Calculation of Friction Factor through Colebrook Equation with using Newton-Raphson Method for Re = 1e6 Flow with varying Relative Roughness Values

Heat Transfer
Project Report

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Contents

1 Introduction

This report describes how a code was written to find the friction coefficient, f,with the Colebrook equation using Newton-Raphson iteration method, based on the relative roughness, ϵ/D,values given in the table below. The flow is assumed to have Reynold number of 10^6 which is turbulent flow.

Relative Roughness ϵ/D
0.0
0.0001
0.0005
0.001
0.005
0.01
0.05

2 Mathematical Background

Colebrook equation for turbulent flow can be defined as

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \tag{2.1}$$

Colebrook equation can be rewritten as a function in terms of friction coefficient, f

$$\mathcal{F}(f) = \frac{1}{\sqrt{f}} + 2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \tag{2.2}$$

Derivative of the function can be determined as

$$\mathcal{F}'(f) = -\frac{1}{2}\frac{1}{(\sqrt{f})^3} + \frac{0.868}{\dfrac{\epsilon/D}{3.7} + \dfrac{2.51}{Re\sqrt{f}}}\left(-\frac{2.51}{2Re(\sqrt{f})^3}\right) \tag{2.3}$$

$$\mathcal{F}'(f) = -\frac{1}{(\sqrt{f})^3}\left[\frac{1}{2} + \frac{1.09008}{Re\left(\dfrac{\epsilon/D}{3.7} + \dfrac{2.51}{Re\sqrt{f}}\right)}\right] \tag{2.4}$$

Newton-Raphson method for the given problem can be constructed generally as

$$f_{i+1} = f_i - \frac{\mathcal{F}(f)}{\mathcal{F}'(f)} \tag{2.5}$$

Before initiating the Newton-Raphson iteration, an initial value must be determined for friction factor, f. There are approximation to the Colebrook/Moody chart relationship that does not require an iterative scheme for calculating friction factor, f. For this project, an alternate form by Haaland[3], which is easier to use, is chosen and given by,

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right] \tag{2.6}$$

We can rearrange this equation for an initial friction factor expression as,

$$f_0 \approxeq \frac{1}{3.24\left\{\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]\right\}^2} \tag{2.7}$$

$$\%\text{ error percentage} = \frac{\text{Moody chart value - Colebrook method value}}{\text{Moody chart value}} \cdot \% \tag{2.8}$$

After computations, friction factor values with respect to relative roughness values can be plotted for numerical results from Colebrook equation and values that read from Moody chart cen be plotted as

