The complete tree-level result up to order $1/Q$
for polarized deep-inelastic leptoproduction
[Nucl.Phys. B461 (1996) 197]

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We present the results of the tree-level calculation of deep-inelastic leptoproduction, including polarization of target hadron and produced hadron. We also discuss the dependence on transverse momenta of the quarks, which leads to azimuthal asymmetries for the produced hadrons.

I. INTRODUCTION

In recent years several possible ways to probe the structure of hadrons in hard scattering processes have been pointed out. Most well-known are the inclusive lepton-hadron (ℓH) scattering experiments that provide detailed information on the unpolarized and polarized quark distributions. This information is valuable as it, within the framework of Quantum Chromodynamics (QCD), can be expressed as well-defined matrix elements of quark and gluon operators within the nucleon. For instance, the operator product expansion relates moments of quark distributions to expectation values of local operators that represent static properties of the nucleon such as its electric or axial charge, baryon number, etc. Some of these matrix elements can also be determined in an independent way, such as the axial charge of the nucleon from neutron decay. We will consider in this paper matrix elements of nonlocal combinations of quark fields [1,2]. They represent forward quark-target scattering amplitudes that can be interpreted as quark momentum distributions or multi-parton distributions [3].

In inclusive ℓH scattering one can determine combinations of the quark distributions $f_1^a(x)$ and $g_1^a(x)$ where x is the fractional lightcone momentum $x = p^+/P^+$ of a quark with momentum p in a hadron with momentum P. The index a refers to the flavor of the quark. The antiquark distributions found in inclusive ℓH scattering are denoted $f_1^{\bar{a}}(x)$ or $\bar{f}_1^a(x)$, etc. These quark distributions appear in the structure functions, which in the Bjorken limit scale in the variable $x_B = Q^2/2P \cdot q$, where $Q^2 = -q^2$ with q the (large) momentum of the virtual photon. Restricting ourselves to the exchange of photons, the experimentally accessible leading structure functions are

$$\frac{F_2(x_B, Q^2)}{x_B} = 2\, F_1(x_B, Q^2) = \sum_a e_a^2 \left(f_1^a(x_B) + \bar{f}_1^a(x_B) \right), \tag{1}$$

$$2\, g_1(x_B, Q^2) = \sum_a e_a^2 \left(g_1^a(x_B) + \bar{g}_1^a(x_B) \right), \tag{2}$$

where e_a are the quark charges. The functions appear in leading order in $1/Q$ and are referred to as twist-two functions since their moments are matrix elements of local operators of twist two. In naming the quark distributions we extend the scheme proposed by Jaffe and Ji [4,5]. For the unpolarized quark distributions, often denoted as $f^a(x)$ or $a(x)$, the notation becomes $f_1^a(x)$. For the polarized quark distributions, for which one finds $\Delta f^a(x)$ or $\Delta a(x)$, the notation becomes $g_1^a(x)$,

The only other distribution function that can be obtained in inclusive deep-inelastic ℓH scattering up to order $1/Q$ is the twist-three function $g_T^a(x)$. It appears in the structure function

$$2\, g_2(x_B, Q^2) = \sum_a e_a^2 \left(g_2^a(x_B) + \bar{g}_2^a(x_B) \right), \tag{3}$$

where $g_2^a(x) = -g_1^a(x) + g_T^a(x)$. The functions g_T^a can be expressed as matrix elements of quark and gluon field operators [6,7] and their moments involve local operators of twist three [8].

In processes involving at least two hadrons, it is possible to measure one more twist-two quark distribution function. This function, the transverse spin distribution $h_1^a(x)$ (also known as $\Delta_T a(x)$) and the antiquark distribution $\bar{h}_1^a(x)$ are chirally odd, hence they must be combined with some other chirally odd structure [9,10]. This is possible in Drell-Yan scattering or in semi-inclusive ℓH scattering. In the latter case the chirally odd structure is found in the quark $\rightarrow$ hadron fragmentation process. At the twist-three level, two more chirally odd quark distributions, $e^a(x)$ and $h_L^a(x)$ appear [4,5].

