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A Dynamic Interval Goal Programming Approach to the Regulation of a Lake–River System

RAIMO P. HÄMÄLÄINEN* and JUHA MÄNTYSAARI
Systems Analysis Laboratory, Helsinki University of Technology, Helsinki, Finland

ABSTRACT

This paper describes a model and a decision support tool for the regulation of a lake–river system using both goal sets and goal points for the water level over the year. The inflow forecast is updated periodically, which results in a series of dynamic rolling horizon goal programming problems. This involves heavy computation, and yet it can be successfully done with the spreadsheet program. The related decision support tool with a graphical user interface is called Interactive analysis of dynamic water regulation Strategies by Multi-criteria Optimization (ISMO). The Finnish Environment Institute (FEI) actively uses it in the generation of regulation policy alternatives when considered from the different perspectives of the stakeholders. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: dynamic systems; environment; goal programming; multi-objective optimization; spreadsheets; water resources

1. INTRODUCTION

Multi-criteria problems with a time-dependent structure have started to receive increasing attention in the literature (Kornbluth, 1992; Trzaskalik, 1997; Caballero *et al*., 1998). There are many problem areas, such as water resources management (Haimes, 1977; Szidarovsky and Duckstein, 1986; Rios and Salewicz, 1995; Agrell *et al*., 1998), electric space heating (Hämäläinen and Parantainen, 1995; Hämäläinen *et al*., 2000) and production planning (Changkong *et al*., 1981), in which the system dynamics is an inherent feature. Sometimes also the preferences of the decision-maker can evolve over time (Kornbluth, 1992) or there can be discounting over time (Atherton and French, 1998). In sequential decision problems, the succession of decisions create the dynamics in the form of repeated games (Fudenberg and Tirole, 1991) or bargaining (Ehtamo *et al*., 1989). In each of these settings, different solution methodologies are used. The management of a lake–river system is a dynamic multi-criteria optimization problem, with strong control constraints and essential uncertainties. Yet, in the development of new strategies, one would need to be able to test them interactively. In this paper, we describe an approach implemented in practice to solve such a problem.

We use a dynamic goal programming formulation, which can also be seen as a tracking problem in the control engineering terminology. In multi-criteria control problems (Mäkilä, 1989; Carvalho and Ferreira, 1995), the dynamic equations represent the system being controlled. In our dynamic goal programming model, there are both goal points and goal sets. The sets represent acceptability intervals for the water levels. The definition of goals in terms of sets rather than points is a way to introduce flexibility and robustness into the solution. This is particularly well motivated in dynamic problems, where the achievement of goals at different time points is restricted by the system dynamics. Then, it can be better to softly relax the goal of reaching a number of points and define goal sets around them instead. Allowing such flexibility can make it possible to find a better overall dynamic strategy.

2. THE DYNAMIC MULTI-CRITERIA MULTI-STAKEHOLDER SETTING

In complex environmental decision-making, the understanding and communication of the

* Correspondence to: Systems Analysis Laboratory, Helsinki University of Technology, P.O. Box 1100, 02015 HUT, Helsinki, Finland. E-mail: raimo@hut.fi

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opinions of interest groups is very important (Keeney, 1994; Marttunen and Hämäläinen, 1995; French *et al*., 1998) and multi-criteria decision modeling helps to separate facts from values (Hämäläinen, 1991). In such problems, the evolutionary nature of decision and negotiation support becomes clear. A general framework to approach such multi-party decision problems has been suggested, and it is applied in the present lake regulation project (Hämäläinen *et al*., 2001). In this case, stakeholders whose multi-criteria interests have to be taken into account in the policy-making include, for example, the power production companies (seasonally different value of hydro power), farmers (flooding), environmentalists (nesting of birds and hatching of fish, biodiversity, erosion), owners of lakefront recreational properties (quality of shores, flooding, stability of water level, property values), fishermen (fish spawning), boat owners and lake transportation (water level, flow rate). The goal programming model developed here is used to generate different regulation alternatives and their impacts for the representatives of these stakeholder groups. Decision analytic prioritizations done by the stakeholders are entered into the project's home pages (Hämäläinen, 1999) for the public to see by using the web-Hipre software (Hämäläinen and Mustajoki, 1998). Web-Hipre is the first general purpose multi-attribute decision analysis (MADA) web-based tool running both multi-attribute value theory and of AHP prioritizations.

The strategies are defined in terms of six annual goal points for the water levels, with lower and upper acceptability bounds, called goal intervals, for each point. The problem is to find the regulation strategy that would minimize the deviations from these goals for given inflow data over the planning period. In the goal interval formulation, the distance is measured from the boundaries, and it is zero for all points within the interval. The use of goal sets fits the philosophy of goal programming well, as goal points need or cannot always be reached (Charnes and Cooper, 1961). The set approach allows flexibility in the vicinity of goals, which are the ideals close to which one wants to find the solution.

