

good title!

Characterizing Resistivity, Carrier Mobility, and Carrier Concentration in Germanium via the Hall Effect

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Grade Breakdown

20%	19%	Writing, showing instead of telling, figures well-made and described	Very well written! Nice figures. Watch labels.
20%	18%	Apparatus & measurements understood, explained & sensible	Define coordinates, label sample pic for clarity
20%	18%	Justified quantitative statistical uncertainties, chi^2 & residuals discussed quantitatively	χ^2 shouldn't be figure title.
20%	19%	Justified quantitative systematic uncertainties (not part of first reports)	Some noise ~10°C.
20%	20%	Validity of work and sufficient work completed	Extracted n & μ , R_h . All good!
Bonus	—	Going beyond basic measurements, concise figures, clean writing, etc.	[Power law for $\mu(T)$, thermal ENF, purge to avoid water short, activation energy for high T data] possible extensions.

94%

Great work!

Abstract

*nice abstract. Looks like you took
it quite far!*

This experiment aims to characterize resistivity, carrier mobility, and carrier concentration in a p-doped germanium sample using the Hall effect and a four-point probe measurement technique. We investigated these properties by taking voltage readings across seven points of the crystal under varying temperatures and magnetic field strengths. The Hall voltage measurements [allowed us] *best to not refer to "we, us, etc."* to analyze the temperature dependence of the resistivity. They were used to characterize the Hall effect by calculating the Hall coefficient, which ranged between $R_{H,min} = (5 \pm 2) \times 10^{-6} \text{ m}^3/\text{C}$ at $T = 80.0 \text{ }^\circ\text{C}$ and $R_{H,max} = (1.11 \pm 0.04) \times 10^{-5} \text{ m}^3/\text{C}$ at $T = 16.9 \text{ }^\circ\text{C}$. The positive sign of this coefficient is a direct indicator that the germanium sample is indeed p-doped, meaning it contains excess holes. By plotting the Hall voltage V_H as a function of magnetic field B at a fixed temperature T , we were able to fit for the carrier density n , which gave us values within the range $n_{min} = (7.2 \pm 0.3) \times 10^{20} \text{ m}^{-3}$ to $n_{max} = (1.4 \pm 0.6) \times 10^{21} \text{ m}^{-3}$, corresponding to $-100.0 \text{ }^\circ\text{C} \leq T \leq 80.0 \text{ }^\circ\text{C}$. The average carrier density was calculated to be $n_{avg} = (6.289 \pm 0.008) \times 10^{20} \text{ m}^{-3}$. The resistivity curve was also converted to conductivity σ , which was then used to get the carrier mobility μ as a function of field strength and temperature.

Contents

1	Introduction	1
2	Apparatus	2
2.1	Germanium Sample	2
3	Results	4
3.1	Resistivity	4
3.2	Carrier concentration	6
3.3	Hall coefficient	8
3.4	Carrier mobility	8
4	Conclusion	10

1 Introduction

The Hall effect is a phenomenon that can be observed in (semi)conductive materials when an electric current is passed through the material while subjected to a perpendicular magnetic field. First discovered by Edwin Hall in 1897, the Hall effect results in the generation of a transverse (Hall) voltage which can be measured across the material, perpendicular to the current flow. This comes from the Lorentz force acting on the moving charge carriers - in the case of semiconductors, the charge carriers come from the material being doped with a conductive substance. The carriers thus accumulate on one side of the material, generating an electric field and thus a measurable voltage difference. By investigating the Hall voltage and its dependence on temperature and magnetic field, the type and properties of these dopants can be deduced.

The Hall effect follows the following law:

Can avoid by not putting a line break after the equation.

$$V_H = \frac{BI}{nqt}, \quad (1)$$

where B is the applied magnetic field, I is the current through the material, n is the carrier concentration, q is the elementary charge ($1.602 \times 10^{-19} \text{ C}$), and t is the thickness of the material.

By considering the sign of the Hall voltage, we can determine whether the material is p-doped (excess holes) or n-doped (excess electrons). To characterize the strength of the Hall effect in a semiconductor, we can consider the Hall coefficient:

$$R_H = \frac{V_H t}{BI} = \frac{1}{nq}. \quad (2)$$

It follows that for a p-type semiconductor, the Hall coefficient will be positive, while it will be negative for an n-type material.

