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|  |  |  |
| --- | --- | --- |
| **Number of half-lives elapsed** | **Fraction remaining** | **colspan=2| Percentage remaining** |
| 0 | 1/1 | 100 |  |
| 1 | 1/2 | 50 |  |
| 2 | 1/4 | 25 |  |
| 3 | 1/8 | 12 | .5 |
| 4 | 1/16 | 6 | .25 |
| 5 | 1/32 | 3 | .125 |
| 6 | 1/64 | 1 | .563 |
| 7 | 1/128 | 0 | .781 |
| ... | ... | colspan=2| ... |  |
| *n* | 1/2*n* | colspan=2|100/(2*n*) |  |

**Half-life** (abbreviated **t1⁄2**) is the time required for a quantity to reduce to half its initial value. The term is commonly used in [nuclear physics](/wiki/Nuclear_physics) to describe how quickly unstable atoms undergo, or how long stable atoms survive, [radioactive decay](/wiki/Radioactive_decay). The term is also used more generally to characterize any type of [exponential](/wiki/Exponential_decay) or [non-exponential](/wiki/Rate_law) decay. For example, the medical sciences refer to the [biological half-life](/wiki/Biological_half-life) of drugs and other chemicals in the body. The converse of half-life is [doubling time](/wiki/Doubling_time).

The original term, *half-life period*, dating to [Ernest Rutherford's](/wiki/Ernest_Rutherford) discovery of the principle in 1907, was shortened to *half-life* in the early 1950s.[[1]](#cite_note-1) Rutherford applied the principle of a radioactive [element's](/wiki/Chemical_element) half-life to studies of age determination of [rocks](/wiki/Rock_(geology)) by measuring the decay period of [radium](/wiki/Radium) to [lead-206](/wiki/Isotopes_of_lead#Lead-206).

Half-life is constant over the lifetime of an exponentially decaying quantity, and it is a [characteristic unit](/wiki/Characteristic_unit) for the exponential decay equation. The accompanying table shows the reduction of a quantity as a function of the number of half-lives elapsed.

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## Probabilistic nature of half-life[[edit](/index.php?title=(none)&action=edit&section=1)]

[thumb|right|Simulation of many identical atoms undergoing radioactive decay, starting with either 4 atoms per box (left) or 400 (right). The number at the top is how many half-lives have elapsed. Note the consequence of the](/wiki/File:Halflife-sim.gif) [law of large numbers](/wiki/Law_of_large_numbers): with more atoms, the overall decay is more regular and more predictable.

A half-life usually describes the decay of discrete entities, such as radioactive atoms. In that case, it does not work to use the definition that states "half-life is the time required for exactly half of the entities to decay". For example, if there are 3 radioactive atoms with a half-life of one second, there will not be "1.5 atoms" left after one second.

Instead, the half-life is defined in terms of [probability](/wiki/Probability): "Half-life is the time required for exactly half of the entities to decay [*on average*](/wiki/Expected_value)". In other words, the *probability* of a radioactive atom decaying within its half-life is 50%.

For example, the image on the right is a simulation of many identical atoms undergoing radioactive decay. Note that after one half-life there are not *exactly* one-half of the atoms remaining, only *approximately*, because of the random variation in the process. Nevertheless, when there are many identical atoms decaying (right boxes), the [law of large numbers](/wiki/Law_of_large_numbers) suggests that it is a *very good approximation* to say that half of the atoms remain after one half-life.

There are various simple exercises that demonstrate probabilistic decay, for example involving flipping coins or running a statistical [computer program](/wiki/Computer_program).[[2]](#cite_note-2)[[3]](#cite_note-3)[[4]](#cite_note-4)

## Formulas for half-life in exponential decay[[edit](/index.php?title=(none)&action=edit&section=2)]

[Template:Main](/wiki/Template:Main)

An exponential decay can be described by any of the following three equivalent formulas:

<math>\begin{align}

N(t) &= N\_0 \left(\frac {1}{2}\right)^{\frac{t}{t\_{1/2}}} \\

N(t) &= N\_0 e^{-\frac{t}{\tau}} \\

N(t) &= N\_0 e^{-\lambda t}

\end{align}</math> where

* *N*0 is the initial quantity of the substance that will decay (this quantity may be measured in grams, moles, number of atoms, etc.),
* *N*(*t*) is the quantity that still remains and has not yet decayed after a time *t*,
* [Template:Math](/wiki/Template:Math) is the half-life of the decaying quantity,
* [Template:Mvar](/wiki/Template:Mvar) is a [positive number](/wiki/Positive_number) called the [mean lifetime](/wiki/Mean_lifetime) of the decaying quantity,
* [Template:Mvar](/wiki/Template:Mvar) is a positive number called the [decay constant](/wiki/Decay_constant) of the decaying quantity.

