# Solution of a Heat Transfer Problem using Successive Over-Relaxation(SOR) Method

**Heat Transfer**
**Project Report**

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# Contents

# 1 Introduction

In this report, computation steps of given temperature distribution with successive over-relaxation approach which is a variant of the Gauss–Seidel method are presented. The steady-state temperature distribution over the 5 $cm \times 9$ $cm$ flat plate with a thickness of 0.5 $cm$ is considered. There is a heat generation of $Q = 0.6$ $cal/cm^3s$ in the plate, while there is a heat loss of $\partial T/\partial y = 15$ at the lower edge, the side edges are kept at a constant temperature of 20°$C$. On the upper edge, there is heat exchange with the environment in accordance with the formula

$$-0.16\frac{\partial T}{\partial y} = 0.073(T_o - 25) \tag{1.1}$$

The magnitude $T_o$ indicates the temperatures on the upper edge of the plate. The plate surface is insulated and there is no heat transfer with the environment. Take the step sizes in x and y direction equal and 0.25 $cm$.

It is known that this problem is governed by Poisson equation which is defined as the following differential equation:

$$\nabla^2 T = -\frac{Q}{k\tau} \tag{1.2}$$

Here, $k = 0.16$ $cal/cm°C$ and $\tau = 0.5$ $cm$ respectively.

# 2 Solution Method

## 2.1 SOR Approach to Given Differential Equation

The Poisson equation can be written by discretising the central differences for interior nodes as

$$\frac{T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j}}{h^2} = -\frac{Q}{k\tau} \tag{2.1}$$

And by applying SOR, the equation below can be obtained as

$$T^{k+1}_{i,j} = T^k_{i,j} + \omega\left(\frac{T^{k+1}_{i-1,j} + T^k_{i+1,j} + T^{k+1}_{i,j-1} + T^{k+1}_{i,j+1} - 4T^k_{i,j}}{4} + \frac{Qh^2}{4k\tau}\right) \tag{2.2}$$

## 2.2 Boundary Conditions

There are Dirichlet type boundary conditions on the left and right edges of the plate where $T = 20°C$. For all further nodes are assumed to be at a distance h from the plate edge in the surrounding environment. A boundary condition in terms of temperature gradient is given so that $\partial T/\partial y = 15$ at the lower edge of the plate. This gradient will be included in the processes by calculating formula of;

$$\frac{\partial T}{\partial y} = \frac{T_{i,end-2} - T_{i,end}}{2h} = 15 \tag{2.1}$$

$$T_{i,end} = T_{i,end-2} - 30h \tag{2.2}$$

For lower edge of plate, we obtain a relation by applying SOR method as

$$T_{i,end-1}^{k+1} = T_{i,end-1}^{k} + \omega\left(\frac{T_{i-1,end-1}^{k+1} + T_{i+1,end-1}^{k} + T_{i,end}^{k+1} + T_{i,end-2}^{k+1} - 4T_{i,end-1}^{k}}{4} + \frac{Qh^2}{4k\tau}\right) \tag{2.3}$$

$$T_{i,end}^{k+1} = T_{i,end-2}^{k} \tag{2.4}$$

Boundary condition of the upper edge of the plate is given as

$$-0.16\frac{\partial T}{\partial y} = 0.073(T_o - 25) \tag{2.5}$$

The temperature gradient here is calculated and used in the boundary condition.

$$\frac{\partial T}{\partial y} = \frac{T_{i,1} - T_{i,3}}{2h} \tag{2.6}$$

$$-k\frac{\partial T}{\partial y} = -k\frac{T_{i,1} - T_{i,3}}{2h} = H(T_{i,2} - T_o) \tag{2.7}$$

$$T_{i,1} = T_{i,2} - \frac{2hH}{k}\left(T_{i,2} - T_o\right) \tag{2.8}$$

(Using the nodes adjacent to the upper edge of the plate and the imaginary nodes at a distance h from the plate in the surrounding environment). Applying SOR formula for upper edge of plate;

$$T_{i,2}^{k+1} = T_{i,2}^{k} + \omega\left(\frac{T_{i-1,2}^{k+1} + T_{i+1,2}^{k} + T_{i,3}^{k+1} + T_{i,1}^{k+1} - 4T_{i,2}^{k}}{4} + \frac{Qh^2}{4k\tau}\right) \tag{2.9}$$

