Oscar Perez Cytron

# N-Body Simulation Figures

**1. Validation test:**

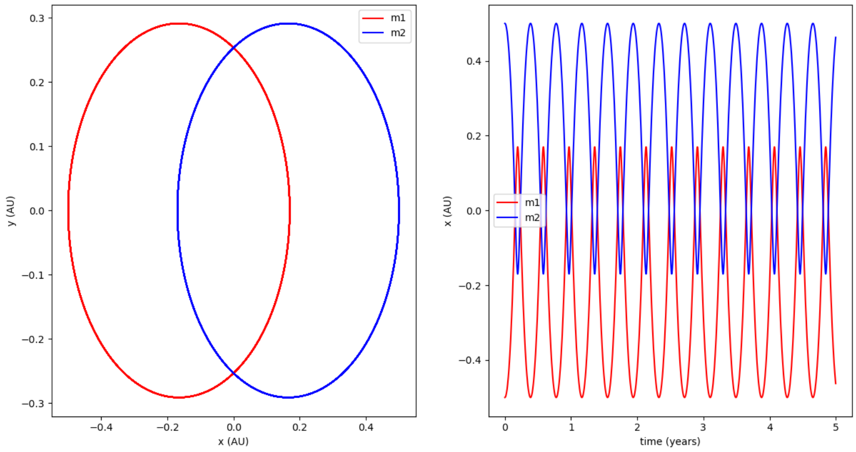


Figure 1
Two figures that represent the positions of two objects of equal mass separated 1 AU over 5 years. The first graph shows the positions on the xy plane, while the second represents the position x of each object over time. Our closed elliptical orbits coincide with the predicted results.

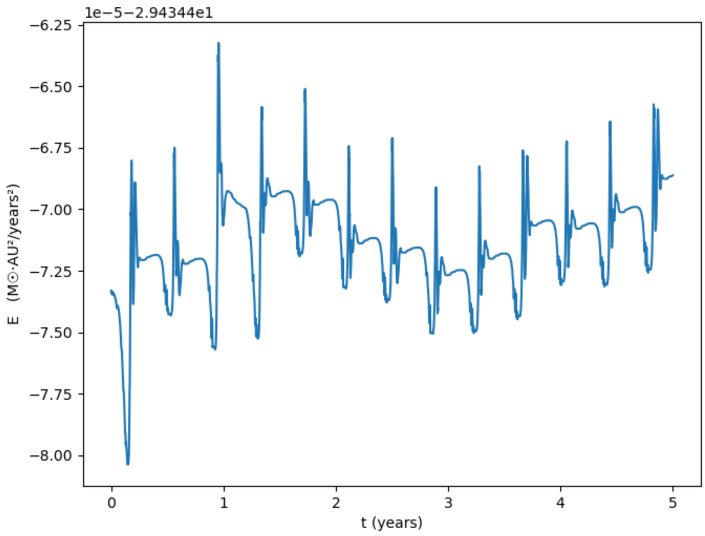


Figure 2
This figure represents the change in the total energy of the system over time. The total energy remains practically unchanged, with a maximum variation of about $2 \cdot 10^{-5}$ and $\Delta E = \frac{E_f - E_0}{E_0} = -1.6 \cdot 10^{-7}$ for a timestep $dt = 0.001\ years$.

## 2. Sun, Earth, Mars three-body problem:

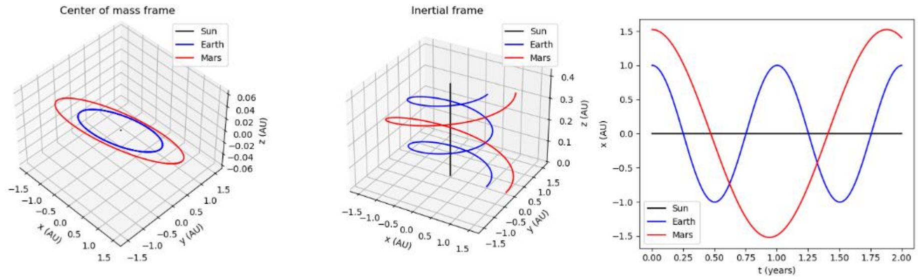


Figure 3
The Earth and Mars orbiting the Sun. The speed of the sun is set to 1 km/s on the z axis to test our center of mass coordinate system. The first two figures appear to coincide with expectations as both orbits are closed and retain approximately the same distance from the Sun. From the third figure representing the position on the x axis over time, we can see that the period of the earth is exactly one year (as one would expect) and that of Mars is a little under two years (1.88 in theory).

