# Q5 - Of part

(a) $W^{(1)} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$

$W^{(2)} = \begin{bmatrix} W_{31} \\ W_{32} \end{bmatrix}$

(b) First, lets define $V_1, V_2$:

$V_1 = W_{11} \cdot 1 + W_{12} \cdot X$

$V_2 = W_{21} \cdot 1 + W_{22} \cdot X$

Now let's define $o_1, o_2$:

$o_1 = \varphi(V_1) = \varphi(W_{11} + W_{12} \cdot X)$

$o_2 = \varphi(V_2) = \varphi(W_{21} + W_{22} \cdot X)$

So, $\hat{y}(x) = W_{31} \cdot o_1 + W_{32} \cdot o_2 = W_{31} \cdot \varphi(W_{11} + W_{12} \cdot X) + W_{32} \cdot \varphi(W_{21} + W_{22} \cdot X)$.

(c) The function $y(x)$ is:

$\cancel{y = \begin{cases} -x-1 ; x<0 \\ x-1 ; x \geq 0 \end{cases}}$ $y = \begin{cases} \max(0, \frac{-x-1}{2}) ; & x<0 \\ \max(0, x-1) ; & x \geq 0 \end{cases}$

And $\varphi(x) = \max(0, x)$, so: $\varphi(\frac{-x-1}{2}) = \max(0, \frac{-x-1}{2})$

$\varphi(x-1) = \max(0, x-1)$

$y = \begin{cases} \varphi(\frac{-x-1}{2}), & x<0 \\ \varphi(x-1), & x \geq 0 \end{cases}$

When will $\hat{y} = y$?

When:

$W_{31} \cdot \varphi(W_{11} + W_{12} \cdot x) + W_{32} \cdot \varphi(W_{21} + W_{22} \cdot x) = \begin{cases} \varphi(\frac{-x-1}{2}), & x<0 \\ \varphi(x-1), & x \geq 0 \end{cases}$

(c) continued:

With set of weights:

$W_{11} = -\frac{1}{2}$, $W_{12} = -\frac{1}{2}$

$W_{21} = 1$, $W_{22} = -1$

$W_{31} = 1$, $W_{32} = 1$

We get:

for $x \geq 0$:

$\hat{y} = 1 \cdot \varphi(-\frac{1}{2} - \frac{x}{2}) + 1 \cdot \varphi(1+x)$

$\varphi(x-1) = \max(x-1, 0) = 0$

$\hat{y} = \varphi(\frac{-x-1}{2}) = y(x)$

for $x<0$:

$\hat{y} = 1 \cdot \varphi(\frac{-x-1}{2}) + 1 \cdot \varphi(x-1)$

$\varphi(\frac{-x-1}{2}) = \max(\frac{-x-1}{2}, 0) = 0$

$\hat{y} = y$

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(d) $L = (y - \hat{y}(x))^2$

$\vec{\nabla} L$ will have 6 elements, one for each weight.

$$\nabla L = \left[ \frac{\partial L}{\partial w_{11}}, \frac{\partial L}{\partial w_{12}}, \ldots \frac{\partial L}{\partial w_{32}} \right]$$

A common term that can help us would be:

$\frac{\partial L}{\partial \hat{y}} = \delta = 2 \cdot (\hat{y}(x) - y(x))$, and we will apply the chain rule each time.

$$\frac{\partial L}{\partial w_{11}} = \delta \cdot \frac{\partial \hat{y}(x)}{\partial w_{11}} = \delta \cdot \frac{\partial (w_{31} \cdot \varphi(w_{11} + w_{12}x) + w_{32} \cdot \varphi(w_{21} + w_{22}x))}{\partial w_{11}} =$$

$$= \delta \cdot (w_{31} \cdot \varphi'(w_{11} + w_{12}x) \cdot 1 + w_{32} \cdot 0) = \delta \cdot w_{31} \cdot \varphi(w_{11} + w_{12}x)$$

$$\frac{\partial L}{\partial w_{12}} = \delta \cdot \frac{\partial (w_{31} \cdot \varphi(w_{11} + w_{12}x) + w_{32} \cdot \varphi(w_{21} + w_{22}x))}{\partial w_{12}} = \delta \cdot w_{31} \cdot \varphi'(w_{11} + w_{12}x) \cdot X$$

$$\frac{\partial L}{\partial w_{21}} = \delta \cdot \frac{\partial (w_{31} \cdot \varphi(w_{11} + w_{12}x) + w_{32} \cdot \varphi(w_{21} + w_{22}x))}{\partial w_{21}} = \delta \cdot w_{32} \cdot \varphi'(w_{21} + w_{22}x)$$

$$\frac{\partial L}{\partial w_{22}} = \delta \cdot w_{32} \cdot \varphi'(w_{21} + w_{22}X) \cdot X$$

$$\frac{\partial L}{\partial w_{31}} = \delta \cdot \varphi(w_{11} + w_{12}x)$$

$$\frac{\partial L}{\partial w_{32}} = \delta \cdot \varphi(w_{21} + w_{22}x)$$

$$\boxed{\text{So } \nabla L = [\delta \cdot w_{31} \cdot \varphi'(w_{11} + w_{12}x), \delta \cdot w_{31} \cdot \varphi'(w_{11} + w_{12}x) \cdot X, \delta \cdot w_{32} \cdot \varphi'(w_{21} + w_{22}x), \delta \cdot w_{32} \cdot \varphi'(w_{21} + w_{22}x) \cdot X, \delta \cdot \varphi(w_{11} + w_{12}x), \delta \cdot \varphi(w_{21} + w_{22}x)]}$$

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(e) $w_{11}' = -\eta \cdot \delta \cdot w_{31} \cdot \varphi'(w_{11} + w_{12}x) + w_{11}$

$w_{12}' = -\eta \cdot \delta \cdot w_{31} \cdot \varphi'(w_{11} + w_{12}x) \cdot x + w_{12}$

$w_{21}' = -\eta \cdot \delta \cdot w_{32} \cdot \varphi'(w_{21} + w_{22}x) + w_{21}$

$w_{22}' = -\eta \cdot \delta \cdot w_{32} \cdot \varphi'(w_{21} + w_{22}x) \cdot x + w_{22}$

$w_{31}' = -\eta \cdot \delta \cdot \varphi(w_{11} + w_{12}x) + w_{31}$

$w_{32}' = -\eta \cdot \delta \cdot \varphi(w_{21} + w_{22}x) + w_{32}$

(f) Given: $x=0$, $y=0$, $w_{11}=1$, $w_{12}=1$, $w_{21}=-1$, $w_{22}=-1$, $w_{31}=1$, $w_{32}=1$

$L = (y - \hat{y})^2$

$y = 0$

$\hat{y} = 1 \cdot \varphi(1 + 1 \cdot 0) + 1 \cdot \varphi(-1 - 1 \cdot 0) = 1 \cdot 1 + 1 \cdot (0) = 1$

~~$L = (0-0)^2 = 0$~~

$L = (0-1)^2 = 1$