In processes involving at least two hadrons or two jets, one also has the possibility to investigate azimuthal dependences. This involves a momentum that is transverse to the large momentum flow, e.g., the azimuthal dependence of hadrons around the (spacelike) direction defined by the virtual photon. This is the specific case investigated in this paper. Other examples are the azimuthal dependence of the produced muon pair in Drell-Yan scattering or the azimuthal dependence of hadrons around the jet-jet axis in two-jet events in e^+e^- annihilation.

As has been discussed first by Ralston and Soper [11] and recently by us [12], one has the possibility to measure in total six quark distribution functions depending on the lightcone fraction x and the transverse momentum squared $\boldsymbol{p}_T^2$, which depends on x and the quark virtuality p^2. In this paper we investigate the full tree-level result for semi-inclusive ℓH scattering up to order $1/Q$ which in addition to the six twist-two distribution functions involves eight twist-three functions. After $\boldsymbol{p}_T$-integration only the three twist-two and three twist-three (x-dependent) functions discussed above remain. For semi-inclusive scattering one also needs to study the quark fragmentation process involving a number of twist-two and twist-three fragmentation functions. The treatment of the fragmentation functions proceeds along the same lines as the distribution functions with the exception that one also can have some time-reversal odd structure. This structure emerges as the outgoing hadron in ℓH scattering is not a plane wave state in contrast to the incoming hadron.

An important motivation to study quark and gluon distribution and fragmentation functions in hard scattering processes are the existence of factorization theorems in QCD that imply the universality of these functions, notwithstanding their (logarithmic) scale dependence. Such factorization theorems exist for the p_T-integrated functions [13]. They do not exist for the unintegrated distributions [14,15]. Nevertheless, we consider the present complete tree-level result up to (subleading) order $1/Q$ as an important step. It represents the extension of the naive parton model result to subleading order and shows the dominant structures to be expected in the cross section although QCD corrections, such as Sudakov effects, may affect the asymmetries [16]. At the tree-level the only QCD input at order $1/Q$ is the use of the equations of motion which assures the electromagnetic gauge invariance. In view of possible future measurements with electrons or muons at DESY, CERN or Fermilab it is important to be aware of possible azimuthal asymmetries, not only because they contain valuable new information on the quark and gluon structure of hadrons but also in order to estimate corrections to p_T-integrated quark distributions coming from variations in angular acceptance.

The structure of this paper is as follows. In section 2 we present the formalism of ℓH scattering followed in section 3 by the analysis of quark distribution and fragmentation functions. This is followed by the full tree-level result up to order $1/Q$ in section 4, the basic result of this paper. The observable consequences of this result are outlined in the next two sections, starting with the full p_T-integrated result for $\ell H \rightarrow hX$ in section 5. The cross sections are given including polarization of lepton and target hadron and including measured final state polarization, all of this for the case of spin $1/2$ particles. In section 6, the result for azimuthal asymmetries involving unintegrated distribution and fragmentation functions is presented, including only polarizations of initial state particles. Finally the results are summarized in section 7.

II. FORMALISM

We consider deep-inelastic semi-inclusive lepton-hadron scattering, $\ell + H \rightarrow \ell' + h + X$, in which a lepton with momentum l scatters off a hadron H with momentum P and one hadron h with momentum P_h belonging to the current jet is measured in coincidence with the scattered lepton with momentum l'. The momentum of the exchanged virtual photon $q = l - l'$ is spacelike with $-q^2 \equiv Q^2 \rightarrow \infty$. We will use the invariants $x_B \equiv Q^2/2P \cdot q$ $\approx -P_h \cdot q/P_h \cdot P$, $y \equiv P \cdot q/P \cdot l \approx Q^2/x_B s$ (where s is the invariant mass squared of the photon-hadron system), and $z_h \equiv P \cdot P_h/P \cdot q \approx -2P_h \cdot q/Q^2$, the approximate sign indicating relations valid up to order $1/Q^2$ corrections. Furthermore, we use a transverse (two-component) vector $\boldsymbol{q}_T$ which is small (i.e., of order M). In the frame where h and H are collinear it is precisely the transverse component of q, while in the frame where P and q are collinear it is up to $1/Q^2$ corrections equal to $\boldsymbol{q}_T \approx -\boldsymbol{P}_{h\perp}/z_h$. Note that we will henceforth systematically neglect $1/Q^2$ corrections even if we use equal signs.

