Modified Rodrigues Parameters Representation for Small Satellite Using Quaternions

Attitude Dynamics and Control
Project Report

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Contents

1 Modified Rodrigues Parameters Representation for Small Satellite Using Quaternions

The subject of the homework is to characterize the attitude dynamics of the Low Earth Orbit Satellite in terms of Modified Rodrigues Parameters. To investigate the behaviors of satellite, the graphs will include the propagation of Modified Rodrigues Parameters with time.
The terms of the modified Rodrigues parameters are:

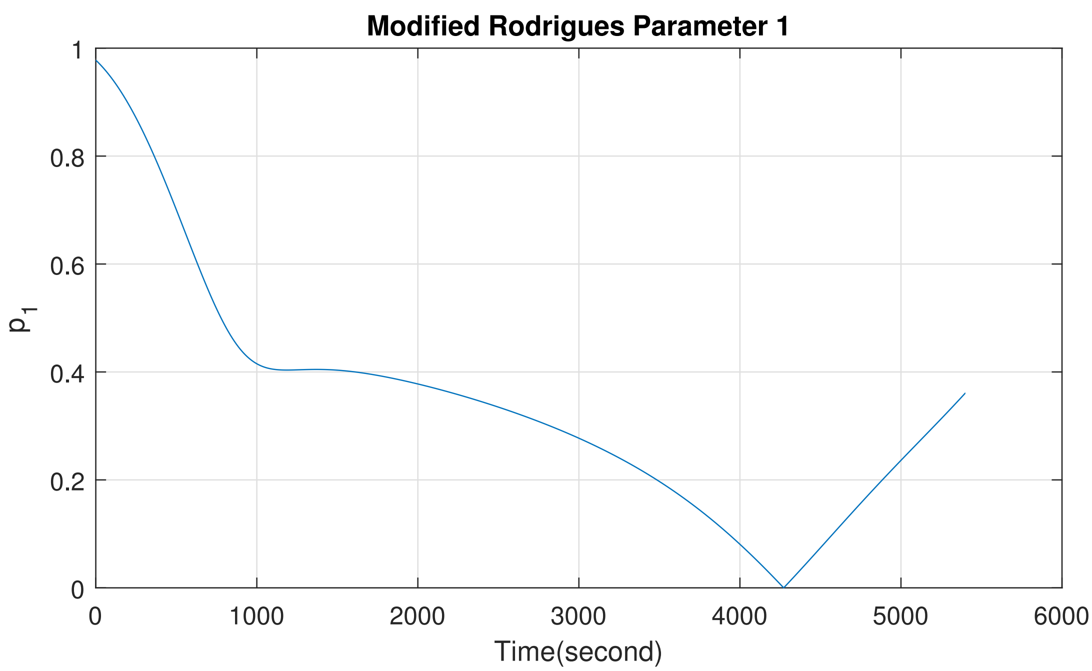
$$p_1 = \frac{q_1}{1 + q_4} \tag{1}$$

$$p_2 = \frac{q_2}{1 + q_4} \tag{2}$$

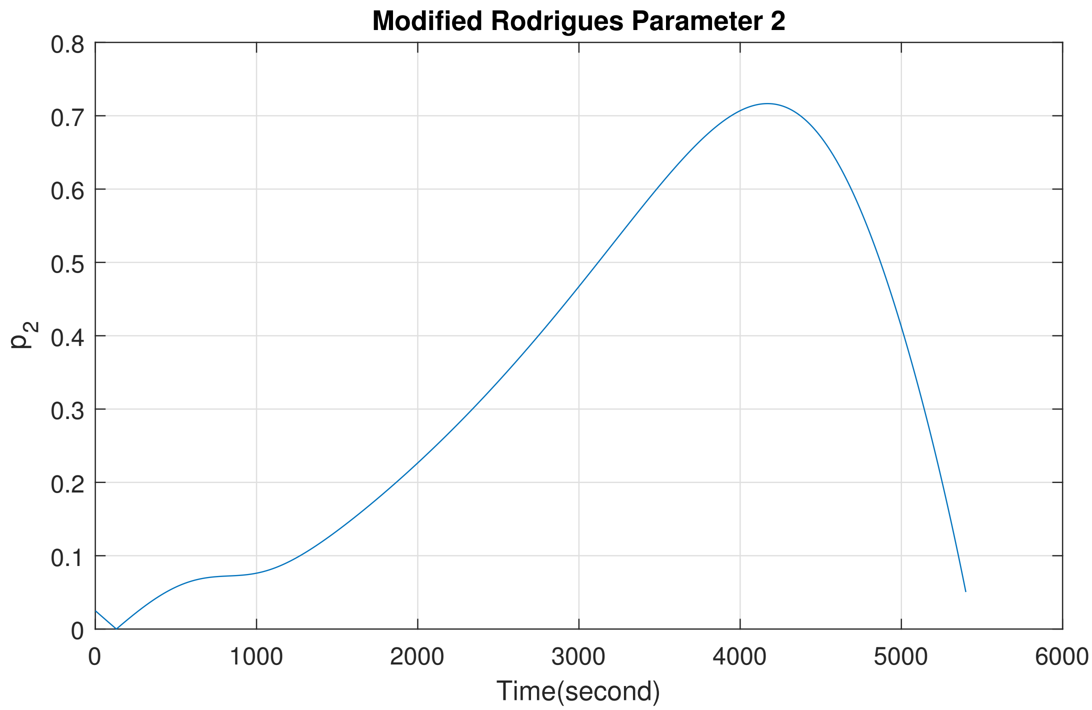
$$p_3 = \frac{q_3}{1 + q_4} \tag{3}$$

where:
q_1 is first quaternion
q_2 is second quaternion
q_3 is third quaternion
q_4 is fourth quaternion
The Modified Rodrigues Parameters should be found and plotted (3 figures).

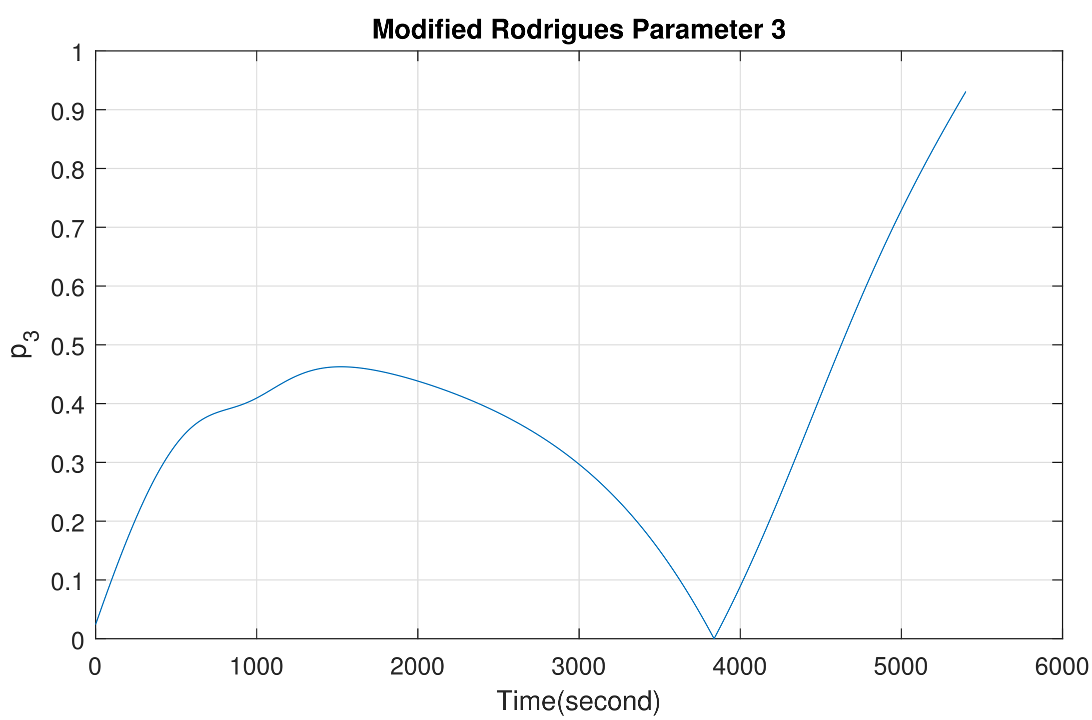
Graph of 1^{st} the Modified Rodrigues Parameter(p_1):