The resistivity can also be considered by measuring the voltage drop across the material, given by:

$$\rho = \frac{V_x A}{IL}, \quad (3)$$

where V_x is the voltage drop along the direction of current flow, and A is the cross-sectional area of the sample. The resistivity is inversely proportional to the conductivity σ , given by $\sigma = 1/\rho$. This lends itself to a parametrization of the mobility (μ) of the charge carriers (electrons or holes) in the sample:

$$\mu = \frac{\sigma}{nq}. \quad (4)$$

Conductivity and mobility are directly proportional, so by measuring the conductivity, we can characterize the dopants in terms of their mobility within the material.

2 Apparatus

2.1 Germanium Sample

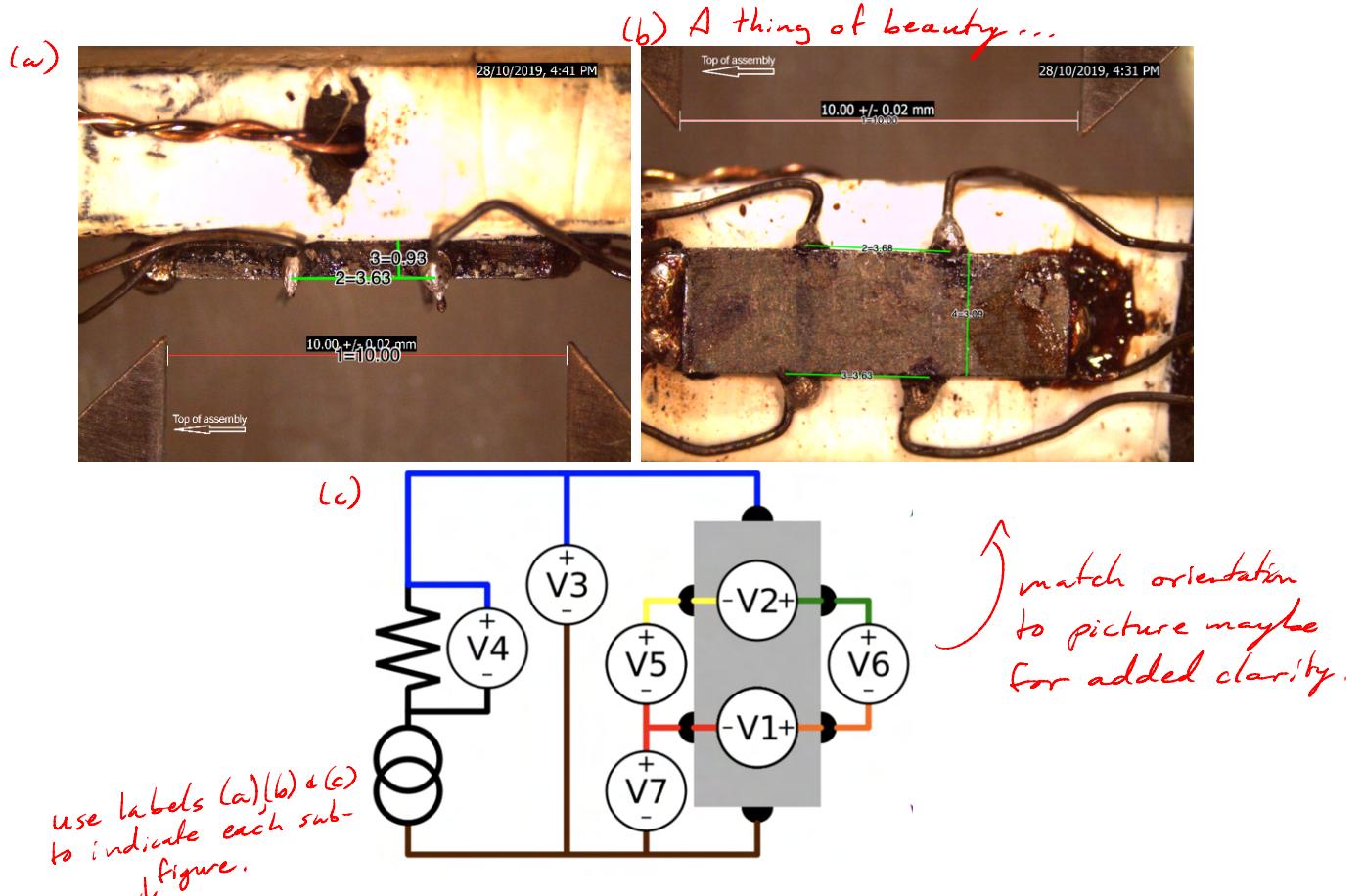
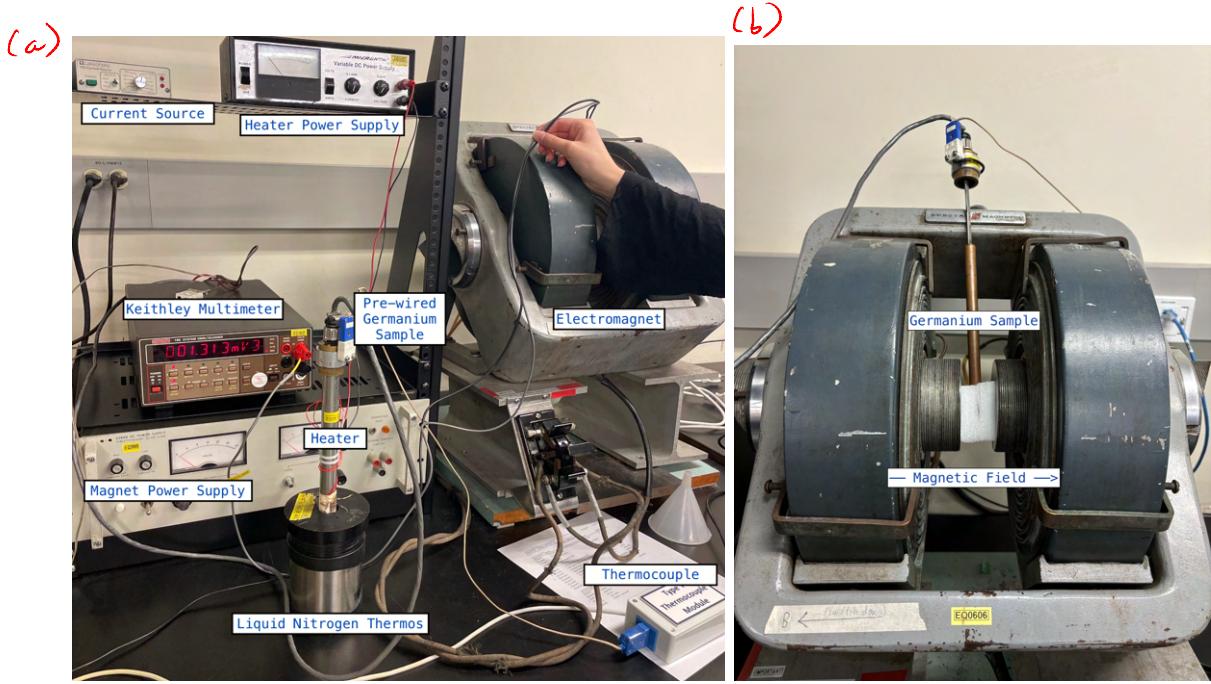


