Simulation of Magnetometer Measurements

Attitude Dynamics and Control
Project Report

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Contents

1 Simulation of Magnetometer Measurements

Three onboard magnetometers of satellite measures the components of the magnetic field vector in the body frame. Therefore for the measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A . Overall measurement model may be given as

$$\begin{bmatrix} H_x(\phi,\theta,\psi,t) \\ H_y(\phi,\theta,\psi,t) \\ H_z(\phi,\theta,\psi,t) \end{bmatrix} = A \cdot \begin{bmatrix} H_1(t) \\ H_2(t) \\ H_3(t) \end{bmatrix} + \eta_1 \tag{1}$$

where,

- $H_1(t)$,$H_2(t)$ and $H_1(t)$ represent the Earth magnetic field vector components in the orbit frame as a function of time
- $H_x(\phi,\theta,\psi,t)$,$H_y(\phi,\theta,\psi,t)$ and $H_z(\phi,\theta,\psi,t)$ show the measured Earth magnetic field vector components in body frame as a function of time and varying Euler angles
- η_1 is the zero mean Gaussian white noise with the characteristic of

  $$E[\eta_{1_k}\eta_{1_j}^T] = I_{3x3}\sigma_m^2\delta_{kj} \tag{2}$$

  where

  - I_{3x3} is the identity matrix with the dimension of 3 x 3
  - σ_m^2 is the standard deviation of each magnetometer error
  - δ_{kj} is the Kronecker symbol.

Overall measurement model of magnetometers may be given as
- In terms of direction cosines

$$B_m(k) = A(k)H_0(k) + b_c(k) + \nu_c(k) \tag{3}$$

- In terms of projections

$$B_m(k) = A(k)H(k)(10^6) + b(k) + \nu(k) \tag{4}$$

where

- A(k) is the direction cosines in terms of Euler angles

$$A = \begin{bmatrix} cos(\theta)cos(\psi) & cos(\theta)sin(\psi) & -sin(\theta) \\ -cos(\phi)sin(\psi) + sin(\phi)sin(\theta)cos(\psi) & cos(\phi)cos(\psi) + sin(\phi)sin(\theta)sin(\psi) & sin(\phi)cos(\theta) \\ sin(\phi)sin(\psi) + cos(\phi)sin(\theta)cos(\psi) & -sin(\phi)cos(\psi) + cos(\phi)sin(\theta)sin(\psi) & cos(\phi)cos(\theta) \end{bmatrix} \tag{5}$$

- b(k) is the magnetometer bias vector

$$b = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T \tag{6}$$

where $b_x = 3\mu T$; $b_y = 5\mu T$; $b_z = 6\mu T$

$$b_c = \begin{bmatrix} b_{x_c} & b_{y_c} & b_{z_c} \end{bmatrix}^T \tag{7}$$

where $b_{x_c} = 0.04$; $b_{y_c} = 0.06$; $b_{z_c} = 0.08$

- $\nu(k)$ is the zero mean Gaussian white noise

$$\nu_c = \begin{bmatrix} \sigma_{m_c} randn & \sigma_{m_c} randn & \sigma_{m_c} randn \end{bmatrix}^T \tag{8}$$

$$\nu = \begin{bmatrix} \sigma_m randn & \sigma_m randn & \sigma_m randn \end{bmatrix}^T \tag{9}$$