In the development of new lake regulation strategies, one needs to be able to quickly test the effects of changing the goal points. The model is implemented in an Excel spreadsheet program with a customized user interface. This software is called Interactive analysis of dynamic water regulation Strategies by Multi-criteria Optimization (ISMO) (Hämäläinen and Mäntysaari, 1998a). Figure 1 shows the basic window for setting the

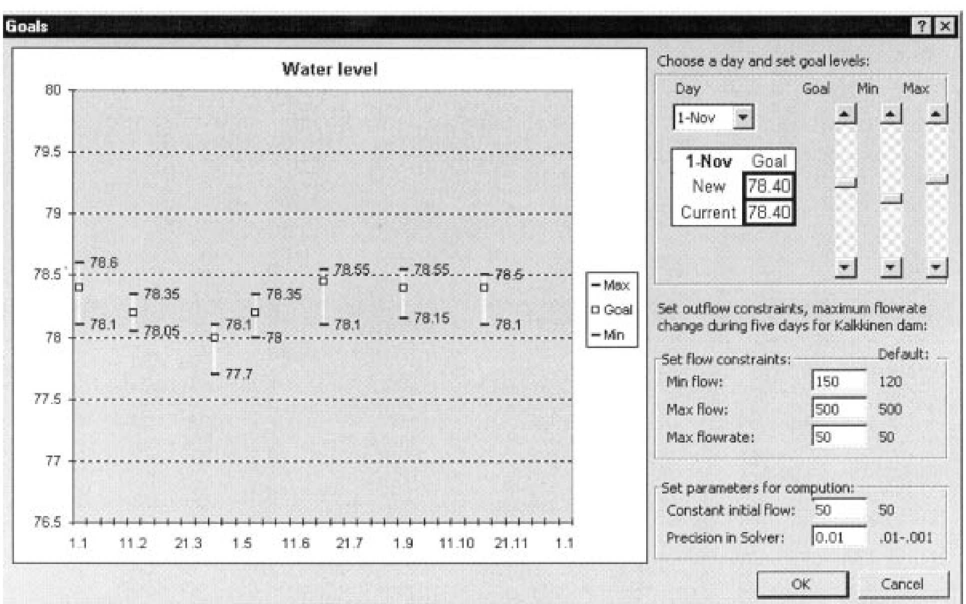

Figure 1. Setting of goal points and acceptability intervals in ISMO spreadsheet program. (Reproduced by permission of Helsinki University of Technology).

goals. The specification of goals as intervals or sets reflects the satisfying approach also used by the lake regulators in practice. The model development was founded by the Finnish Environment Institute (FEI) as part of the large, overall regulation development project described in Marttunen *et al.* (1999).

3. MODELING THE SYSTEM DYNAMICS AND GOAL SETS

The system consists of a series of four lakes and a river connecting them into the sea (see Figure 2). Only Lake Päijänne is dynamically controlled. The flow rate in the connecting river is strictly constrained by lower and upper bounds. Thus, the problem is a constrained dynamic control problem. The major inflow effecting the dynamics is the inflow into Lake Päijänne, $q^{\text{in}}(t)$. The rate of incoming water varies greatly both seasonally and annually. The seasonal drainage for which the regulation policy can be planned with ISMO is a period of maximum 4 years. Moreover, impact models are needed because they are functions of the water level in the lake and the flow rate in the river at different times of year. For example, the price of electricity is highest in the winter time,

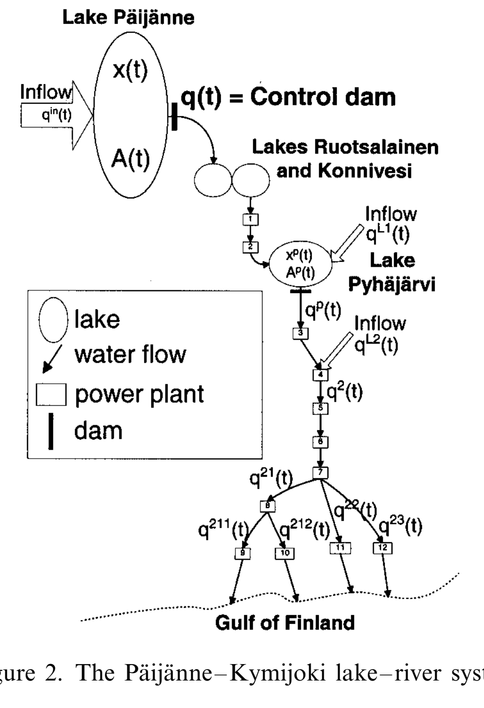

Figure 2. The Päijänne–Kymijoki lake–river system.

and thus, the power companies want to collect water by raising the levels toward fall, and then run the lakes into a low water level by the following spring. ISMO includes a number of specific submodels used to compute the impacts on, for example, fish spawning and loon nesting.

The control variable, $q(t)$, is the total outflow from Lake Päijänne. Only part of the total outflow can be controlled, as there is also an uncontrollable natural channel, and the total outflow consists of the outflow through the natural channel and controlled dam. The natural channel has a piecewise linear discharge function depending on the water level in the lake. The second dam, denoted by the black bar seen in the Figure 2 right after Lake Pyhäjärvi, is not dynamically controlled, but it has a rule-based regulation, which is included in the dynamics. The power plants have their own flow-dependent power production curves. However, they do not affect the dynamics of the system. The lower part of the river has a number of bifurcations which all have fixed flow distributions. A discrete time model with observations on the days 1, 11 and 21 of each month is used. For this technical reason, the time intervals in the model are not of the same length.

