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Radiative electroweak effects in deep inelastic scattering of polarized leptons by polarized nucleons

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Abstract. The one-loop electroweak radiative correction of the lowest order to lepton current for deep inelastic scattering of longitudinally polarized leptons by polarized nucleons is obtained in a model-independent way. Detailed numerical analysis within the kinematical requirements of future polarization collider experiments is performed. The possibility of reducing radiative effects by detection of hard photons is studied.

1. Introduction

Deep inelastic lepton–hadron scattering, since the discovery of Bjorken scaling at the end of the 1960s, has played a crucial role in the development of our present understanding of the nature of particle interaction. The appearance of the first data on polarized deep inelastic scattering (DIS) opened a new field of experimental and theoretical investigation and as a result led to the exciting results of EMC in 1988 [1].

Naturally, the data processing of modern experiments on DIS of a polarized lepton on a polarized nuclear target requires correct account of the radiative corrections (RC). A series of works have taken RC into account in the frame of QED theory for polarization experiments (see e.g. [2–5]). Those results were for experiments on a fixed target. However polarization DIS experiments at colliders will be possible in future [6].

For the calculation of RC for such experiments we cannot restrict our consideration only to γ-exchange graphs, because here the squared transfer momentum Q^2 is so high that weak effects begin to play an essential role in the total cross section and spin asymmetries. At the same time the ratio m^2/Q^2 (where m is the mass of a scattering lepton) becomes so small that it could be restricted to non-vanishing terms for $m \to 0$. Such calculation has already been done by us [7], although only in the case of the naive parton model. In this paper we present explicit expressions for a model-independent part of the one-loop lowest-order RC (figure 1) to polarized lepton–nucleon scattering described in terms of electroweak structure functions (SF). Moreover, the target has arbitrary polarization and the lepton has a longitudinal one. The contributions appearing from additional virtual particles (V-contribution) in the on-mass renormalization scheme and t'Hooft–Feynman gauge are presented.

The results of calculations for the unitary gauge can be found in [8]. For collider experiments the only RC to difference of the cross sections with opposite proton polarizations in the longitudinal case has been studied numerically there. This RC factor is found to be in good agreement with our result. Our numerical analysis is performed for

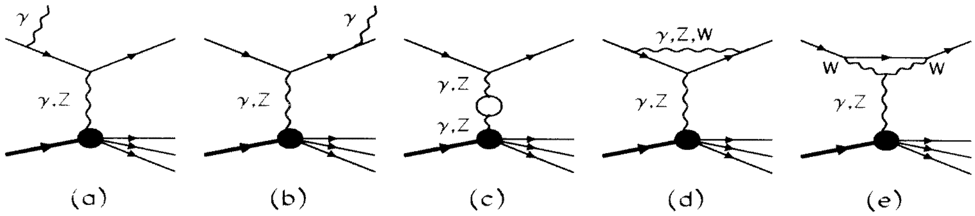

Figure 1. Full set of Feynman graphs contributing to the model-independent electroweak correction of the lowest order. All possible graphs, which give the contribution to vacuum polarization, are designated by an open circle [12].

different observables in collider experiments with and without using experimental cuts. The separation of variables in accordance with [3] allows us to write all formulae in a more compact form than in [8].

This paper is organized as follows. In the section 2 the Born contribution and all necessary kinematic invariants are presented. Section 3 is devoted to the electroweak one-loop correction. Numerical analysis of kinematical conditions for collider experiments is presented in section 4 and in section 5 we conclude.

2. Born contribution

Here we consider the deep inelastic polarized lepton–hadron scattering

$$\ell(k_1, \xi) + N(p, \eta) \to \ell(k_2) + X \tag{1}$$

in the frame of electroweak standard theory. The quantities in brackets k_1, k_2 and p, define the momenta of an initial lepton, final lepton and proton respectively ($k_1^2 = k_2^2 = m^2$, $p^2 = M^2$). ξ and η are the polarization vectors of the scattering particles.

Since the lepton is considered to be longitudinally polarized, its polarization vector has the form [3]:

$$\xi = \frac{S}{m\sqrt{\lambda_s}} k_1 - \frac{2m}{\sqrt{\lambda_s}} p = \xi_0 + \xi'. \tag{2}$$

The kinematical invariants are defined in a standard way:

$$\begin{array}{lll} S = 2k_1 p & X = 2k_2 p = (1-y)S & Q^2 = -q^2 = -(k_1 - k_2)^2 = xyS \\ S_x = S - X & S_p = S + X & \lambda_s = S^2 - 4m^2 M^2 \end{array} \tag{3}$$

where x and y are the usual scaling variables.

The double-differential cross section of lepton–nucleon scattering on the Born level ($\mathrm{d}\sigma^B/\mathrm{d}x\mathrm{d}y \equiv \sigma^B$) in the frame of electroweak theory reads

$$\sigma^B = \frac{4\pi\alpha^2 S_x S}{\lambda_s Q^4} [L_{\mu\nu}^{\gamma\gamma} W_{\mu\nu}^{\gamma}(p,q) + \tfrac{1}{2}(L_{\mu\nu}^{\gamma Z} + L_{\mu\nu}^{Z\gamma}) W_{\mu\nu}^{\gamma Z}(p,q)\chi + L_{\mu\nu}^{ZZ} W_{\mu\nu}^{Z}(p,q)\chi^2] \tag{4}$$

where $\chi = Q^2/(Q^2 + M_Z^2)$ (M_Z is the Z-boson mass). The lepton tensors $L_{\mu\nu}^{mn}$ can be given as

$$L_{\mu\nu}^{mn} = \tfrac{1}{4} Sp\gamma_\mu (v^m - a^m\gamma_5)(\hat{k}_1 + m)(1 - P_L\gamma_5\hat{\xi})\gamma_\nu(v^n - a^n\gamma_5)(\hat{k}_2 + m) \tag{5}$$

where

$$\begin{array}{ll} v^\gamma = 1 & v^Z = (-1 + 4s_w^2)/4s_w c_w \\ a^\gamma = 0 & a^Z = -1/4s_w c_w \end{array} \tag{6}$$

are the standard electroweak coupling constants, c_w and s_w are the cosine and sine of Weinberg's angle respectively and P_L is the degree of lepton polarization.

