Student: hania rifai **Instructor:** Hyunkyoung Yoon **Date:** 08/08/21	**Course:** MAT 211B – Mathematics for Business Analysis Summer 2021 **Book:** Bittinger: Calculus and Its Applications, 2e **Time:** 19:26

Of the entering class at a college, 62% attended public high school, 29% attended private high school, and 9% were home schooled. Of those who attended public high school, 13% made the Dean's list, 12% of those who attended private high school made the Dean's list, and 21% of those who were home schooled made the Dean's list. A student is randomly chosen.
a) Find the probability that the student made the Dean's list.
b) Find the probability that the student came from a private high school, given that the student made the Dean's list.
c) Find the probability that the student was not home schooled, given that the student did not make the Dean's list.

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a) Let D be the event that the student made the Dean's list, E_1 be the event that the student went to public school, E_2 be the event that the student went to private school, and E_3 be the event that the student was home schooled. Then D is the union of three mutually exclusive sets, $D \cap E_1$, $D \cap E_2$, and $D \cap E_3$. In this situation, P(D) is defined by the following formula.

$$P(D) = P\left(D \mid E_1\right) \cdot P\left(E_1\right) + P\left(D \mid E_2\right) \cdot P\left(E_2\right) + P\left(D \mid E_3\right) \cdot P\left(E_3\right)$$

To find P(D), start by identifying $P(E_1)$, $P(E_2)$, and $P(E_3)$.

$P(E_1) = 0.62$, $P(E_2) = 0.29$, and $P(E_3) = 0.09$

Now, find $P\left(D \mid E_1\right)$, $P\left(D \mid E_2\right)$, and $P\left(D \mid E_3\right)$.

$P\left(D \mid E_1\right) = 0.13$, $P\left(D \mid E_2\right) = 0.12$, and $P\left(D \mid E_3\right) = 0.21$

Find P(D).

$$P(D) = P\left(D \mid E_1\right) \cdot P\left(E_1\right) + P\left(D \mid E_2\right) \cdot P\left(E_2\right) + P\left(D \mid E_3\right) \cdot P\left(E_3\right)$$
$$P(D) = (0.13)(0.62) + (0.12)(0.29) + (0.21)(0.09)$$
$$P(D) = 0.1343$$

Thus, the probability that the student made the Dean's list is 0.1343.

b) The probability that the student came from a private high school, given that the student made the Dean's list, is $P\left(E_2 \mid D\right)$ according to the event definitions from part (a).

Let sample space S be partitioned into n subsets $E_1, E_2, \ldots, E_n$, and let D be any event in S. Bayes Theorem states that the probability of E_i given D is given by the formulas below.

$$P\left(E_i \mid D\right) = \frac{P\left(E_i \cap D\right)}{P(D)}$$
$$P\left(E_i \mid D\right) = \frac{P\left(E_i \cap D\right)}{P\left(E_1 \cap D\right) + P\left(E_2 \cap D\right) + \ldots + P\left(E_n \cap D\right)}$$
$$P\left(E_i \mid D\right) = \frac{P\left(D \mid E_i\right) \cdot P\left(E_i\right)}{P\left(D \mid E_1\right) \cdot P\left(E_1\right) + P\left(D \mid E_2\right) \cdot P\left(E_2\right) + \ldots + P\left(D \mid E_n\right) \cdot P\left(E_n\right)}$$

Substitute the known values into the equation and simplify, rounding to four decimal places. Note that the denominator of Bayes theorem is P(D), as found in part A.

$$P\left(E_2 \mid D\right) = \frac{P\left(D \mid E_2\right) \cdot P\left(E_2\right)}{P\left(D \mid E_1\right) \cdot P\left(E_1\right) + P\left(D \mid E_2\right) \cdot P\left(E_2\right) + P\left(D \mid E_3\right) \cdot P\left(E_3\right)}$$
$$P\left(E_2 \mid D\right) = \frac{(0.12)(0.29)}{0.1343}$$
$$P\left(E_2 \mid D\right) = 0.2591$$

Thus, the probability that the student came from a private high school, given that the student made the Dean's list, is about 0.2591.

c) The probability that the student was not home schooled, given that the student did not make the Dean's list, is $P\left(E_3{}' \mid D'\right)$ according to the event definitions from part (a).

Note that $P\left(E_3{}' \mid D'\right) = 1 - P\left(E_3 \mid D'\right)$, and Bayes theorem indicates that $P\left(E_3 \mid D'\right) = \dfrac{P\left(D' \mid E_3\right) \cdot P\left(E_3\right)}{P\left(D'\right)}$.

Note that $P\left(E_3\right) = 0.09$. Find $P\left(D' \mid E_3\right)$ and P(D'). Note that $P\left(D' \mid E_3\right) = 1 - P\left(D \mid E_3\right)$. Find $P\left(D' \mid E_3\right)$.

$$P\left(D' \mid E_3\right) = 1 - 0.21 = 0.79$$

Next, note that $P(D') = 1 - P(D)$. Find $P(D')$.

$$P(D') = 1 - 0.1343 = 0.8657$$

Substitute the computed values into the equation and simplify to find $P\left(E_3{}' \mid D'\right)$, rounding to four decimal places.

$$P\left(E_3{}' \mid D'\right) = 1 - P\left(E_3 \mid D'\right)$$
$$P\left(E_3{}' \mid D'\right) = 1 - \frac{P\left(D' \mid E_3\right) \cdot P\left(E_3\right)}{P\left(D'\right)}$$
$$P\left(E_3{}' \mid D'\right) = 1 - \frac{(0.79)(0.09)}{0.8657}$$
$$P\left(E_3{}' \mid D'\right) = 0.9179$$

Thus, the probability that the student was not home schooled, given that the student did not make the Dean's list, is about 0.9179.