## 2.3 Computation of Relaxation Factor

Before the calculation using SOR method with these formulas, we need to calculate relaxation factor with solving the formula below

$$\left[\cos\left(\frac{\pi}{p}\right) + \cos\left(\frac{\pi}{q}\right)\right]^2 \omega^2 - 16\omega + 16 = 0 \tag{2.1}$$

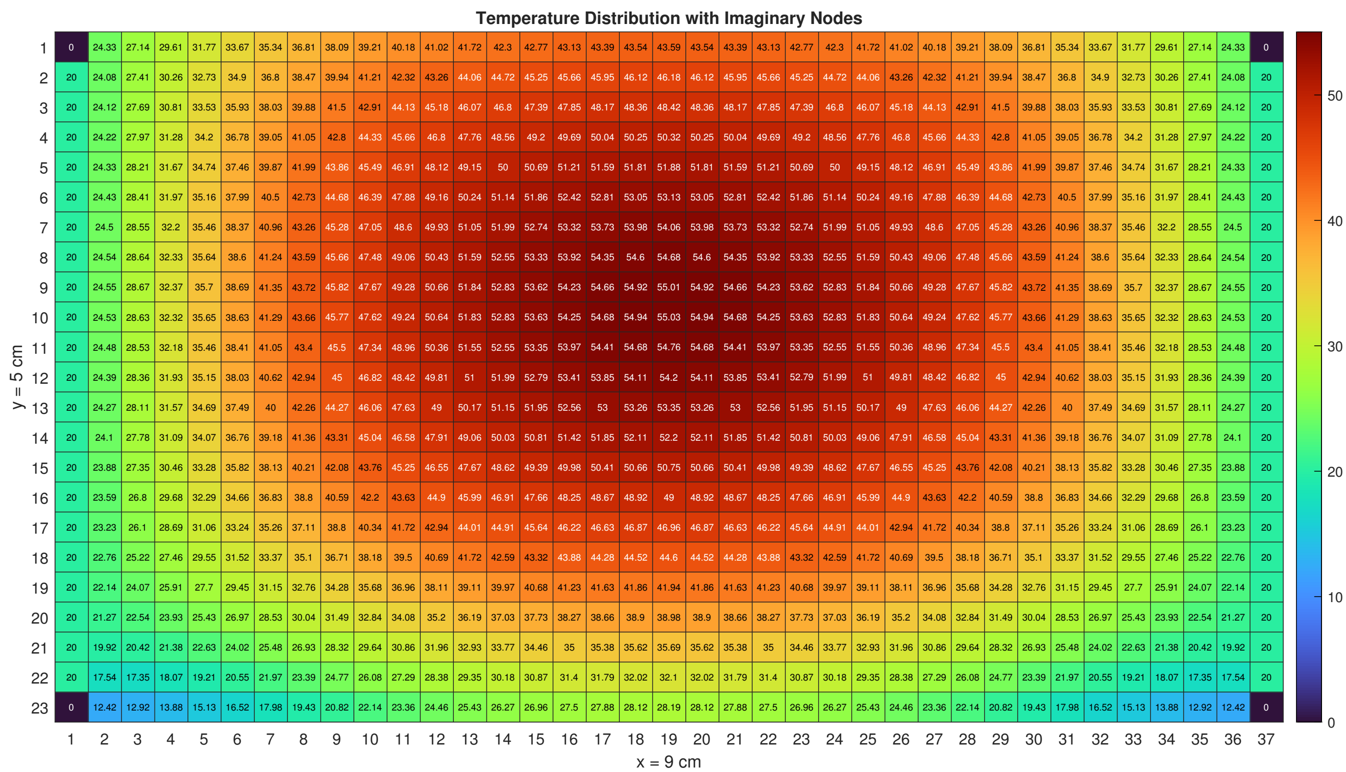
