Comparison of Radionavigation Methods

Avionics Systems
Project Report

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Contents

1 Introduction

In this homework, position and distance of aircraft will be determined using methods of range measurement, range difference and sum of range difference. Initial values for the given methods are presented below:

- Measured distances from ground station to aircraft:
  - $D_1 = 63,4 + n \times 0.1\ km$
  - $D_2 = 69,2 + n \times 0.1\ km$
  - $D_3 = 44,8 + n \times 0.1\ km$

  where $n = 15$ from the student sequence number

- Coordinates of ground stations
  - Ground station 1: $O_1(100.8, 1.1, 30.3)\ km$
  - Ground station 2: $O_2(-20.2, 0.5, 19.8)\ km$
  - Ground station 3: $O_3(40.4, 0.86, -38.5)\ km$

2 Theoretical Background

2.1 Range Measurement Method

In the range measurement method, the position surface is a sphere with radius D which is the range from the station to the aircraft. Thus, an position of aircraft in a space can be found by intersecting three D-constant position surfaces.

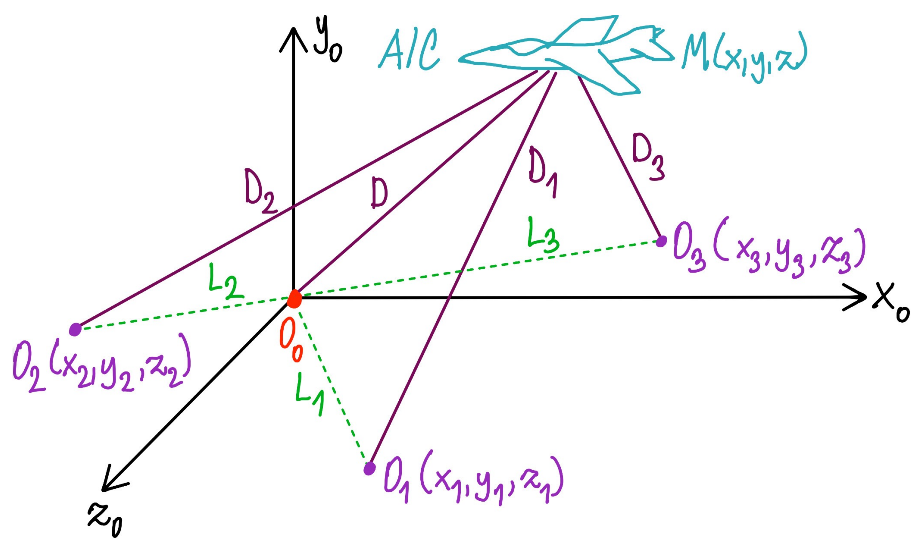

Figure 1: Nomenclature scheme of determination of position of the aircraft

To find the position of aircraft, three measurement stations which measure distance are located on the ground.. To find position parameters of aircraft, $M(x, y, z)$, let us consider that the ground stations are located on the points of $O_1(x_1, y_1, z_1)$, $O_2(x_2, y_2, z_2)$, and $O_3(x_3, y_3, z_3)$. L_1, L_2 and L_3 show the distance between the positions of ground stations and the origin of $O_0X_0Y_0Z_0$ coordinate system and D_1, D_2, and D_3 show the distances to the aircraft at $M(x, y, z)$ point. So, Eq.[2.1.1,2.1.2,2.1.3] can be written for ground station 1, $O_1(x_1, y_1, z_1)$. Distance between aircraft and ground station 1

$$D_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} \tag{2.1.1}$$

Distance between origin and ground station 1

$$L_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} \tag{2.1.2}$$

Distance between aircraft and origin of coordinate system

$$D = \sqrt{x^2 + y^2 + z^2} \tag{2.1.3}$$

with similar analogy, D_2,D_3,L_2 and L_3can be calculated.
To find coordinates of aircraft, $M(x, y, z)$, we need to express D in terms of D_1, D_2, D_3 and L_1, L_2, L_3. With using these expressions, we can write

$$D_1 = \sqrt{L_1^2 + D^2 - 2(x_1x + y_1y + z_1z)} \tag{2.1.4}$$

with an appropriate change of indices, we can obtain expressions for D_2 and D_3 as

