

## Fwd: Scan to My University Email address

Euan Spence

Mon 13/05/2019 22:04

To: Owen Pembroly <orp20@bath.ac.uk>;

Cc: Ivan Graham <masigg@bath.ac.uk>;

1 attachments (4 MB)

2019-05-13T213241.pdf;

Hi Owen (cc Ivan),

Attached are my comments on your draft.

If you don't understand something I've written, or want to discuss something, just give me a call. (On Tuesday and Wednesday I'll mostly be at home.)

In one of the comments (about the definition of triangulation) I mention "Ciarlet" - I mean Page 61 of this <https://www.sciencedirect.com/science/article/pii/S1570865905800390> (If you can't access it, let me know and I'll e-mail it to you.)

Here are some more-detailed comments about the literature review on the bounds.

a) I understand why you made the distinction between "obstacle scattering" and "scattering by the medium", but it has the following drawbacks:

- A and  $n$  varying with a jump is how one models penetrable obstacles (so, at the least, "obstacle scattering" should be changed to "impenetrable-obstacle scattering"),
- The example of exponential growth by Simon et al is in the context of obstacles.
- Theorems 2.6 and 2.7 include impenetrable (star-shaped) obstacles, so you kind of need to talk a bit about obstacles for the reader to understand this aspect of the theorems (e.g. why the star-shaped assumption is needed).

b) I also thought that the distinction between the EDP and the TEDP was a bit artificial, e.g. the example of exponential growth also holds for the TEDP (since the quasimodes have compact support).

c) If it were me, I would structure the discussion in the following way. (However, this is only a suggestion, and what you have already is pretty good already.)

- Want to prove bound 2.7 (with  $g_D=0$ ) and understand  $k$  dependence of  $k$  (will discuss  $A$  and  $n$  dependence later).
- General obstacle upper bound (2.9) by Burq in case of smooth medium and smooth impenetrable obstacle, Bellasoued when  $A$  and  $n$  jump.
- (2.9) sharp (through a sequence) in obstacle case by Simon et al. Superalgebraic growth (through a sequence) in penetrable obstacle case (for "bad" jump) by Popov and Vodev.
- Key geometric property: trapping/nontrapping. Easy to define when medium and impenetrable obstacle are smooth, harder otherwise.
- If nontrapping, have  $C \sim 1/k$  - Melrose - Sjostrand + Vainberg/Lax-Philips, refined by Burq (replacing propagation of singularities by propagation of defect measure [just for info: don't need to say this]).
- Will call cases when  $C \sim 1/k$  "nontrapping", even if don't strictly meet definition.
- In some nontrapping cases can prove bound using Morawetz multiplier
  - star-shaped impenetrable obstacles (Morawetz and Ludwig), nontrapping impenetrable obstacles (Morawetz, Ralston, Strauss),  $A$  and  $n$  varying smoothly - Bloom, Kazinaroff,  $A$  and  $n$  jumping (in "good" way) Moiola Spence, with then both these last two cases superceded by results in hetero paper.
- Impedance boundary condition - goal is to mimic radiation condition, therefore expect  $C$  for TEDP to have same behaviour for EDP.
- Indeed, have  $C \sim 1/k$  for star-shaped  $\Gamma_D$  and  $\Gamma_I$  by Cummings and Feng (no obstacle by Melenk).
- Have  $C \sim 1/k$  for IIP with smooth boundary by Baskin, Spence, Wunsch. Open problem to prove this for Lipschitz boundary -best is Spence 2014, following on from bounds by Esterhazy and Melenk (using method Feng and Sheen used for square).
- Variable  $A$  and  $n$  for TEDP/IIP - Brown + Gallistl, Feng, Lin, Lorton, Ohlberger +Verfurth, and then in 1-d by Chaumont-Frelet, Graham + Sauter, Sauter + Torres.
- Expect PML to also have  $C \sim 1/k$  - proved for problem with impenetrable obstacle in <https://hal.archives-ouvertes.fr/hal-01887267> and for no obstacle in Li and Wu.
- Then talk about the bounds explicit in  $A$  and  $n$  exactly as you do (and then also the "Dependence on trapping behaviour..." exactly as you do).

I hope these comments make sense, and are useful.

As I said above, if you'd like to discuss, give me a call!

Euan

----- Forwarded message -----

From: Euan Spence <eas25@bath.ac.uk>

Date: Mon, 13 May 2019 at 21:34

Subject: Scan to My University Email address

To: Euan Spence <eas25@bath.ac.uk>

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