The cross section for semi-inclusive lepton-hadron scattering is given by

$$\frac{d\sigma}{dx_B\, dy\, dz_h\, d^2\boldsymbol{q}_T} = \frac{\pi\,\alpha^2}{2Q^4}\, y\, z_h\, L_{\mu\nu}\, 2M\mathcal{W}^{\mu\nu}. \tag{4}$$

The leptonic tensor is standard and is for polarized leptons (neglecting the lepton masses) given by

$$L_{\mu\nu}(l\lambda;l'\lambda') = \delta_{\lambda\lambda'}\left(2l_\mu l'_\nu + 2l_\nu l'_\mu - Q^2 g_{\mu\nu} + 2i\lambda\,\epsilon_{\mu\nu\rho\sigma}q^\rho l^\sigma\right), \tag{5}$$

with the lepton helicity being $\lambda = \pm 1$. The hadronic tensor is given by

$$2M\mathcal{W}_{\mu\nu}(q;PS;P_hS_h) = \frac{1}{(2\pi)^4}\sum_X \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X-P_h)$$
$$\times \langle PS|J_\mu(0)|P_X;P_hS_h\rangle\langle P_X;P_hS_h|J_\nu(0)|PS\rangle. \tag{6}$$

The role of the polarization vectors S and S_h is discussed in Appendix A. For the inclusive process one finds

$$\frac{d\sigma}{dx_B\, dy} = \frac{\pi\,\alpha^2}{Q^4}\, y\, L_{\mu\nu}\, 2MW^{\mu\nu}, \tag{7}$$

with the hadronic tensor given by

$$2M\, W_{\mu\nu}(q;PS) = \frac{1}{2\pi}\sum_X \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X)\langle PS|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|PS\rangle$$
$$= \frac{1}{2\pi}\int d^4x\; e^{iq\cdot x}\,\langle PS|[J_\mu(x), J_\nu(0)]|PS\rangle. \tag{8}$$

In order to expand the leptonic and hadronic tensors it is convenient to work with vectors orthogonal to q. A spacelike vector is defined by q^μ and a timelike vector defined by $\tilde{P}^\mu \equiv P^\mu - (P\cdot q/q^2)\,q^\mu$,

$$Z^\mu \equiv -q^\mu, \tag{9}$$

$$T^\mu \equiv -\frac{q^2}{P\cdot q}\,\tilde{P}^\mu \;=\; 2x_B\, P^\mu + q^\mu, \tag{10}$$

These vectors satisfy $T^2 = Q^2$, $Z^2 = -Q^2$. We will consider normalized vectors $\hat{t}^\mu = T^\mu/Q$, $\hat{z}^\mu = Z^\mu/Q = -\hat{q}^\mu = -q^\mu/Q$. In the space orthogonal to $\hat{z}$ and $\hat{t}$ one has the tensors

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} + \hat{q}^\mu\hat{q}^\nu - \hat{t}^\mu\hat{t}^\nu, \tag{11}$$

$$\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma}\hat{t}_\rho\hat{q}_\sigma = \frac{1}{P\cdot q}\,\epsilon^{\mu\nu\rho\sigma}P_\rho q_\sigma. \tag{12}$$

One vector in the perpendicular space is $P_{h\perp}^\mu = g_\perp^{\mu\nu}P_{h\nu}$. We will frequently use the normalized vector $\hat{h}^\mu = P_{h\perp}^\mu/|\boldsymbol{P}_{h\perp}|$.

Also the lepton momenta l and $l' = l - q$ can be expanded in $\hat{t}$, $\hat{z}$ and a perpendicular component,

$$l^\mu = \frac{2-y}{y}\,T^\mu - \frac{1}{2}\,Z^\mu + l_\perp^\mu \;=\; \frac{Q}{2}\,\hat{q}^\mu + \frac{(2-y)Q}{2y}\,\hat{t}^\mu + \frac{Q\sqrt{1-y}}{y}\,\hat{x}^\mu, \tag{13}$$

where $\hat{x}^\mu = l_\perp^\mu/|\boldsymbol{l}_\perp|$, is a spacelike unit-vector in the perpendicular direction lying in the (lepton) scattering plane. The kinematics in the frame where virtual photon and target are collinear (including target rest frame) is illustrated in Fig. 1. With the definition of $\hat{x}$, we have for the leptonic tensor