$$D_2 = \sqrt{L_2^2 + D^2 - 2(x_2x + y_2y + z_2z)} \tag{2.1.5}$$

$$D_3 = \sqrt{L_3^2 + D^2 - 2(x_3x + y_3y + z_3z)} \tag{2.1.6}$$

Using differences $D_1^2 - D_2^2$, $D_1^2 - D_3^2$ and $D_2^2 - D_3^2$ will lead to the x,y and z formulas. The equations for the coordinate determination can be written as

$$(x_1 - x_2)x + (y_1 - y_2)y + (z_1 - z_2)z = c_1 \tag{2.1.7}$$

$$(x_1 - x_3)x + (y_1 - y_3)y + (z_1 - z_3)z = c_2 \tag{2.1.8}$$

$$(x_2 - x_3)x + (y_2 - y_3)y + (z_2 - z_3)z = c_3 \tag{2.1.9}$$

Here,

$$c_1 = \frac{1}{2}(D_2^2 - D_1^2 + L_1^2 - L_2^2) \tag{2.1.10}$$

$$c_2 = \frac{1}{2}(D_3^2 - D_1^2 + L_1^2 - L_3^2) \tag{2.1.11}$$

$$c_3 = \frac{1}{2}(D_3^2 - D_2^2 + L_2^2 - L_3^2) \tag{2.1.12}$$

The solution of Eq.[2.1.7,2.1.8,2.1.9] for aircraft coordinate determination can be found as follows

$$x = e_1 - ay \tag{2.1.13}$$

$$z = e_2 - by \tag{2.1.14}$$

where

$$e_1 = \frac{c_1(z_2 - z_3) - c_3(z_1 - z_2)}{(x_1 - x_2)(z_2 - z_3) - (x_2 - x_3)(z_1 - z_2)} \tag{2.1.15}$$

$$e_2 = \frac{c_1(x_1 - x_3) - c_2(x_1 - x_2)}{(x_1 - x_3)(z_1 - z_2) - (x_1 - x_2)(z_1 - z_3)} \tag{2.1.16}$$

$$a = \frac{(y_1 - y_2)(z_2 - z_3) - (y_2 - y_3)(z_1 - z_2)}{(x_1 - x_2)(z_2 - z_3) - (x_2 - x_3)(z_1 - z_2)} \tag{2.1.17}$$

$$b = \frac{(y_1 - y_2)(x_1 - x_3) - (y_1 - y_3)(x_1 - x_2)}{(x_1 - x_3)(z_1 - z_2) - (x_1 - x_2)(z_1 - z_3)} \tag{2.1.18}$$

For determine y-coordinate, Eq.[2.1.19] must be solved.

$$(a^2+b^2+1)y^2+2(x_1a-y_1+z_1b-ae_1-be_2)y-(D_1^2-L_1^2-e_1^2+2x_1e_1-e_2^2+2z_1e_2) = 0 \tag{2.1.19}$$

Eq.[2.1.19] yields two solution which one is positive and one is negative. Since aircraft cruises in air, positive y value should be taken as result. After obtaining value of all required variables, distance between origin and aircraft, D, is calculated via the Eq.[2.1.3]

2.2 Range Difference Method

In range difference measurement, radio-navigation systems, the distances from two ground stations to aircraft are taken as navigation parameters. Measurement of ΔD distance differences allow to determine that the aircraft locates on $\Delta D = constant$ state surface. This surface is in the form of rotating hyperboloids of the two-leaf stage.

