLI \begin{cases} \max_x \; z = 8x_1 + 7x_2 + 12x_3 + 5x_4 + 9x_5 + 3x_6 \\ 4x_1 + 2x_2 + 3x_3 + 5x_4 + 2x_5 + 7x_6 \leq 45 \\ x \geq 0, \; x \; \underline{INT} \end{cases}$$

$$x^*_{PLI} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 21 \\ 0 \end{bmatrix}$$

RISCRITTO $\bar{y}^T$ IN MODO ORDINATO

$\frac{8}{4}$ $\frac{7}{2}$ $\frac{12}{3}$ $\frac{5}{5}$ $\frac{9}{2}$ $\frac{3}{7}$

2 3.5 4 1 4.5 0.43

$y_4$ $y_3$ $y_2$ $y_5$ $y_1$ $y_6$

$$PL \begin{cases} \max_y \; z = 9y_1 + 12y_2 + 7y_3 + 8y_4 + 5y_5 + 3y_6 \\ 2y_1 + 4y_2 + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 \\ y \geq 0 \end{cases}$$

$a_1$

$b$

VARIABILE DI SLACK

$$y^*_{PL} \begin{cases} y_1^* = \frac{b_1}{a_1} \rightarrow \frac{c_1}{a_1} \geq \frac{c_2}{a_2} \geq \dots \geq \frac{c_m}{a_m} \\ y_2^* = 0 = \dots = y_m \end{cases}$$

ALL'INIZIO SI PONE

$\bar{y}^T = \left[ \left\lfloor \frac{b}{a_1} \right\rfloor, 0, \dots, 0 \right]$

$\bar{z}$ = FUNZIONE OBIETTIVO IN $y$

→ È UN VALORE FINITO, A DIFFERENZA DI UN BRANCH AND BOUND CLASSICO

$y_1^* = \left\lfloor \frac{b}{a_1} \right\rfloor = \left\lfloor \frac{45}{2} \right\rfloor = \lfloor 22.5 \rfloor = 22$

$y_2^* = \dots = y_7^* = 0$

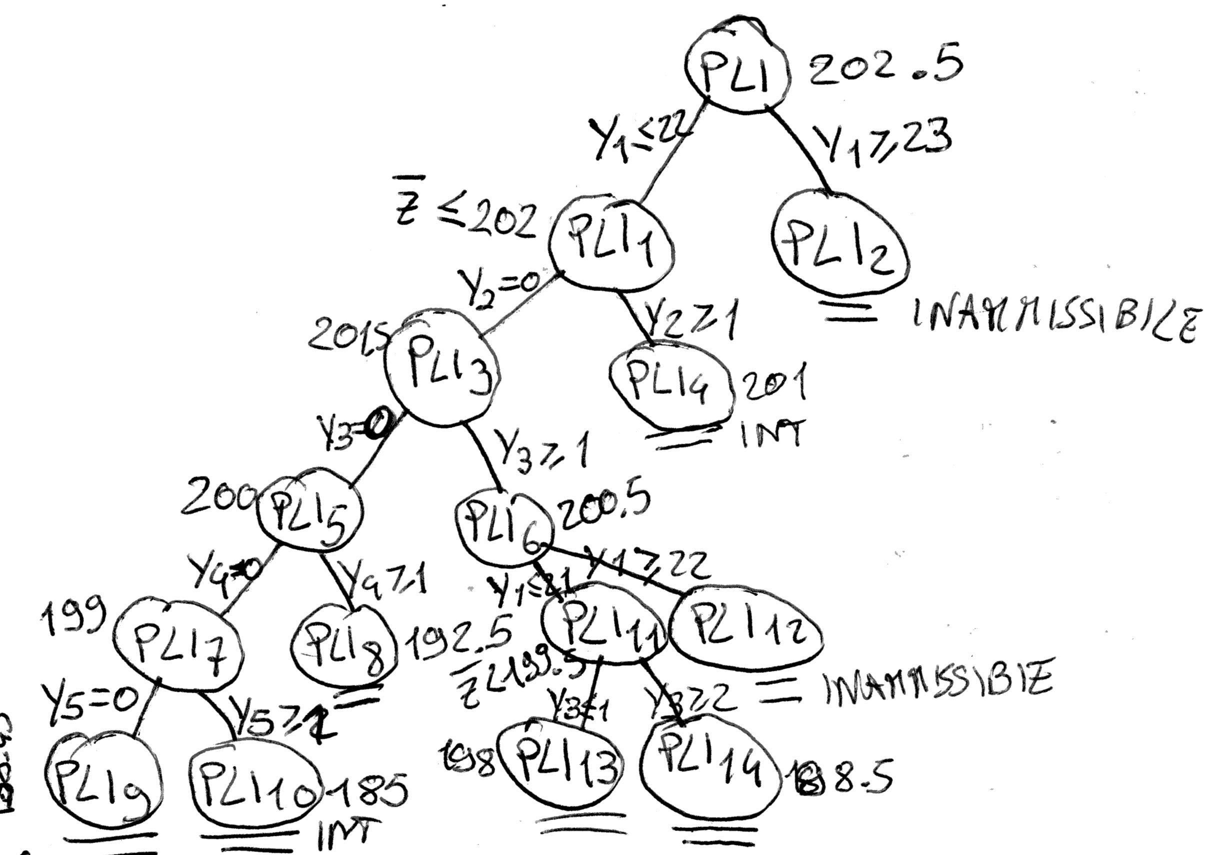
$z^*_{PL} = 9 \cdot \frac{45}{2} = 202.5$ NON INTERA → BRANCH AND BOUND