Figure 1: Dimensions and wiring diagram for a p-typed doped germanium sample [1]. The sample was oriented in the magnetic field such that V_1 and V_2 represent the Hall voltages.



*(a) refer to
in caption*

Figure 2: Experimental setup. The germanium sample was wired to a thermocouple for continuous temperature readings, and placed within a copper casing wired to a powered heater. This was in turn placed in a thermos containing liquid nitrogen. The electromagnet was attached to a power supply that allowed for an adjustable magnetic field. The current was supplied to the germanium via a Lakeshore power supply. All readings were taken with a Keithley multimeter via a Python interface.

A doped germanium sample - $(10.00 \times 3.09 \times 0.93) \pm 0.02$ mm - was prepared for this experiment by wiring it according to the diagram shown in Figure 1. The sample was connected to a 10 mA current source, enabling a controlled current to flow through it. To allow for temperature variation, the sample was placed inside a copper tube, which was heated using a resistor connected to a power supply. This setup allowed the temperature of the sample to be raised to approximately 80°C. For low-temperature measurements, the copper tube containing the germanium sample was submerged in a Thermos filled with liquid nitrogen, cooling the sample to around -100°C (refer to Figure 2 for the setup).

Temperature sweeps were performed at fixed magnetic field strengths. The magnetic field was adjusted by varying the current to the electromagnet, and each field strength was precisely measured using a magnetic field probe before each run. Voltage drops along the sample, parallel to the current direction, were measured using a 4-point probe technique to minimize the impact of contact resistances, ensuring accurate resistivity measurements [2].