All of the hadronic tensors $W_{\mu\nu}^{\gamma,\gamma Z,Z}(p,q)$ can be expressed in terms of eight electroweak SF [9, 10]:

$$
\begin{aligned}
W_{\mu\nu}^I(p,q) = \sum_{i=1}^{8} w_{\mu\nu}^i \bar{F}_i^I &= -\tilde{g}_{\mu\nu}\bar{F}_1^I + \frac{1}{M^2}\tilde{p}_\mu\tilde{p}_\nu\bar{F}_2^I \\
&+ \mathrm{i}\epsilon_{\mu\nu\lambda\sigma}\left[\frac{p^\lambda q^\sigma}{2M^2}\bar{F}_3^I + q^\lambda\eta^\sigma\frac{1}{M}\bar{F}_4^I - q^\lambda p^\sigma\frac{\eta q}{M^3}\bar{F}_5^I\right] \\
&- \frac{1}{2M}[\tilde{p}_\mu\tilde{\eta}_\nu + \tilde{p}_\nu\tilde{\eta}_\mu]\bar{F}_6^I + \frac{\eta q}{M^3}\tilde{p}_\mu\tilde{p}_\nu\bar{F}_7^I + \frac{\eta q}{M}\tilde{g}_{\mu\nu}\bar{F}_8^I
\end{aligned}
\tag{7}
$$

where $I = \gamma, \gamma Z, Z$, $\bar{F}$ are defined as ($\epsilon = M^2/pq$)

$$
\bar{F}_1^I = F_1^I,\ \bar{F}_{2,3}^I = \epsilon F_{2,3}^I,\ \bar{F}_4^I = \epsilon P_N(g_1^I + g_2^I),\ \bar{F}_{5,7}^I = \epsilon^2 P_N g_{2,4}^I,\ \bar{F}_{6,8}^I = \epsilon P_N g_{3,5}^I \tag{8}
$$

and

$$
\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{q_\mu q_\mu}{Q^2} \qquad \tilde{p}_\nu = p_\nu + \frac{pq}{Q^2}q_\nu \qquad \tilde{\eta}_\nu = \eta_\nu + \frac{\eta q}{Q^2}q_\nu. \tag{9}
$$

Hadronic tensors in the form (7–9) correspond to the definition of electroweak SF given in (2.2.7) of [9]. For contraction of the leptonic tensors (5) with $w_{\mu\nu}^i$ we have

$$
\begin{aligned}
L_{\mu\nu}^{mn} w_{\mu\nu}^i &= \theta_i^B R_V^{mn} \qquad i = 1, 2, 6\text{–}8 \\
L_{\mu\nu}^{mn} w_{\mu\nu}^i &= \theta_i^B R_A^{mn} \qquad i = 3\text{–}5.
\end{aligned}
\tag{10}
$$

The quadratic combinations of the electroweak coupling constants are defined as

$$
\begin{aligned}
R_V^{mn} &= (v^m v^n + a^m a^n) - P_L(v^m a^n + v^n a^m) \\
R_A^{mn} &= (v^m a^n + a^m v^n) - P_L(v^m v^n + a^n a^m).
\end{aligned}
\tag{11}
$$

The quantities θ_i^B depend only on the target polarization vector and kinematical invariants:

$$
\begin{aligned}
\theta_1^B &= Q^2 & \theta_5^B &= \eta q Q^2 S_p/2M^3 \\
\theta_2^B &= (SX - M^2Q^2)/2M^2 & \theta_6^B &= -(X\eta k_1 + S\eta k_2)/2M \\
\theta_3^B &= Q^2 S_p/4M^2 & \theta_7^B &= \eta q(SX - M^2Q^2)/2M^3 \\
\theta_4^B &= -Q^2\eta(k_1 + k_2)/M & \theta_8^B &= -\eta q Q^2/M.
\end{aligned}
\tag{12}
$$

Covariant representation of the proton polarization vector both for longitudinally and transversely polarized nucleons η can be found in appendix A of [3]. It does not contain a nucleon polarization degree P_N as in SF definition (8).

Note that only the leading part of ξ (ξ_0) contributes to θ_i^B in the ultrarelativistic approximation. The second term (ξ') gives non-vanishing correction to the cross section of radiated process which is considered below.

Using the hadronic tensor (7) and the results of contractions (10) the cross section (4) can be rewritten in simple form

$$
\sigma^B = \frac{4\pi\alpha^2 y}{Q^4}\sum_{i=1}^{8}\theta_i^B \mathcal{F}_i. \tag{13}
$$

Both for the Born cross section and for RC considered in the next section the electroweak SF are gathered in eight combinations. So it is convenient to define the generalized SF:

$$
\begin{aligned}
\mathcal{F}_i &= R_V^\gamma \bar{F}_i^\gamma + \chi R_V^{\gamma Z}\bar{F}_i^{\gamma Z} + \chi^2 R_V^Z \bar{F}_i^Z \qquad i = 1, 2, 6\text{–}8 \\
\mathcal{F}_i &= R_A^\gamma \bar{F}_i^\gamma + \chi R_A^{\gamma Z}\bar{F}_i^{\gamma Z} + \chi^2 R_A^Z \bar{F}_i^Z \qquad i = 3\text{–}5.
\end{aligned}
\tag{14}
$$

3. The lowest order radiative correction

The RC of the lowest order appears as the result of one-loop effects and the process with a real photon radiation:

$$\ell(k_1, \xi) + N(p, \eta) \to \ell(k_2) + \gamma(k) + X. \tag{15}$$

The cross section of the radiated process ($\mathrm{d}\sigma^R/\mathrm{d}x\mathrm{d}y \equiv \sigma^R$) can be presented as the sum of two parts:

$$\sigma^R = \bar{\sigma}^R + \hat{\sigma}^R. \tag{16}$$

The first ($\bar{\sigma}^R$) is a part of the cross section independent of the leptonic polarization vector (2) and the contribution of its leading term ξ_0. The second term ($\hat{\sigma}^R$) comes from ξ' only. We can rewrite them in terms of leptonic and hadronic tensors:

$$\begin{aligned}\bar{\sigma}^R = -\frac{\alpha^3 S_x S}{\pi\lambda_s}\int \frac{\mathrm{d}^3k}{k_0}\frac{1}{Q_h^4}[&\bar{\mathcal{L}}^{\gamma\gamma}_{\mu\nu}W^{\gamma}_{\mu\nu}(p,q_h) + \tfrac{1}{2}(\bar{\mathcal{L}}^{\gamma Z}_{\mu\nu} + \bar{\mathcal{L}}^{Z\gamma}_{\mu\nu})W^{\gamma Z}_{\mu\nu}(p,q_h)\chi_h \\ &+\bar{\mathcal{L}}^{Z}_{\mu\nu}W^{Z}_{\mu\nu}(p,q_h)\chi_h^2]\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{\sigma}^R = -\frac{\alpha^3 S_x S}{\pi\lambda_s}\int \frac{\mathrm{d}^3k}{k_0}\frac{1}{Q_h^4}[&\hat{\mathcal{L}}^{\gamma\gamma}_{\mu\nu}W^{\gamma}_{\mu\nu}(p,q_h) + \tfrac{1}{2}(\hat{\mathcal{L}}^{\gamma Z}_{\mu\nu} + \hat{\mathcal{L}}^{Z\gamma}_{\mu\nu})W^{\gamma Z}_{\mu\nu}(p,q_h)\chi_h \\ &+\hat{\mathcal{L}}^{Z}_{\mu\nu}W^{Z}_{\mu\nu}(p,q_h)\chi_h^2].\end{aligned} \tag{18}$$

Here $\chi_h = Q_h^2/(Q_h^2 + M_Z^2)$ and $Q_h^2 = -q_h^2 = -(k_1 - k - k_2)^2$ are variables dependent on the momentum of a real photon k.

The hadronic tensors $W^{\gamma,\gamma Z,Z}_{\mu\nu}(p,q_h)$ are defined by (7). The leptonic tensors in (17) include spin-averaged and leading spin-dependent parts

$$\bar{\mathcal{L}}^{mn}_{\mu\nu} = \tfrac{1}{4}Sp\Gamma^m_{\mu\alpha}(\hat{k}_1 + m)(1 - P_L\gamma_5\hat{\xi}_0)\bar{\Gamma}^n_{\alpha\nu}(\hat{k}_2 + m) \tag{19}$$

and $\hat{\sigma}^R$ (18) comes from the contribution of ξ' only:

$$\hat{\mathcal{L}}^{mn}_{\mu\nu} = P_L\tfrac{1}{4}Sp\Gamma^m_{\mu\alpha}(\hat{k}_1 + m)\gamma_5\hat{\xi}'\bar{\Gamma}^n_{\alpha\nu}(\hat{k}_2 + m) \tag{20}$$

where

$$\begin{aligned}\Gamma^m_{\mu\alpha} &= \left[\left(\frac{k_{1\alpha}}{kk_1} - \frac{k_{2\alpha}}{kk_2}\right)\gamma_\mu - \frac{\gamma_\mu\hat{k}\gamma_\alpha}{2kk_1} - \frac{\gamma_\alpha\hat{k}\gamma_\mu}{2kk_2}\right](v^m - a^m\gamma_5) \\ \bar{\Gamma}^n_{\alpha\nu} &= \left[\left(\frac{k_{1\alpha}}{kk_1} - \frac{k_{2\alpha}}{kk_2}\right)\gamma_\nu - \frac{\gamma_\alpha\hat{k}\gamma_\nu}{2kk_1} - \frac{\gamma_\nu\hat{k}\gamma_\alpha}{2kk_2}\right](v^n - a^n\gamma_5).\end{aligned} \tag{21}$$

The results of the contraction of these tensors with $w^i_{\mu\nu}$ (see (7)) can be presented using notations of [3, 11]:

$$\begin{aligned}\frac{1}{\pi}\int \frac{\mathrm{d}^3k}{k_0}\bar{\mathcal{L}}^{mn}_{\mu\nu}w^i_{\mu\nu} &= R^{mn}_V \int \mathrm{d}R\,\mathrm{d}\tau \sum_{j=1}^{k_i} R^{j-2}\theta_{ij}(\tau) \qquad i = 1, 2, 6\text{–}8 \\ \frac{1}{\pi}\int \frac{\mathrm{d}^3k}{k_0}\bar{\mathcal{L}}^{mn}_{\mu\nu}w^i_{\mu\nu} &= R^{mn}_A \int \mathrm{d}R\,\mathrm{d}\tau \sum_{j=1}^{k_i} R^{j-2}\theta_{ij}(\tau) \qquad i = 3\text{–}5\end{aligned} \tag{22}$$

where j runs from 1 to $k_i = (3, 3, 4, 4, 5, 3, 4, 4)$ and the quantities $\theta_{ij}(\tau)$ are independent of R. Arguments of SF x and Q^2 in the case of a radiated process acquire dependence on two photonic variables $R = 2kp$ and $\tau = 2kq_h/R$:

$$Q^2 \to Q^2 + R\tau \qquad x \to \frac{Q^2 + R\tau}{S_x - R}. \tag{23}$$

Integration over the third photonic variable is performed analytically. Then the cross section $\bar{\sigma}^R$ can be obtained in the form

$$\bar{\sigma}^R = -\alpha^3 y \int_{\tau_{\min}}^{\tau_{\max}} \mathrm{d}\tau \sum_{i=1}^{8} \sum_{j=1}^{k_i} \theta_{ij}(\tau) \int_0^{R_{\max}} \mathrm{d}R \frac{R^{j-2}}{(Q^2 + R\tau)^2} \mathcal{F}_i(R,\tau) \tag{24}$$

where integration limits are:

$$\tau_{\max,\min} = \frac{S_x \pm \sqrt{S_x^2 + 4M^2Q^2}}{2M^2} \qquad R_{\max} = \frac{W^2 - (M + m_\pi)^2}{1+\tau}. \tag{25}$$

Here m_π is the pion mass and $W^2 = S_x - Q^2 + M^2$ is the squared mass of final hadrons. Summing up over $i = 1, \ldots, 8$ corresponds to the contribution of the generalized SF $\mathcal{F}_i(R,\tau)$ defined by (14) with the replacement of arguments (23). The infrared divergence occurs in the integral for $R \to 0$ in the term where $j = 1$ (and only in it). Explicit expressions for $\theta_{ij}(\tau)$ are given in the appendix.

