Compressor Aerodynamics

Aircraft Engine Turbomachinery
Project Report

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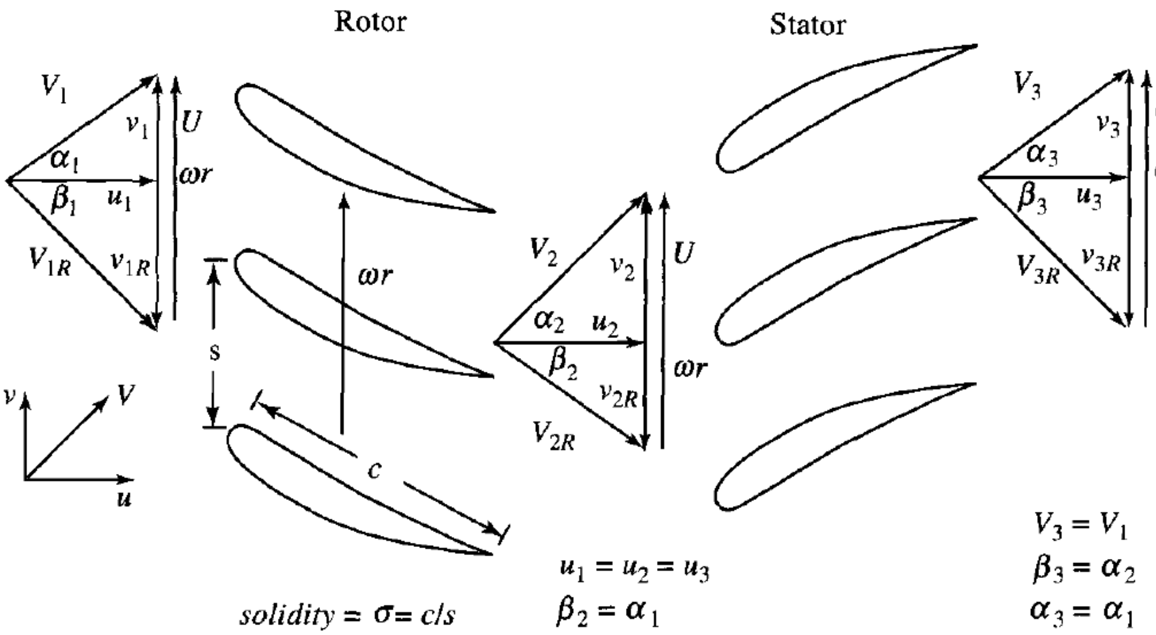
1 Introduction

Compressors are essential in aircraft engines for compressing the breathed air properly to combust with fuel in combustion chamber. In this report, low and high pressure compressor designed for a low bypass ratio supersonic aircraft engine.

1.1 Compressor Aerodynamics

Turbomachinary and aerodynamic properties of compressors are essential for providing flow which have no turbulence into combustion chamber. We decided to use axial flow, constant axial velocity, repeating stage, repeating row, mean-line compressor design.



Velocities:

$$V_1 = V_{1R} + U \quad V_2 = V_{2R} + U$$
$$u_1 = V_1 \cos\alpha_1 \quad u_2 = V_2 \cos\alpha_2$$
$$v_1 = V_1 \sin\alpha_1 \quad v_2 = V_2 \sin\alpha_2$$
$$u_1 = V_{1R} \cos\beta_1 \quad u_2 = V_{2R} \cos\beta_2$$
$$v_{1R} = V_{1R} \sin\beta_1 = u_1 \tan\beta_1 \quad v_{2R} = V_{2R} \sin\beta_2 = u_2 \tan\beta_2$$
$$v_1 + v_{1R} = \omega r = U \quad v_2 + v_{2R} = \omega r = U$$

Figure 1: Repeating row compressor stage nomenclature

1.1.1 Assumptions

1. Repeating row/repeating airfoil cascade geometry ($\alpha_1 = \beta_2 = \alpha_3, \beta_1 = \alpha_2 = \beta_3$).
2. Two-dimensional flow (that is, no property variation or velocity component normal to the flow).
3. Constant axial velocity ($u_l = u_2 = u_3$).
4. Stage polytropic efficiency e_c represents stage losses.
5. Constant mean radius $\left(r_m = \frac{r_h + r_i}{2}\right)$.
6. Calorically perfect gas with known γ_c and R_c.