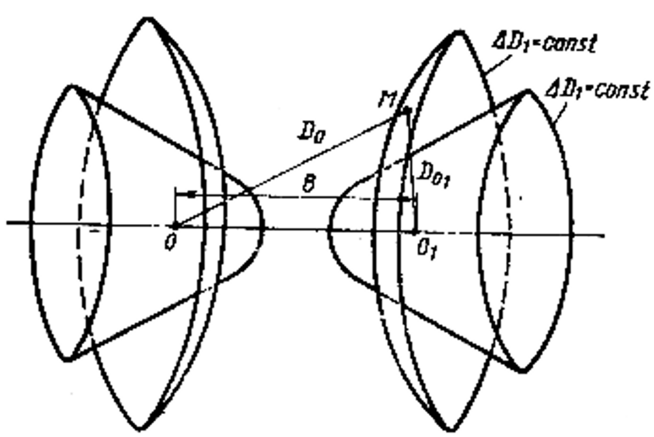

Figure 2: State surfaces of range difference method

To find the position of aircraft, three measurement stations which measure distance are located on the ground. To find position parameters of aircraft, $M(x, y, z)$, let us consider that the main ground station is located on $O_0(x_0, y_0, z_0)$ and assistant ground stations are located on the points of $O_1(x_1, y_1, z_1)$, $O_2(x_2, y_2, z_2)$, and $O_3(x_3, y_3, z_3)$. L_1, L_2 and L_3 show the distance between the positions of ground stations and the origin of $O_0X_0Y_0Z_0$ coordinate system and D_1, D_2, and D_3 show the distances to the aircraft at $M(x, y, z)$ point. The introduced nomenclature can be seen in Fig.[1].

The position of aircraft can be determined as the intersection point of three state surfaces. Therefore, the range difference measurement radio navigation system includes four ground stations. One of these stations is the main station and the others are assistant stations. With the help of signals sent by the main station, three assistant stations are provided to work synchronously. With the range difference method, we derive the following expressions in order to obtain position of an aircraft

$$x_1x + y_1y + z_1z - D\Delta D_1 = f_1 \tag{2.2.1}$$

$$x_2x + y_2y + z_2z - D\Delta D_2 = f_2 \tag{2.2.2}$$

$$x_3x + y_3y + z_3z - D\Delta D_3 = f_3 \tag{2.2.3}$$

where the range differences are defined as

$$\Delta D_1 = D - D_1; \tag{2.2.4}$$

$$\Delta D_2 = D - D_2; \tag{2.2.5}$$

$$\Delta D_3 = D - D_3; \tag{2.2.6}$$

and assistant variables of f_1, f_2 and f_3 are determined as

$$f_1 = \frac{1}{2}(L_1^2 - \Delta D_1^2) \tag{2.2.7}$$

$$f_2 = \frac{1}{2}(L_2^2 - \Delta D_2^2) \tag{2.2.8}$$

$$f_3 = \frac{1}{2}(L_3^2 - \Delta D_3^2) \tag{2.2.9}$$

Via solving these equations, position elements of aircraft can be determined as,

$$x = \frac{1}{2}\left[f_1\left(z_3\Delta D_2 - z_2\Delta D_3\right) - z_1\left(f_3\Delta D_2 - f_2\Delta D_3\right) - \Delta D_1\left(f_2z_3 - f_3z_2\right)\right] \tag{2.2.10}$$

$$z = \frac{1}{A}\left[x_1\left(f_3\Delta D_2 - f_2\Delta D_3\right) - f_1\left(x_3\Delta D_2 - x_2\Delta D_3\right) - \Delta D_1\left(x_2f_3 - x_3f_2\right)\right] \tag{2.2.11}$$

$$y = \sqrt{D^2 - x^2 - z^2} \tag{2.2.12}$$

where

$$A = x_1(z_3\Delta D_2 - z_2\Delta D_3) - z_1(x_3\Delta D_2 - x_2\Delta D_3) - \Delta D_1(x_2z_3 - x_3z_2) \tag{2.2.13}$$

$$D = \frac{1}{A}\left[x_1(z_2f_3 - z_3f_2) - z_1(x_2f_3 - x_3f_2) + f_1(x_2z_3 - x_3z_2)\right] \tag{2.2.14}$$

2.3 Sum of Difference Method

In sum of range difference method, sum of the ranges from ground stations to the aircraft are taken as navigation parameters. Rotating ellipsoid will be the state surface for sum of range difference method. Ellipsoid has a feature that the sum of ranges to centers of the ellipsoid and any point on ellipsoid surface remains constant.