The 4-point method was critical in reducing errors from the electrical contacts, which can introduce resistance artifacts into the measurement.

The magnet was oriented such that the magnetic field pointed from the left to the right side of the magnet, while the sample was positioned so that the magnetic field was directed perpendicular to the current, out of the top of the sample, as shown in Figure 2. The sample was also placed such that the field was perpendicular to the Hall voltage axis, meaning it would be coming out of the page in the Figure 1 circuit diagram. In this configuration, negative Hall voltages were observed at contacts V1 and V2 (see Figure 1), implying a buildup of positive charge carriers (holes) at the negative terminal of the voltmeter. This behaviour is consistent with the Lorentz force acting in the opposite direction to the applied electric field, causing holes to accumulate at the measured terminals. Based on this observation, we can conclude that the germanium sample is p-type, indicating that holes are the majority charge carriers [3].

Magnetic field readings were calibrated using the provided calibration curves for our sensor [1], and readings were taken with an already calibrated Keithley multimeter.

3 Results

For each experimental run, the magnetic field was held constant while a temperature sweep was performed, taking readings one channel at a time. To ensure that data points from a single run could be compared across the different time intervals, interpolation was applied to align the data over time, essentially matching all the voltage channels to the temperature readings. Afterward, the voltages for each channel were further interpolated over temperature, ensuring that the voltages for each channel aligned on a common temperature axis across multiple runs. As a result, we obtained a consistent dataset with voltage measurements by channel, all matched to a uniform set of magnetic field and temperature values.

3.1 Resistivity

The resistivity was measured by taking the average of the voltage drop across V_5 and V_7 , which measure longitudinal voltage differences along the sample (see Figure 1) to minimize

systematic uncertainties from the electrical contacts. The length of the sample was taken to be $(10.00 \pm 0.02) \times 10^{-3}$ m, and the cross sectional area was calculated as $(3.09 \pm 0.02) \times 10^{-3}$ m \cdot $(0.92 \pm 0.02) \times 10^{-3}$ m $= (2.86 \pm 0.06) \times 10^{-6}$ m². The current source provided a current of 10.000 ± 0.001 mA [4].

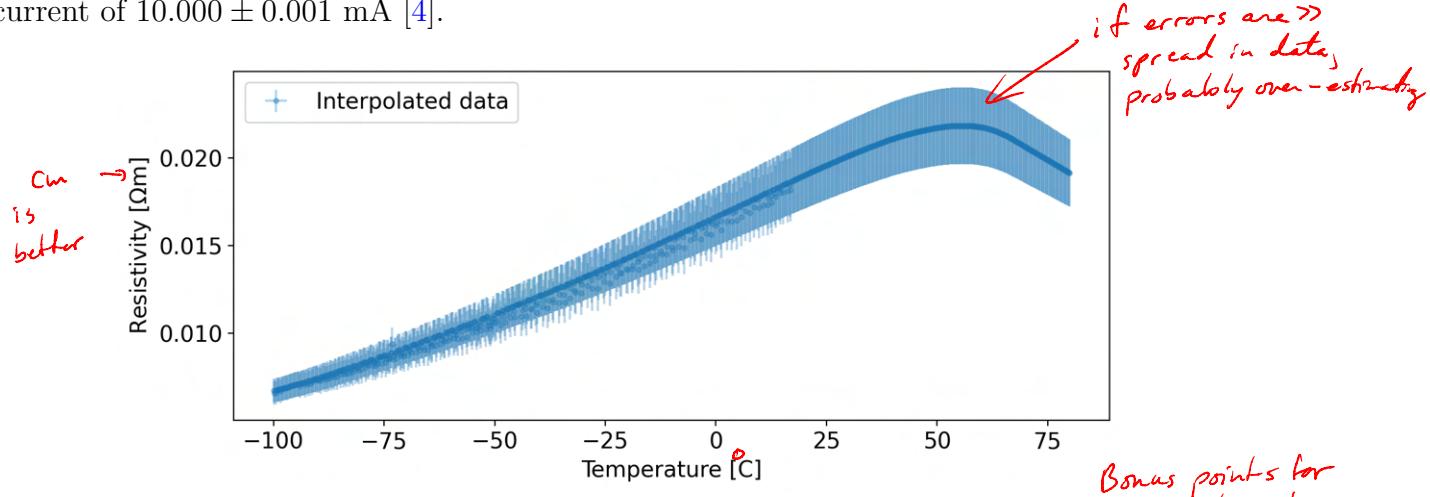


Figure 3: Temperature dependence of the resistivity for the germanium sample at an ambient magnetic field of 0.6 mT. The resistivity increases with temperature in the low-temperature regime, consistent with phonon scattering dominating carrier mobility. Error bars represent the uncertainties in temperature and resistivity measurements given by the digital readings, and are propagated according to equation 3.