The cross section $\hat{\sigma}^R$ can be found in the following way. For the contraction we have

$$\begin{aligned} &\frac{1}{\pi}\int \frac{\mathrm{d}^3k}{k_0} \hat{\mathcal{L}}^{mn}_{\mu\nu} w^i_{\mu\nu} = P_L(v^n a^m + a^n v^m) \int \mathrm{d}R\,\mathrm{d}\tau\, R\hat{\theta}_i(R,\tau) \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} \qquad i = 1, 2, 6\text{–}8 \\ &\frac{1}{\pi}\int \frac{\mathrm{d}^3k}{k_0} \hat{\mathcal{L}}^{mn}_{\mu\nu} w^i_{\mu\nu} = P_L(v^n v^m + a^n a^m) \int \mathrm{d}R\,\mathrm{d}\tau\, R\hat{\theta}_i(R,\tau) \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} \qquad i = 3\text{–}5 \end{aligned} \tag{26}$$

where $C_1(\tau)$ and $B_1(\tau)$ are given in the appendix (49). The integrand of (26) has a peak coming from the region $\tau \sim \tau_s \equiv -Q^2/S$, where $C_1(\tau) \sim m^2$. In the ultrarelativistic approximation the integration over τ can be carried out analytically

$$\int_{\tau_{\min}}^{\tau_{\max}} \mathrm{d}\tau \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} \mathcal{G}(\tau) = \mathcal{G}(\tau_s) + \int_{\tau_{\min}}^{\tau_{\max}} \mathrm{d}\tau \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} [\mathcal{G}(\tau) - \mathcal{G}(\tau_s)] \tag{27}$$

where

$$\int_{\tau_{\min}}^{\tau_{\max}} \mathrm{d}\tau \frac{m^2 B_1(\tau)}{C_1^{3/2}(\tau)} = 1 \tag{28}$$

was used. The quantity

$$\mathcal{G}(\tau) = \int_0^{R_{\max}} \frac{R\mathrm{d}R}{(Q^2 + R\tau)^2} \sum_{i=1}^{8} \hat{\theta}_i(R,\tau) \mathcal{F}_i^{pl}(R,\tau) \tag{29}$$

is a function over τ regular in the ultrarelativistic approximation. The second term in (27) $\sim m^2$ and has to be dropped in the approximation considered. The SF $\mathcal{F}_i^{pl}(R,\tau)$ are parts of the generalized SF containing P_L. The quantities $\hat{\theta}_i(R,\tau_s)$ can be obtained from Born ones (12) by the following replacements:

$$\hat{\theta}_i(R,\tau_s) = \frac{4}{S(S-R)} \theta_i^B \left(k_1 \to \left(1 - \frac{R}{S}\right) k_1 \right). \tag{30}$$

As a result the contribution $\hat{\sigma}_R$ can be expressed in terms of the Born cross section.

Below we give an explicit result for the total one-loop lowest order correction which includes the contribution from the radiation of a real photon (σ_R, see figures 1(*a*) and (*b*)) and from additional virtual particles (σ_V, see figures 1(*c*)–(*e*)) and can be presented as the sum of four infrared free terms:

$$\sigma_V + \sigma_R = \frac{\alpha}{\pi} \delta_{VR} \sigma^B + \sigma_V^r + \sigma_R^F + \hat{\sigma}_R. \tag{31}$$

The factor

$$\delta_{VR} = \left(\ln\frac{Q^2}{m^2} - 1\right)\ln\frac{(W^2-(M+m_\pi)^2)^2}{(X+Q^2)(S-Q^2)} + \frac{3}{2}\ln\frac{Q^2}{m^2} - 2$$
$$-\frac{1}{2}\ln^2\frac{X+Q^2}{S-Q^2} + \mathrm{Li}_2\frac{SX-Q^2M^2}{(X+Q^2)(S-Q^2)} - \frac{\pi^2}{6} \qquad (32)$$

appears in front of the Born cross section after cancellation of infrared divergence by summing an infrared part separated from σ_R and the so-called 'QED part' of the V-contribution which arises from lepton vertex graphs including an additional virtual photon.

The contribution from electroweak loops with the exception of the 'QED part' can be written in terms of the Born cross section with the following replacement:

$$\sigma_V^r = \sigma^B(R^{mn}_{V,A} \to \delta R^{mn}_{V,A}) \qquad (33)$$

where

$$\delta R^{\gamma\gamma}_{V,A} = -2\Pi^\gamma R^{\gamma\gamma}_{V,A} - 2\Pi^{\gamma Z}\chi R^{Z\gamma}_{V,A}$$
$$+\frac{\alpha}{4\pi}\left[2R^{ZZ}_{V,A}\Lambda_2(-Q^2, M_Z) + (1-P_L)\frac{3}{2s_w^2}\Lambda_3(-Q^2, M_W)\right]$$
$$\delta R^{\gamma Z, Z\gamma}_{V,A} = -2(\Pi^\gamma + \Pi^Z)R^{\gamma Z}_{V,A} - 2\Pi^{\gamma Z}(R^{\gamma\gamma}_{V,A} + \chi R^{ZZ}_{V,A})$$
$$+\frac{\alpha}{4\pi}\left[2(v^Z R^{ZZ}_{V,A} + a^Z R^{ZZ}_{A,V})\Lambda_2(-Q^2, M_Z) + (1-P_L)\right.$$
$$\left.\times\left\{\frac{1}{8s_w^3 c_w}\Lambda_2(-Q^2, M_W) + \frac{3}{4s_w^2}\left(v^Z + a^Z - \frac{c_w}{s_w}\right)\Lambda_3(-Q^2, M_W)\right\}\right]$$
$$\delta R^{ZZ}_{V,A} = -2\Pi^Z R^{ZZ}_{V,A} - 2\Pi^{\gamma Z}R^{\gamma Z}_{V,A}$$
$$+\frac{\alpha}{4\pi}\left[(2\left((v^Z)^2 + (a^Z)^2\right)R^{ZZ}_{V,A} + 4v^Z a^Z R^{ZZ}_{A,V})\Lambda_2(-Q^2, M_Z) + (1-P_L)\right.$$
$$\left.\times(v^Z + a^Z)\left\{\frac{1}{4s_w^3 c_w}\Lambda_2(-Q^2, M_W) - 3\frac{c_w}{2s_w^3}\Lambda_3(-Q^2, M_W)\right\}\right]. \qquad (34)$$

Here $M_{Z,W}$ are the masses of Z and W bosons and

$$\Pi^\gamma = -\frac{\hat\Sigma^\gamma(-Q^2)}{Q^2} \qquad \Pi^Z = -\frac{\hat\Sigma^Z(-Q^2)}{Q^2+M_Z^2} \qquad \Pi^{\gamma Z} = -\frac{\hat\Sigma^{\gamma Z}(-Q^2)}{Q^2}. \qquad (35)$$

Quantities $\hat\Sigma^{\gamma,\gamma Z,Z}$ are defined by the formulae (A.2), (3.17), (B.2)–(B.5) of [12] and $\Lambda_{2,3}$ by (B.4), (B.6) of [13].