$\bar{y}^T = [22\ 0\ 0\ 0\ 0\ 0]$ agg $\bar{y}^T = [2\ 1\ 1\ 0\ 0\ 0\ 0\ 0]$

$\bar{z} = 22 \cdot 9 = 198 \geq \bar{z}$ quindi si continua

aggiornato a 201

PROFONDITÀ




COEFFICIENTI DI COSTO SONO INTERI QUINDI NON SI TROVA MEGLIO DI 198

SI RISCRIVE IL VINCOLO

$$3y_2 + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 - (22 \cdot 2) = 45 - 44 = 1 \to \hat{b}$$

$$PL_1 \begin{cases} y_1 \le 22 \\ y_1^* = 22 \end{cases}$$

$$y_2^* = \frac{1 \to \hat{b}}{3 \to a_2} \Rightarrow y_3 = 0 = \dots = y_7$$

$$PL_3 \begin{cases} y_2 = 0 \\ y_1 \le 22 \end{cases}$$

$y_1^* = 22$

$2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 1$

$$y_3^* = \frac{\hat{b}}{a_3} = \frac{1}{2} \qquad y_4^* = \dots = y_7^* = 0$$

$z^*_{PL3} = 201.5 \ge \bar{z}$ quindi si continua

$$y^{*\prime}_{PL3} = \left[22 \ 0 \ \tfrac{1}{2} \ 0 \ 0 \ 0 \ 0\right] \qquad z^*_{PL3} = 201.5$$

$PL_5 \begin{cases} y_3 = y_2 = 0 \\ y_1 \leq 22 \end{cases}$

$y_1^* = 22$

$4y_4 + 5y_5 + 7y_6 + y_7 = 1$

$y_4 = \frac{1}{4} \Rightarrow y_5 = y_6 = y_7 = 0$

$y^{*T}_{PL5} = [22 \; 0 \; 0 \; \frac{1}{4} \; 0 \; 0 \; 0]$ $z^*_{PL5} = 200 > \bar{z}$

$PL_7 \begin{cases} y_4 = y_3 = y_2 = 0 \\ y_1 \leq 22 \end{cases}$

$y_1^* = 22 \Rightarrow 5y_5 + 7y_6 + y_7 = 1$

$y_5^* = \frac{1}{5}$ $y_6^* = 0 = y_7^*$

$y^{*T}_{PL7} = [22 \; 0 \; 0 \; 0 \; \frac{1}{5} \; 0 \; 0]$

$z^*_{PL7} = 199 > \bar{z}$ quindi si continua

$\begin{cases} y_5 = y_4 = y_3 = y_2 = 0 \\ y_1 \leq 22 \end{cases}$

$y_1^* = 22$ $7y_6 + y_7 = 45 - 44 = 1$

$y_6 = \frac{1}{7}$ $y_7 = 0$

$y^{*T}_{PL9} = [22 \; 0 \; 0 \; 0 \; 0 \; \frac{1}{7} \; 0]$

$z^*_{PL9} = 198.43$

$PL_{10} \begin{cases} y_5 \geq 1 \\ y_4 = y_3 = y_2 = 0 \\ y_1 \leq 22 \end{cases}$