As shown in figure 3, there is a non-uniform temperature dependence observed depending on the temperature regime the germanium is operating in. At low temperatures, the resistivity is approximately linear with respect to temperature, reflecting that the dopants are less resistive, confirming the presence of dopants as suggested in [5]. One could expect that as temperature increases, the dopants gain more energy to become ionized, and thus would cause a decrease in resistivity in the material. In our case, the increase in resistivity with temperature could be due to increased scattering causing less efficient flow of electrons [3]. This means that between the two competing effects, ionization of the dopants and phonon scattering, the latter is dominating (although minimal) at low temperatures due to insufficient charge carriers. Further experiments at even lower temperatures could potentially reach a regime where scattering becomes more minimal, allowing the low ionization of dopants to dominate and reduce resistivity. The increasing resistivity trend follows until around 50°C is reached when it begins dropping as the temperature continues to increase. We theorize that this is due to the ionization of the dopants dominating over the effects of scattering,

typically Kelvin is used for absolute scale.

thus contributing more charge carriers to the system. Since the temperatures remain below a maximum of roughly 350 K, this behaviour change is not from the band gap energy of germanium itself being reached, as the literature suggests this would only occur for germanium at temperatures above approximately 500 K [3]. *good!*

3.2 Carrier concentration

This is weird [100: Chi-square: 7.7736076268 -60: Chi-square: 11.6344668884 0: Chi-square: 16.4251159060
60: Chi-square: 3.6152993599 80: Chi-square: 227.9656098341]

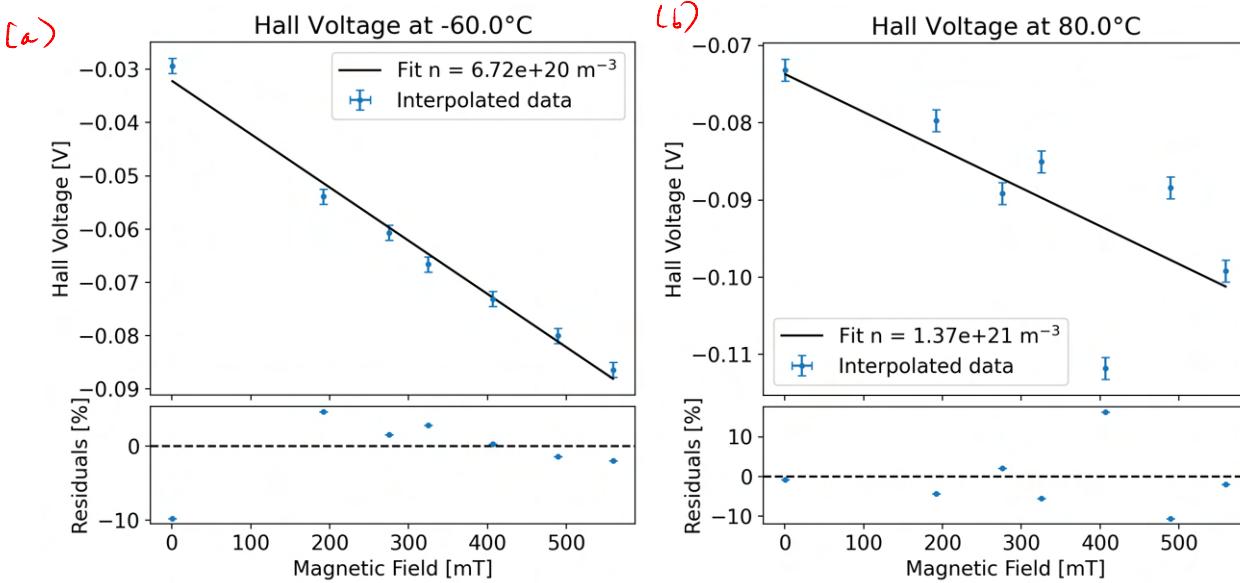


Figure 4: Hall voltage at varied magnetic fields for the germanium sample measured at -60°C and 80°C . The solid lines show the linear fit used to extract the carrier concentration n . At -60°C , the Hall voltage data is well-constrained, resulting in a more precise fit ($\chi^2 = 11.63$), while the fit at 80°C shows larger deviations and residuals ($\chi^2 = 227.96$), reflecting increased scatter in the data. The difference in fit quality between the two temperatures suggests temperature-dependent changes in carrier dynamics.