Graph of 2^{nd} the Modified Rodrigues Parameter(p_2):



Graph of 3^{rd} the Modified Rodrigues Parameter(p_3):



2 Appendix: MATLAB Code

```
clc;clear;close all;

n = 20; %sequence number

%initial data of the attitude angles(rad)
phi = -0.01 - 0.002 * n;
theta = 0.01 + 0.002 * n;
psi = 0.005 + 0.002 * n;

%initial data of angular velocities of the satellite
wX = 0.0002 + 0.00001 * n;
wY = 0.0003 + 0.00001 * n;
wZ = 0.0004 + 0.00001 * n;
w = [wX;wY;wZ];

%initial moments of inertia of the satellite(m^4)
Jx = 2.1*(10^(-3));
Jy = 2*(10^(-3));
Jz = 1.9*(10^(-3));

w_orbit = 0.0011; %The angular orbit velocity of satellite(rad/s)
N_T = 3.6*(10^(-10)); %The disturbance torque acting on the satellite(Nm)
dt = 0.1; %the sample time(s)
N = 54000; %iteration number
for k = 1:N
%the angular velocities
w(1) = w(1) + ((dt/Jx)*(Jy - Jz)*w(3)*w(2)) + ((dt/Jx)*N_T);
w(2) = w(2) + ((dt/Jy)*(Jz - Jx)*w(1)*w(3)) + ((dt/Jy)*N_T);
w(3) = w(3) + ((dt/Jz)*(Jx - Jy)*w(1)*w(2)) + ((dt/Jz)*N_T);

%the euler angles
phi = phi + dt*((((w(2)*sin(phi)) + (w(3)*cos(phi)))*tan(theta)) + w(1));
theta = theta + dt*((w(2)*cos(phi)) - (w(3)*sin(phi)) + (w_orbit));
psi = psi + dt*(((w(2)*sin(phi)) + (w(3)*cos(phi)))*sec(theta));

%quaternions
q1 = cos(phi/2)*cos(theta/2)*cos(psi/2)+sin(phi/2)*sin(theta/2)*sin(psi/2);
q2 = sin(phi/2)*cos(theta/2)*cos(psi/2)-cos(phi/2)*sin(theta/2)*sin(psi/2);
q3 = cos(phi/2)*sin(theta/2)*cos(psi/2)+sin(phi/2)*cos(theta/2)*sin(psi/2);
```

```
q4 = cos(phi/2)*cos(theta/2)*sin(psi/2)+sin(phi/2)*sin(theta/2)*cos(psi/2);
q = [abs(q1);abs(q2);abs(q3);abs(q4)];

%modified Rodrigues parameters
p1 = q(1)/(1+q(4));
p2 = q(2)/(1+q(4));
p3 = q(3)/(1+q(4));
p = [p1;p2;p3];

%arranging arrays for plotting
P(:,k) = p;
end

t = 0:dt:(N-1)*dt;

figure(1);
plot(t,P(1,:));
title('Modified Rodrigues Parameter 1');
xlabel('Time(second)');
ylabel('p_{1}');
grid on;

figure(2);
plot(t,P(2,:));
title('Modified Rodrigues Parameter 2');
xlabel('Time(second)');
ylabel('p_{2}');
grid on;

figure(3);
plot(t,P(3,:));
title('Modified Rodrigues Parameter 3');
xlabel('Time(second)');
ylabel('p_{3}');
grid on;
```

3 References

[1] Prof. Dr. Cengiz Hacızade, *Istanbul Technical University UCK421E Lecture Notes*, 2021.

[2] J.R.Wertz., *Space Attitude Determination and Control*, D.Reidel Publishing Company, Dordrecht, Holland, 2002.

[3] Hajiyev, C., & Soken, H.E., *Fault Tolerant Attittude Estimation for Small Satellites*, 1^{st} Ed., CRC Press, 2021.