Simulation of The Low Earth Orbit Satellite's Attitude Dynamics

Attitude Dynamics and Control Project Report

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Contents

1 Simulation of The Low Earth Orbit Attitude Dynamics of Satellite

The subject of this project is to characterise the attitude dynamics of the Low Earth Orbit satellite in the simulation environment. To investigate the behaviours of satellite, the figures will contain the changes of angular velocities and Euler angles. The mathematical model of the satellite's rotational motion will be written in terms of the Euler angles and its angular velocities and it is solved by an iterative approach with using the initial values of these parameters. One of the commercial engineering software will be used to plot the figures. The necessary algorithm for programming is derived according to the mathematical model of the satellite's rotational motion.

The notations going to be used in the project is given below:

- ϕ is the roll angle
- θ is the pitch angle
- ψ is the yaw angle
- w_x, w_y and w_z are the angular velocities of the satellite
- J_x, J_y and J_z are the moments of inertia of the satellite
- w_{orbit} is the angular orbit velocity of the satellite
- N_T is the disturbance torque acting on the satellite
- N is the iteration number
- Δt is the sample time
- C and A are the transformation matrices

The necessary initial conditions of the satellite are listed below. The sequence number is $n = 20$ for this project.

- Initial data of the attitude angles (rad)

$$\phi_0 = -0.01 - 0.002 * n \tag{1}$$

$$\theta_0 = 0.01 + 0.002 * n \tag{2}$$

$$\psi_0 = 0.005 + 0.002 * n \tag{3}$$

- The initial data of angular velocities of satellite(rad/s)

$$w_{x_0} = 0.0002 + 0.0001 * n \tag{4}$$

$$w_{y_0} = 0.0003 + 0.0001 * n \tag{5}$$

$$w_{z_0} = 0.0004 + 0.0001 * n \tag{6}$$

- The initial momments of inertia of the satellite(m^4)

$$J_x = 2.1(10^{-3}) \tag{7}$$
$$J_y = 2(10^{-3}) \tag{8}$$
$$J_z = 1.9(10^{-3}) \tag{9}$$

- The angular orbit velocity of the satellite(rad/s)

$$w_{orbit} = 0.0011 \tag{10}$$

- The disturbance torque acting on the satellite($N \cdot m$)

$$N_T = 3.6(10^{-10}) \tag{11}$$

- The iteration number

$$N = 54000 \tag{12}$$

- The sample time(s)

$$\Delta t = 0.1 \tag{13}$$

The mathematical model of the satellite's rotational motion about its center of mass is given as:

The Euler angles:

$$\phi_{i+1} = \phi_i + \Delta t[(\omega_{y_i} sin\phi_i + \omega_{z_i} cos\phi_i)tan\theta_i + \omega_{x_i}] \tag{14}$$
$$\theta_{i+1} = \theta_i + \Delta t(\omega_{y_i} cos\phi_i - \omega_{z_i} sin\phi_i + \omega_{orbit}) \tag{15}$$
$$\psi_{i+1} = \psi_i + \Delta t[(\omega_{y_i} sin\phi_i + \omega_{z_i} cos\phi_i)sec\theta_i \tag{16}$$

The angular velocities:

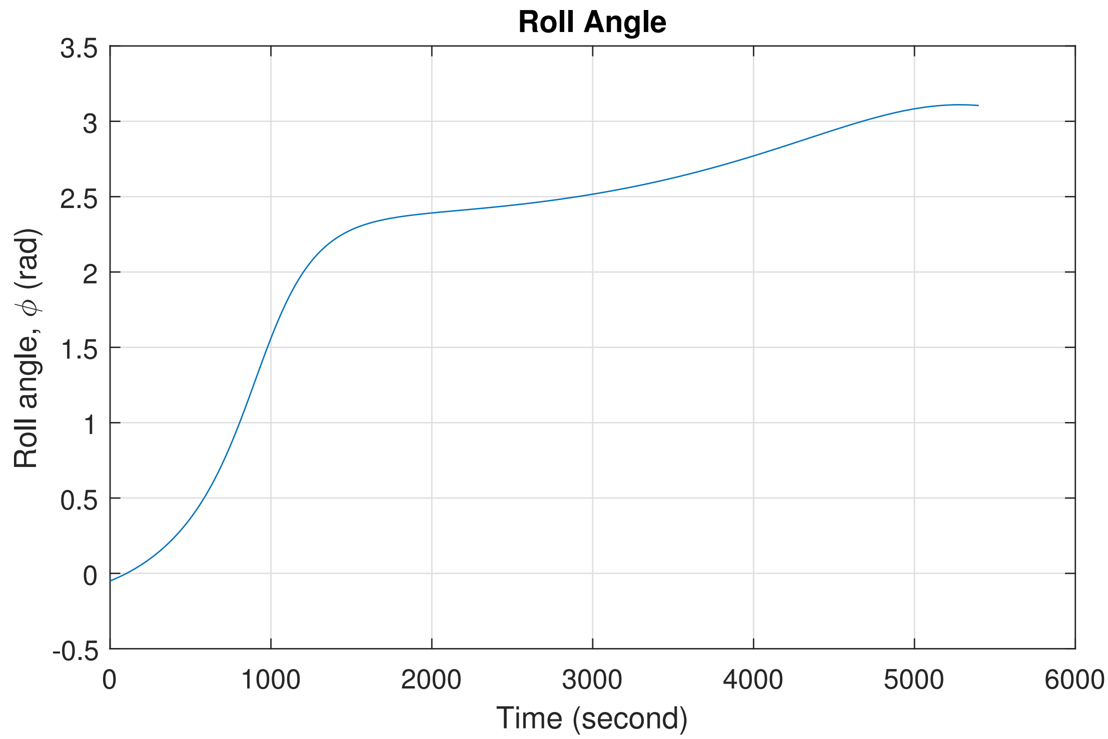
$$\omega_{x_{i+1}} = \omega_{x_i} + \frac{\Delta t}{J_x}(J_y - J_z)\omega_{z_i}\omega_{y_i} + \frac{\Delta t}{J_x}N_T \tag{17}$$
$$\omega_{y_{i+1}} = \omega_{y_i} + \frac{\Delta t}{J_y}(J_z - J_x)\omega_{x_i}\omega_{z_i} + \frac{\Delta t}{J_y}N_T \tag{18}$$
$$\omega_{z_{i+1}} = \omega_{z_i} + \frac{\Delta t}{J_z}(J_x - J_y)\omega_{x_i}\omega_{y_i} + \frac{\Delta t}{J_z}N_T \tag{19}$$

The transformation matrix:

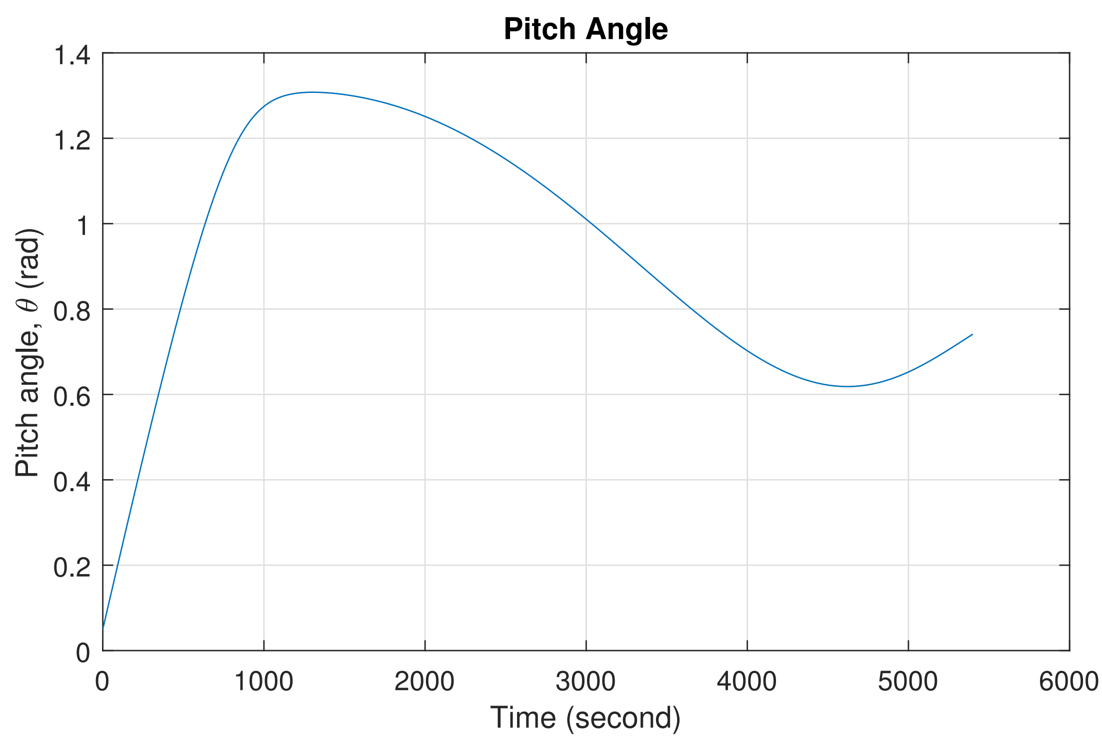
$$A = \begin{bmatrix} cos(\theta)cos(\psi) & cos(\theta)sin(\psi) & -sin(\theta) \\ -cos(\phi)sin(\psi) + sin(\phi)sin(\theta)cos(\psi) & cos(\phi)cos(\psi) + sin(\phi)sin(\theta)sin(\psi) & sin(\phi)cos(\theta) \\ sin(\phi)sin(\psi) + cos(\phi)sin(\theta)cos(\psi) & -sin(\phi)cos(\psi) + cos(\phi)sin(\theta)sin(\psi) & cos(\phi)cos(\theta) \end{bmatrix} \tag{20}$$