t-dependent? *What about your measurement would add noise @ higher temperature?*
Sohnson noise?

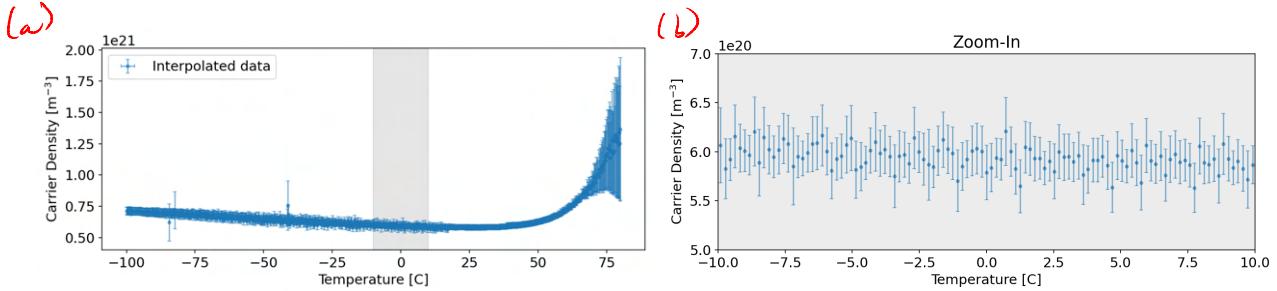


Figure 5: Carrier concentration n for the germanium sample, extracted from Hall voltage measurements in Figure 1. Error bars represent uncertainties in the fits for n , which grow significantly at higher temperatures. The right panel provides a zoomed-in view of the small temperature range (shaded in grey).

Similar to our resistivity calculation, we also averaged over the two Hall voltage readings, V_1 and V_2 (see Figure 1) to improve systematics, and we used the thickness measurement of $(0.92 \pm 0.02) \times 10^{-3}$ m, with q taken to be constant. Hall voltage was compared against magnetic field strength (see Figure 1) and fitted to equation 1 to get a value for the carrier concentration n . Across the temperatures, the concentration remained within the range $n_{min} = (7.2 \pm 0.3) \times 10^{20}$ m $^{-3}$ to $n_{max} = (1.4 \pm 0.6) \times 10^{21}$ m $^{-3}$. By repeating the fit at all temperature points, we were able to take a weighted average of the values of n obtained, resulting in a final value of $n = (6.289 \pm 0.008) \times 10^{20}$ m $^{-3}$, which can roughly be approximated as 1000 mole per cubic meter of charge carriers, in agreement with the literature on other p-type samples [3, 6].

The trends in carrier density at low temperatures can be attributed to the limited amount of thermal energy available to ionize carriers. In this regime, the carrier density decreases as recombination dominates. However, around 50°C, we see the carrier density begin to increase rapidly as there is more energy available to ionize carriers. This is consistent with the resistivity trend seen in Figure 3. At high temperatures, the fit for the carrier density has higher values of χ^2 ($\chi^2 = 227.96$ at 80 °C), indicating that the fits have higher uncertainty. This can be seen in the residuals of Figure 4 and in the error bars of Figure 5. This is due to the Hall voltage not varying with magnetic field strength in the same way as for low temperatures. One possible explanation is that, at higher temperatures, increased phonon scattering could disrupt the charge carrier motion, leading to non-linear behaviour in the Hall effect.

