4.

1. Given that , its covariance matrix is:
2. First we'll find the eigenvalues of the matrix, using the characteristic polynomial:

We can find its roots:

Which are the eigenvalues of C:

Now we'll find the matching eigenvectors. We need to find the solution to the homogeneous equation (x is 2x1 vector):

For each .

First:

The rows are dependent, so we remain with one dimension in the null space solution:

Same for the other vector:

So in general, the eigen vectors are the columns of U the diagonalize matrix of C. moreover, because that U columns form an orthonormal basis is equivalent to that U is unitary matrix, so and the final eigen-decomposition is:

1. The main component is the eigen-vector of C that is matching the biggest eigen-value. In our case it is:

Most of the variance of data is spread across this axis.

1. The main eigen-value is the biggest eigen-value and represents the amount of variability (variance) in that axis (eigen-vector).
2. To project new data point on the main component:

This is the projection of the 2D sample on the main component axis