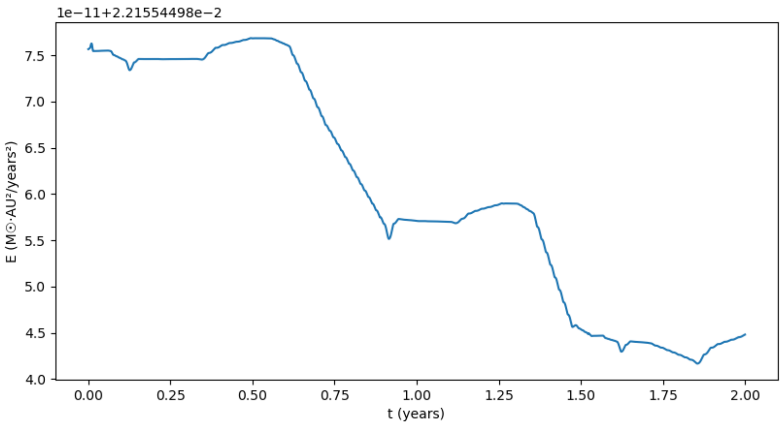


Figure 4
Variation of the total energy of the system over time. The variation appears to be of the order of $10^{-11}$, with $\Delta E = -1.4 \cdot 10^{-9}$ for a timestep $dt = 0.0001\ s$. Our integration method for this system is incredibly precise and quite speedy, taking 1,25 $s$ to work through 20,000 points.

### 3. Full Solar System (9 bodies):

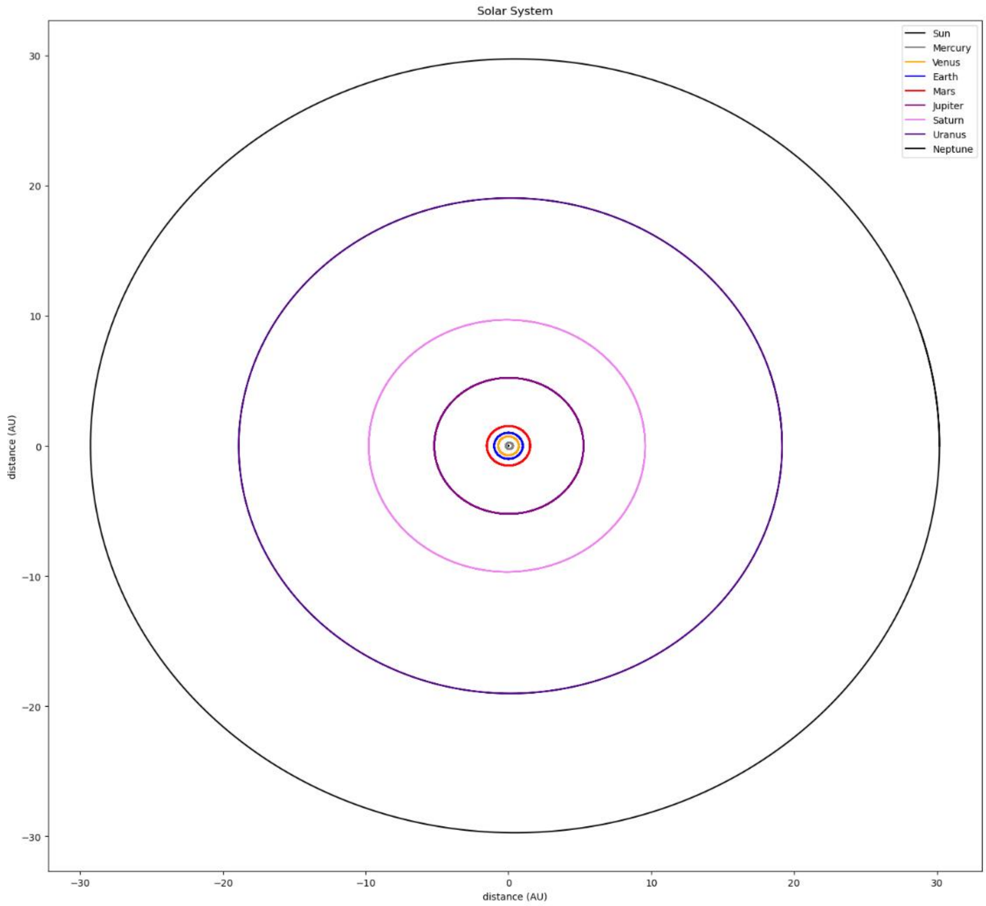


Figure 5
Full solar system model in AU. The orbits all appear elliptical (practically spherical). We can draw more information by plotting against time.