1.1.2 Design Method

For design the compressor aerodynamically, diffusion factor(D), inlet velocity in Mach number(M_1), specific heat(γ), solidity($\sigma = \frac{\bar{c}}{s}$) and stage polytropic efficiency(e_c)
Constant axial velocity assumption leads great simplification because it provides every velocity triangle in Figure[1] has the same dimensional properties
1) Conservation of mass:

$$\frac{dm}{dt} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 = \rho_3 u_3 A_3 \tag{1}$$

$$\rho_1 A_1 = \rho_2 A_2 = \rho_3 A_3 \tag{2}$$

2) Repeating row constraint:
Since $\beta_2 = \alpha_1$, then

$$\vartheta_{2R} = \vartheta_1 = \omega r - \vartheta_2 \tag{3}$$

$$\omega r = \vartheta_1 + \vartheta_2 \tag{4}$$

Incidentally, since $\beta_3 = \alpha_2$,then $\vartheta_{3R} = \vartheta_2$, and

$$\vartheta_3 = \omega r - \vartheta_{3R} = \omega r - \vartheta_2 \tag{5}$$

And these calculations shows us that the velocity conditions at the stage entrance and stage exit are equal as:

$$\vartheta_3 = \vartheta_1 \tag{6}$$

3) Diffusion factor(D):

Diffusion factor is an analytical expression showing the magnitude of the reverse pressure gradient encountered by the boundary layer on the suction side of the blade. It depends on only flow geometry, hence it does not express anything about cascade airfoil profile.

$$D \doteq \left(1 - \frac{cos\alpha_2}{cos\alpha_1}\right) + \left(\frac{tan\alpha_2}{-} tan\alpha_1 2\sigma\right) cos\alpha_2 \tag{7}$$

Therefore for a given value of α_1, σ and D there is only one corresponding α_2 value [5], which can be solved as:

$$\alpha_2 = arccos\left(\frac{2\sigma\left(1-D\right)\Gamma + \sqrt{\Gamma^2 + 1 - 4\sigma^2\left(1-D\right)^2}}{\Gamma^2+1}\right) \tag{8}$$

where,

$$\Gamma = \frac{2\sigma + sin\alpha_1}{cos\alpha_1} \tag{9}$$

4) Degree of reaction($^\circ R_c$):

Degree of reaction is a measure of good compressor stage design which determined as:

$$^\circ R_c = \frac{\text{rotor static temperature rise}}{\text{stage static temperature rise}} = \frac{T_2 - T_1}{T_3 - T_1} \tag{10}$$

For perfect gas assumption with constant density($\rho \approx$ constant), equation[10] becomes:

$$^\circ R_c \approx \left[\frac{P_2 - P_1}{P_3 - P_1}\right]_{\rho \approx const} \tag{11}$$

A special and valuable characteristic of repeating stage, repeating row compressor stages is that $^\circ R_c$ must be exactly 0.5 because of the forced similarity of the rotor and stator velocity triangles. When $^\circ R_c$ is 0.5 the stator and rotor rows will carry the loads of the stage static temperature rise, and neither will benefit at the expense of the other.

5) Stage total temperature increase (ΔT_t):

Temperature rise accross the stage can be calculated from the total enthalpy change. We assumpted the gas is calorifically perfect and there is no chemical reaction, then $\Delta h = c_p \Delta T$

$$\Delta T_t = T_{t3} - T_{tl} = \frac{V_2^2 - V_1^2}{c_p} = \frac{V_1^2}{c_p} = \left(\frac{cos^2\alpha_1}{cos^2\alpha_2} - 1\right) \tag{12}$$

6) Stage temperature ratio (τ_s):

$$\tau_s = \frac{T_{t3}}{T_{t1}} = \frac{(\gamma-1)\,M_1^2}{1+(\gamma-1)\,\dfrac{M_1^2}{2}}\left(\frac{cos^2\alpha_1}{cos^2\alpha_2}-1\right)+1 \tag{13}$$

7) Stage pressure ratio:

$$\pi_s = \frac{P_{t3}}{P_{t1}} = \left(\frac{T_{t3}}{T_{t1}}\right)^{\dfrac{\gamma e_c}{\gamma-1}} = \tau_s^{\frac{\gamma e_c}{\gamma-1}} \tag{14}$$