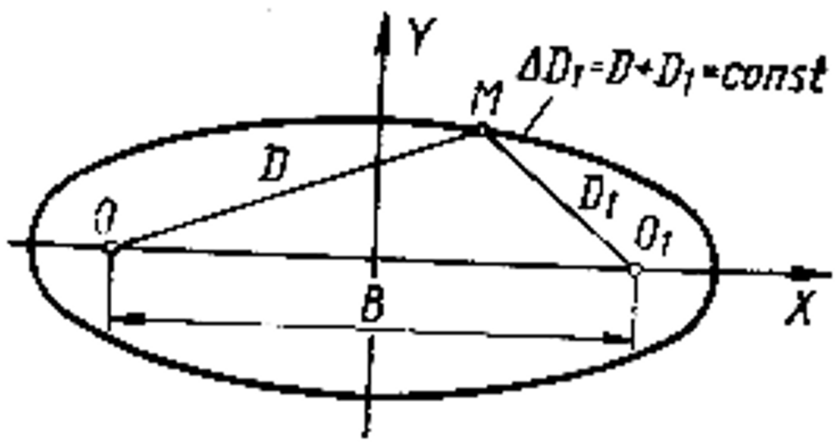

Figure 3: State surfaces of sum of range differences method

To find the position of aircraft, three measurement stations which measure distance are located on the ground. To find position parameters of aircraft, $M(x, y, z)$, let us consider that the main ground station is located on $O_0(x_0, y_0, z_0)$ and assistant ground stations are located on the points of $O_1(x_1, y_1, z_1)$, $O_2(x_2, y_2, z_2)$, and $O_3(x_3, y_3, z_3)$. L_1, L_2 and L_3 show the distance between the positions of ground stations and the origin of $O_0X_0Y_0Z_0$ coordinate system and D_1, D_2, and D_3 show the distances to the aircraft at $M(x, y, z)$ point. The introduced nomenclature can be seen in Fig.[1].
The position of aircraft can be determined as the intersection point of three state surfaces. Therefore, the sum of range difference measurement radio navigation system includes four ground stations. One of these stations is the main station and the others are assistant stations. With the help of signals sent by the main station, three assistant stations are provided to work synchronously.

With the sum of range difference method, we derive the following expressions in order to obtain position of an aircraft

$$x_1x + y_1y + z_1z - D\Delta D_1 = f_1 \tag{2.3.1}$$

$$x_2x + y_2y + z_2z - D\Delta D_2 = f_2 \tag{2.3.2}$$

$$x_3x + y_3y + z_3z - D\Delta D_3 = f_3 \tag{2.3.3}$$

where the sum of range differences are defined as

$$\Delta D_1 = D + D_1; \tag{2.3.4}$$

$$\Delta D_2 = D + D_2; \tag{2.3.5}$$

$$\Delta D_3 = D + D_3; \tag{2.3.6}$$

and assistant variables of f_1, f_2 and f_3 are determined as

$$f_1 = \frac{1}{2}(L_1^2 - \Delta D_1^2) \tag{2.3.7}$$

$$f_2 = \frac{1}{2}(L_2^2 - \Delta D_2^2) \tag{2.3.8}$$

$$f_3 = \frac{1}{2}(L_3^2 - \Delta D_3^2) \tag{2.3.9}$$

Via solving these equations, position elements of aircraft can be determined as,

$$x = \frac{1}{2}\left[f_1\left(z_3\Delta D_2 - z_2\Delta D_3\right) - z_1\left(f_3\Delta D_2 - f_2\Delta D_3\right) - \Delta D_1\left(f_2z_3 - f_3z_2\right)\right] \tag{2.3.10}$$

$$z = \frac{1}{A}\left[x_1\left(f_3\Delta D_2 - f_2\Delta D_3\right) - f_1\left(x_3\Delta D_2 - x_2\Delta D_3\right) - \Delta D_1\left(x_2f_3 - x_3f_2\right)\right] \tag{2.3.11}$$

$$y = \sqrt{D^2 - x^2 - z^2} \tag{2.3.12}$$

where

$$A = x_1(z_3\Delta D_2 - z_2\Delta D_3) - z_1(x_3\Delta D_2 - x_2\Delta D_3) - \Delta D_1(x_2z_3 - x_3z_2) \tag{2.3.13}$$

$$D = \frac{1}{A}\left[x_1(z_2f_3 - z_3f_2) - z_1(x_2f_3 - x_3f_2) + f_1(x_2z_3 - x_3z_2)\right] \tag{2.3.14}$$

3 Results and Conclusion

For obtaining results, computer code shown in appendix is used. For every introduced method, the results found are shown in Table[1].