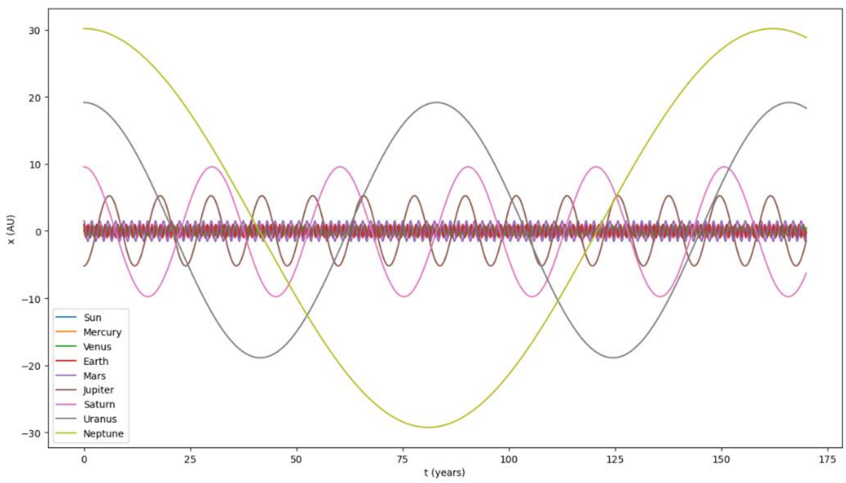


Figure 6
Positions of planets ove time. Neptune's period appears to be about 165 years, which is consistent with its real value.

## 4. Burrau's Problem

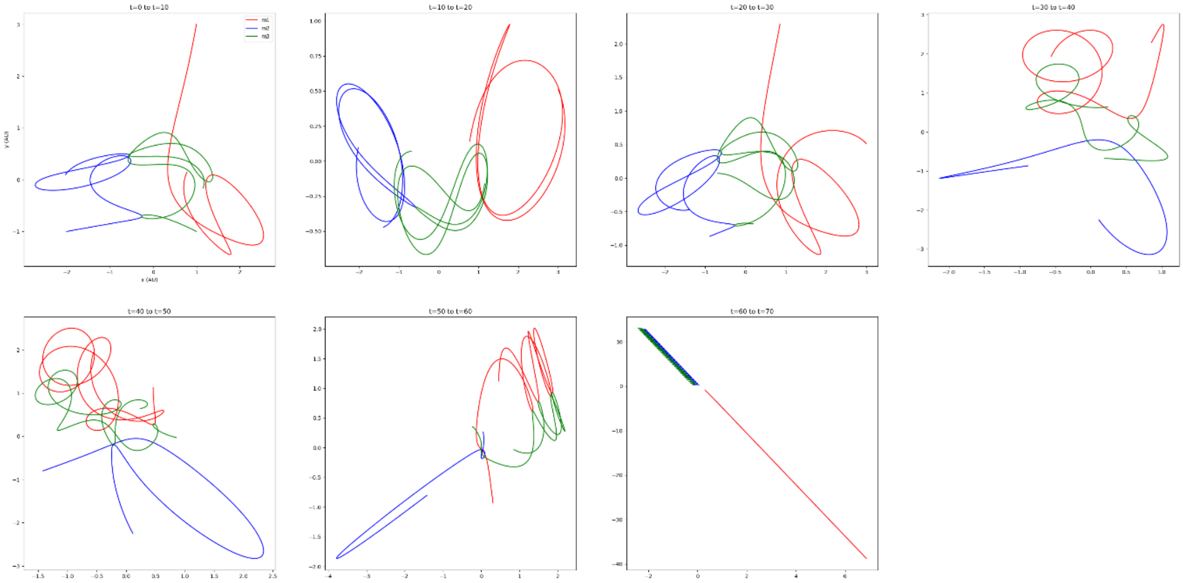

Figure 7

Burrau's problem from t=0 to t=70 time units with 1 time unit= $\sqrt{\frac{1AU}{G \cdot M_{\odot}}} \approx 5 \cdot 10^6 \, years$.