σ_m is the standard deviation of each magnetometer error

$$\sigma_{m_c} = 0.008$$
$$\sigma_m = 1.66\mu T$$

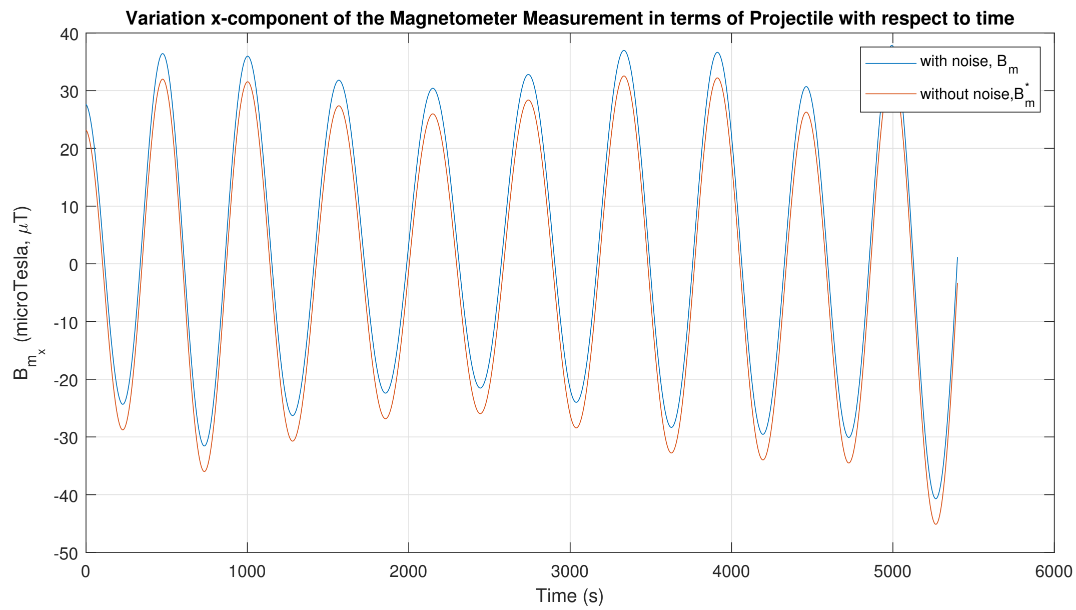
$(1W/m^2 = 1T = 10^6\mu T)$

Satellite Dynamics will be calculated 54000 iterations with the time step dt=0.1s

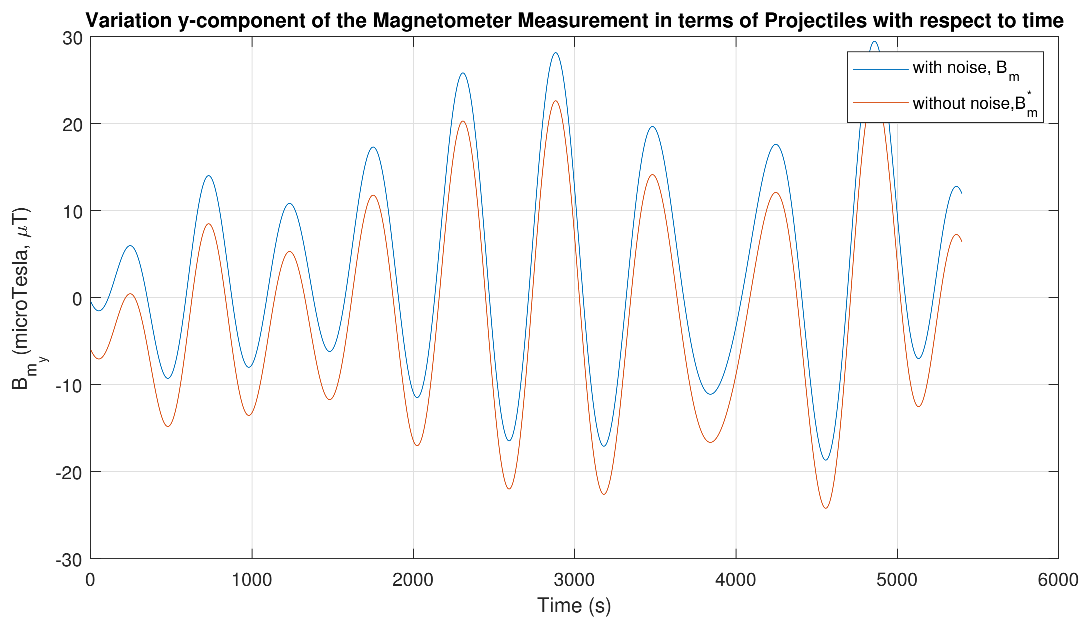
Explanation: Graphs which are with noise and without noise are plotted in a one figure in order to comparing them. So, there are 6 plots instead of 12 plots.