The main dynamic component is the water level, x_i, in Lake Päijänne. It is measured as meters above the Normal Null (NN) sea level. The index i refers to the discretized time interval. The water level is driven by the difference between the inflow and outflow, $\Delta q_i = q_i^{\text{in}} - q_i$. The water level also depends on the surface area of the lake, $A(x)$, the previous water level and time interval. The area of the lake is approximated from historical data by a piecewise linear function (Figure 3).

The modeling of the change in the water level, given the inflow and outflow difference, takes place in two phases, as the surface area of the lake depends on the water level. First an initial change, $\Delta\tilde{x}_i$, is calculated using the current area of the lake:

$$\Delta\tilde{x}_i = \frac{\Delta q_i \Delta t_i}{A(\tilde{x}_{i-1})}$$

$$\tilde{x}_i = \tilde{x}_{i-1} + \Delta\tilde{x}_i \tag{1}$$

and then we use this initial change to generate the actual change:

$$\Delta x_i = \frac{\Delta q_i \Delta t_i}{A\left(\dfrac{x_{i-1} + \tilde{x}_i}{2}\right)}$$

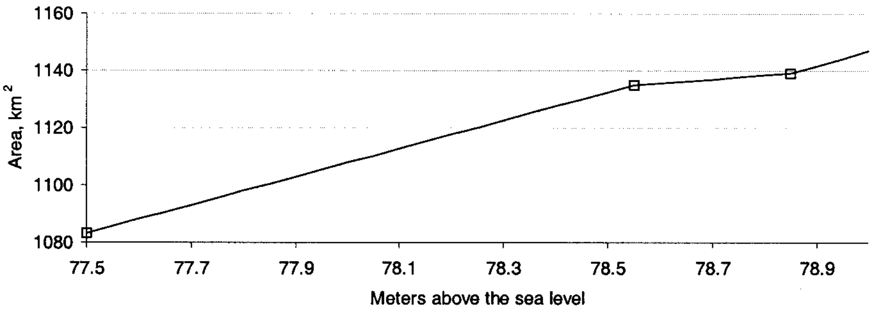

Figure 3. The surface area of Lake Päijänne as a function of the water level.

$$x_i = x_{i-1} + \Delta x_i \quad (2)$$

The initial condition, x_0, is taken from the historical data and at the beginning we set $\tilde{x}_0 = x_0$. This procedure of generating the dynamics of a lake is illustrated in Figure 4.

3.1. Hard constraints

The regulation strategy includes lower and upper bounds for the outflow from Lake Päijänne over the planning period, i.e.

$$q_{\min} \leq q_i \leq q_{\max} \quad (3)$$

These are each hard constraints in the model and they can be easily changed for sensitivity analysis purposes. For Lake Pyhäjärvi, located downstream of Lake Päijänne, a separate fixed regulation policy is used. It also includes hard water level constraints.

3.2. Goal sets and soft constraints

In addition to the above hard constraints, the regulation policy also includes an upper limit for the change in the flow rate:

$$|q_i - q_{i-1}| \leq \Delta q_{\max} \quad (4)$$

This constraint cannot always be met, and thus it is considered as a soft constraint by taking it into account through a penalty function. The penalty term, p_{i1}, is a quadratic deviation from the maximum change of outflow outside the flow rate bound (4):

$$p_{i1} = \begin{cases} (|\Delta q_i| - \Delta q_{\max})^2 & \text{if } |\Delta q_i| > \Delta q_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

There is also a small quadratic penalty term, $p_{i2} = c_2(q_i - q_{i-1})^2$, for any change in outflow. This additional penalty is only a modeling technique used to help to avoid rapid changes in the outflow.

The deviation from the goal point is measured by a quadratic distance between the goal, x_k^{goal}, and the true water level:

$$g_k = (x_k^{\text{goal}} - x_k)^2 \quad (6)$$

The deviation from the goal set, which is the acceptability interval, is also measured by a quadratic distance from the lower, l_k, and upper, u_k, bound of the goal interval:

$$I_k = \begin{cases} (l_k - x_k)^2 & \text{if } x_k < l_k \\ (x_k - u_k)^2 & \text{if } x_k > u_k \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

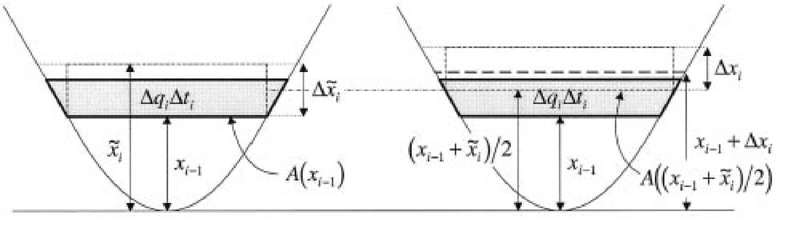

Figure 4. Modeling the change in the water level.