The infrared free part of the cross section of the process (15) has the form

$$\sigma_R^F = -\alpha^3 y\int_{\tau_{\min}}^{\tau_{\max}}\mathrm{d}\tau\sum_{i=1}^{8}\left\{\theta_{i1}(\tau)\int_0^{R_{\max}}\frac{\mathrm{d}R}{R}\left[\frac{\mathcal{F}_i(R,\tau)}{(Q^2+R\tau)^2} - \frac{\mathcal{F}_i(0,0)}{Q^4}\right]\right.$$
$$\left.+\sum_{j=2}^{k_i}\theta_{ij}(\tau)\int_0^{R_{\max}}\mathrm{d}R\frac{R^{j-2}}{(Q^2+R\tau)^2}\mathcal{F}_i(R,\tau)\right\}. \qquad (36)$$

As was shown in (26)–(30) the last term of (31) is obtained in terms of the Born cross section

$$\hat\sigma_R = \frac{\alpha y}{\pi S}\int_0^{R^s_{\max}}\frac{R\mathrm{d}R}{(S_x - R)}\tilde\sigma^B_{pl} \qquad (37)$$

where the upper limit $R^s_{\max} = S(W^2-(M+m_\pi)^2)/(S-Q^2)$, and $\tilde\sigma^B_{pl}$ is the lepton polarization part of the Born cross section with the following replacement of kinematical variables: $S \to S-R$, $Q^2 \to Q^2(1-R/S)$ and $k_1\eta \to k_1\eta(1-R/S)$.

4. Numerical analysis

In this section the RC to different observable quantities in deep inelastic electron–proton scattering at collider are studied numerically.

The double-differential cross section as a function of the polarization characteristics of the scattering particles can be presented as the sum of four terms:

$$\sigma = \sigma^u + P_L\sigma^\xi + P_N\sigma^\eta + P_N P_L\sigma^{\xi\eta}. \qquad (38)$$

The first term is an unpolarized cross section and the other three characterize the polarized contributions independent of polarization degree. There are no problems with luminosity measurement in current collider experiments, so apart from the usual measurement of polarized asymmetries the absolute measurement of cross sections with different polarization configurations of beam and target will probably be possible in future polarization experiments at collider. Besides, new methods of data processing, where experimental information of spin observables is extracted directly from the polarized part of the cross section [15, 16] are actively being developed. In [16] it is shown how to separate completely unpolarized and polarized cross sections from a sample of experimental data using a special likelihood procedure and a binning of polarization degrees. All of the above, as well as the fact that RC to asymmetry is always constructed from RC to parts of the cross section, allows us to restrict our consideration to the numerical studying of RC for all of the cross sections in equation (38) and their combinations.

RC to these cross sections defined as a ratio of the cross section including one-loop RC only to the Born one

$$\delta^a = \frac{\sigma^a_{RC}}{\sigma^a_B} = \frac{\sigma^a_{\mathrm{tot}}}{\sigma^a_B} - 1 \qquad (a = u, \xi, \eta, \xi\eta) \qquad (39)$$

is presented on figure 2 as a function of the scaling variables x and y. The kinematical region corresponds to the present unpolarized collider experiment at HERA [14]. The results were obtained using the GRV-parton model [17, 18] for the electroweak SF $F_i^{\gamma,\gamma Z,Z}$ and $g_i^{\gamma,\gamma Z,Z}$ which are defined by the formulae (23)–(25) of [19] (see also [9]).

In experiments at the collider, detection of a hard photon by a calorimeter is used to reduce radiative effects. The broken curves in figure 2 demonstrate the influence of an experimental cut on the RC. One of the simplest variants of the cut when events having a radiative photon energy $E_\gamma > 10$ GeV are rejected from analysis, is considered.

From these plots one can see that the relative RC for the polarized part of the cross section has the same behaviour as the unpolarized one: it goes up when y tends to kinematical boards ($y \to 0, y \to 1$) and when x goes down. At the same time the correction to polarized parts can exceed the correction to unpolarized ones several times over. Use of the cut on photon energy does not influence RC in the region of small y and suppresses RC in the rest of the range. More detailed discussion of RC at collider with and without experimental cuts can be found in [20].

The simplest case for absolute measurements in polarization experiments is the observation of the difference of cross section for opposite configurations of proton spin

$$\Delta\sigma_\parallel = \sigma^{\uparrow\Uparrow} - \sigma^{\uparrow\Downarrow} \qquad \Delta\sigma_\perp = \sigma^{\uparrow\Leftarrow} - \sigma^{\uparrow\Rightarrow}. \qquad (40)$$

The first and second arrows correspond to lepton and proton polarization degrees equal to ± 1. The main contribution to $\Delta\sigma$ comes from the electromagnetic structure functions g_1^γ and g_2^γ. The values of RC to $\Delta\sigma$ defined the same as in (39) are presented in figure 3 for longitudinally and transversely polarized protons. The RC to $\Delta\sigma_\parallel$ cross section calculation