$2y_1 + 5y_5 + 7y_6 + y_7 \leq 45$

$y_5 \geq 1 \rightarrow y_5 - s_5 = 1$ con $s_5 \geq 0$

$y_5 = 1 + s_5$

$2y_1 + 5(1 + s_5) + 7y_6 + y_7 = 45$

$2y_1 + 5 + 5s_5 + 7y_6 + y_7 = 45$

$2y_1 + 5s_5 + 7y_6 + y_7 = 40$

$y_1 = \frac{40}{2} = 20$ e lo zaino è pieno

$s_5 = 0 = y_6 = y_7$

$y_5 = 1 + s_5 = 1$

$\begin{cases} y_1 \leq 22 \text{ non va più bene perché si avrebbe} \\ y^*_{1PL10} = 2 \cdot 22 = 44 > 40 \end{cases}$

$y^{*T}_{PL10} = [20 \; 0 \; 0 \; 0 \; 1 \; 0 \; 0]$

$z^*_{PL10} = 185$

$$PL_{13}\begin{cases} y_3 \le 1 \\ y_2 = 0 \\ \cancel{y_1 \le 22} \\ y_3 \ge 1 \\ y_1 \le 21 \end{cases} \qquad \begin{cases} y_3 = 1 \\ y_2 = 0 \\ y_1 \le 21 \end{cases}$$

$$2y_1 + \cancel{3y_2} + 3y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 \qquad 2+s_3$$

$$2y_1 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 - 2 = 43$$

$$y_1^* = 21 \longrightarrow 4y_4 + 5y_5 + 7y_6 + y_7 = 1$$

$$y_4 = \frac{1}{4} \Rightarrow y_5 = y_6 = y_7 = 0$$

$$y^{*T}_{PL_{13}} = \left[21 \ 0 \ 1 \ \frac{1}{4} \ 0 \ 0 \ 0\right]$$

$$z^*_{PL_{13}} = 198 = \bar{z}$$ si chiude

$$PL_{14}\begin{cases} y_3 \ge 2 \\ y_1 \le 21 \\ \cancel{y_3 \ge 1} \\ y_2 = 0 \\ \cancel{y_1 \le 22} \end{cases} \qquad \begin{cases} y_3 \ge 2 \\ y_1 \le 21 \\ y_2 = 0 \end{cases}$$

$$2y_1 + \cancel{3y_2} + 3y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45$$

$$2y_1 + 2(2+s_3) + 4y_4 + 5y_5 + 7y_6 + y_7 = 45$$

$$2y_1 + 2s_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 - 4 = 41$$

$$y_1 = \frac{41}{2} = 20.5 \Rightarrow s_3 = y_4 = y_5 = y_6 = y_7 = 0$$

$$y_3 = 2 + s_3 = 2$$

$$y^{*T}_{PL_{14}} = \left[\frac{41}{2} \ 0 \ 2 \ 0 \ 0 \ 0 \ 0\right] \to \text{INTERO}$$

$$z^*_{PL_{14}} = 198.5$$

MA NON SCENDE SOTTO 198, QUINDI SI TERMINA

$$PL_{13}\begin{cases} y_1 \ge 22 \\ y_3 \ge 1 \\ y_2 = 0 \\ y_1 \le 22 \end{cases} \qquad \begin{cases} y_1 \ge 22 \\ y_3 \ge 1 \\ y_2 = 0 \end{cases} \to y_3 - s_3 = 1 \qquad y_3 = 1 + s_3$$

$$2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 - 44$$

$$2(1+s_3) + 4y_4 + 5y_5 + 7y_6 + y_7 = 1$$

$$2s_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = -1 \to \hat{b} < 0$$

INAMMISSIBILE

$$PL_4\begin{cases} Y_1 \le 22 \\ Y_2 \ge 1 \rightarrow Y_2 - S_2 = 1 \qquad Y_2 = 1 + S_2 \end{cases}$$

$$2y_1 + 3y_2 + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45$$

$$2y_1 + 3(1+S_2) + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45$$

$$2y_1 + 3S_2 + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45$$

$$2y_1 + 3S_2 + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45 - 3 = 42$$

$$Y_1^* = \frac{42}{2} = 21 \geq 7 \quad S_2 = Y_3 = Y_4 = Y_5 = Y_6 = 0 = Y_7$$

$$Y_2 = 1 + S_2 = 1$$

$$Y^{*T}_{PL4} = [21\ 1\ 0\ 0\ 0\ 0\ 0]$$

$$Z^{*T}_{PL4} = 201 \quad \text{INT} \quad \text{soluzione migliore di } \bar{z}$$

$$PL_2\begin{cases} Y_1 \ge 23 \end{cases}$$

$$2y_1 + 3y_2 + 2y_3 + 4y_4 + 5y_5 + 7y_6 + y_7 = 45$$

$$2 \cdot 23 = \underbrace{46 > 45}_{\text{INAMMISSIBILE}}$$