The Magnetometer Measurement in terms of Projectiles:

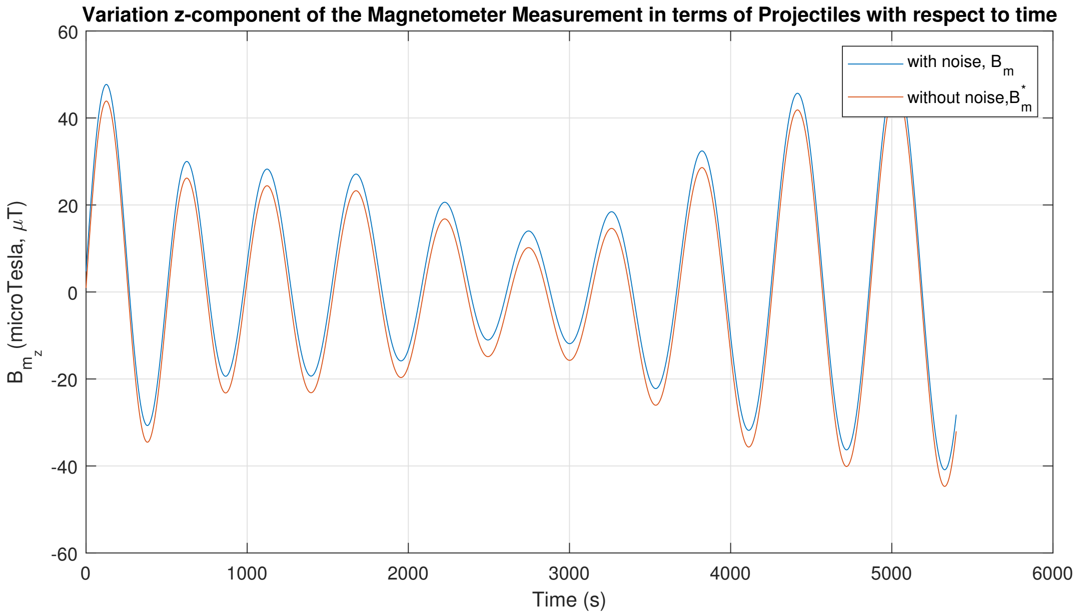
Graph of x-component of the magnetometer measurement in terms of projectiles:



Graph of y-component of the magnetometer measurement in terms of projectiles:

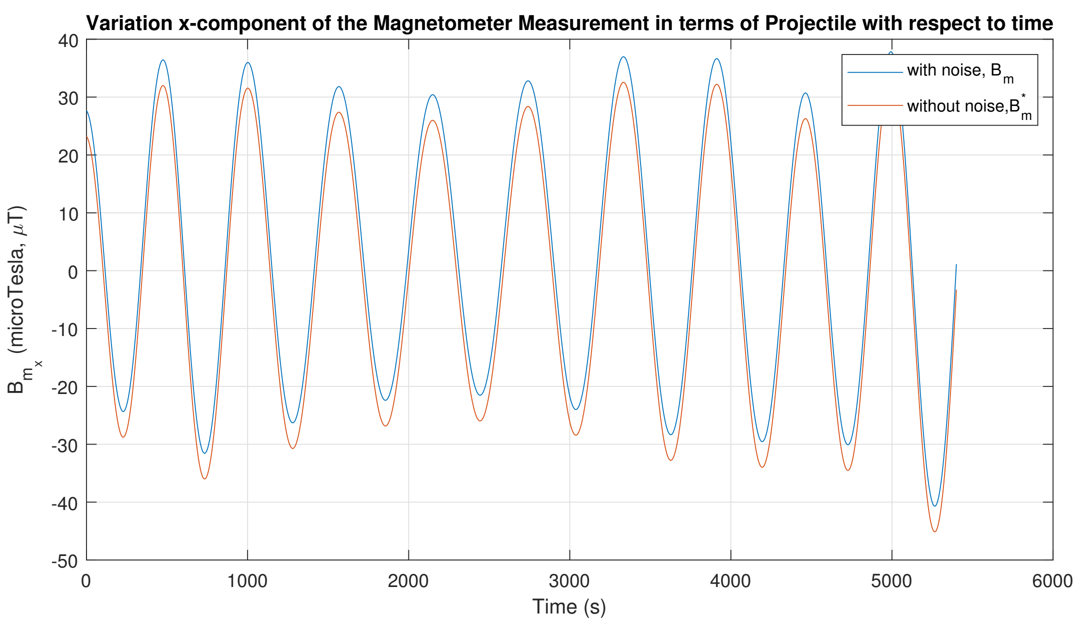


Graph of z-component of the magnetometer measurement in terms of projectiles:

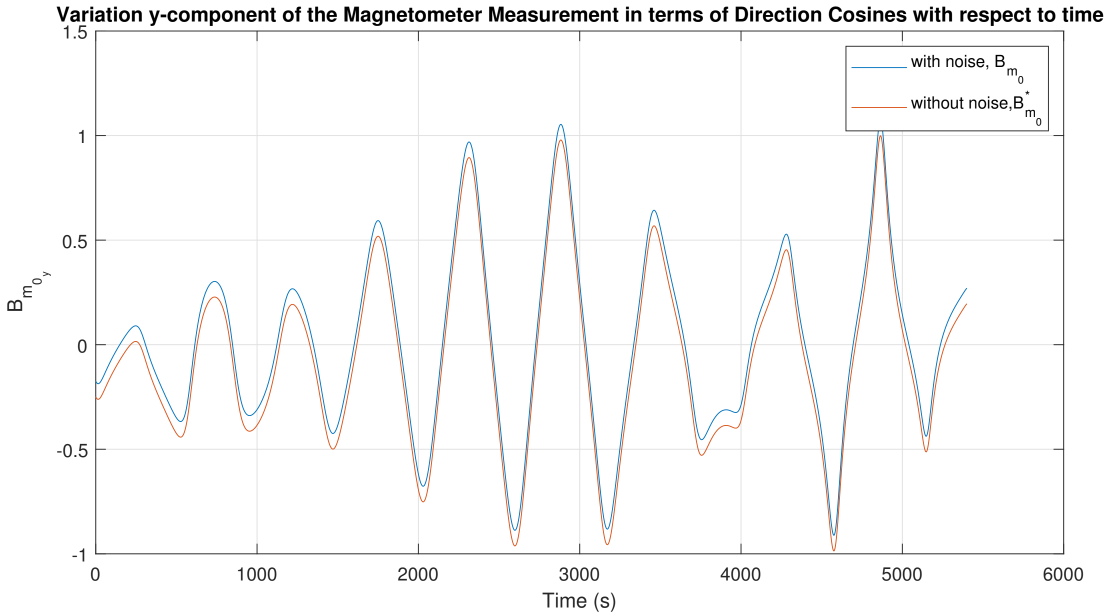


The Magnetometer Measurement in terms of Direction Cosines:

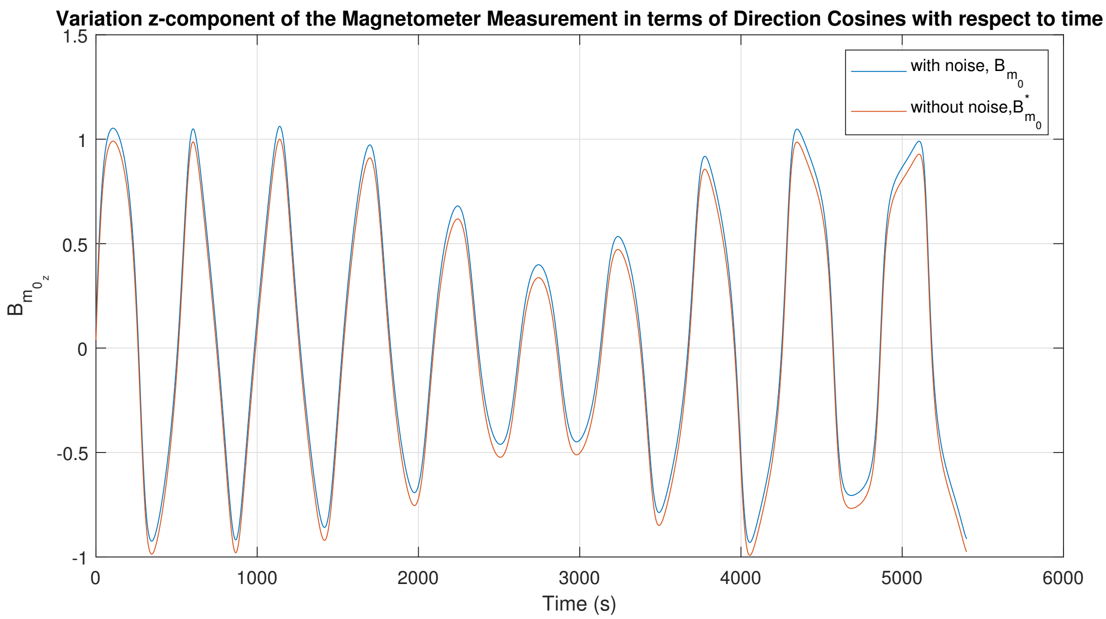
Graph of x-component of the magnetometer measurement in terms of direciton cosines:



Graph of y-component of the magnetometer measurement in terms of direciton cosines:



Graph of z-component of the magnetometer measurement in terms of direciton cosines:



2 Appendix: MATLAB Code

```
clc;clear;close all;

n = 20; %sequence number

%initial data of the attitude angles(rad)
phi = -0.01 - 0.002 * n;
theta = 0.01 + 0.002 * n;
psi = 0.005 + 0.002 * n;

%initial data of the angular velocities of satellite
wX = 0.0002 + 0.00001 * n;
wY = 0.0003 + 0.00001 * n;
wZ = 0.0004 + 0.00001 * n;
w = [wX;wY;wZ];

%initial moments of inertia of the satellite(m^4)
Jx = 2.1*(10^(-3));
Jy = 2*(10^(-3));
Jz = 1.9*(10^(-3));

w_orbit = 0.0011; %The angular orbit velocity of satellite(rad/s)
N_T = 3.6*(10^(-10)); %The disturbance torque acting on the satellite(Nm)

Me = 7.943e15; %the magnetic dipole moment of the Earth in Wb.m
mu = 3.98601e14; %the Earth Gravitational constant in m^3/s^2
i = (80 + (0.5*n))*(pi/180); %inclination converted from deg to rad
we = 7.29e-5; %the spin rate of the Earth in rad/s
epsilon = 11.7*(pi/180); %the magnetic dipole tilt converted to rad
r0 = (6378.14 + 500 + (2*n))*1000; %the distance between the center of ...
    mass of the satellite and the Earth in m
w0 = sqrt(mu/r0^3); %the angular velocity of the orbit with respect to ...
    the inertial frame in rad/s

dt = 0.1; %the sample time(s)
N = 54000; %iteration number

% pre-allocation
Bm0=zeros(3,N);
Bm0_star=zeros(3,N);
```

```
Bm=zeros(3,N);
Bm_star=zeros(3,N);

for k = 1:N
% the angular velocities
w(1) = w(1) + ((dt/Jx)*(Jy - Jz)*w(3)*w(2)) + ((dt/Jx)*N_T);
w(2) = w(2) + ((dt/Jy)*(Jz - Jx)*w(1)*w(3)) + ((dt/Jy)*N_T);
w(3) = w(3) + ((dt/Jz)*(Jx - Jy)*w(1)*w(2)) + ((dt/Jz)*N_T);

% the euler angles
phi = phi + dt*((((w(2)*sin(phi)) + (w(3)*cos(phi)))*tan(theta)) + w(1));
theta = theta + dt*((w(2)*cos(phi)) - (w(3)*sin(phi)) + (w_orbit));
psi = psi + dt*(((w(2)*sin(phi)) + (w(3)*cos(phi)))*sec(theta));

% transformation matrix A
A = [cos(theta)*cos(psi) cos(theta)*sin(psi) ...
    -sin(theta);((-cos(phi)*sin(psi)) + (sin(phi)*sin(theta)*cos(psi))) ...
    ((cos(phi)*cos(psi)) + (sin(phi)*sin(theta)*sin(psi))) ...
    (sin(phi)*cos(theta));((sin(phi)*sin(psi))+(cos(phi)*sin(theta)*cos(psi))) ...
    ((-sin(phi)*cos(psi))+(cos(phi)*sin(theta)*sin(psi))) ...
    (cos(phi)*cos(theta))];

% components of Earth Magnetic Field
hx = ...
    (Me/r0^3)*((cos(w0*k)*((cos(epsilon)*sin(i))-(sin(epsilon)*cos(i)*cos(we*k)))-(sin(w
hy = (-(Me/r0^3))*((cos(epsilon)*cos(i))+(sin(epsilon)*sin(i)*cos(we*k)));
hz = ...
    ((2*Me)/r0^3)*((sin(w0*k)*((cos(epsilon)*sin(i))-(sin(epsilon)*cos(i)*cos(we*k)))-(2
h =[hx;hy;hz]*10^6; %converted to microTesla

% components of Direction Cosine Matrix
hx0 = hx/(sqrt(hx^2+hy^2+hz^2));
hy0 = hy/(sqrt(hx^2+hy^2+hz^2));
hz0 = hz/(sqrt(hx^2+hy^2+hz^2));
h0=[hx0;hy0;hz0];

% standard deviation of each magnetometer errors
sigma_mC = 0.008; % in terms of direction cosine
sigma_m = 1.66; % in terms of projectiles

% magnetometer bias vectors