*Might be thermal EMF contributing to measured voltages.
Typically one reverses current + measures again. Signal should
flip sign, thermal EMF 7
stays the same. Can subtract to eliminate thermal EMF.
This is important when measuring small e or light T.*

3.3 Hall coefficient

The trends observed in the Hall coefficient R_H align with the expected behaviour of a p-doped semiconductor (CITE), as we observe a positive Hall coefficient (see Figure 2). Temperature and Hall coefficient increase based on the trends described for the carrier concentration in Section 3.2, since R_H and n are inversely proportional, given by equation 2. The coefficient was found to range between $R_{H,min} = (5 \pm 2) \times 10^{-6} \text{ m}^3/\text{C}$ at $T = 80.0 \text{ }^\circ\text{C}$ and $R_{H,max} = (1.11 \pm 0.04) \times 10^{-5} \text{ m}^3/\text{C}$ at $T = 16.9 \text{ }^\circ\text{C}$.

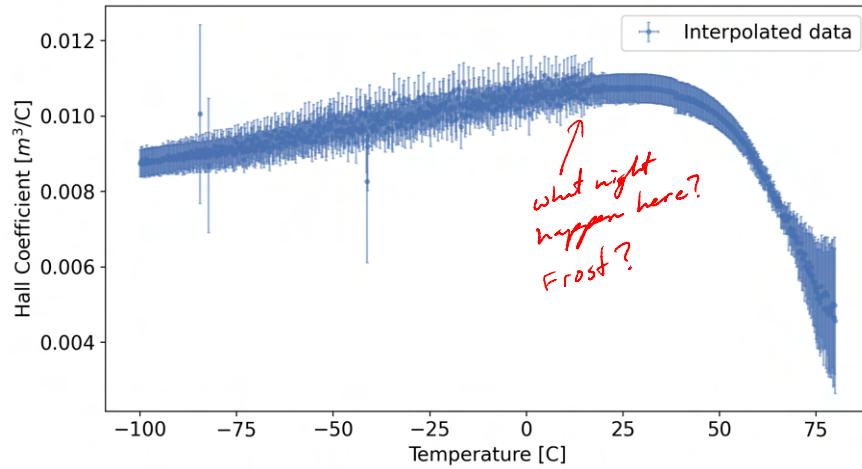


Figure 6: Hall coefficient R_H for the germanium sample at ambient magnetic field (0.6mT), The Hall coefficient decreases with increasing temperature, following the inverse trend of the carrier concentration n. Error bars represent uncertainties in the Hall voltage measurements and carrier concentration fits.

3.4 Carrier mobility

To analyze carrier mobility in our doped germanium sample, we first convert our resistivity from Section 3.1 to conductivity, as they are inversely related. We then solve for the carrier mobility as in equation 4, where the temperature trends in our resistivity lend themselves to trends observed in our mobility. At higher temperatures, resistivity increases due to frequent phonon scattering and collisions between carriers, relating to a decrease in mobility as seen in Figure 7(b). Additionally, we observed a slight decrease in mobility as a function of magnetic field strength (see Figure 7(c)), which could be attributed to the magnetoresistance coming from the Hall effect. Since the carriers are undergoing a Lorentz force which affects their

trajectory, it increases their path within the sample. This effect is heightened with higher magnetic field strengths, resulting in the observed decrease in mobility.

The statistical uncertainties in our mobility measurement primarily come from uncertainties in resistivity and carrier concentration discussed above, both of which depended on small variations in voltage and magnetic field measurements.