The optimal regulation policy is derived by minimizing, F, the weighted sum of the goal deviations and penalties, i.e.

$$F = \sum_{k \in K} d_k + \sum_{i=1}^{N} p_i \tag{8}$$

where

$$d_k = c_g g_k + c_I I_k$$
$$= \begin{cases} c_g(x_k^{\text{goal}} - x_k)^2 + c_I(l_k - x_k)^2 & \text{if } x_k < l_k \\ c_g(x_k^{\text{goal}} - x_k)^2 + c_I(x_k - u_k)^2 & \text{if } x_k > u_k \\ c_g(x_k^{\text{goal}} - x_k)^2 & \text{otherwise} \end{cases}$$

and

$$p_i = c_1 p_{i1} + c_2 p_{i2} = \begin{cases} c_1(|q_i - q_{i-1}| - \Delta q_{\max})^2 + c_2(q_i - q_{i-1})^2 & \text{if } |q_i - q_{i-1}| > \Delta q_{\max} \\ c_2(q_i - q_{i-1})^2 & \text{otherwise} \end{cases}$$

subject to the dynamics (1)–(2) and flow constraints (3), where K is the set of goal observation indexes of the planning period. The coefficients c_g, c_I, c_1 and c_2 are adjustable design parameters in ISMO. Here, the values used are $c_g = 10$ L/m^2, $c_I = 100$ L/m^2, $c_1 = 100$ s^2/m^6 and $c_2 = 0.00001$ s^2/m^6. In the goal point formulation, the coefficient c_I is zero. We get the goal interval formulation without weighting on the individual goal points when coefficient c_g is zero.

4. GENERATION OF REGULATION STRATEGIES

The users give the desired target water levels, goal points, at six different points during one year and/or an acceptability interval for each point (see Figure 1). The control problem is to stay within the intervals and/or track the goal points as closely as possible. The maximum planning period is 4 years. Naturally, the solution depends on the inflows, that is, the weather pattern assumed. Annual inflows vary considerably and the consequences, that is impacts, such as floods and power production, strongly depend on the true water levels and flow rates. Thus, the use of average inflows as design data does not make sense. In averaged data, the peak flows always disappear, as indicated by Figures 5 and 6. The planning is done in practice with respect of normal, dry and wet inflow year patterns taken from the historical data (see Figure 9).

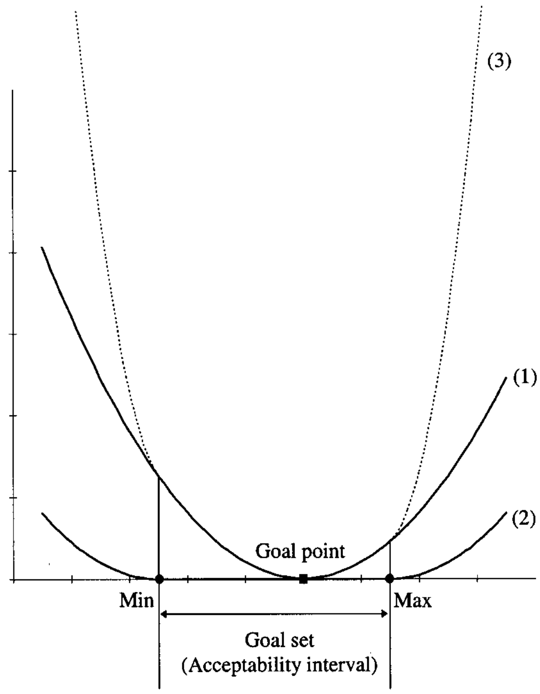

Figure 5. The goal deviation functions for goal point (1), goal set (2) and the combined model (3).

An essential point is that in real life perfect information on the inflows will not be available. Therefore, the generation of the realistic solution for the regulation strategy needs to be based on a model of the real operational regulator and his weather forecasting horizon and data. In this case, the operator of the lake–river system has a policy of using a rolling two goal points time window. Moreover, he updates his decisions at each point (approximately 10 days interval) based on the observed inflow data (see Figure 7). From the computational point of view, this means solving a dynamic optimization problem whenever the rolling time window of planning period is changed, that is for each month, and making heuristic corrections to outflow decisions within the month. In ISMO, the solution time on a Pentium Pro 200 MHz is in the range of 10–15 min for a 4 year period (Hämäläinen and Mäntysaari, 1998b). This is somewhat too long for extensive demonstrations online, together with the stakeholders, but still, it is acceptable for experts when they develop alternative policies.

The practical decision support sessions are based on a series of scenarios where a set of

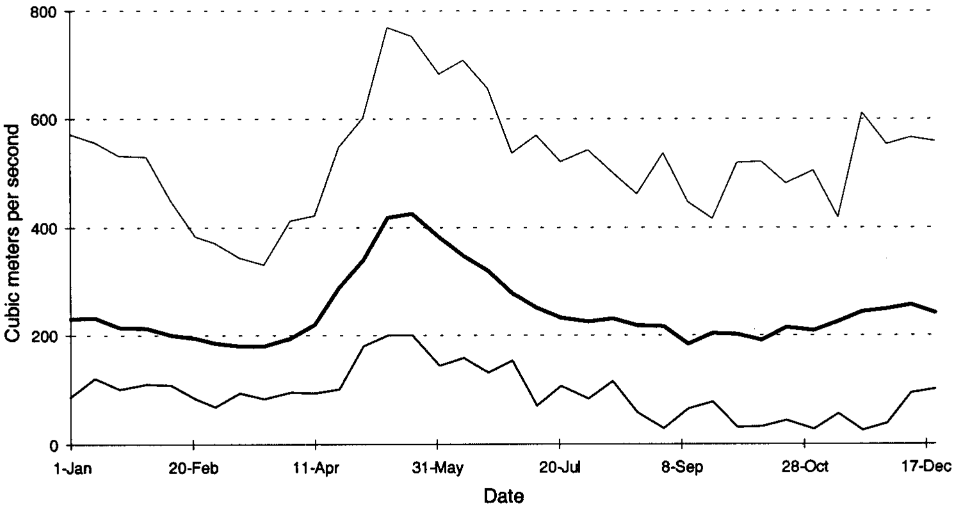

Figure 6. Averaging strongly flattens the peak and low flows: see the maximum, average and minimum inflows during a year at Lake Päijänne according to historical data from years 1970–1995.

