Earth's Magnetic Field Vector through the Orbit of the Satellite Using Dipole Model

Attitude Dynamics and Control
Project Report

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Contents

1 Earth's Magnetic Field Vector through the Orbit of the Satellite Using Dipole Model

Aim of this homework is to study the behavior of the magnetic field vector of Earth, while spacecraft's position changes through orbit. This is an important data; because Earth's magnetic field vector directly influences the attitude of the spacecraft.
As the satellite navigates along its orbit, magnetic field vector differs in a relevant way with the orbital parameters. If those parameters are known, then, magnetic field tensor vector that affects satellite can be shown as a function of time analytically. Note that, these terms are obtained in the orbit reference frame.

$$H_1(t) = \frac{M_e}{r_0^3}\{cos(\omega_0 t)[cos(\varepsilon)sin(i) - sin(\varepsilon)cos(i)cos(\omega_e t)] - sin(\omega_0 t)sin(\varepsilon)sin(\omega_e t)\} \quad (1)$$

$$H_{12}(t) = -\frac{M_e}{r_0^3}[cos(\varepsilon)cos(i) - sin(\varepsilon)sin(i)cos(\omega_e t)] \quad (2)$$

$$H_3(t) = \frac{2M_e}{r_0^3}\{sin(\omega_0 t)[cos(\varepsilon)sin(i) - sin(\varepsilon)cos(i)cos(\omega_e t)] - 2sin(\omega_0 t)sin(\varepsilon)sin(\omega_e t)\} \quad (3)$$

- $M_e = 7.943\ x\ 10^{15}\ Wb.m$, the magnetic dipole moment of the Earth
- $\mu = 3.98601\ x\ 10^{14}\ m^3/s^2$, the Earth Gravitational constant
- $i = 80 + (0.5\ x\ n)\ deg$, the orbit inclination
- $\omega_e = 7.29\ x\ 10^{-5}\ rad/s$
- $\varepsilon = 11.7$, the magnetic dipole tilt (Inclination of Geomagnetic axis relative to the Earth's daily rotating axis)
- $r_0 = (6378.14 + 500 + (2\ x\ n))\ x\ 1000\ m$, the distance between the center of mass of the satellite and the Earth
- $\omega_0 = (\mu/r_0^3)^{1/2}$, the angular velocity of the orbit with respect to the inertial frame
- $n = 20$, the student sequence number for this homework.

To find the direction of the magnetic fie vector we can track its direction cosines.
Its components are computed as:

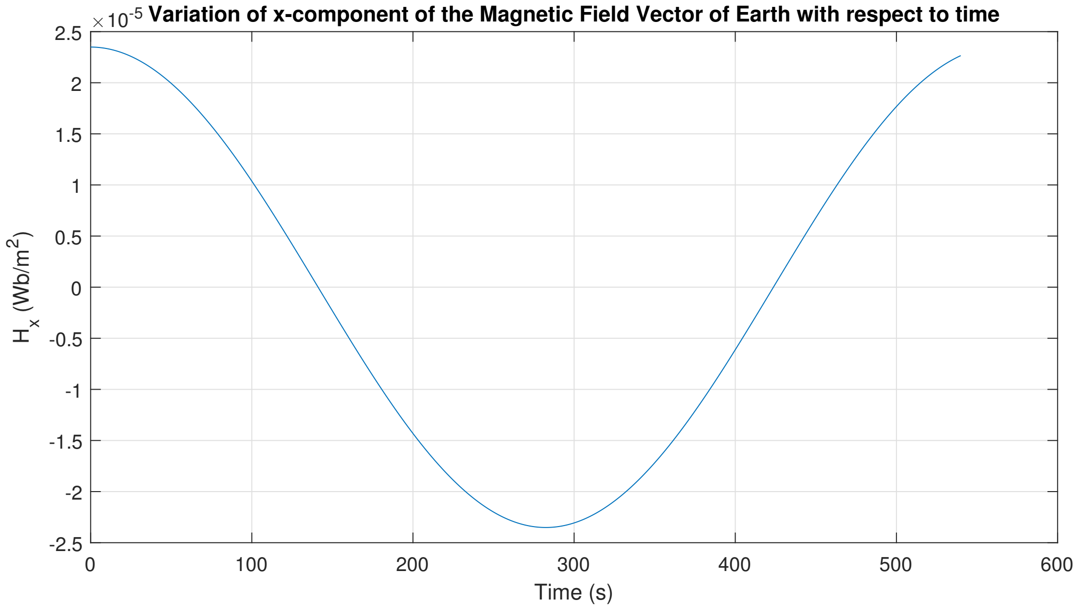
$$H_0 = \begin{bmatrix} H_{x_0} \\ H_{y_0} \\ H_{z_0} \end{bmatrix} = \frac{1}{\sqrt{H_x^2 + H_y^2 + H_z^2}} \cdot \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \quad (4)$$