$$L_{(\ell H)}^{\mu\nu} = \frac{Q^2}{y^2}\left[-2\left(1-y+\frac{1}{2}y^2\right)g_\perp^{\mu\nu} + 4(1-y)\hat{t}^\mu\hat{t}^\nu\right.$$

$$+4(1-y)\left(\hat{x}^\mu\hat{x}^\nu + \frac{1}{2}g_\perp^{\mu\nu}\right) + 2(2-y)\sqrt{1-y}\,\hat{t}^{\{\mu}\hat{x}^{\nu\}}$$

$$\left.-i\lambda\, y(2-y)\,\epsilon_\perp^{\mu\nu} - 2i\lambda\, y\sqrt{1-y}\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}\hat{x}_\rho\right]. \tag{14}$$

Theoretically, it is often convenient to work in the frame in which the two hadrons H and h are collinear. Except for corrections proportional to M^2/Q^2 the vectors P^μ/Q and P_h^μ/Q determine two lightlike directions n_+^μ and n_-^μ, respectively. Using for momenta the representation $p = [p^-, p^+, \boldsymbol{p}_T]$, where $p^\pm = (p^0 \pm p^3)/\sqrt{2}$, one obtains in a frame in which H and h are collinear

$$P = \left[\frac{x_B M^2}{A\sqrt{2}}, \frac{A}{x_B\sqrt{2}}, \boldsymbol{0}_T \right] \equiv \frac{Q}{x_B\sqrt{2}} n_+ + \frac{x_B M^2}{Q\sqrt{2}} n_-, \tag{15}$$

$$P_h = \left[\frac{z_h Q^2}{A\sqrt{2}}, \frac{A M_h^2}{z_h Q^2\sqrt{2}}, \boldsymbol{0}_T \right] \equiv \frac{z_h Q}{\sqrt{2}} n_- + \frac{M_h^2}{z_h Q\sqrt{2}} n_+, \tag{16}$$

$$q = \left[\frac{Q^2}{A\sqrt{2}}, -\frac{A}{\sqrt{2}}, \boldsymbol{q}_T \right] = \frac{Q}{\sqrt{2}} n_- - \frac{Q}{\sqrt{2}} n_+ + q_T, \tag{17}$$

where A fixes the frame and $n_+ \cdot n_- = 1$. In a collinear frame as given here, the photon acquires a transverse momentum q_T with length $q_T^2 = -\boldsymbol{q}_T^2 = -Q_T^2$. It is easy to see that $P_{h\perp}^\mu = -z_h q_T^\mu$ and thus $Q_T = |\boldsymbol{P}_{h\perp}|/z_h$. Transverse projection operators are given by

$$g_T^{\mu\nu} \equiv g^{\mu\nu} - n_+^{\{\mu} n_-^{\nu\}}, \tag{18}$$

$$\epsilon_T^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma}, \tag{19}$$

where the brackets around the indices indicate symmetrization of these indices. Note that these *transverse* projectors are not identical to the *perpendicular* ones defined above if the transverse momentum of the outgoing hadron does not vanish. The lightlike directions, however, can easily be expressed in $\hat{t}$, $\hat{z}$ and a perpendicular component,

$$n_+^\mu = \frac{1}{\sqrt{2}} \left[\hat{t}^\mu + \hat{z}^\mu \right], \tag{20}$$

$$n_-^\mu = \frac{1}{\sqrt{2}} \left[\hat{t}^\mu - \hat{z}^\mu - 2 \frac{q_T}{Q} \right], \tag{21}$$

showing that the differences are of order $1/Q$. Especially for the treatment of azimuthal asymmetries, it is important to keep track of these differences.

III. QUARK CORRELATION FUNCTIONS

In this section we investigate the 'soft' hadronic matrix elements that appear in the diagrammatic expansion of a hard scattering amplitude [17]. The most important matrix elements that can be measured in deep-inelastic semi-inclusive lepton-hadron scattering are of two kinds. The first [1–3] is

$$\Phi_{ij}(P, S; p) = \frac{1}{(2\pi)^4} \int d^4x \, e^{i\,p\cdot x} \langle P, S | \overline{\psi}_j(0) \mathcal{L}(0, x; \text{path}) \psi_i(x) | P, S \rangle, \tag{22}$$