Table 1: Results of Position and Distance

Method		
Range Measurement Method	**Position,** $M(x, y, z)$ **[km]**	(45.36806664, 20.88165831, 2.95047059)
	Distance, D **[km]**	50.03009496
Range Differences Method	**Position,** $M(x, y, z)$ **[km]**	(45.36806664, 20.88165831, 2.95047059)
	Distance, D **[km]**	50.03009496
Sum of Range Differences Method	**Position,** $M(x, y, z)$ **[km]**	(45.36806664, 20.88165831, 2.95047059)
	Distance, D **[km]**	50.03009496

According to the results, all methods gave similar results as expected. The differences caused by computation, truncation and rounding errors. But for compare these methods further, absolute and relative error values within an simulation should be calculated.

4 Appendix: Computation Code

radioNavigation.m

```
clc;clear;close all;
syms yyy
%% Initial Values
n = 15; % student sequence number
% position of ground stations
O1 = [100.8;1.1;30.3]; % ground station 1
O2 = [-20.2;0.5;19.8]; % ground station 2
O3 = [40.4;0.86;-38.5]; % ground station 3
% distances of A/C between ground stations
D1 = 63.4 + n*0.1;
D2 = 69.2 + n*0.1;
D3 = 44.8 + n*0.1;
% position elements of ground stations
x1 = O1(1);y1 = O1(2);z1 = O1(3);
x2 = O2(1);y2 = O2(2);z2 = O2(3);
x3 = O3(1);y3 = O3(2);z3 = O3(3);
%% Theoretical Equations
% distances of ground stations from the origin
```

```
L1 = sqrt(x1^2+y1^2+z1^2);
L2 = sqrt(x2^2+y2^2+z2^2);
L3 = sqrt(x3^2+y3^2+z3^2);
% assistant variables
c1 = 0.5*(D2^2 - D1^2 + L1^2 - L2^2);
c2 = 0.5*(D3^2 - D1^2 + L1^2 - L3^2);
c3 = 0.5*(D3^2 - D2^2 + L2^2 - L3^2);
e1 = ((c1*(z2-z3))-(c3*(z1-z2))) / (((x1-x2)*(z2-z3))-((x2-x3)*(z1-z2)));
e2 = ((c1*(x1-x3))-(c2*(x1-x2))) / (((x1-x3)*(z1-z2))-((x1-x2)*(z1-z3)));
a = (((y1-y2)*(z2-z3))-((y2-y3)*(z1-z2))) / ...
    (((x1-x2)*(z2-z3))-((x2-x3)*(z1-z2)));
b = (((y1-y2)*(x1-x3))-((y1-y3)*(x1-x2))) / ...
    (((x1-x3)*(z1-z2))-((x1-x2)*(z1-z3)));
%% Position Elements of A/C
eqn = (a^2 + b^2 + 1)*yyy*yyy + 2*(x1*a - y1 + z1*b - a*e1 - b*e2)*yyy ...
    - (D1^2 - L1^2 - e1^2 + (2*x1*e1) - e2^2 + (2*z1*e2)) == 0;
yy = vpa(solve(eqn,yyy));
% y coordinate must be positive since aircraft has positive altitude ...
    (over the ground)
for i = 1:length(yy)
if yy(i) > 0
y = yy(i);
end
end
x = e1 - a*y;
z = e2 - b*y;
M = [x;y;z];
% Distance of A/C with Range Measurement Method
D = sqrt(x^2+y^2+z^2);
%% Range Differences
ΔD1R = D - D1;
ΔD2R = D - D2;
ΔD3R = D - D3;
% assistant variables
f1 = 0.5*(L1^2-ΔD1R^2);
f2 = 0.5*(L2^2-ΔD2R^2);
f3 = 0.5*(L3^2-ΔD3R^2);