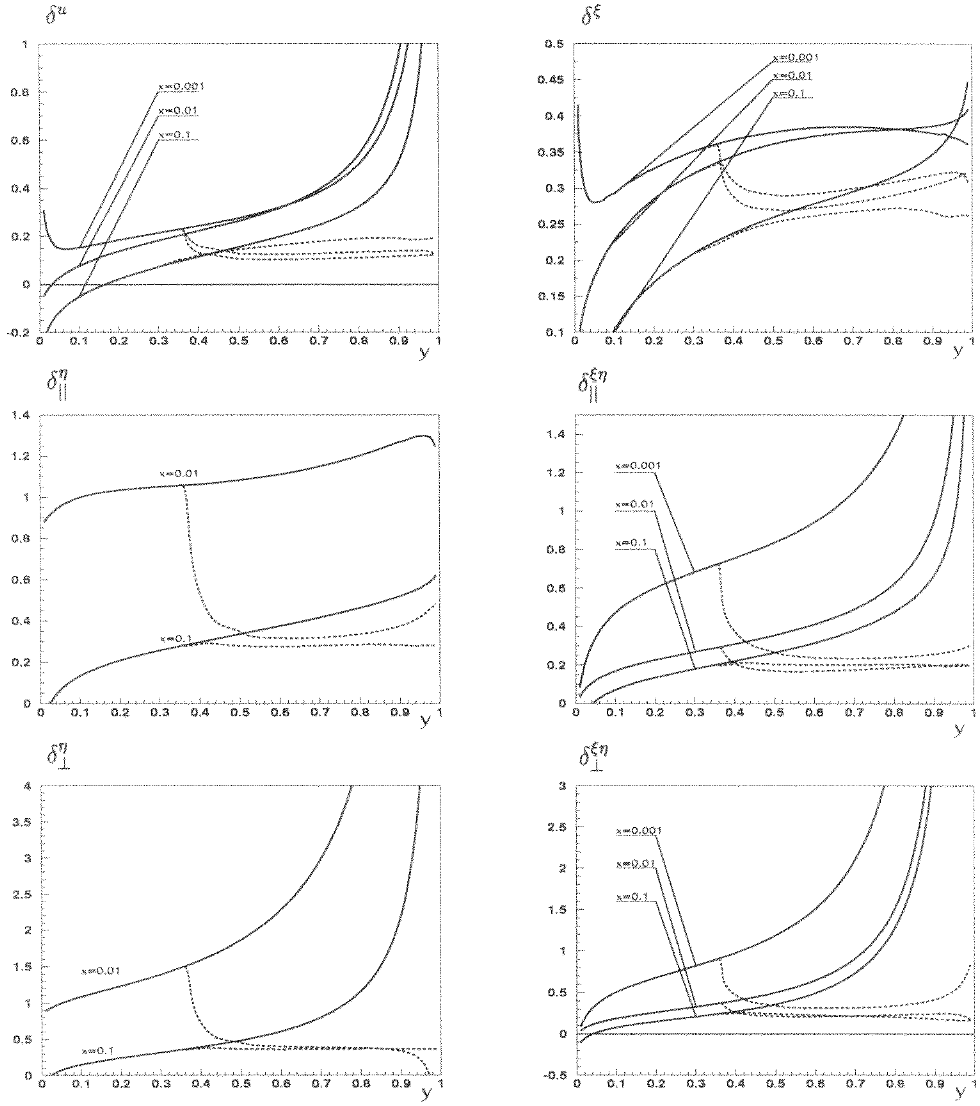

Figure 2. Radiative correction to σ^u, σ^ξ, σ^η and $\sigma^{\xi\eta}$ defined in (38) with (broken curves) and without cut (full curves). The down indexes $\perp$ and $||$ correspond to longitudinally and transversely polarized proton beam respectively. $\delta^\eta_\|$ and $\delta^\eta_\perp$ are singular for $x = 0.001$ due to the Born cross section σ^η crossing zero, so the corresponding curves δ^η are rejected.

in unitary gauge is studied numerically in [8] for kinematics of collider experiments. This result and those presented in figure 3 are in good agreement.

This result was obtained using the usual approximation $g_2^\gamma = 0$. Yet there is no reason to be restricted to such a case, especially when dealing with transversely polarized protons. One alternative model is the well known Wandzura–Wilcheck formula [21]

$$g_2^{\mathrm{WW}}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(\xi, Q^2)\frac{\mathrm{d}\xi}{\xi}. \tag{41}$$

The quantity characterizing the influence of a model on the cross section (40) can be defined

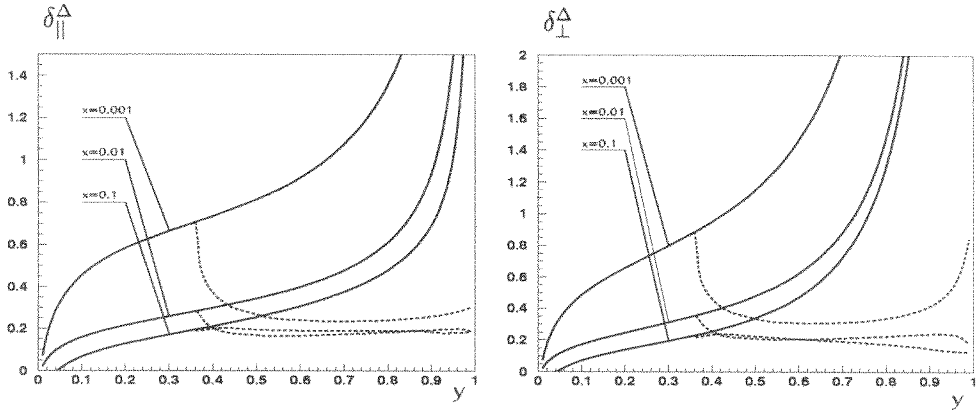

Figure 3. Radiative correction to $\Delta\sigma$ defined in (40) with (broken curves) and without cut (full curves). The down indexes $\perp$ and $||$ correspond to longitudinally and transversely polarized proton beam respectively.

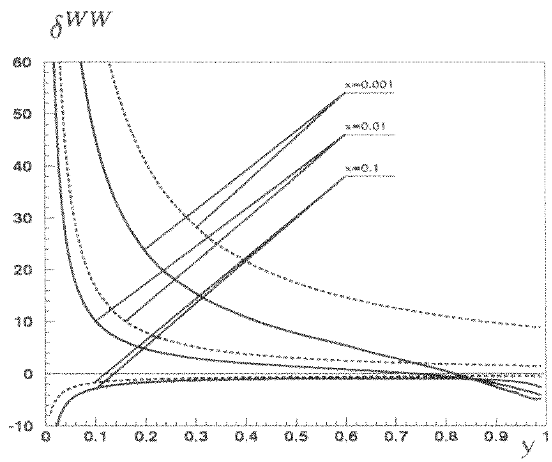

Figure 4. The quantity δ^{WW} on the Born level (broken curves) and on the level of radiative correction (full curves).

as

$$\delta^{\mathrm{WW}} = \frac{\Delta\sigma_{\perp}(g_2^{\gamma} = g_2^{\mathrm{WW}}) - \Delta\sigma_{\perp}(g_2^{\gamma} = 0)}{\Delta\sigma_{\perp}(g_2^{\gamma} = 0)}. \tag{42}$$

Figure 4 shows that the influence is important and the cross sections (40) calculated with $g_2^{\gamma} = g_2^{\mathrm{WW}}$ and with $g_2^{\gamma} = 0$ differ by some dozen times in the region of small x and y.