The three parameters [Template:Math](/wiki/Template:Math), [Template:Mvar](/wiki/Template:Mvar), and [Template:Mvar](/wiki/Template:Mvar) are all directly related in the following way:

<math>t\_{1/2} = \frac{\ln (2)}{\lambda} = \tau \ln(2)</math>

where ln(2) is the [natural logarithm](/wiki/Natural_logarithm) of 2 (approximately 0.693).

|  |
| --- |
| **Click "show" to see a detailed derivation of the relationship between half-life, decay time, and decay constant.** |
| Start with the three equations  <math>\begin{align}  N(t) &= N\_0 \left(\frac{1}{2}\right)^{t/t\_{1/2}} \\  N(t) &= N\_0 e^{-\frac{t}{\tau}} \\  N(t) &= N\_0 e^{-\lambda t}  \end{align}</math>  We want to find relationships among [Template:Math](/wiki/Template:Math), [Template:Mvar](/wiki/Template:Mvar), and [Template:Mvar](/wiki/Template:Mvar) such that these three equations describe exactly the same exponential decay process. Comparing the equations, we find the following conditions,  <math>\left(\frac {1}{2}\right)^{t/t\_{1/2}} = e^{-\frac{t}{\tau}} = e^{-\lambda t} .</math>  Next, we'll take the [natural logarithm](/wiki/Natural_logarithm) of each of these quantities.  <math>\ln\left(\left(\frac{1}{2}\right)^{t/t\_{1/2}}\right) = \ln\left(e^{-\frac{t}{\tau}}\right) = \ln\left(e^{-\lambda t}\right)</math>  Using the properties of logarithms, this simplifies to the following:  <math>\frac{t}{t\_{1/2}} \ln\left(\frac{1}{2}\right) = \left(-\frac{t}{\tau}\right)\ln(e) = (-\lambda t)\ln(e)</math>  Since the natural logarithm of *e* is 1, we get:  <math>\frac{t}{t\_{1/2}} \ln\left(\frac{1}{2}\right) = -\frac{t}{\tau} = -\lambda t</math>  Canceling the factor of *t* and plugging in <math>\ln\left(\frac {1}{2}\right) = -\ln(2)</math>, the final result is:  <math>t\_{1/2} = \tau \ln 2 = \frac{\ln 2}{\lambda}.</math> |

By plugging in and manipulating these relationships, we get all of the following equivalent descriptions of exponential decay, in terms of the half-life:

<math>\begin{align}

N(t) &= N\_0 \left(\frac{1}{2}\right)^{\frac{t}{t\_{1/2}}} = N\_0 2^{-t/t\_{1/2}} \\

&= N\_0 e^{-t\ln(2)/t\_{1/2}} \\

t\_{1/2} &= \frac{t}{\log\_2(N\_0/N(t))} = \frac{t}{\log\_2(N\_0) - \log\_2(N(t))} \\

&= \frac{1}{\log\_{2^t}(N\_0) - \log\_{2^t}(N(t))} = \frac{t\ln(2)}{\ln(N\_0) - \ln(N(t))}

\end{align}</math>

Regardless of how it's written, we can plug into the formula to get

* <math>N(0) = N\_0</math> as expected (this is the definition of "initial quantity")
* <math>N\left(t\_{1/2}\right) = \frac{1}{2} N\_0</math> as expected (this is the definition of half-life)
* <math>\lim\_{t\to \infty} N(t) = 0</math>; i.e., amount approaches zero as *t* [approaches infinity](/wiki/Limit_of_a_function) as expected (the longer we wait, the less remains).

### Decay by two or more processes[[edit](/index.php?title=(none)&action=edit&section=3)]

Some quantities decay by two exponential-decay processes simultaneously. In this case, the actual half-life [Template:Math](/wiki/Template:Math) can be related to the half-lives *t*1 and *t*2 that the quantity would have if each of the decay processes acted in isolation:

<math>\frac{1}{T\_{1/2}} = \frac{1}{t\_1} + \frac{1}{t\_2}</math>

For three or more processes, the analogous formula is:

<math>\frac{1}{T\_{1/2}} = \frac{1}{t\_1} + \frac{1}{t\_2} + \frac{1}{t\_3} + \cdots</math>

For a proof of these formulas, see [Exponential decay § Decay by two or more processes](/wiki/Exponential_decay#Decay_by_two_or_more_processes).

### Examples[[edit](/index.php?title=(none)&action=edit&section=4)]

[thumb|Half life demonstrated using dice in a](/wiki/File:Dice_half-life_decay.jpg) [classroom experiment](/wiki/V:Physics_and_Astronomy_Labs/Radioactive_decay_with_dice) [Template:Further](/wiki/Template:Further) There is a half-life describing any exponential-decay process. For example:

* The current flowing through an [RC circuit](/wiki/RC_circuit) or [RL circuit](/wiki/RL_circuit) decays with a half-life of *RC*ln(2) or ln(2)*L/R*, respectively. For this example, the term [half time](/wiki/Half_time_(physics)) might be used instead of "half life", but they mean the same thing.
* In a first-order [chemical reaction](/wiki/Chemical_reaction), the half-life of the reactant is ln(2)/[Template:Mvar](/wiki/Template:Mvar), where [Template:Mvar](/wiki/Template:Mvar) is the [reaction rate constant](/wiki/Reaction_rate_constant).
* In [radioactive decay](/wiki/Radioactive_decay), the half-life is the length of time after which there is a 50% chance that an atom will have undergone [nuclear](/wiki/Atomic_nucleus) decay. It varies depending on the atom type and [isotope](/wiki/Isotope), and is usually determined experimentally. See [List of nuclides](/wiki/List_of_nuclides).