bc = [0.04;0.06;0.08]; % in terms of direction cosine
b = [3;5;6]; % in terms of projectiles
```

```

% zero mean Gaussian white noises
rng(0);
vc = [sigma_mC*randn;sigma_mC*randn;sigma_mC*randn]; %in terms of ...
    direction cosine
v = [sigma_m*randn;sigma_m*randn;sigma_m*randn];

%magnetometer measurements in terms of projectiles
Bm(:,k) = ((A*h)+b+v); % with bias and noise
Bm_star(:,k) = (A*h); % without bias and noise

%magnetometer measurements in terms of direction cosines
Bm0(:,k)=(A*h0)+bc+vc; % with bias and noise
Bm0_star(:,k)=(A*h0); % without bias and noise
end

t = 0:dt:(N-1)*dt; %constructing time axis

figure(1)
bmx = plot(t,Bm(1,:));
grid on
hold on
bmsx = plot(t,Bm_star(1,:));
hold off
title('Variation x-component of the Magnetometer Measurement in terms ...
    of Projectile with respect to time');
xlabel('Time (s)');
ylabel('B_{m_{x}} (microTesla, \muT)');
legend([bmx,bmsx],'with noise, B_{m}','without noise,B_{m}^{*}');

figure(2)
bmy = plot(t,Bm(2,:));
grid on
hold on
bmsy = plot(t,Bm_star(2,:));
hold off
title('Variation y-component of the Magnetometer Measurement in terms ...
    of Projectiles with respect to time');
xlabel('Time (s)');
ylabel('B_{m_{y}}(microTesla, \muT)');
legend([bmy,bmsy],'with noise, B_{m}','without noise,B_{m}^{*}');

figure(3)
```

```
bmz = plot(t,Bm(3,:));
grid on
hold on
bmsz = plot(t,Bm_star(3,:));
hold off
title('Variation z-component of the Magnetometer Measurement in terms ...
    of Projectiles with respect to time');
xlabel('Time (s)');
ylabel('B_{m_{z}}(microTesla, \muT)');
legend([bmz,bmsz],'with noise, B_{m}','without noise,B_{m}^{*}');

figure(4)
bm0x = plot(t,Bm0(1,:));
grid on
hold on
bm0sx = plot(t,Bm0_star(1,:));
hold off
title('Variation x-component of the Magnetometer Measurement in terms ...
    of Direction Cosines with respect to time');
xlabel('Time (s)');
ylabel('B_{m_{0_{x}}}');
legend([bm0x,bm0sx],'with noise, B_{m_{0}}','without noise,B_{m_{0}}^{*}');

figure(5)
bm0y = plot(t,Bm0(2,:));
grid on
hold on
bm0sy = plot(t,Bm0_star(2,:));
hold off
title('Variation y-component of the Magnetometer Measurement in terms ...
    of Direction Cosines with respect to time');
xlabel('Time (s)');
ylabel('B_{m_{0_{y}}}');
legend([bm0y,bm0sy],'with noise, B_{m_{0}}','without noise,B_{m_{0}}^{*}');

figure(6)
bm0z = plot(t,Bm0(3,:));
grid on
hold on
bm0sz = plot(t,Bm0_star(3,:));
hold off
title('Variation z-component of the Magnetometer Measurement in terms ...
    of Direction Cosines with respect to time');
```

```
xlabel('Time (s)');
ylabel('B_{m_{0_{z}}}');
legend([bm0z,bm0sz],'with noise, B_{m_{0}}','without noise,B_{m_{0}}^{*}');
```

3 References

[1] Prof. Dr. Cengiz Hacızade, *Istanbul Technical University UCK421E Lecture Notes*, 2021.

[2] J.R.Wertz., *Space Attitude Determination and Control*, D.Reidel Publishing Company, Dordrecht, Holland, 2002.

[3] Hajiyev, C., & Soken, H.E., *Fault Tolerant Attittude Estimation for Small Satellites*, 1^{st} Ed., CRC Press, 2021.