8)Stage efficiency:

$$\eta_s = \frac{T_{t3i-T_{t1}}}{T_{t3}-T_{t1}} = \frac{\pi_s^{(\gamma-1)/\gamma}-1}{\tau_s-1} = \frac{\tau_s^{e_c}-1}{\tau_s-1} \tag{15}$$

9) Stage exit Mach number:

$$\frac{M_3}{M_1} = \sqrt{\frac{T_1}{T_3}} = \sqrt{\frac{1}{\tau_s\left[1+(\gamma-1)\,\dfrac{M_1^2}{2}\right]-(\gamma-1)\,\dfrac{M_1^2}{2}}} \tag{16}$$

Since $T_3/T_1 > 1$, then $M_3/M_1 < 1$, and the Mach number decreases stage by stage as the flow progresses through the compressor, causes compressibility effects to become less important.

10) Wheel speed/inlet velocity ratio ($\omega r/V_1$):

$$\frac{\omega r}{V_1} = cos\alpha_1\,(tan\alpha_1 + tan\alpha_2) \tag{17}$$

11) Inlet relative Mach number(M_{1R}):

$$\frac{M_{1R}}{M_1} = \frac{cos\alpha_1}{cos\alpha_2} \tag{18}$$

Since $\alpha_2 > \alpha_1$, then $M_{1R} > M_1$ and M_1 must be chosen carefully in order to avoid excessively high inlet relative Mach numbers.

1.2 Blade Geometry Features

1.2.1 Airfoil Geometry

After the repeating row compressor stage flow field geometry has been selected, it remains to design the orientation of physical airfoils that will realizes it.
The necessary nomenclature for this step is shown in figure[2] with the inlet flow angle shown at other than the design point.

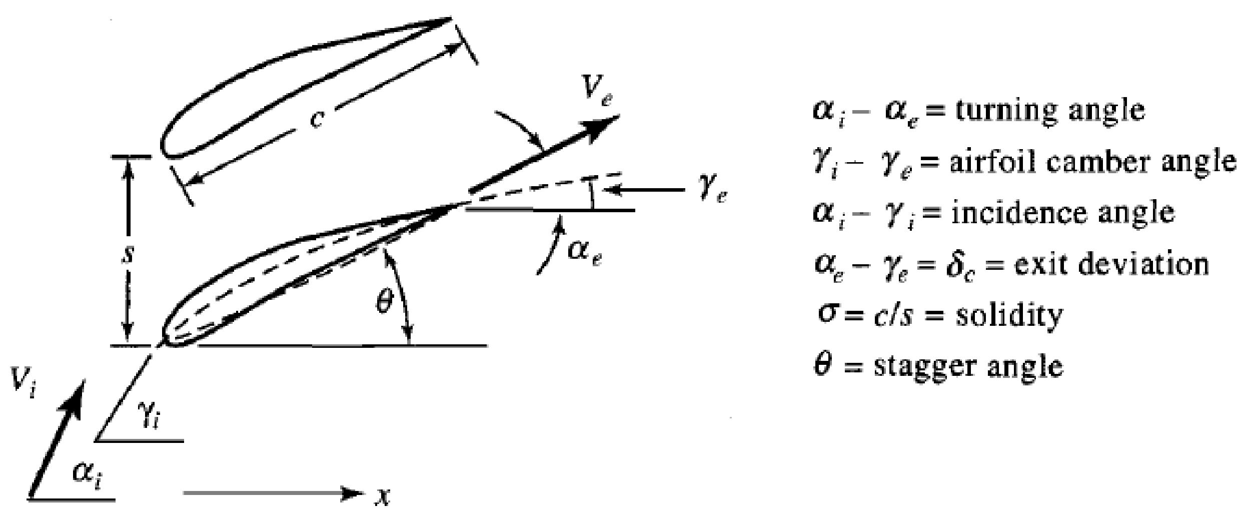


Figure 2: Cascade airfoil nomenclature

Carter rule: Carter's rule for compressors can be determined as:

$$\delta_c = \frac{\gamma_1 - \gamma_2}{4\sigma} \tag{1}$$

It is a useful method for determining the metal angles γ_l and γ_2 at thedesign point is to take $\gamma_l = \alpha_l$. With solving Carter's rule $\gamma_2 = \alpha_2 - \delta_c$ can be computed. Equation[] can be manipulated as:

$$\delta_c = \frac{\alpha_1 - \alpha_2}{4\sigma - 1} \tag{2}$$

1.2.2 Fan Structure

We decided to use structure of NASA Rotor 67 fan. NASA Rotor 67 is a rotor with a low-aspect-ratio, and it is the first-stage rotor of a two-stage fan.