The half life of a species is the time it takes for the concentration of the substance to fall to half of its initial value.

## Half-life in non-exponential decay[[edit](/index.php?title=(none)&action=edit&section=5)]

[Template:Main](/wiki/Template:Main)

The decay of many physical quantities is not exponential—for example, the evaporation of water from a puddle, or (often) the chemical reaction of a molecule. In such cases, the half-life is defined the same way as before: as the time elapsed before half of the original quantity has decayed. However, unlike in an exponential decay, the half-life depends on the initial quantity, and the prospective half-life will change over time as the quantity decays.

As an example, the radioactive decay of [carbon-14](/wiki/Carbon-14) is exponential with a half-life of 5,730 years. A quantity of carbon-14 will decay to half of its original amount ([on average](/wiki/#Probabilistic_nature_of_half-life)) after 5,730 years, regardless of how big or small the original quantity was. After another 5,730 years, one-quarter of the original will remain. On the other hand, the time it will take a puddle to half-evaporate depends on how deep the puddle is. Perhaps a puddle of a certain size will evaporate down to half its original volume in one day. But on the second day, there is no reason to expect that one-quarter of the puddle will remain; in fact, it will probably be much less than that. This is an example where the half-life reduces as time goes on. (In other non-exponential decays, it can increase instead.)

The decay of a mixture of two or more materials which each decay exponentially, but with different half-lives, is not exponential. Mathematically, the sum of two exponential functions is not a single exponential function. A common example of such a situation is the waste of nuclear power stations, which is a mix of substances with vastly different half-lives. Consider a mixture of a rapidly decaying element A, with a half-life of 1 second, and a slowly decaying element B, with a half-life of 1 year. In a couple of minutes, almost all atoms of element A will have decayed after repeated halving of the initial number of atoms, but very few of the atoms of element B will have done so as only a tiny fraction of its half-life has elapsed. Thus, the mixture taken as a whole will not decay by halves.

## Half-life in biology and pharmacology[[edit](/index.php?title=(none)&action=edit&section=6)]

[Template:Main](/wiki/Template:Main) A biological half-life or elimination half-life is the time it takes for a substance (drug, radioactive nuclide, or other) to lose one-half of its pharmacologic, physiologic, or radiological activity. In a medical context, the half-life may also describe the time that it takes for the concentration of a substance in [blood plasma](/wiki/Blood_plasma) to reach one-half of its steady-state value (the "plasma half-life").

The relationship between the biological and plasma half-lives of a substance can be complex, due to factors including accumulation in [tissues](/wiki/Tissue_(biology)), active [metabolites](/wiki/Metabolite), and [receptor](/wiki/Receptor_(biochemistry)) interactions.[[5]](#cite_note-5) While a radioactive isotope decays almost perfectly according to so-called "first order kinetics" where the rate constant is a fixed number, the elimination of a substance from a living organism usually follows more complex chemical kinetics.

For example, the biological half-life of water in a [human being](/wiki/Human_being) is about 9 to 10 days,[Template:Citation needed](/wiki/Template:Citation_needed) though this can be altered by behavior and various other conditions. The biological half-life of [cesium](/wiki/Cesium) in human beings is between one and four months.

## See also[[edit](/index.php?title=(none)&action=edit&section=7)]

* [Half time (physics)](/wiki/Half_time_(physics))
* [List of isotopes by half-life](/wiki/List_of_isotopes_by_half-life)
* [Mean lifetime](/wiki/Mean_lifetime)

## References[[edit](/index.php?title=(none)&action=edit&section=8)]

[Template:Reflist](/wiki/Template:Reflist)

## External links[[edit](/index.php?title=(none)&action=edit&section=9)]

[Template:Wiktionary](/wiki/Template:Wiktionary) [Template:Commons category](/wiki/Template:Commons_category)

* [Nucleonica.net](http://www.nucleonica.net), Nuclear Science Portal
* [Nucleonica.net](http://www.nucleonica.net/wiki/index.php/Help:Decay_Engine), wiki: Decay Engine
* [Bucknell.edu](http://www.facstaff.bucknell.edu/mastascu/elessonshtml/SysDyn/SysDyn3TCBasic.htm), System Dynamics – Time Constants
* [Subotex.com](http://www.subotex.com/SuboxoneTaperChart.aspx), Half-Life elimination of drugs in blood plasma – Simple Charting Tool

[Template:Radiation](/wiki/Template:Radiation)

[Template:Authority control](/wiki/Template:Authority_control)

[Template:DEFAULTSORT:Half-Life](/wiki/Template:DEFAULTSORT:Half-Life) [Category:Radioactivity](/wiki/Category:Radioactivity) [Category:Exponentials](/wiki/Category:Exponentials) [Category:Chemical kinetics](/wiki/Category:Chemical_kinetics)