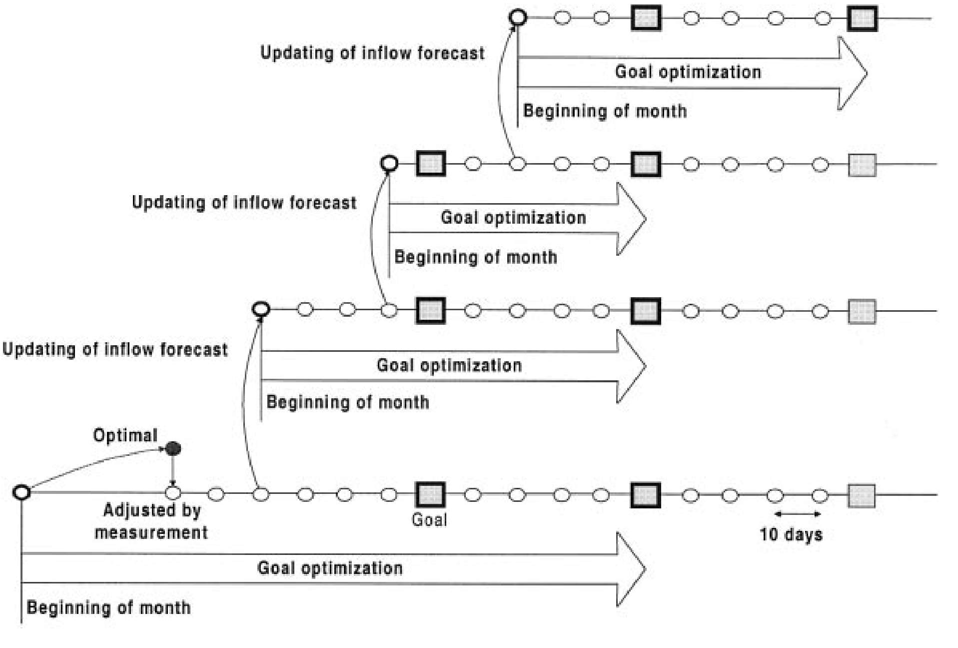

Figure 7. Generation of the realistic solution.

presolved cases are analyzed by the stakeholders. The stakeholder can also ask for changes in an individual goal point at a time that provides a simple means to understand the dynamic multi-criteria nature of the problem.

The real need for this decision support tool can easily be demonstrated by comparing the utopia solution based on perfect weather information with a 1 year look ahead time window to the realistic solution which has the operational regulation practice with incomplete weather forecasts, as its basis shows a case run where the goal points have been set so that the water level would remain as constant as possible over the period. One easily sees, for example, that in the utopia solution, the high inflow in the middle of the period has been anticipated and compensated by increasing the outflow. Thus, the resulting water levels do not change as much as they would in the practical operation under incomplete forecasts about the inflow. Stakeholders often expect that such utopia solutions could be achieved in real life too. The