H_0: Direction cosine elements of the magnetic field vector, ($H_x = H_1$; $H_y = H_2$; $H_z = H_3$)

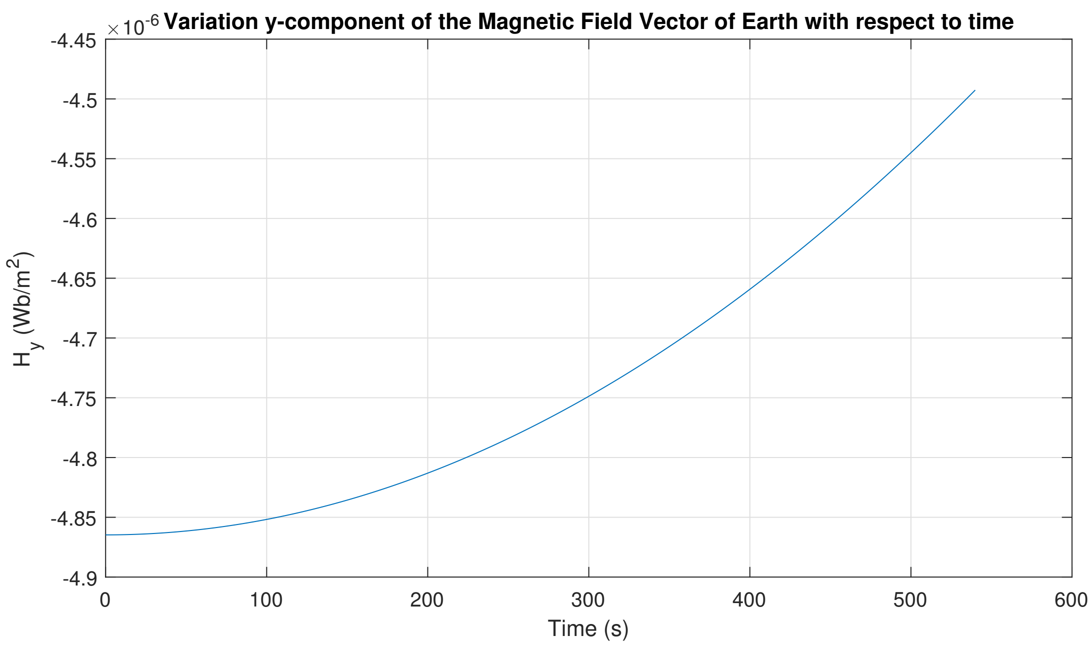
The Earth's Magnetic Field vector $\overrightarrow{H}$ and then direction cosines will be calculated 54000 iterations with time step $\Delta t = 0.1\ s$ by using first 3 equation, we can compute the elements of the Earth's Magnetic Field vector H and then direction cosines can be calculated from equation (4).

Components of the Magnetic Field of Earth:

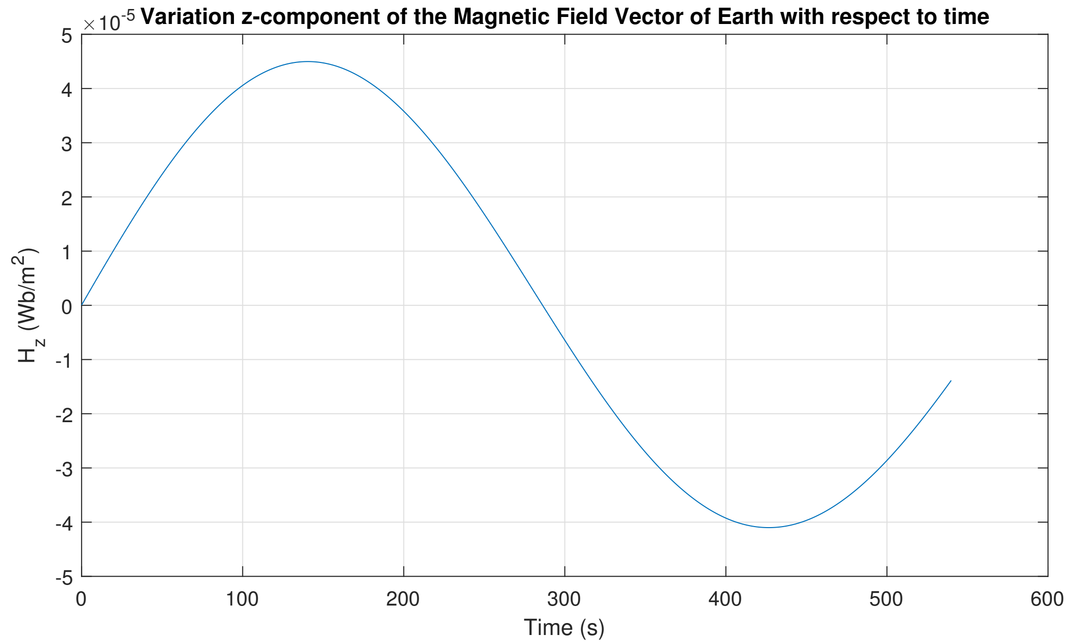
Graph of x-component of the Magnetic Field Vector of Earth,H_x:



Graph of y-component of the Magnetic Field Vector of Earth,H_y:



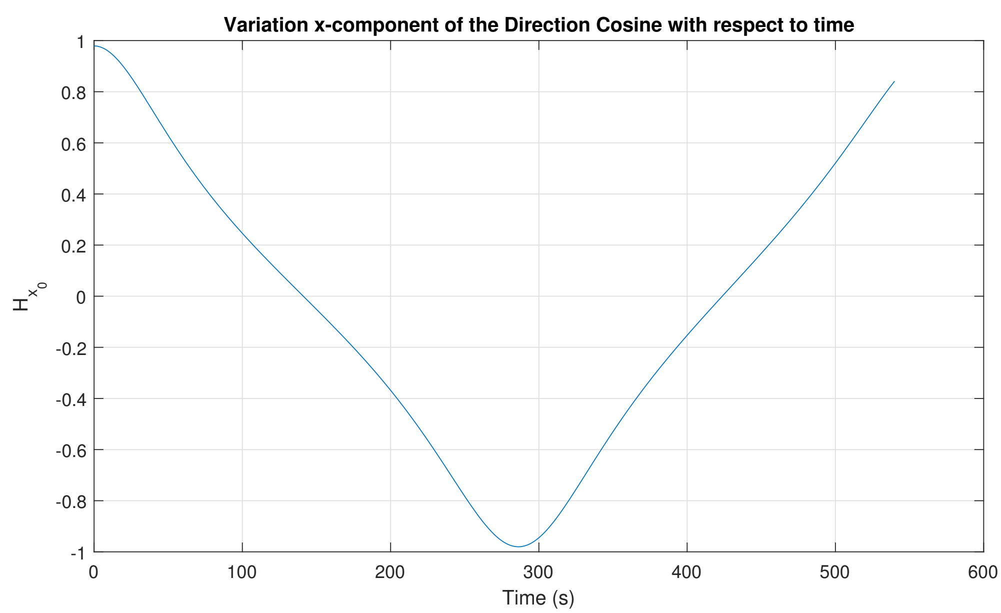
Graph of z-component of the Magnetic Field Vector of Earth,H_z:



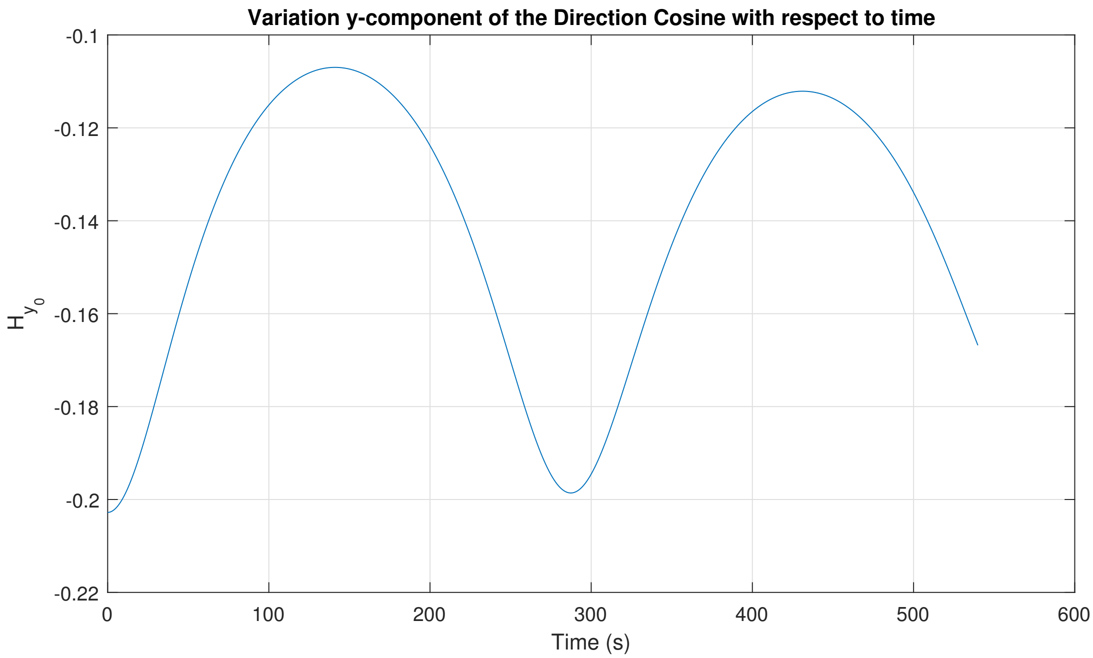
Components of the Direction Cosine:

Direction cosines are components of unit vector of magnetic field on x, y and z axes. Note that, squares of direction of cosines should be equal to 1, since they are components of a unit vector. Therefore, direction cosines should vary in the interval [-1,+1].

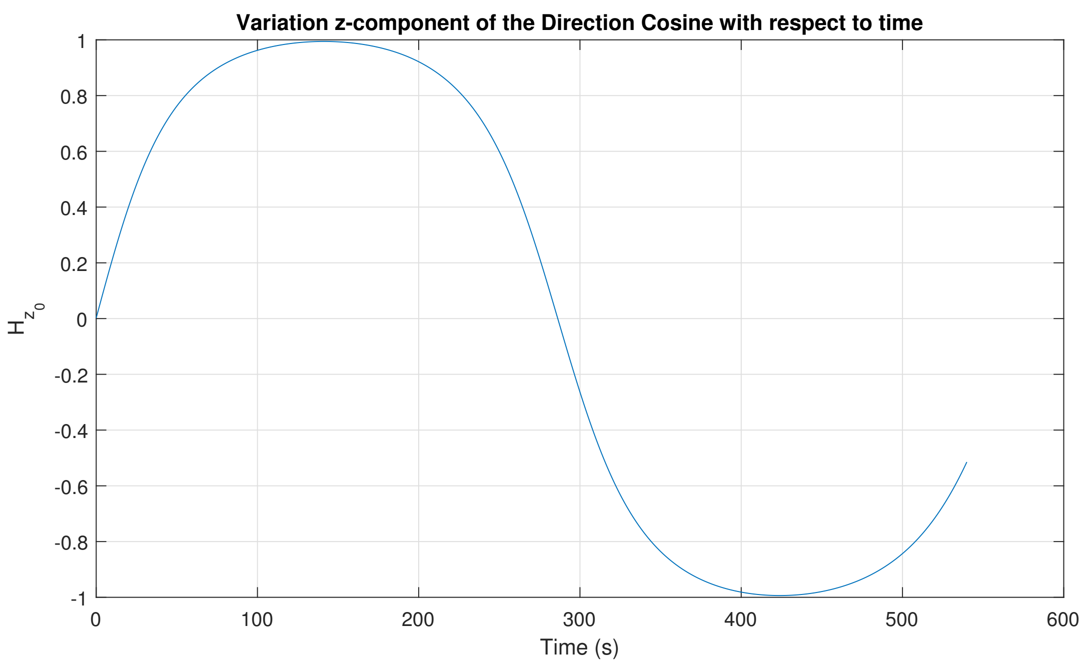
Graph of x-component of the Direction Cosine Matrix,H_{x_0}:



Graph of y-component of the Direction Cosine Matrix,H_{y_0}:



Graph of z-component of the Direction Cosine Matrix,H_{z_0}:



2 Appendix: MATLAB Code

```
clc;clear;close all;

n=20;%sequence number

Me = 7.943e15; %the magnetic dipole moment of the Earth in Wb.m
mu = 3.98601e14; %the Earth Gravitational constant in m^3/s^2
i = (80 + (0.5*n))*(pi/180); %inclination converted from deg to rad
we = 7.29e-5; %the spin rate of the Earth in rad/s
epsilon = 11.7*(pi/180); %the magnetic dipole tilt converted to rad
r0 = (6378.14 + 500 + (2*n))*1000; %the distance between the center of ...
    mass of the satellite and the Earth in m
w0 = sqrt(mu/r0^3); %the angular velocity of the orbit with respect to ...
    the inertial frame in rad/s

dt = 0.1; %the sample time(s)
N = 54000*dt; %iteration

for k = 1:N
%Components of Earth Magnetic Field (Wb/m^2(Tesla))
hx = ...
    (Me/r0^3)*((cos(w0*k)*((cos(epsilon)*sin(i))-(sin(epsilon)*cos(i)*cos(we*k)))-(sin(w
hy = (-(Me/r0^3))*((cos(epsilon)*cos(i))+(sin(epsilon)*sin(i)*cos(we*k)));
hz = ...
    ((2*Me)/r0^3)*((sin(w0*k)*((cos(epsilon)*sin(i))-(sin(epsilon)*cos(i)*cos(we*k)))-(2

%Components of Direction Cosine Matrix
hx0 = hx/(sqrt(hx^2+hy^2+hz^2));
hy0 = hy/(sqrt(hx^2+hy^2+hz^2));
hz0 = hz/(sqrt(hx^2+hy^2+hz^2));

%arranging arrays for plotting
Hx(:,k) = hx;
Hy(:,k) = hy;
Hz(:,k) = hz;
Hx0(:,k) = hx0;
Hy0(:,k) = hy0;
Hz0(:,k) = hz0;
end

```

```
t = 0:dt:(N-1)*dt;%constructing time axis

figure(1);
plot(t,Hx);
grid on;
title('Variation of x-component of the Magnetic Field Vector of Earth ...
    with respect to time');
xlabel('Time (s)');
ylabel('H_{x} (Wb/m^{2})');

figure(2);
plot(t,Hy);
grid on;
title('Variation y-component of the Magnetic Field Vector of Earth with ...
    respect to time');
xlabel('Time (s)');
ylabel('H_{y} (Wb/m^{2})');

figure(3);
plot(t,Hz);
grid on;
title('Variation z-component of the Magnetic Field Vector of Earth with ...
    respect to time');
xlabel('Time (s)');
ylabel('H_{z} (Wb/m^{2})');

figure(4);
plot(t,Hx0);
grid on;
title('Variation x-component of the Direction Cosine with respect to ...
    time');
xlabel('Time (s)');
ylabel('H_{x_{0}}');

figure(5);
plot(t,Hy0);
grid on;
title('Variation y-component of the Direction Cosine with respect to ...
    time');
xlabel('Time (s)');
ylabel('H_{y_{0}}');

figure(6);
```

```
plot(t,Hz0);
grid on;
title('Variation z-component of the Direction Cosine with respect to ...
    time');
xlabel('Time (s)');
ylabel('H_{z_{0}}');
```

3 References

[1] Prof. Dr. Cengiz Hacızade, *Istanbul Technical University UCK421E Lecture Notes*, 2021.

[2] J.R.Wertz., *Space Attitude Determination and Control*, D.Reidel Publishing Company, Dordrecht, Holland, 2002.

[3] Hajiyev, C., & Soken, H.E., *Fault Tolerant Attittude Estimation for Small Satellites*, 1^{st} Ed., CRC Press, 2021.