where a summation over color indices is implicit and flavor indices are omitted. This expression, referred to as a quark-quark correlation function, is a forward matrix elements of quark fields with explicit Dirac indices i and j in a hadronic state characterized by its momentum P and spin vector S. The quark fields are accompanied by a path ordered exponential (link operator),

$$\mathcal{L}(0, x; \text{path}) = \mathcal{P} \exp\left(-ig \int_0^x ds^\mu \, A_\mu(s) \right), \tag{23}$$

necessary to render the definition color gauge-invariant. It introduces, however, a path-dependence. In order to obtain a simple parton interpretation (Fig. 2) for the correlation functions the choice of path is important, as we will see later. Technical details will be discussed in Appendix B. The other type of matrix elements measured in semi-inclusive lepton-hadron scattering are the quark decay functions [2] (Fig. 2)

$$\Delta_{ij}(P_h, S_h; k) = \sum_X \frac{1}{(2\pi)^4} \int d^4x \, e^{ik\cdot x} \langle 0 | \mathcal{L}(0, x; \text{path}) \psi_i(x) | P_h, S_h; X \rangle \langle P_h, S_h; X | \overline{\psi}_j(0) | 0 \rangle$$

$$= \frac{1}{(2\pi)^4} \int d^4x \, e^{ik\cdot x} \langle 0 | \mathcal{L}(0, x; \text{path}) \psi_i(x) a_h^\dagger a_h \overline{\psi}_j(0) | 0 \rangle, \tag{24}$$

where an averaging over color indices is implicit. Again a quark link operator accompanies the quark fields. The second way of writing using hadron creation and annihilation operators is to be considered as a shorthand of the first expression, involving (out-)states $|P_h, S_h; X\rangle$ where P_h and S_h are the momentum and spin vectors for the produced hadron.

The correlation functions are constrained by the hermiticity properties of the fields, and by parity and time reversal invariance. Without constraints from time reversal invariance one has [11,12]

$$
\begin{aligned}
\Phi(P, S; p) = {} & A_1 + A_2\,\slashed{P} + A_3\slashed{p} + A_4\,\sigma^{\mu\nu}P_\mu p_\nu + i\,A_5\,(p\cdot S)\gamma_5, + A_6\,\slashed{S}\gamma_5 \\
& + A_7\,(p\cdot S)\,\slashed{P}\gamma_5 + A_8\,(p\cdot S)\,\slashed{p}\gamma_5 + i\,A_9\,\sigma^{\mu\nu}\gamma_5\,S_\mu P_\nu + i\,A_{10}\,\sigma^{\mu\nu}\gamma_5\,S_\mu p_\nu \\
& + i\,A_{11}\,(p\cdot S)\,\sigma^{\mu\nu}\gamma_5\,p_\mu P_\nu + A_{12}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu p^\rho S^\sigma,
\end{aligned} \tag{25}
$$

where the first four terms do not involve the hadron polarization vector. Hermiticity requires all the amplitudes $A_i = A_i(p\cdot P, p^2)$ to be real. The amplitudes A_4, A_5 and A_{12} vanish when also time reversal invariance applies. A similar decomposition in 12 amplitudes can be made for Δ, but because of the fact that out-states appear in the definition of the correlation functions time reversal invariance cannot be used and none of the amplitudes vanish. Because the time-reversal condition for A_4, A_5 and A_{12} reads $A_i^* = -A_i$ we refer to these amplitudes as time-reversal odd.

The above matrix elements are assumed to be nonzero only for quark momenta limited to a characteristic hadronic scale, which is much smaller than Q^2. This means that in the above matrix elements p^2, k^2, $p\cdot P$ and $k\cdot P_h \ll Q^2$. With the definition of momenta in the collinear frame (Eqs 15 - 17) one then arrives at the Sudakov decomposition, for the quark momenta,

$$
p = \left[\frac{(p^2 + \boldsymbol{p}_T^2)}{2x\,P^+},\ x\,P^+,\ \boldsymbol{p}_T\right] \equiv \frac{x\,Q}{x_B\sqrt{2}}\,n_+ + \frac{x_B(p^2 + \boldsymbol{p}_T^2)}{x\,Q\sqrt{2}}\,n_- + p_T \approx x\,P + p_T, \tag{26}
$$

