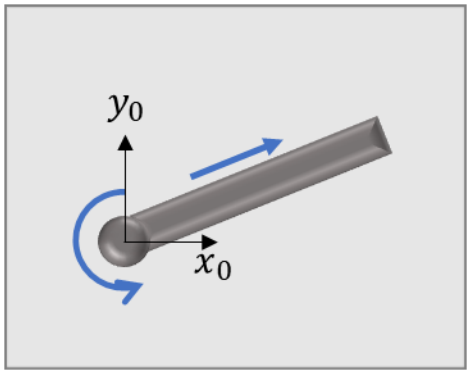
Homework Exercise 5

Submission is in pairs only.

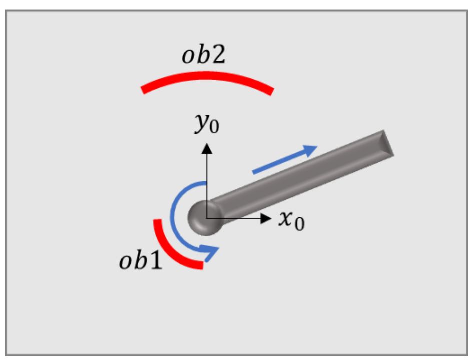
Submission deadline: 1/7/2021, 23:59

Question 1

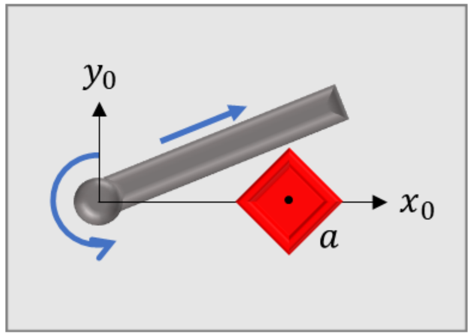
The planar revote-prismatic arm below is parametrized by $[\vartheta_1 \quad d_2]^T$. Any angle of the revolute joint is allowed; the prismatic joint has mechanical limits: $d_2 \in [l_1, l_2]$. Assume the width of the prismatic joint is infinitesimally small.



1. Two arch-shaped obstacles are placed in the arm's workspace. Both arches are parts of circles that are centered in the arm's base frame. The radius of *ob1* is $r_1 < l_1$ and its angular range is $[180°, 270°]$ relative to the x_0. The radius of *ob2* is $l_1 < r_2 < l_2$ and its angular range is $[70°, 110°]$ relative to x_0. Draw the C-space.



2. The arches are now removed from the C-space and instead, a square obstacle whose side is a is inserted. The square is centered at $(x_{ob}, y_{ob}) = \left(\frac{l_1+l_2}{2}, 0\right)$ and is rotated 45° relative to the horizon. Assume that $x_{ob} - \frac{a\sqrt{2}}{2} > l_1$. Find the analytic expressions of the C-space obstacle boundaries.



Question 2

Consider the lower bound of PRM's success probability that was discussed in class:

$$\Pr[(a, b)\ success] \geq 1 - \left\lceil \frac{2L}{\rho} \right\rceil e^{-\sigma\rho^d n}$$

Prove or disprove the following statements:

1. In a given C-space, n samples yielded success. $2n$ samples in the same C-space will also necessarily yield success.
2. In a given C-space, choosing a shorter path will necessarily result in higher lower bound.

Question 3

Develop a controller for a one-dof mass-spring-damper system of the form $m\ddot{x} + b\dot{x} + kx = f$, where f is the control force and $m = 4\ kg$, $b = 2\ Ns/m$, and $k = 0.1\ N/m$.

1. What is the damping ratio of the uncontrolled system? Is the uncontrolled system overdamped, underdamped, or critically damped? If it is underdamped, what is the damped natural frequency? What is the time constant of convergence to the origin?
2. Choose a P controller $f = K_p x_e$, where $x_e = x_d - x$ is the position error and $x_d = 0$. What value of K_p yields critical damping?
3. Choose a D controller $f = K_d \dot{x}_e$, where $\dot{x}_d = 0$. What value of K_d yields critical damping?
4. For the PD controller above, if $x_d = 1$ and $\dot{x}_d = \ddot{x}_d = 0$, what is the steady-state error $x_e(t)$ as t goes to infinity? What is the steady-state control force?
5. Now insert a PID controller for f. Assume $x_d \neq 0$ and $\dot{x}_d = \ddot{x}_d = 0$. Write the error dynamics in terms of $\ddot{x}_e, \dot{x}_e, x_e$, and $\int x_e(t)dt$ on the left-hand side and a constant forcing term on the right-hand side. (Hint: You can write kx as $-k(x_d - x) + kx_d$.) Take the time derivative of this equation and give the conditions on K_p, K_i, K_d for stability (e.g., using the Routh Hurwitz criterion from signals and systems). Show that zero steady-state error is possible with a PID controller.