

①

$$P(0,1 \leq X \leq 0,5) = \int_{0,1}^{0,5} 3x^2 dx = x^3 \Big|_{0,1}^{0,5} = (0,5)^3 - (0,1)^3$$

②

$$X \sim U[a, b]$$

$$F_X(x) = P(X \leq x) = \begin{cases} 0 & x < a \\ \int_a^x \frac{1}{b-a} dy & a \leq x \leq b \\ 1 & x > b \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

③

$$T \sim \text{EXP}(1) \Leftrightarrow F_T(t) = 1 - e^{-t} \quad t \geq 0$$

$$(a) P(T < 2) = F_T(2) = 1 - e^{-2}$$

$$(b) P(2 < T < 8) = F_T(8) - F_T(2) = 1 - e^{-8} - (1 - e^{-2}) = e^{-2} - e^{-8}$$

$$(c) P(T > t) = 0,25$$

$$e^{-t} = 0,25 \Leftrightarrow \ln(e^{-t}) = \ln(0,25) \Leftrightarrow t = -\ln(0,25)$$

⑤

$$X \sim \text{EXP}(0,00001)$$

$$P(X < X_{0,2}) = 0,2 \Leftrightarrow e^{-0,00001 \cdot X_{0,2}} = 0,2$$

$$X_{0,2} = 100000 \ln(0,2)$$

$$X_{0,4} = 100000 \ln(0,4)$$

$$P(X < X_{0,4}) = 0,4$$

$$X_{0,6} = 100000 \ln(0,6)$$

$$X_{0,8} = 100000 \ln(0,8)$$

$$⑥ \quad X \sim N(5, (10)^2)$$

$$① \quad P(X < 0) = P_{\text{norm}}(0, 5, 10)$$

$$P(X > 10) = 1 - P_{\text{norm}}(10, 5, 10)$$

$$P(X \geq 15) = 1 - P_{\text{norm}}(15, 5, 10)$$

$$② \quad P(-20 < X < 15) = P(X < 15) - P(X < -20) \\ = P_{\text{norm}}(15, 5, 10) - P_{\text{norm}}(-20, 5, 10)$$

$$P(-5 \leq X \leq 30) = P_{\text{norm}}(30, 5, 10) - P_{\text{norm}}(-5, 5, 10)$$

$$③ \quad P(X > x) = 0,05 \Leftrightarrow x = q_{\text{norm}}(0,95, 5, 10)$$

$$④ \quad P(X < x) = 0,23 \Leftrightarrow x = q_{\text{norm}}(0,23, 5, 10)$$

$$⑦ \quad ① \quad X = \{0, 1, 2\} \quad P(X=0) = P(0 \leq T < 1) = 1 - e^{-2}$$

$$P(X=1) = P(1 \leq T < 2) = F_T(2) - F_T(1) = e^{-2} - e^{-4}$$

$$P(X=2) = P(T > 2) = e^{-4}$$

$$② \quad Y = \{0, 1, 2, \dots\} \quad P(Y=k) = e^{-2k} - e^{-2(k+1)}$$

$$P(Y=k) = e^{-2k} (1 - e^{-2}) = (e^{-2})^k (1 - e^{-2})$$

$$Y = \cancel{X} X - 1 \quad \text{com } X \sim \text{Geométrica} \left(\frac{1 - e^{-2}}{e^{-2}} \right)_{\text{PERIGO}}$$

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b

$$0,1 = P(X > x) = 1 - F_X(x) = 1 - \left(\frac{1}{2} + \frac{1}{\pi} \arctan(x) \right)$$

$$\frac{1}{\pi} \arctan(x) = 0,4 \quad \Leftrightarrow \quad \arctan(x) = \pi \cdot 0,4$$

$$\tan(\arctan(x)) = \tan(\pi \cdot 0,4)$$

$$x = \tan(\pi \cdot 0,4)$$