Experimental information on electroweak structure functions can be obtained using data of electron and positron scattering with different spin configurations both of leptons and protons. Corresponding combinations of the cross sections were offered in [19] and discussed in [9]. Here we consider two of them, which allow us to extract SF $g_1^{\gamma\mathrm{Z,Z}}$ and $g_5^{\gamma\mathrm{Z,Z}}$. On the Born level they read

$$\begin{aligned} &\sigma_-^{\uparrow\Uparrow} - \sigma_-^{\uparrow\Downarrow} + \sigma_+^{\uparrow\Uparrow} - \sigma_+^{\uparrow\Downarrow} + \sigma_-^{\downarrow\Uparrow} - \sigma_-^{\downarrow\Downarrow} + \sigma_+^{\downarrow\Uparrow} - \sigma_+^{\downarrow\Downarrow} \\ &\qquad = \frac{32\pi\alpha^2 S}{Q^4} x(2 - 2y - y^2)[v^{\mathrm{Z}}\chi g_5^{\gamma\mathrm{Z}} + ((a^{\mathrm{Z}})^2 + (v^{\mathrm{Z}})^2)\chi^2 g_5^{\mathrm{Z}}] \end{aligned} \tag{43}$$

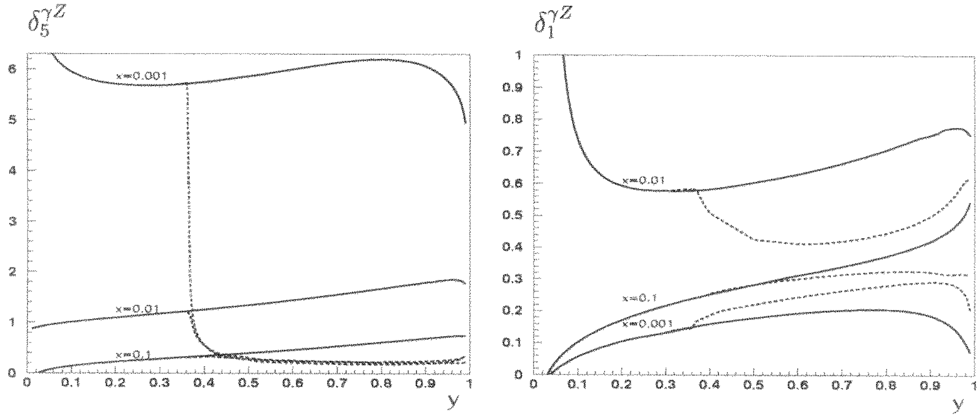

Figure 5. Radiative correction to quantities defined in (43) and (44) with (broken curves) and without cut (full curves).

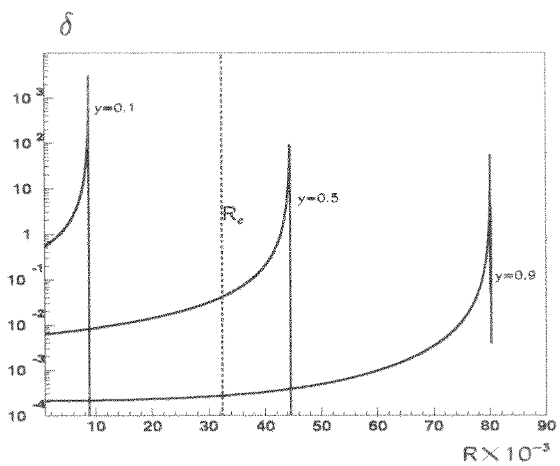

Figure 6. Integrand of radiative correction to the cross section (36) over R normalized to the Born one in the case of combination of the cross section (43). The broken line shows the used cut of photon energy ($E_\gamma < 10$ GeV).

and

$$\sigma_-^{\uparrow\Uparrow} - \sigma_-^{\uparrow\Downarrow} - \sigma_+^{\uparrow\Uparrow} - \sigma_+^{\uparrow\Downarrow} + \sigma_-^{\downarrow\Uparrow} - \sigma_-^{\downarrow\Downarrow} - \sigma_+^{\downarrow\Uparrow} - \sigma_+^{\downarrow\Downarrow}$$
$$= \frac{32\pi\alpha^2 S}{Q^4} xy(2-y)[a^Z \chi g_1^{\gamma Z} + 2v^Z a^Z \chi^2 g_1^Z] \tag{44}$$

where the lower index $-(+)$ corresponds to the electron(positron)–proton scattering. These expressions were obtained using model [9, 19] where $g_2^{\gamma Z} = g_4^{\gamma Z} = g_4^Z = 0$ and $g_3^{\gamma Z, Z} = 2xg_5^{\gamma Z, Z}$.

From figure 5 one can see that in the region of small x the RC cross section (43) exceeds the Born one several times over. The main effect comes from σ_R^F (36), the integrand of which over R for the sum of the cross section (43) is sketched on figure 6. It can be understood from the analysis of this figure that using the experimental cut as discussed above allows one essentially to reduce radiative effects for high y and does not influence the magnitude of RC for $y < 0.3$.

5. Conclusion

We have obtained compact and transparent formulae for the lowest order model-independent electroweak RC within t'Hooft–Feynman gauge (31)–(37). These formulae can be applied to data processing of experiments at collider. Numerical analysis of the obtained formulae for kinematics of collider experiments shows that RC to the unpolarized and polarized parts of the cross section have the same behaviour, however polarized correction can exceed the unpolarized one several times over. The detection of a hard photon in calorimeter allows one to reduce the radiative effects in the region $y > 0.3$. Comparison of figure 3 with the corresponding figure of [8] has shown good agreement.