Properties of NASA Rotor 67 is given as:

Solidity	1.61
Thickness/Chord Ratio	0.05
Inlet tip relative Mach number	1.38
Hub to tip ratio at leading edge	0.375
Hub to tip ratio at trailing edge	0.478

Table 1: Properties of NASA 67 rotor

1.2.3 Cascade Airfoil

Cascede airfoil used in aerodynamic definition calculations is chosen among NACA 65 series circular arc airfoils because of their driving the flow as laminar as possible and transonic characteristics. The chosen cascade airfoil is NACA 65009 airfoil. Because its manufacturing is simple and it have transonic characteristics comparable with circular arc airfoils.

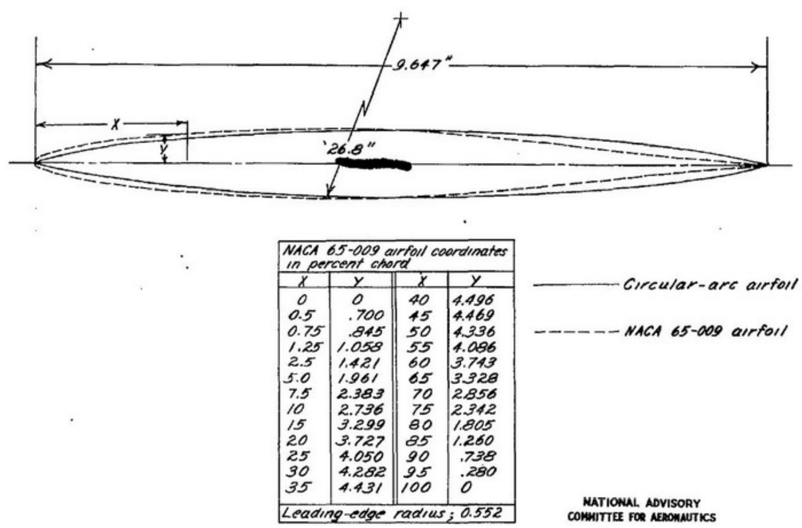


Figure 3: Sketch and airfoil coordinates of NACA 65-009 airfoil

Chord length	0,492 ft
Solidity, σ	0.134
Thickness/Chord Ratio, t/c	0.06
Tip clearance, ε/c	0.02

Table 2: Properties of NACA 65-009 airfoil

2 Compressor Aerodynamic Definitions

2.1 Low Pressure Compressor(LPC - Fan)

With determined values of

- $D = 0.5$
- $M_1 = 1.6$
- $\sigma = 1.61$
- $e_c = 0.9039$

the behavior of every stage of repeating row compressor can be computed wtih using the method introduced in section 1.1.3. With these parameters are determined, we obtain this graph for various α_1 values as:

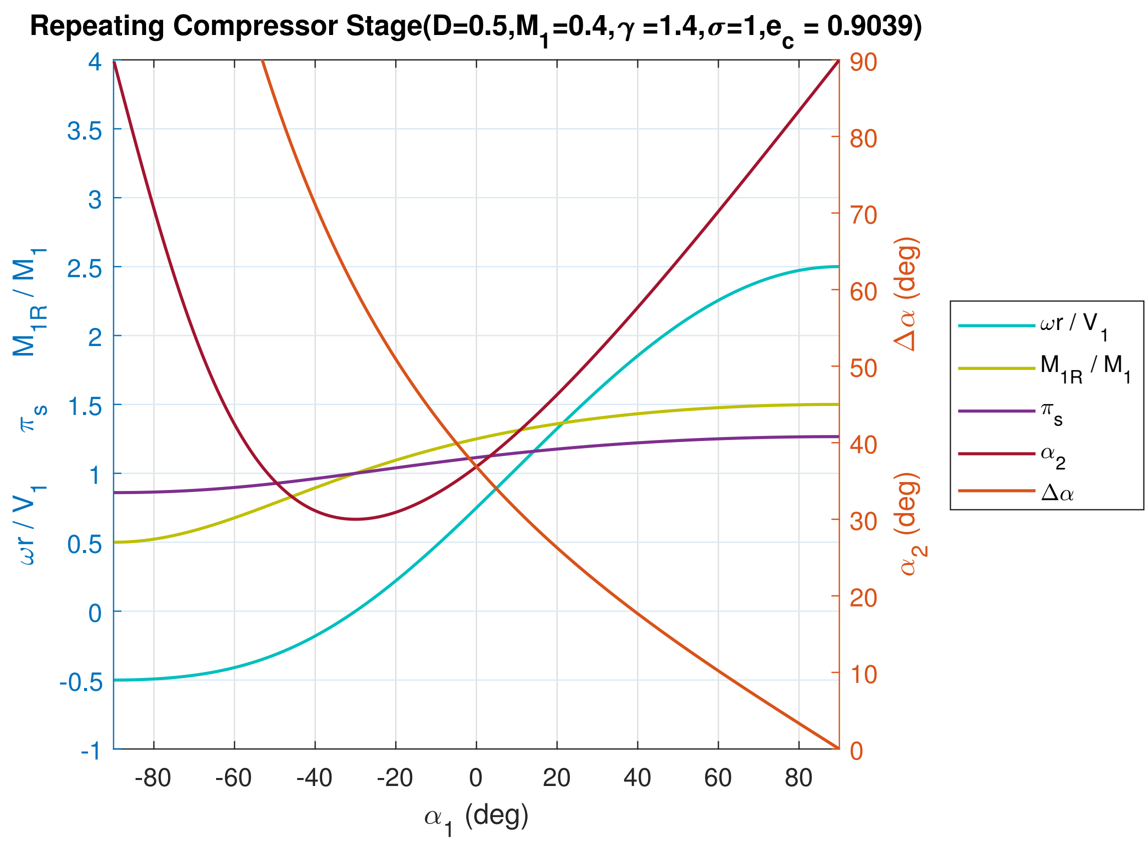


Figure 4: Repeating Compressor Stage for Fan ($D = 0.5, \sigma = 1.61, e_c = 0.9039$)

2.2 High Pressure Compressor(HPC) Design

With determined values of

- $D = 0.5$
- $M_1 = 0.52$
- $\gamma = 1.4$
- $\sigma = 0.134$
- $e_c = 0.9039$

the behavior of every stage of repeating row compressor can be computed wtih using the method introduced in section 1.1.3. With these parameters are determined, we obtain this graph for various α_1 values as:

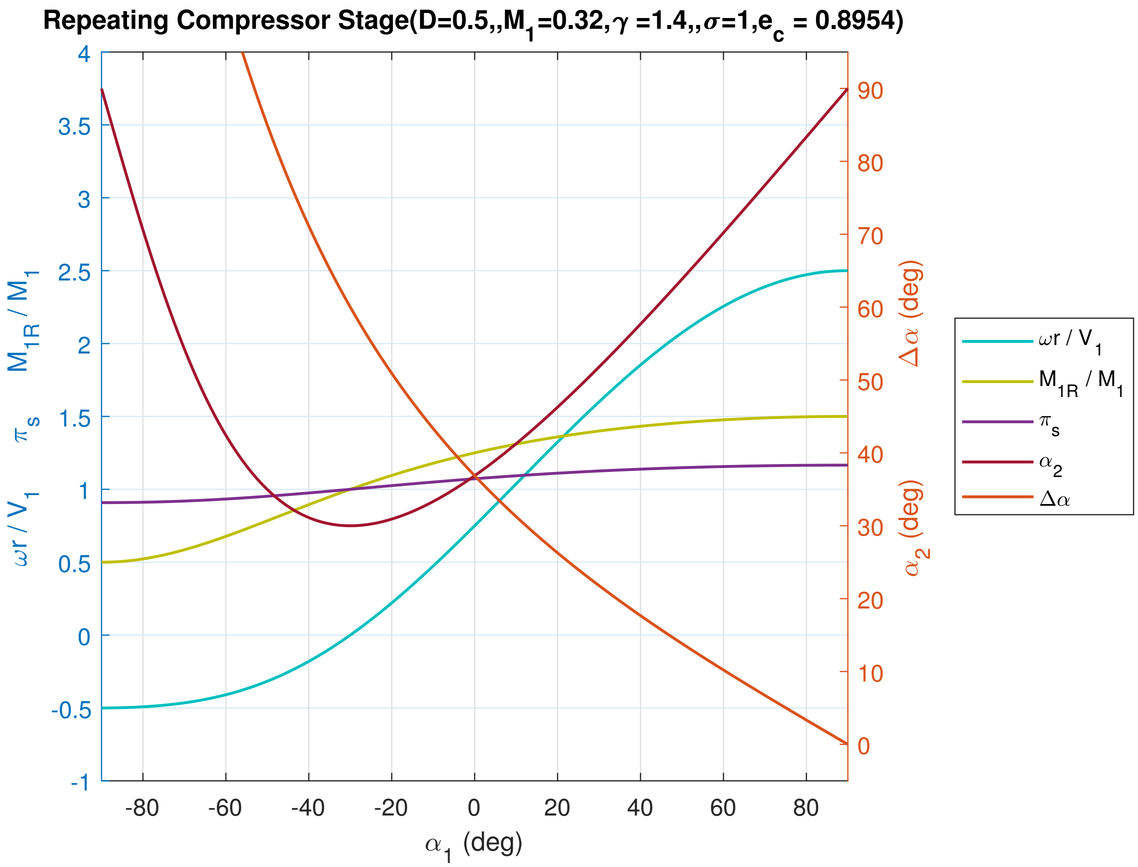


Figure 5: Repeating Compressor Stage for HPC ($D = 0.5, \sigma = 0.134, e_c = 0.8954$)

3 Appendix A: MATLAB Codes

3.1 Appendix A-1: Low Pressure Compressor Aerodynamic Definition Code

LPC.m file :

```
clc;clear;close all;

%% Aerodynamic Definition Calculations of LPC

D=0.5; % Diffusion factor
M1 = 0.4; % Fan input velocity in Mach #
gamma = 1.4; % Ratio of specific heats
sigma = 1; % Solidity of fan
ec = 0.9039; % Polytropic efficiency of fan
alpha1 = -89.9:0.1:89.9; % blade angle
Gamma = ((2*sigma)+(sind(alpha1)))./(cosd(alpha1));
alpha2 = ...
    acosd(((2*sigma.*Gamma*(1-D))+(sqrt((Gamma.^2)+1-((4*sigma^2)*(1-D)^2))))./((Gamma.^
∆Alpha = alpha2 - alpha1; % Turning angle
wr_V1=(cosd(alpha1)).*((tand(alpha1))+((tand(alpha2)))); % Rotational ...
    speed-inlet velocity ratio
M1R_M1=(cosd(alpha1))./(cosd(alpha2)); % relative velocity - inlet ...
    velocity ratio
tauS = ...
    ((((gamma-1)*M1^2)/(1+((gamma-1)*((M1^2)/2))))*(((cosd(alpha1)).^2./(cosd(alpha2)).^
    % Stage temperature ratio
piS = tauS.^((gamma*ec)/(gamma-1)); % Stage pressure ratio
∆Tt = (((cos(alpha1)).^2)./((cos(alpha2)).^2))-1; % Stage total ...
    temperature increase
Tt1 = ∆Tt./(tauS-1);
M1R = M1R_M1.*M1; % Inlet relative Mach number

%blade angles
dc = (alpha1-alpha2)./((4*sqrt(sigma))-1);
gamma1 = alpha1;
gamma2 = alpha2 - dc;

%plot
figure(1)
yyaxis left
```

```
p1 = plot(alpha1,wr_V1,'Color', [0, 0.75, ...
    0.75],'LineStyle','-','LineWidth',1.2);
hold on;
p2 = plot(alpha1,M1R_M1,'Color',[0.75, 0.75, ...
    0],'LineStyle','-','LineWidth',1.2);
p3 = plot(alpha1,piS,'Color',[0.4940, 0.1840, ...
    0.5560],'LineStyle','-','LineWidth',1.2);
xlabel('\alpha_{1} (deg)');
ylabel('\omegar / V_{1}       \pi_{s}       M_{1R} / M_{1}');
axis([-90 90 -1 4]);
yyaxis right
p4 = plot(alpha1,alpha2,'Color',[0.6350, 0.0780, ...
    0.1840],'LineStyle','-','LineWidth',1.2);
p5 = plot(alpha1,ΔAlpha,'Color',[0.8500, 0.3250, ...
    0.0980],'LineStyle','-','LineWidth',1.2);
ylabel('\alpha_{2} (deg)                \Delta\alpha (deg)');
grid on;
title('Repeating Compressor Stage(D=0.5,M_{1}=0.4,\gamma ...
    =1.4,\sigma=1,e_{c} = 0.9039)');
legend('\omegar / V_{1}','M_{1R} / ...
    M_{1}','\pi_{s}','\alpha_{2}','\Delta\alpha','Location','eastoutside');
axis([-90 90 0 90]);
hold off;
```