where

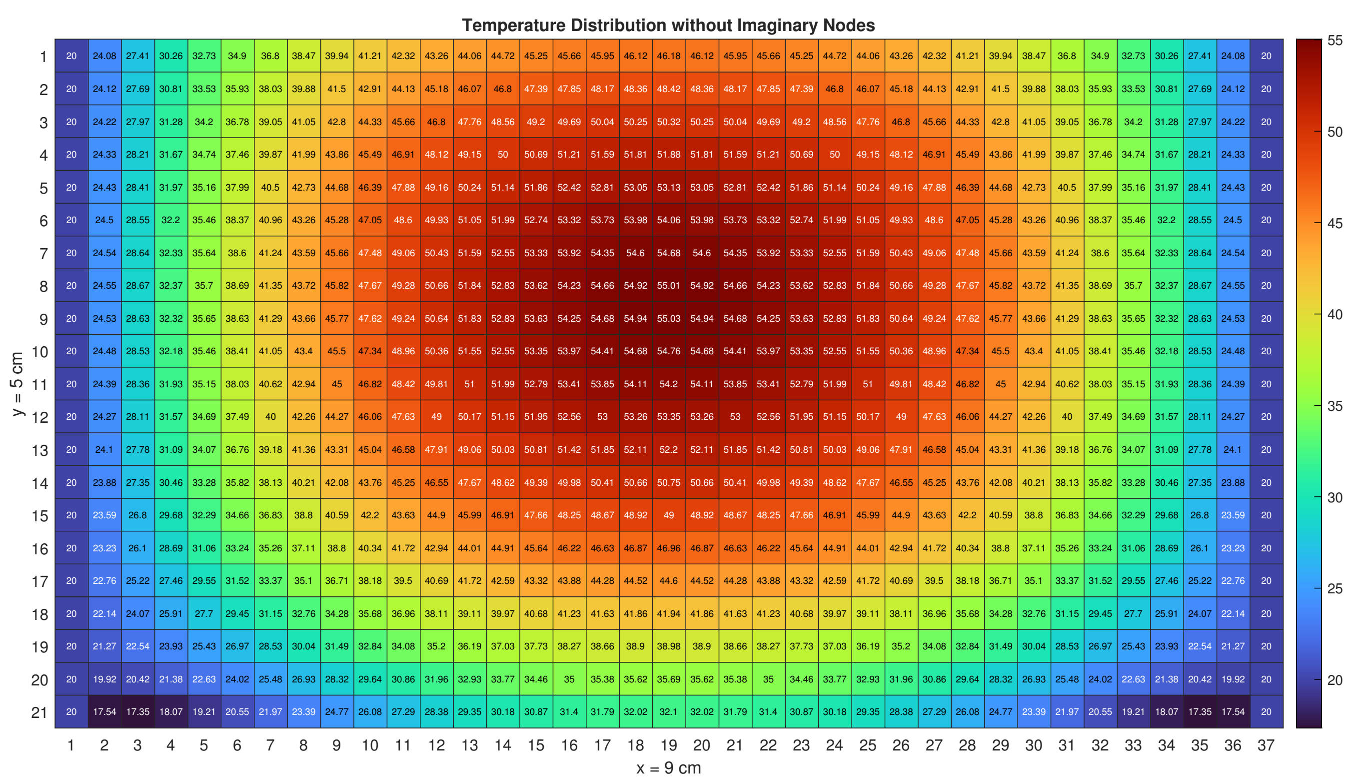
- $p = \dfrac{\text{Length in x direction}}{\text{step size}, h}$
- $q = \dfrac{\text{Length in y direction}}{\text{step size}, h}$

Between solutions, the value that beetween 0 and 2 is taken as relaxation factor, $\omega$ because relaxation factor varies between 0 and 2.