Our used time step was $dt = 10^{-5}$ and a tolerance for error was $10^{-19}$. Our graphed results are accurate compared to those in the reference papers up to t=50. After this our results gradually diverge, with similar motion but in the opposite direction for t=60 to t=70. Using a larger time step, a more valid solution may be found, but the code is unable to allocate the space for such large arrays.

## 5. Randomly generated particles:

Particles are uniformly randomly distributed between -1 and 1. Masses are twice a normal distribution centered at 0. The initial velocities are all zero. Time units are the same as in prior problem.

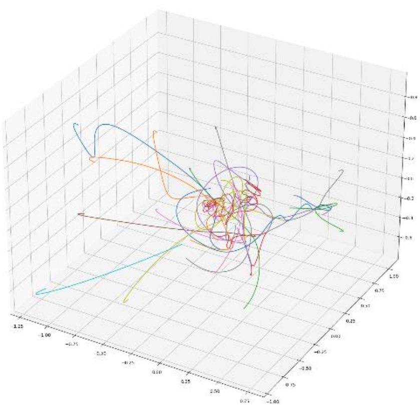

Figure 8
40 bodies, final time 0.25

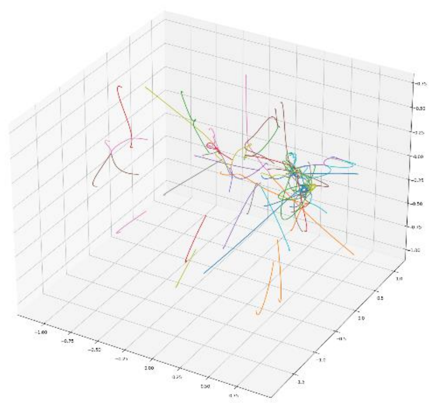

Figure 9
20 bodies, final time 0.5

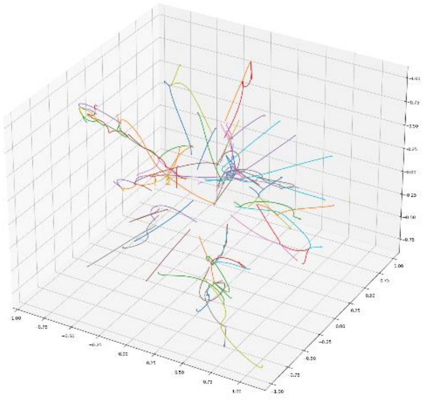

Figure 10
60 bodies, final time 0.18

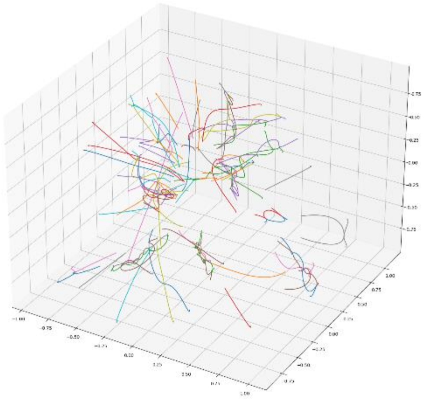

Figure 11
80 bodies, final time 0.14

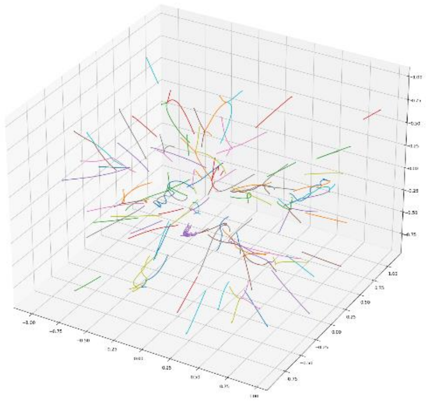

Figure 12
100 bodies, final time 0.09

This process helped see the limits of the program. For a higher number of bodies smaller time steps are required or the code blows up. The final two programs took over 6 minutes to run.