A = x1*(z3*ΔD2R-z2*ΔD3R) - z1*(x3*ΔD2R-x2*ΔD3R) - ΔD1R*(x2*z3-x3*z2);
% Position of A/C with Range Difference Method
x = (1/A)*(f1*(z3*ΔD2R-z2*ΔD3R)-z1*(f3*ΔD2R-f2*ΔD3R)-ΔD1R*(f2*z3-f3*z2));
z = (1/A)*(x1*(f3*ΔD2R-f2*ΔD3R)-f1*(x3*ΔD2R-x2*ΔD3R)-ΔD1R*(x2*f3-x3*f2));
y = sqrt(D^2-x^2-z^2);
MR = [x;y;z];
```

```
DR = (1/A)*(x1*(z2*f3-z3*f2) - z1*(x2*f3-x3*f2) + f1*(x2*z3 - x3*z2));
syms xxxx yyyy zzzz
eqn1 = x1*xxxx + y1*yyyy + z1*zzzz - D*∆D1R == f1;
eqn2 = x2*xxxx + y2*yyyy + z2*zzzz - D*∆D2R == f2;
eqn3 = x3*xxxx + y3*yyyy + z3*zzzz - D*∆D3R == f3;
[x,y,z] = solve(eqn1,eqn2,eqn3,xxxx,yyyy,zzzz);
DR = sqrt(x^2+y^2+z^2);
MR = [x,y,z];
%% Sum of Range Differences
∆D1 = D + D1;
∆D2 = D + D2;
∆D3 = D + D3;
% assistant variables
f1 = 0.5*(L1^2-∆D1^2);
f2 = 0.5*(L2^2-∆D2^2);
f3 = 0.5*(L3^2-∆D3^2);
A = x1*(z3*∆D2-z2*∆D3) - z1*(x3*∆D2-x2*∆D3) - ∆D1*(x2*z3-x3*z2);
% Position of A/C with Sum of Range Difference Method
x = (1/A)*(f1*(z3*∆D2-z2*∆D3)-z1*(f3*∆D2-f2*∆D3)-∆D1*(f2*z3-f3*z2));
z = (1/A)*(x1*(f3*∆D2-f2*∆D3)-f1*(x3*∆D2-x2*∆D3)-∆D1*(x2*f3-x3*f2));
y = sqrt(D^2-x^2-z^2);
MSR = [x;y;z];
DSR = (1/A)*(x1*(z2*f3-z3*f2) - z1*(x2*f3-x3*f2) + f1*(x2*z3 - x3*z2));
syms xxxx yyyy zzzz
eqn1 = x1*xxxx + y1*yyyy + z1*zzzz - D*∆D1 == f1;
eqn2 = x2*xxxx + y2*yyyy + z2*zzzz - D*∆D2 == f2;
eqn3 = x3*xxxx + y3*yyyy + z3*zzzz - D*∆D3 == f3;
[x,y,z] = solve(eqn1,eqn2,eqn3,xxxx,yyyy,zzzz);
DSR = sqrt(x^2+y^2+z^2);
MSR = [x,y,z];
%% Printing Results
fprintf("Range Measurement Method:\n")
fprintf("Position of aircraft: \nM(%.8f,%.8f,%.8f) km\n",M(1),M(2),M(3));
fprintf("Distance of aircraft to origin:\nD = %.8f km\n\n", D);
fprintf("Range Difference Method:\n")
fprintf("Position of aircraft: \nM(%.8f,%.8f,%.8f) ...
    km\n",MR(1),MR(2),MR(3));
fprintf("Distance of aircraft to origin:\nD = %.8f km\n\n", DR);
fprintf("Sum of Range Differences Method:\n")
fprintf("Position of aircraft: \nM(%.8f,%.8f,%.8f) ...
    km\n",MSR(1),MSR(2),MSR(3));
fprintf("Distance of aircraft to origin:\nD = %.8f km\n", DSR);
```

5 References

[1] Haciyev, C. Avionic Systems Lecture Notes. Istanbul Technical University. 2023.

[2] Haciyev, C. (1999). Radyo Navigasyon (Radio Navigation). 1st edition. Istanbul Technical University. Istanbul.

[3] Haciyev, C., & Sametoglu, A. M. (2021). Accuracy in determining aircraft position by Terrestrial Radio Navigation. Accuracy in Determining Aircraft Position by Terrestrial Radio Navigation, vm02(is02), 49–56. https://doi.org/10.23890/ijast.vm02is02.0201