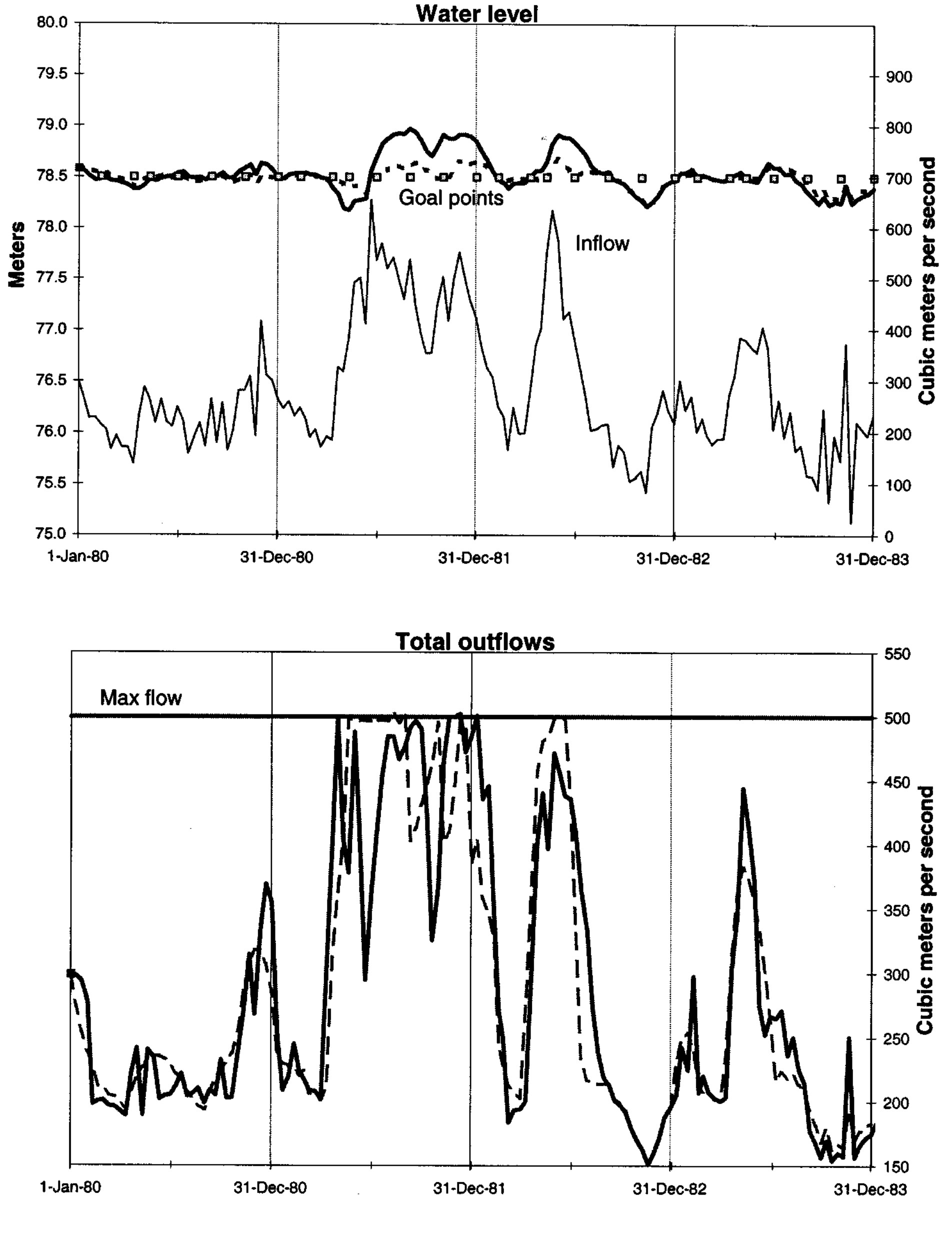

Figure 8. Comparison of policies: utopia solution (dashed line) with perfect knowledge of inflows and one year look ahead optimization; realistic solution (solid line) with true updated inflow forecasts and optimization by a rolling two goal points time window following the real operational regulation practice (see also Figure 7).

model proved very useful in revealing such misconceptions.

5. COMPARISON OF TWO REGULATION STRATEGIES FOR DIFFERENT GOAL FORMULATIONS

The recreation strategy aims at keeping the water level nearly constant during the entire year. The power strategy aims at maximizing the profit from the high cost electricity in the wintertime by collecting a large water reserve during the summertime and releasing it in the wintertime. These strategies can be developed by using different goal formulations. When we only use goal points, that is, the target water levels, the coefficient c_I is set to zero. When we relax the goal points to goal intervals, where interval bounds are formed by increasing and decreasing each goal by 10 cm, coefficient c_g is set to zero. Finally, we could combine these two by using goal points inside the goal intervals, that is, by setting the coefficients c_g and c_I to a positive value.

An important use of this regulation model is to demonstrate what really is possible to achieve under different weather patterns (see Figures 8 and 9). However, here we only focus on the effects related to the different goal formulations. Therefore, we use one reference year (1992) with perfect knowledge of inflows. For computational reasons, the inflow of the following year is assumed identical to the inflow of year 1992.

The strategies do, indeed, reach the goal points quite well during the year (see Figures 10 and 11). The relaxation of the model by replacing the goal points by goal intervals makes the outflows much smoother. The interval solution takes full advantage of the acceptability bounds by running the water level at the lower or upper bound at the end of the year. For the parameters used, the results for combined point and interval goals become almost the same as those for the goal points.

The transients at the end of the year are modeling artefacts owing to the endpoint optimization effect. This phenomenon could be avoided, for example, by solving a series of identical years and extending the planning period up to 3 years. The second year could then be used as the basic solution.

6. DISCUSSION

The early literature on goal programming and multi-criteria decision-making originated from managerial or business applications (Charnes and Cooper, 1961). Today, however, it looks that environmental problems are becoming one of the most fruitful areas of multi-criteria decision analysis.

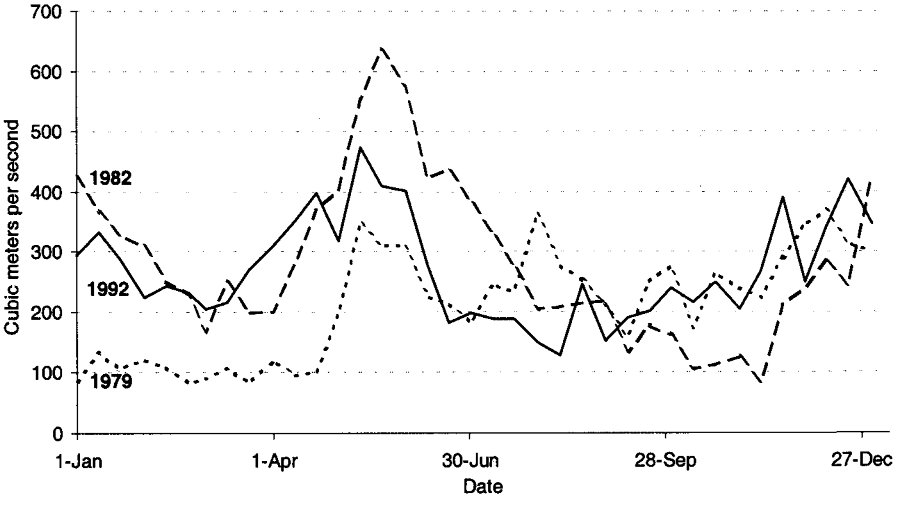

Figure 9. Examples of inflows: normal (1992), wet (1982) and dry (1979) years.