Appendix

In this appendix the explicit expressions for quantities $\theta_{ij}(\tau)$ are presented. Due to the factorization of infrared terms all θ_{i1} are proportional to Born contributions:

$$\theta_{i1}(\tau) = 4F_{\mathrm{IR}}\theta_i^B. \tag{45}$$

The other $\theta_{ij}(\tau)$ functions are given

$$\begin{aligned}
\theta_{12} &= 4\tau F_{\mathrm{IR}}\\
\theta_{13} &= -2(2F + F_d\tau^2)\\
\theta_{22} &= (F_{1+}S_xS_p - F_dS_p^2\tau + 2m^2F_{2-}S_p - 2(2M^2\tau - S_x)F_{\mathrm{IR}})/2M^2\\
\theta_{23} &= (4FM^2 + 2F_dM^2\tau^2 - F_dS_x\tau - F_{1+}S_p)/2M^2\\
\theta_{32} &= (F_{1+}S_xQ^2 + 2m^2F_{2-}Q^2 - m^2F_{2+}S_p\tau + 3F_{\mathrm{IR}}S_p\tau)/2M^2\\
\theta_{33} &= (2m^2F_{2-}\tau - F_dS_p\tau^2 - 2F_{1+}Q^2)/2M^2\\
\theta_{34} &= -\tau F_{1+}/2M^2\\
\theta_{42} &= 2(\eta\mathcal{K}\tau(m^2F_{2+} - 3F_{\mathrm{IR}}) - \eta qF_{1+}Q^2 - 2m^2F_{2-}^\eta Q^2)/M\\
\theta_{43} &= 2(\eta\mathcal{K}F_d\tau^2 + 2F_{1+}^\eta Q^2 - 2m^2F_{2-}^\eta\tau)/M\\
\theta_{44} &= 2\tau F_{1+}^\eta/M\\
\theta_{62} &= (\eta\mathcal{K}(F_{1+}S_x - 2F_dS_p\tau + 2m^2F_{2-}) + \eta q(F_{1+}S_p + 2F_{\mathrm{IR}})\\
&\qquad +2F_{\mathrm{IR}}^\eta S_x + 2m^2F_{2-}^\eta S_p)/M\\
\theta_{63} &= -(\eta\mathcal{K}F_{1+} + \eta qF_d\tau + F_d^\eta S_x\tau + F_{1+}^\eta S_p)/M
\end{aligned} \tag{46}$$

and for $i = 5, 7, 8$:

$$\begin{array}{lll}
\theta_{51} = 2\eta q\theta_{31}/M & \theta_{71} = \eta q\theta_{21}/M & \theta_{81} = -\eta q\theta_{11}/M\\
\theta_{52} = 2(\eta q\theta_{32} - \theta_{31}^\eta)/M & \theta_{72} = (\eta q\theta_{22} - \theta_{21}^\eta)/M & \theta_{82} = -(\eta q\theta_{12} - \theta_{11}^\eta)/M\\
\theta_{53} = 2(\eta q\theta_{33} - \theta_{32}^\eta)/M & \theta_{73} = (\eta q\theta_{23} - \theta_{22}^\eta)/M & \theta_{83} = -(\eta q\theta_{13} - \theta_{12}^\eta)/M\\
\theta_{54} = 2(\eta q\theta_{34} - \theta_{33}^\eta)/M & \theta_{74} = -\theta_{23}^\eta/M & \theta_{84} = \theta_{13}^\eta/M\\
\theta_{55} = -2\theta_{34}^\eta/M & &
\end{array} \tag{47}$$

where $\mathcal{K} = k_1 + k_2$. The following equalities define the functions F:

$$\begin{aligned}
F &= \lambda_Q^{-1/2} \\
F_{\mathrm{IR}} &= m^2 F_{2+} - Q^2 F_d \\
F_d &= \tau^{-1}(C_2^{-1/2}(\tau) - C_1^{-1/2}(\tau)) \\
F_{1+} &= C_2^{-1/2}(\tau) + C_1^{-1/2}(\tau) \\
F_{2\pm} &= B_2(\tau)C_2^{-3/2}(\tau) \mp B_1(\tau)C_1^{-3/2}(\tau) \\
F_i &= -\lambda_Q^{-3/2} B_1(\tau).
\end{aligned} \tag{48}$$

Here $\lambda_Q = S_x^2 + 4M^2Q^2$ and

$$\begin{aligned}
B_{1,2}(\tau) &= -\tfrac{1}{2}\,(\,\lambda_Q \tau \pm S_p(S_x\tau + 2Q^2)\,) \\
C_1(\tau) &= (S\tau + Q^2)^2 + 4m^2(Q^2 + \tau S_x - \tau^2 M^2) \\
C_2(\tau) &= (X\tau - Q^2)^2 + 4m^2(Q^2 + \tau S_x - \tau^2 M^2).
\end{aligned} \tag{49}$$

For all ij, θ_{ij}^{η} in (47) are defined as

$$\theta_{ij}^{\eta} = \theta_{ij}(F_{\mathrm{All}} \to F_{\mathrm{All}}^{\eta}) \tag{50}$$

where

$$\begin{aligned}
2F^{\eta} &= F(r_\eta - \tau s_\eta) + 2F_i s_\eta \\
2F_{2+}^{\eta} &= (2F_{1+} + \tau F_{2-})s_\eta + F_{2+}r_\eta \\
2F_{2-}^{\eta} &= (2F_d + F_{2+})\tau s_\eta + F_{2-}r_\eta \\
2F_d^{\eta} &= F_{1+}s_\eta + F_d r_\eta \\
2F_{1+}^{\eta} &= (4F + \tau^2 F_d)s_\eta + F_{1+}r_\eta.
\end{aligned} \tag{51}$$

The quantities

$$s_\eta = a_\eta + b_\eta, r_\eta = \tau(a_\eta - b_\eta) + 2c_\eta \tag{52}$$

are the combination of coefficients of the polarization vector η expansion over the basis (see also appendix A of [3])

$$\eta = 2(a_\eta k_1 + b_\eta k_2 + c_\eta p). \tag{53}$$

For example in the case of the longitudinally polarized beam they read:

$$a_\eta = \frac{M}{\sqrt{\lambda_s}}, b_\eta = 0 \qquad c_\eta = -\frac{S}{2M\sqrt{\lambda_s}}. \tag{54}$$

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