# 3 Results

Temperature distribution results are plotted as heat map for both with imaginary nodes and without imaginary nodes below.



# 4 Appendix: MATLAB Code

```
clc;clear;close all;
%% Given Properties
Q = 0.6; % Heat Generation Rate [cal/cm^3s]
k = 0.16; % Thermal Conductivity [cal/cmC]
H = 0.073; % Heat Transfer Coefficient [cal/cm^2C]
tau = 0.5; % Plate Thickness [cm]
T0 = 25; % Surrounding Temperature [C]
%% Preallocation
xlength = 9; % x length [cm]
ylength = 5; % y length [cm]
h = 0.25; % cm, Interval
xnode = xlength/h + 1; % Number of Nodes on x axis
ynode = ylength/h + 3; % Number of Nodes on y axis
T = zeros(ynode,xnode); % Final Preallocated Array for Temperature
%% SOR Factor Calculations
syms w
p = xlength/h; % x size divided by step size
q = ylength/h; % y size divided by step size
F = (((cos(pi/q))+(cos(pi/p)))^2)*w^2-16*w+16==0; % SOR factor equation
w = double(vpasolve(F,w)); % Solution of the equation
for i=1:1:length(w)
if 0<w(i) && w(i)<2 % SOR factor changes between 0 and 2
omega=w(i);
end
end
%% Drichlet Boundary Conditions
Tleft = 20; % Left Side Temperatures
Tright = Tleft; % Right Side Temperatures
T(2:end-1,1) = Tleft;
T(2:end-1,end) = Tright;
%% SOR Iteration Calculations
for kiter = 1:100000000
a = T(2,2);
for i=2:xnode-1
% Lower Edge Points
T(end,i) = T (end-2,i)-30*h; % Lower Imaginary Line
T(end-1,i) = T(end-1,i) + ...
    omega*((T(end-1,i-1)+T(end,i)+T(end-1,i+1)+T(end-2,i)-4*T(end-1,i))/4 + ...
    (Q*h^2)/(4*k*tau)); % Lower Boundary Line
% Upper Edge Points
T(1,i) = T(3,i) - 2*h*H/k*(T(2,i)-T0); % Upper Imaginary Line
```

```
T(2,i) = T(2,i) + omega*((T(2,i-1)+T(3,i)+T(2,i+1)+T(1,i)-4*T(2,i))/4 ...
    +(Q*h^2)/(4*k*tau)); % Upper Boundary Line
% Interior Points
for j = 2:ynode-1
T(j,i) = T(j,i) + omega*((T(j,i-1)+T(j-1,i)+T(j,i+1)+T(j+1,i)-4*T(j,i))/4 + ...
    (Q*h^2)/(4*k*tau));
end
end
b = T(2,2);
err = abs((a-b)/b)*100;
if err < 10^-6
break
end
end
%% Heat Map Plotting
% Temperature Distribution with Imaginary Nodes
figure(1)
Timaginary = T;
heatmap(T,'colormap',turbo);
title('Temperature Distribution with Imaginary Nodes');
xlabel('x = 9 cm');
ylabel('y = 5 cm');
% Temperature Distribution without Imaginary Nodes
figure(2)
T = T(2:end-1,:);
heatmap(T,'colormap',turbo);
title('Temperature Distribution without Imaginary Nodes');
xlabel('x = 9 cm');
ylabel('y = 5 cm');
```

# 5 References

[1] Gerald C. F. & Wheatley P. O. (2003). *Applied Numerical Analysis*. 7th ed. Addison-Wesley.

[2] Çengel, Y. A., Ghajar, A. J. *Heat and Mass Trasfer*, 5th Ed., McGraw-Hill, 2015.