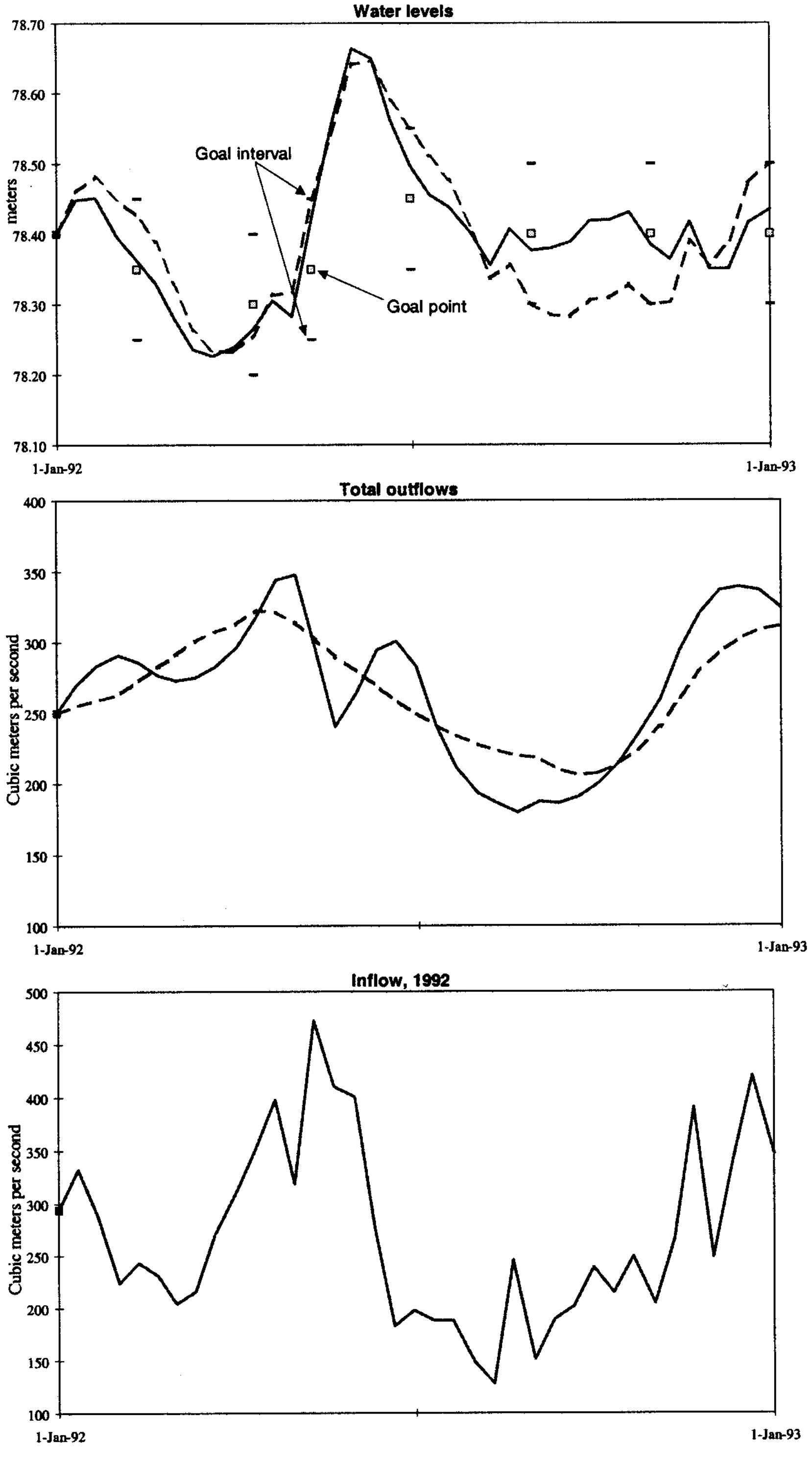

Figure 10. Recreation strategy for a normal inflow year 1992 with different goal formulations; goal point (solid line) and goal interval (dashed line).