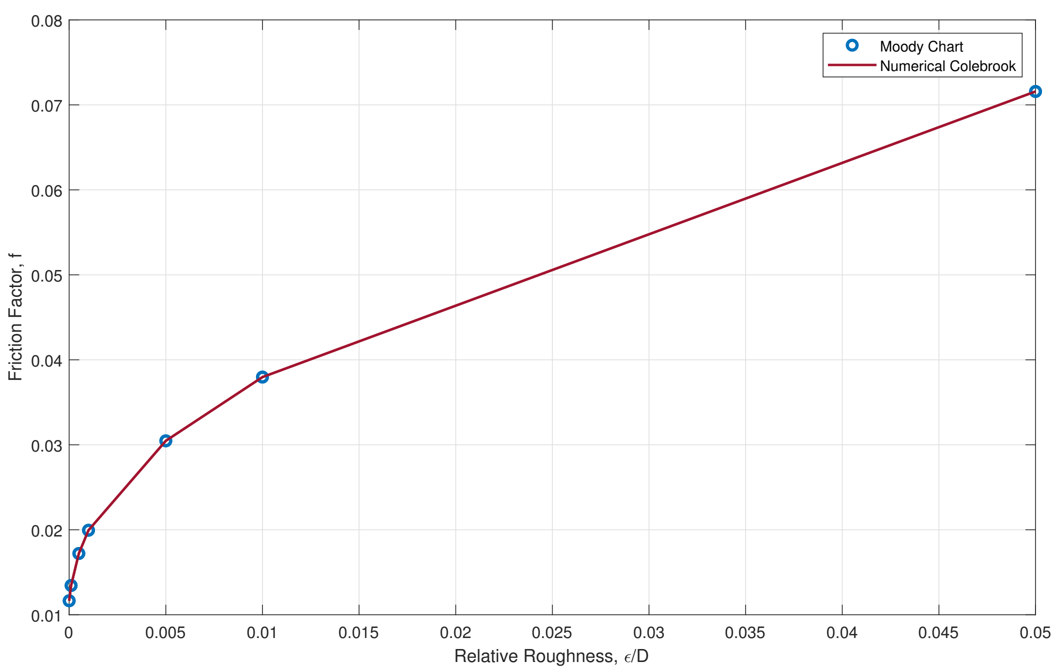

Figure 1: Friction factor values with respect to relative roughness

And the table below can be obtained from written code,

Relative roughness (ϵ/D)	**Friction factor (f)**	**Moody Chart value**	**Percentage error (%)**
0	0.011645041	0.011645041	5.59303e-09
0.0001	0.013441438	0.013441438	1.07651e-08
0.0005	0.01720673	0.01720673	1.16902e-08
0.001	0.019943466	0.019943466	1.14943e-08
0.005	0.030465026	0.030465026	1.11864e-08
0.01	0.037964742	0.037964742	1.11412e-08
0.05	0.071573754	0.071573754	1.1111e-08

Conclusion: Calculating with Colebrook equation yields friction factor values very similar to the values that read from Moody chart with small iteration numbers. Hence, Colebrook equation is a strong candidate for calculating friction factor values in numerical fluid mechanics computer programs in spite of reading from Moody chart.

3 References

[1] Çengel, Y. A., Ghajar, A. J. *Heat and Mass Trasfer*, 5th Ed., McGraw-Hill, 2015.

[2] Munson B. R. Huebsch W. W. Okiishi T. H. & Young D. F. (2012). *Fundamentals of Fluid Mechanics*, 7th edition. John Wiley & Sons.

[3] Haaland, S.E., "Simple and Explicit Formulas for the Friction-Factor in Turbulent Pipe Flow," Transactions of the ASME, Journal of Fluids Engineering, Vol. 105, 1983.

4 Appendix: MATLAB Code

```
clear;clc;close all;
%% Main Calculation Section
Re = 1e6; % Reynolds number of the flow

F = @(x,y) (1/sqrt(x)) + 2*log10(y/3.7 + 2.51/(Re*sqrt(x))); % ...
    Colebrook equation
dF = @(x,y) (-1/(sqrt(x)*sqrt(x)*sqrt(x)))*(0.5 + 1.09008/(Re*(y/3.7 + ...
    2.51/(Re*sqrt(x))))); % Derivative of Colebrook equation

ed = [0.0,0.0001,0.0005,0.001,0.005,0.01,0.05]; % Relative roughness values
```

```
md = ...
    [0.011645040997991622,0.013441437692508499,0.017206729844406813,0.019943465840476866,
    % Corresponding Friction factor values red from Moody chart

for i = 1:7
f0 = 1 / ( 3.24*(log(6.9/Re + (ed(i)/3.7)^1.11))^2); % Initial value
for j = 1:6
f0 = f0 - F(f0,ed(i))/dF(f0,ed(i)); % Newton-Raphson method
end
f0array(i)=f0;
error(i) = 100*(md(i)-f0array(i))/md(i); % Percentage error calculation
end

%% Plotting Section
figure(1)
plot(ed,md,'o','Color',[0, 0.4470, 0.7410],'LineWidth',2);
hold on;
plot(ed,f0array,'Color',[0.6350, 0.0780, 0.1840],'LineWidth',1.5);
xlabel('Relative Roughness, \epsilon/D');
ylabel('Frictin Factor, f');
legend('Moody Chart','Numerical Colebrook')
grid on;

%% Table Output Section
fg = uifigure;
uit = uitable(fg);
d = {ed(1),f0array(1),md(1),error(1);
    ed(2),f0array(2),md(2),error(2);
    ed(3),f0array(3),md(3),error(3);
    ed(4),f0array(4),md(4),error(4);
    ed(5),f0array(5),md(5),error(5);
    ed(6),f0array(6),md(6),error(6);
    ed(7),f0array(7),md(7),error(7);
};
uit.Data = d;
uit.Position = [20 20 520 400];
uit.ColumnName = {'Relative roughness ($\epsilon/D$)','Friction factor ...
    ($f$)','Moody Chart value','Percentage error (%)'};
```