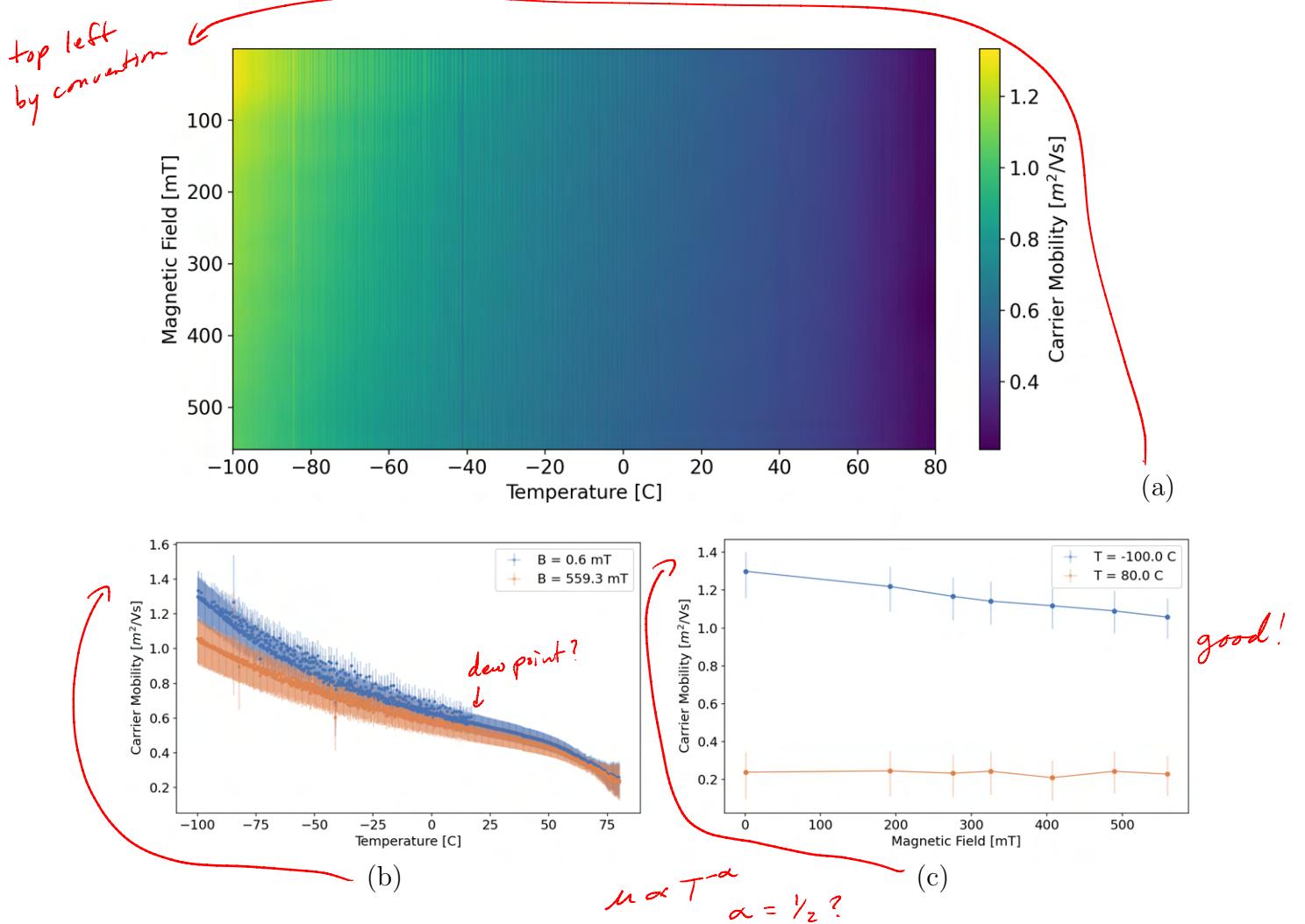


Figure 7: (a) Carrier mobility varied across temperature and magnetic field for the germanium sample. The mobility can be seen to follow a decreasing trend along both axes, with a steeper decrease for the temperature axis. (b) slices of the plot along 2 different magnetic fields. (c) slices of the plot along 2 different temperatures. Error bars come from the propagation of uncertainty on resistivity and concentration fits.

4 Conclusion

In this experiment, we investigated the electronic transport properties of a p-doped germanium sample by analyzing resistivity, carrier concentration, and mobility as functions of temperature and magnetic field strength. The resistivity measurements revealed a distinct temperature dependence, where resistivity initially increased at low temperatures due to phonon scattering dominating carrier behaviour. As the temperature rose, the ionization of dopants led to a decrease in resistivity, illustrating the interplay between phonon scattering and the availability of charge carriers. This behaviour is consistent with the expectations outlined in previous literature, confirming the presence of p-type dopants in our sample.

The Hall coefficient was measured, demonstrating a positive value indicative of a p-type semiconductor, further confirming our characterization of the germanium sample. A direct relationship between the Hall coefficient and carrier concentration was established, with the Hall coefficient decreasing as temperature increased, mirroring the inverse trend observed in carrier concentration. This potentially corroborates the understanding that increased thermal energy facilitates dopant ionization, which could enhance charge carrier availability. We also explored the carrier mobility across temperatures from -100 to 80°C and magnetic fields from 0.6mT to 350mT . The results indicated a decrease in mobility with increasing temperature, attributed to enhanced scattering mechanisms, while mobility also exhibited a decline with higher magnetic field strengths due to magnetoresistance effects. The 3D representation of mobility as a function of both temperature and magnetic field further illustrates these trends, highlighting how complex carrier dynamics can be in semiconductor materials.

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