3.2 Appendix A-2: High Pressure Compressor Aerodynamic Definition Code

HPC.m file :

```
clc;clear;close all;

%% Aerodynamic Definition Calculations of HPC
D=0.5; % Diffusion factor
M1 = 0.32; % Fan input velocity in Mach #
gamma = 1.4; % Ratio of specific heats
sigma = 1; % Solidity of HPC
ec = 0.8954; % Polytropic efficiency of HPC
alpha1 = -90:0.01:90; % blade angle
Gamma = ((2*sigma)+(sind(alpha1)))./(cosd(alpha1));
alpha2 = ...
    acosd(((2*sigma.*Gamma*(1-D))+(sqrt((Gamma.^2)+1-((4*sigma^2)*(1-D)^2))))./((Gamma.^
```

```
ΔAlpha = alpha2 - alpha1; % Turning angle
wr_V1=(cosd(alpha1)).*((tand(alpha1))+((tand(alpha2)))); % Rotational ...
    speed-inlet velocity ratio
M1R_M1=(cosd(alpha1))./(cosd(alpha2)); % relative velocity - inlet ...
    velocity ratio
tauS = ...
    ((((gamma-1)*M1^2)/(1+((gamma-1)*((M1^2)/2))))*(((cosd(alpha1)).^2./(cosd(alpha2)).^
    % Stage temperature ratio
piS = tauS.^((gamma*ec)/(gamma-1)); % Stage pressure ratio
ΔTt = (((cos(alpha1)).^2)./((cos(alpha2)).^2))-1; % Stage total ...
    temperature increase
Tt1 = ΔTt./(tauS-1);
M1R = M1R_M1.*M1; % Inlet relative Mach number

%blade angles
dc = (alpha1-alpha2)./((4*sqrt(sigma))-1);
gamma1 = alpha1;
gamma2 = alpha2 - dc;

%plot
figure(1)
yyaxis left
p1 = plot(alpha1,wr_V1,'Color', [0, 0.75, ...
    0.75],'LineStyle','-','LineWidth',1.2);
hold on;
p2 = plot(alpha1,M1R_M1,'Color',[0.75, 0.75, ...
    0],'LineStyle','-','LineWidth',1.2);
p3 = plot(alpha1,piS,'Color',[0.4940, 0.1840, ...
    0.5560],'LineStyle','-','LineWidth',1.2);
xlabel('\alpha_{1} (deg)');
ylabel('\omegar / V_{1}      \pi_{s}      M_{1R} / M_{1}');
axis([-90 90 -1 4]);
yyaxis right
p4 = plot(alpha1,alpha2,'Color',[0.6350, 0.0780, ...
    0.1840],'LineStyle','-','LineWidth',1.2);
p5 = plot(alpha1,ΔAlpha,'Color',[0.8500, 0.3250, ...
    0.0980],'LineStyle','-','LineWidth',1.2);
ylabel('\alpha_{2} (deg)                \Delta\alpha (deg)');
grid on;
title('Repeating Compressor Stage(D=0.5,,M_{1}=0.32,\gamma ...
    =1.4,,\sigma=1,e_{c} = 0.8954)');
legend('\omegar / V_{1}','M_{1R} / ...
    M_{1}','\pi_{s}','\alpha_{2}','\Delta\alpha','Location','eastoutside');
```

```
axis([-90 90 -5 95]);
hold off;
```

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