Euler angles, angular velocities and the transformation matrix of the small satellite should be found at each step. 6 figures (angles, angular velocities for x, y, z axes) should be plotted.

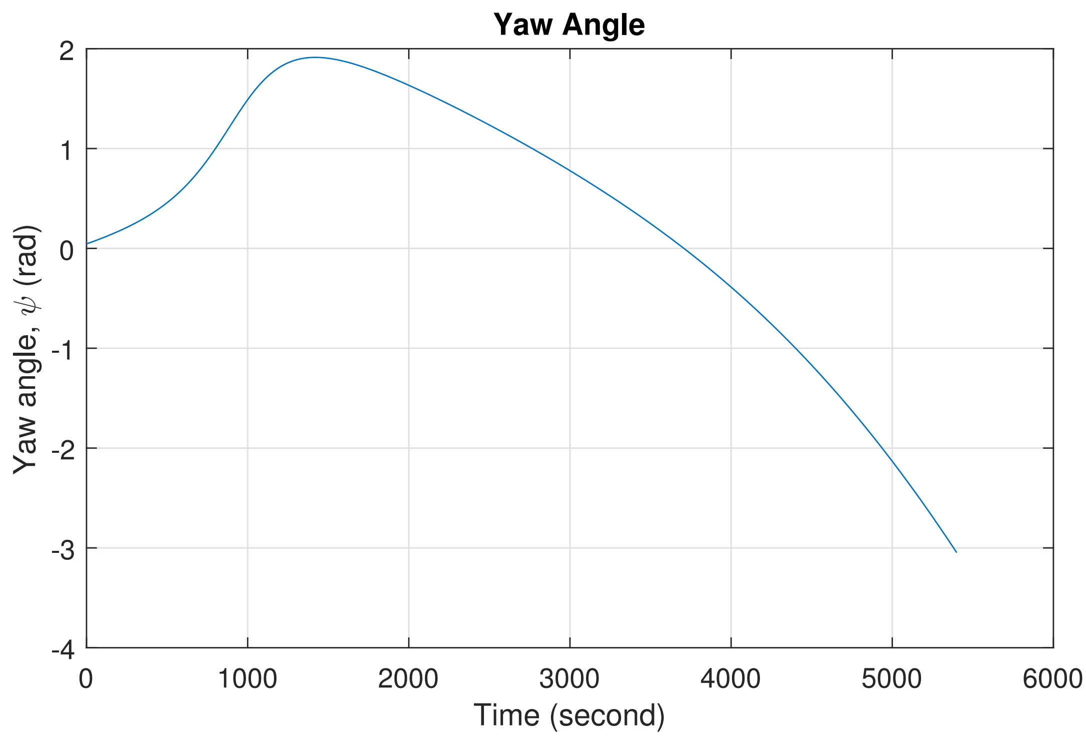
Graph of roll (ϕ) angle:



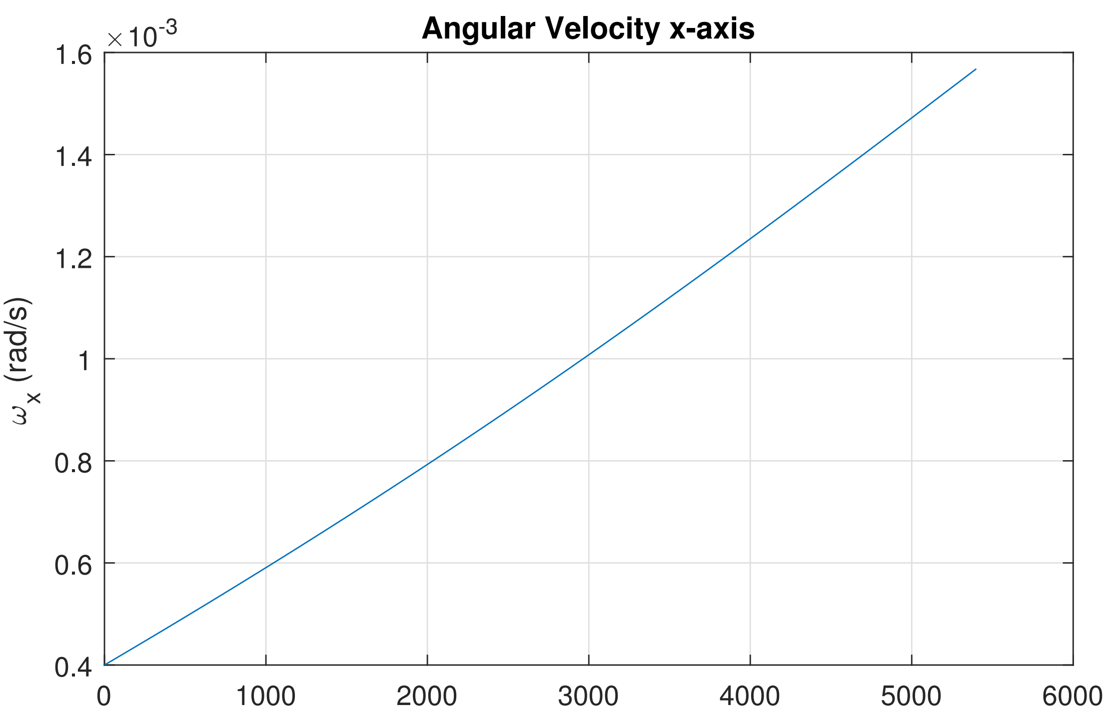
Graph of pitch (θ) angle:



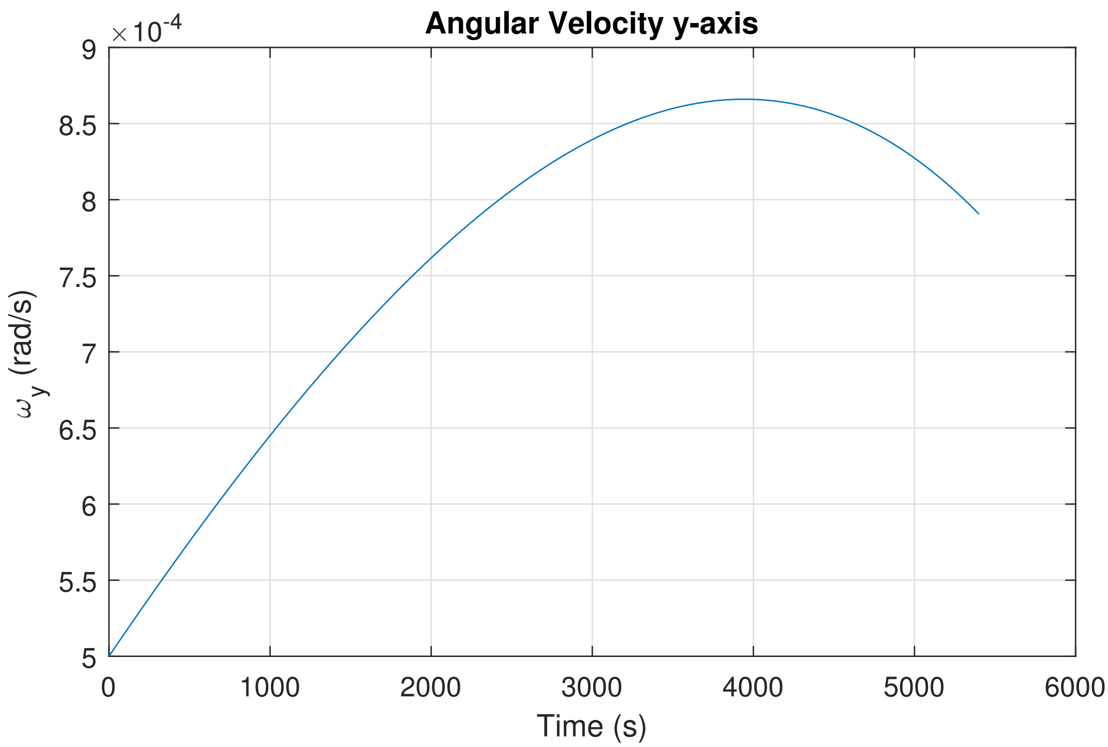
Graph of yaw (ψ) angle:



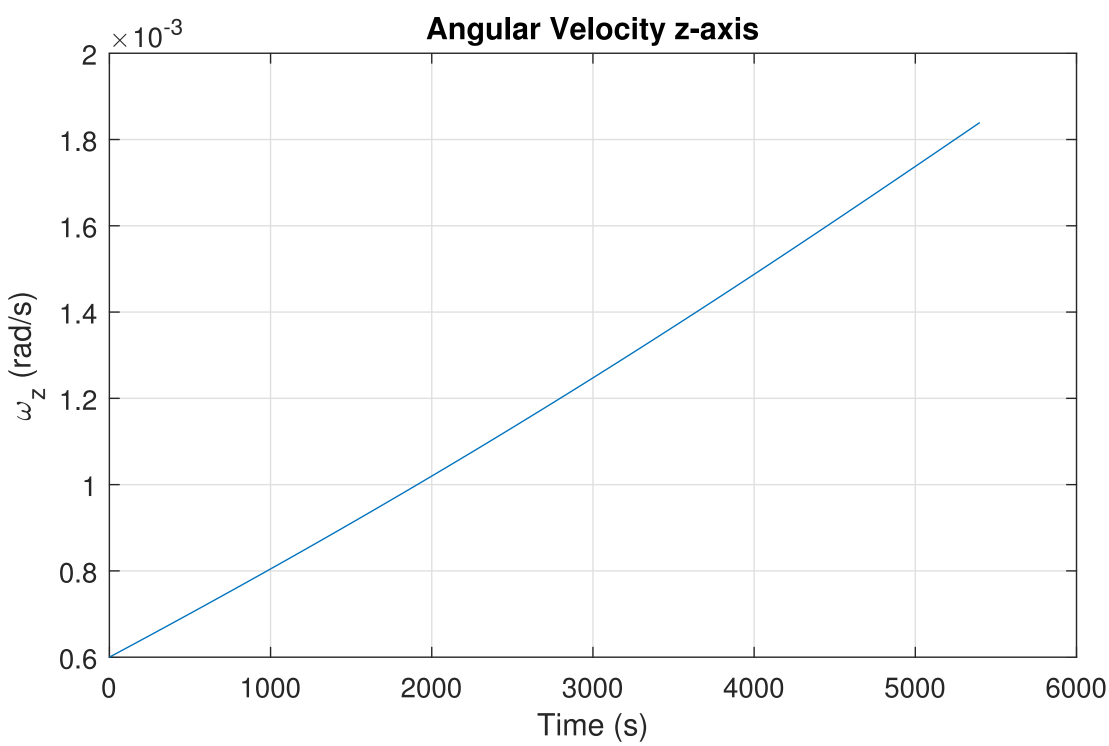
Graph of angular velocity in x direction:



Graph of angular velocity in y direction:



Graph of angular velocity in z direction:



2 Appendix: MATLAB Code

```matlab
n = 20; %sequence number

%initial data of the attitude angles(rad)
phi = -0.01 - 0.002 * n;
theta = 0.01 + 0.002 * n;
psi = 0.005 + 0.002 * n;

%initial data of the angular velocities of satellite
wX = 0.0002 + 0.00001 * n;
wY = 0.0003 + 0.00001 * n;
wZ = 0.0004 + 0.00001 * n;
w = [wX;wY;wZ];

%initial moments of inertia of the satellite(m^4)
Jx = 2.1*(10^(-3));
Jy = 2*(10^(-3));
Jz = 1.9*(10^(-3));

w_orbit = 0.0011; %The angular orbit velocity of satellite(rad/s)
N_T = 3.6*(10^(-10)); %The disturbance torque acting on the satellite(Nm)
dt = 0.1; %the sample time(s)
N = 54000; %iteration number
W = zeros(3,N); %pre-allocation
for k = 1:N
%the angular velocities
w(1) = w(1) + ((dt/Jx)*(Jy - Jz)*w(3)*w(2)) + ((dt/Jx)*N_T);
w(2) = w(2) + ((dt/Jy)*(Jz - Jx)*w(1)*w(3)) + ((dt/Jy)*N_T);
w(3) = w(3) + ((dt/Jz)*(Jx - Jy)*w(1)*w(2)) + ((dt/Jz)*N_T);

%the euler angles
phi = phi + dt*((((w(2)*sin(phi)) + (w(3)*cos(phi)))*tan(theta)) + w(1));
theta = theta + dt*((w(2)*cos(phi)) - (w(3)*sin(phi)) + (w_orbit));
psi = psi + dt*(((w(2)*sin(phi)) + (w(3)*cos(phi)))*sec(theta));

%transformation matrix A
A = [cos(theta)*cos(psi) cos(theta)*sin(psi) ...
    -sin(theta);((-cos(phi)*sin(psi)) + (sin(phi)*sin(theta)*cos(psi))) ...
    ((cos(phi)*cos(psi)) + (sin(phi)*sin(theta)*sin(psi))) ...
    (sin(phi)*cos(theta));((sin(phi)*sin(psi))+(cos(phi)*sin(theta)*cos(psi))) ...
```

```
    ((-sin(phi)*cos(psi))+(cos(phi)*sin(theta)*sin(psi))) ...
    (cos(phi)*cos(theta))];

%arranging arrays for plotting
W(:,k) = w;
Phi(k) = phi;
Theta(k) = theta;
Psi(k) = psi;
end

t = 0:dt:(N-1)*dt;

figure(1);
plot(t,W(1,:));
grid on;
title('Angular Velocity x-axis');
xlabel('Time (s)');
ylabel('Wx (rad/s)');

figure(2);
plot(t,W(2,:));
grid on;
title('Angular Velocity y-axis');
xlabel('Time (s)');
ylabel('Wy (rad/s)');

figure(3);
plot(t,W(3,:));
grid on;
title('Angular Velocity z-axis');
xlabel('Time (s)');
ylabel('Wz (rad/s)');

figure(4)
plot(t,Phi);
title('Roll Angle');
xlabel('Time (second)');
ylabel('Roll angle, \phi (rad)');
grid on;

figure(5)
plot(t,Theta);
title('Pitch Angle');
```

```
xlabel('Time (second)');
ylabel('Pitch angle, \theta (rad)');
grid on;

figure(6)
plot(t,Psi);
title('Yaw Angle');
xlabel('Time (second)');
ylabel('Yaw angle, \psi (rad)');
grid on;
```

3 References

[1] Prof. Dr. Cengiz Hacızade, *Istanbul Technical University UCK421E Lecture Notes*, 2021.

[2] J.R.Wertz., *Space Attitude Determination and Control*, D.Reidel Publishing Company, Dordrecht, Holland, 2002.

[3] Hajiyev, C., & Soken, H.E., *Fault Tolerant Attittude Estimation for Small Satellites*, 1^{st} Ed., CRC Press, 2021.