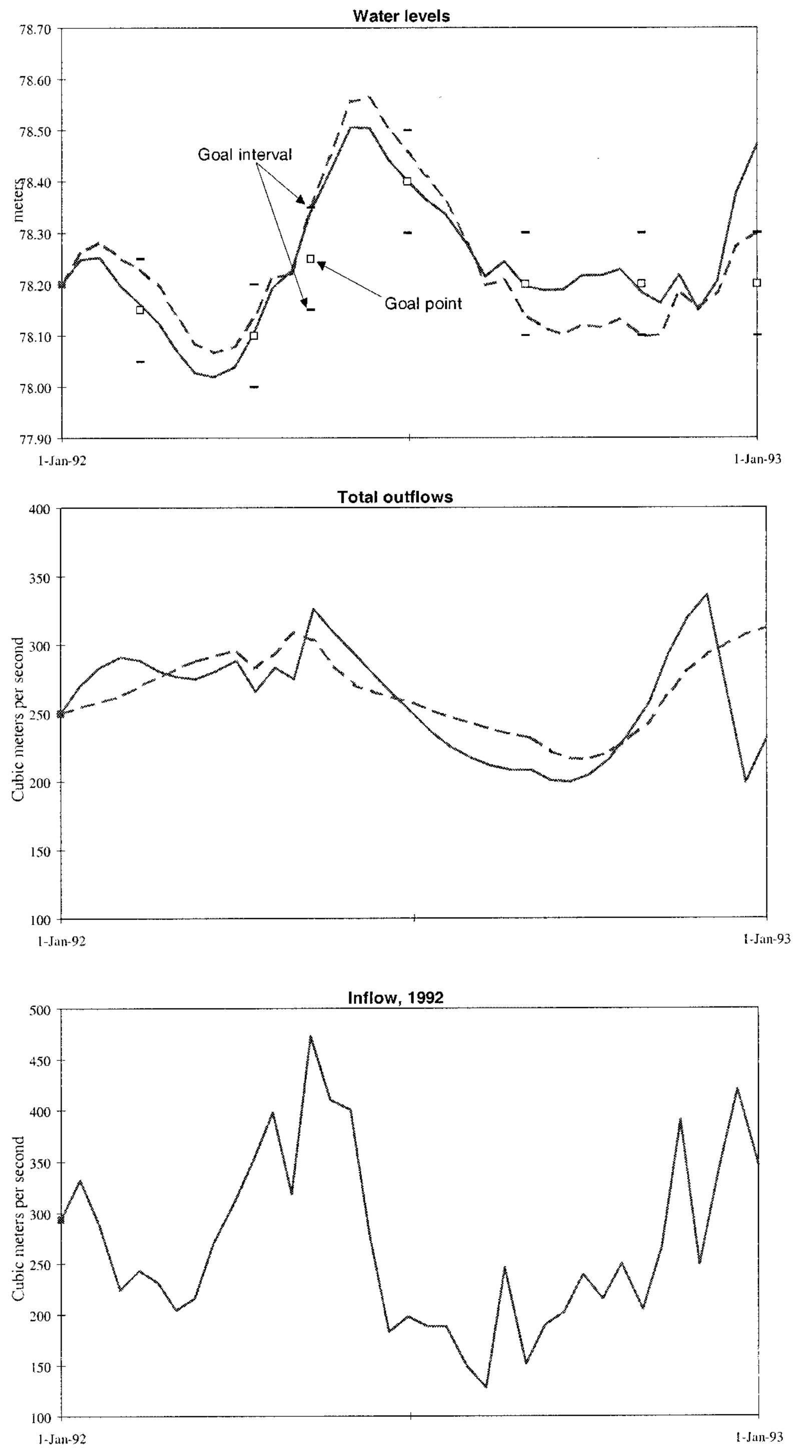

Figure 11. Power strategy for a normal inflow year 1992 with different goal formulations; goal point (solid line) and goal interval (dashed line).

The present project is an example of a methodologically challenging and practically important environmental problem where a dynamic multicriteria model is essential. The need for modeling is easily understood by comparing the lake regulation solution, with perfect knowledge of the inflow, with the solution with realistic forecasting data and operational regulation practice. These solutions differ considerably.

Our MCDM model was first used extensively in FEI for the development of a set of preliminary regulation strategies. Six different strategies were chosen to be presented to the representatives of the interest groups. Owing to the dynamics of the lake river system, one really needs a tool like this, as it would take a very long time to iterate the outflows manually so that the target water levels would be met. The related impact tables were the key information material in the decision analysis interviews and conferences with the stakeholders. The participants also had the opportunity to make individual changes in the goal water levels and the related solutions were computed for them with ISMO. This helped the representatives to create confidence in the process, and to better understand the restrictions posed by the system dynamics.

The possibility of defining goals as intervals, that is, sets, was proposed already in the seminal paper of Charnes and Cooper (1977). Yet, this idea has not received much attention in practical applications. When optimizing a dynamically constrained system like ours, the goal levels set for the different time points can have a strong effect on each other. The reachability of all the goal points at the same time is by no means guaranteed. The use of interval goals gives a convenient way of relaxing the problem formulations. This is one of the benefits of the goal set concept in dynamic problems, as it is a simple way to reach a flexible overall solution.

How the goal points should be specified to optimize a set of given regulation impacts has not been directly approached in this paper. The setting of goal points is, in fact, another multicriteria problem. Here, we have followed the traditional goal programming philosophy by assuming that stakeholders can indeed define them directly. This process could, however, be seen and modeled in a much wider context as a negotiation problem over the goal points between the regulator and the stakeholders (Hämäläinen *et al.*, 2001). Such approaches remain open topics of goal programming applications and research.

Our approach emphasizes that decisions are often evolving over time, and thus, dynamic decision modeling needs to be studied. However, we have not yet considered the behavioral issues related to dynamic multi-criteria decision-making. Human decision-makers easily show biases when using decision models (see, for example, Pöyhönen and Hämäläinen, 1998). So far, we know very little about the possible biases related to dynamic settings, and these are clearly an important topic for future research.

Recently, spreadsheet programs have become an important platform for the development of visual interactive models even for relatively complex problems (see, for example, Mäntysaari and Hämäläinen, 1997). Our ISMO software is clearly a good example of this development.

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