Performance Calculations in Hover Case

Helicopter Theory
Project Report

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1 Introduction

In this project, performance calculations for hover case will be performed with variation with respect to density altitude. Density altitude variation is given as 1 km, 3 km and 5 km for this project. Input values are taken from datasheets Aerospatiale AS 365N helicopter.

1.1 Input Values

Input Type	Symbol	Unit	Value
Density at sea level	ρ_{SL}	kg/m^3	1.225
Engine power at sea level	P_{SL}	kW	2×530=1060
Radius of main rotor	R	m	5.965
Actuator disk area	S	m^2	111.782
Induced power coefficient	$\mathcal{K}$		1.15
Gross takeoff weight	GTOW	kg	4000
Main rotor rotational speed	Ω	rad/sec	31.416
Chord of the main blade	c	m	0.405
Number of main rotor blades	N_b		4
Solidity of main rotor	σ		0.08645
Distance of tail rotor from center of the main rotor	$X_{T/R}$	m	11.63
Radius of tail rotor	$R_{T/R}$	m	0.55
Tail rotor disk area	$S_{T/R}$	m^2	0.9503
Rotational speed of tail rotor	$\Omega_{T/R}$	rad/sec	1500
Number of tail rotor blades	$N_{b,T/R}$		8
Soliditiy of tail rotor blades	$\sigma_{T/R}$		1.505
Drag coefficient	C_{d_0}		0.0012
Gravitational acceleration	g	m/s^2	9.81

2 Performance Calculations in Hover Case

For hover case, $T = W$

2.1 Air Density

Density altitude is defined as

$$h_\rho = \frac{518.4}{0.00357}\left[1 - \left(\frac{\rho}{\rho_0}\right)^{0.235}\right] \cdot 0.3048 \tag{2.1}$$

And if we rearrange the Eq.[2.1], we can determine density in terms of density altitude as

$$\rho = [1 - 0.000022594 h_\rho]^{\dfrac{1}{0.235}} \rho_0 \tag{2.2}$$

2.2 Available Engine Power

$$P_{available} = P_{SL}\frac{\rho}{\rho_0} \tag{2.1}$$

2.3 Required Power and Power Coefficient of the Main Rotor

Induced power in hover case for main rotor is

$$P_i = \frac{\mathcal{K} W^{1.5}}{\sqrt{2\rho A}} \tag{2.1}$$

Profile power in hover case for main rotor is

$$P_0 = \rho A \left(\Omega R\right)^3 \frac{\sigma C_{d_0}}{8} \tag{2.2}$$

$$C_P = \frac{P_i + P_0}{\rho S \left(\Omega R\right)^3} \tag{2.3}$$

2.4 Tail Rotor Thrust

In order to overcome the torque produced by main rotor, tail rotor must also produce same amount of torque as

$$Q_{T/R} = Q_{main} = \frac{P_{main}}{\Omega} \tag{2.1}$$

And thrust produced by tail rotor can be determined as

$$T_{T/R} = \frac{Q_{T/R}}{X_{T/R}} \tag{2.2}$$

2.5 Required Power and Power Coefficient of Tail Rotor

Induced power in hover case for tail rotor is

$$P_{i,T/R} = \frac{\mathcal{K} T_{T/R}^{1.5}}{\sqrt{2\rho A_{T/R}}} \tag{2.1}$$

Profile power in hover case for tail rotor is

$$P_{0,T/R} = \rho A_{T/R} \left(\Omega R_{T/R}\right)^3 \frac{\sigma C_{d_0}}{8} \tag{2.2}$$

$$C_{P,T/R} = \frac{P_{i,T/R} + P_{0,T/R}}{\rho S_{T/R} \left(\Omega_{T/R} R_{T/R}\right)^3} \tag{2.3}$$

2.6 Excess Power

Excess power is defined as

$$\Delta P = P_{available} - \left(P_{main} + P_{T/R}\right) \tag{2.1}$$

2.7 Figure Of Merit

Figure of merit is defined as

$$FM = \frac{P_{ideal}}{P_h} = \frac{\dfrac{T}{\rho \left(\Omega R\right)^2 A}}{P_{main} + P_{T/R}} \tag{2.1}$$

3 Results and Conclusion

Table 1: Effect of Density Altitude

Density Altitude $[km]$	**1 km**	**3 km**	**3 km**
Air density $[km/m^3]$	1.1115	0.9087	0.7355
Available power $[kW]$	961.7724	786.3113	636.4575
Required induced power of main rotor $[kW]$	567.0739	627.1603	697.0937
Required profile power of main rotor $[kW]$	10.6023	8.6681	7.0161
Required induced power of tail rotor $[kW]$	49.7424	63.5255	82.2833
Required profile power of tail rotor $[kW]$	0.1537	0.1257	0.1017
Excess power $[kW]$	334.2000	86.8317	-150.0374
Figure of merit	0.8536	0.8577	0.8609

Conclusion:

- At sea level air density equals 1.225 kg/m^3 as the height increases, temperature and pressure decreases, so air density decreases.
- Helicopters have turboshaft engines. The gas turbine engines breath air and by combusting it with fuel, they produce power. When the density of air which pushed by engine decrease, the total thrust decreaese. As the helicopter goes higher, the power produced by engine decreases.
- As the density decreases, the thrust produced by main rotor decrease but helicopter weight is same. Hence for maintaining hover case, the helicopter will need to procude more power. So required induced power for main rotor increases.
- When the density decreases, required profile power decrease because the drag effect which rotor system need to overcome will be lesser. So required profile power decrease by as the height of the helicopter increases.
- For the tail rotor the explanations are same, total power consumption increase as the height of the helicopter increases. Tail rotor overcomes the torque produced by main rotor.
- As the height increases total power generated by engine decreases and total power consumption increases. Therefore excess power decreases because excess power is difference of these. Excess power is maximum at sea level and excess power is zero at maximum altitude.
- At 5 km density altitude, excess power is seen as negative which means this densitiy altitude represent an altitude higher than service ceiling of the helicopter

4 Appendix: MATLAB Code

```
clc; clear; close all;
g = 9.81; % Gravitional Acceleation [m/sec^2]
rho_SL = 1.225; % Density of air at sea level [kg/m^3]
alt = [1000,3000,5000];% Operating density altitude values [m]
P_SL = 2*530*10^3; % Engine power at sea level [W]
R = 11.93/2;% Main rotor radius [m]
S = pi*R^2;% Actuator disk area [m^2]
K = 1.15;% Induced power coefficient
W = 4000*9.81;% Weight of helicopter [N]
T = W;% Thrust of main rotor in hover case [N]
omega = 300*(1/60)*(2*pi);% Main Rotor Angular Speed [rad/sec]
Cd0 = 0.0012;% Drag coefficient
c = 0.405;% Chord of main rotor blades [m]
Nb = 4;% Number of main rotor blades
solidity = Nb*c/(pi*R);% Solidity of main rotor
X_tr = 11.63; % Distance of tail rotor from center of the main rotor [m]
R_tr = 1.1/2;% Radius of tail rotor blades [m]
S_tr = pi*R_tr^2;% Tail rotor disk area [m^2]
omega_tr = 1500*(1/60)*(2*pi); % Angular velocity of tail rotor [rad/sec]
c_tr = 0.2;% Chord of tail rotor blades [m]
Nb_tr = 13;% Number of tail rotor blades
solidity_t = Nb_tr*c_tr/(pi*R_tr);% Solidity of tail rotor blades
%% Part (a)
A = alt*((0.00357)/(0.3048*518.4));
rho = ((1-A).^(1/0.235))*rho_SL;
%% Part (b)
P = P_SL.*((1-A).^(1/0.235));
%% Part (c)
P_main_i = (K.*T.^1.5)./(sqrt(2.*rho*S));
P_main_0 = S.*rho.*((omega.*R)^3).*solidity.*Cd0/8;
P_main_h = P_main_i + P_main_0;
C_p_main = P_main_h./(rho.*S.*((omega*R)^3));
%% Part (d)
Q_main_h = P_main_h./omega;
T_tr = Q_main_h./X_tr;
%% Part (e)
P_tail_i = (K*T_tr.^1.5)./(sqrt(2.*rho*S_tr));
P_tail_0 = S_tr.*rho.*((omega_tr*R_tr)^3)*solidity_t*Cd0/8;
P_tail_h = P_tail_i + P_tail_0 ;
```

```
C_p_tr = P_tail_h./(rho.*S_tr.*((omega_tr*R_tr)^3));
%% Part (f)
∆_p = P - (P_main_h+P_tail_h); % Excess power at each altitude
%% Part (g):
C_t =T./(rho*S*(omega*R)^2);
Cpi = C_t.^(3/2)./sqrt(2);
FM  = Cpi./(K*Cpi + (solidity*Cd0/8)); % Calculating the Figure of ...
    Merit (FM) at each altitude
%% Part (h)
fig = uifigure;
table = uitable(fig);
d = {'Air density [kg/m^3]',rho(1),rho(2),rho(3);
    'Available power [kW]',P(1)*10^-3,P(2)*10^-3,P(3)*10^-3;
    'Required induced power of main rotor ...
        [kW]',P_main_i(1)*10^-3,P_main_i(2)*10^-3,P_main_i(3)*10^-3;
    'Required profile power of main rotor ...
        [kW]',P_main_0(1)*10^-3,P_main_0(2)*10^-3,P_main_0(3)*10^-3;
    'Required induced power of tail rotor ...
        [kW]',P_tail_i(1)*10^-3,P_tail_i(2)*10^-3,P_tail_i(3)*10^-3;
    'Required profile power of tail rotor ...
        [kW]',P_tail_0(1)*10^-3,P_tail_0(2)*10^-3,P_tail_0(3)*10^-3;
    'Excess power [kW]',∆_p(1)*10^-3,∆_p(2)*10^-3,∆_p(3)*10^-3;
    'Figure of merit',FM(1),FM(2),FM(3);};
table.Data = d;
table.ColumnName = {'Altitude','1 km','3 km','5 km'};
table.Position = [20 20 520 400];
```

5 References

[1] Leishman G., *Principles of Helicopter Aerodynamics*, 2nd Ed., Cambridge University Press, 2006.

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[3] Type-Certificate Data Sheet for SA 365 / AS 365 / EC 155, No. EASA.R.105, 07, European Aviation Safety Agency, 2021

[4] Özdemir Ö., *Istanbul Technical University UCK459E - Helicopter Theory Lecture Notes*, 2022.