$$
k = \left[\frac{P_h^-}{z},\ \frac{z\,(k^2 + \boldsymbol{k}_T^2)}{2\,P_h^-},\ \boldsymbol{k}_T\right] \equiv \frac{z_h\,Q}{z\sqrt{2}}\,n_- + \frac{z\,(k^2 + \boldsymbol{k}_T^2)}{z_h\,Q\sqrt{2}}\,n_+ + k_T \approx \frac{1}{z}\,P_h + k_T, \tag{27}
$$

where in the last expressions for p (k) the terms proportional to n_- (n_+), irrelevant in leading, $\mathcal{O}(1)$, or subleading, $\mathcal{O}(1/Q)$, order have been omitted. It turns out that up to subleading order only the combinations $\int dp^-\,\Phi(P, S; p)$ and $\int dk^+\,\Delta(P_h, S_h; k)$ are important. For cross sections integrated over transverse momenta only the combinations $\int dp^-d^2\boldsymbol{p}_T\,\Phi(P, S; p)$ and $\int dk^+d^2\boldsymbol{k}_T\,\Delta(P_h, S_h; k)$ are of importance at leading order.

The functions that appear in hard scattering processes can be expressed as specific Dirac projections of the correlation functions, integrated over p^-,

$$
\begin{aligned}
\Phi^{[\Gamma]}(x, \boldsymbol{p}_T) &= \frac{1}{2}\int dp^-\,Tr(\Phi\,\Gamma)\Big|_{p^+=xP^+,\ \boldsymbol{p}_T} \\
&= \int \frac{d\xi^-d^2\xi_T}{2\,(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P, S|\overline{\psi}(0)\,\Gamma\,\mathcal{L}(0, \xi; n_-)\,\psi(\xi)|P, S\rangle\Big|_{\xi^+=0}.
\end{aligned} \tag{28}
$$

In general the matrix element contains an infinite tower of operators because of the link operator. Choosing the path in the definition of Φ to lie in the plane $x^+ = 0$ and in essence along the direction x^-, indicated with 'path' $= n_-$ (see Appendix B), one finds that in an appropriately chosen gauge in leading order only one (bilocal) operator combination contributes, which allows a parton interpretation.

The projections $\Phi^{[\Gamma]}$ depend on the fractional momentum $x = p^+/P^+$ and on $\boldsymbol{p}_T$ and furthermore on the hadron momentum P (in essence only P^+ and M). Depending on the Lorentz structure of the Dirac matrix Γ the projections $\Phi^{[\Gamma]}$ can be ordered according to powers of M/P^+ multiplied with a function depending only on x and $\boldsymbol{p}_T^2 = -p_T^2$. It is easy to convince oneself that each factor M/P^+ leads to a suppression with a power M/Q in cross sections [18]. In analogy with inclusive scattering we therefore refer to the projections as having a 'twist' t related to the power $(M/P^+)^{t-2}$ that appears. With this definition the moments (in x) of the $\boldsymbol{p}_T$-integrated functions indeed involve local operators of twist t [2].

It is convenient to use for the spin vector S also a Sudakov decomposition,

$$
S = \left[-\frac{\lambda M}{2P^+},\ \frac{\lambda P^+}{M},\ \boldsymbol{S}_T\right] = \frac{\lambda Q}{Mx_B\sqrt{2}}\,n_+ - \frac{\lambda Mx_B}{Q\sqrt{2}}\,n_- + S_T \approx \lambda\frac{P}{M} + S_T, \tag{29}
$$

with $\lambda^2 + \boldsymbol{S}_T^2 = 1$ and again neglecting the contribution (proportional to n_-) which is irrelevant up to subleading order. The following distribution functions appear at leading order in $(M/P^+)^0$ (twist-two), where as said before we omit flavor indices,

$$\Phi^{[\gamma^+]}(x, \boldsymbol{p}_T) = f_1(x, \boldsymbol{p}_T^2), \tag{30}$$

$$\Phi^{[\gamma^+\gamma_5]}(x, \boldsymbol{p}_T) = \lambda\, g_{1L}(x, \boldsymbol{p}_T^2) + g_{1T}(x, \boldsymbol{p}_T^2)\, \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M}, \tag{31}$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \boldsymbol{p}_T) = S_T^i\, h_{1T}(x, \boldsymbol{p}_T^2) + \frac{p_T^i}{M}\left[\lambda\, h_{1L}^\perp(x, \boldsymbol{p}_T^2) + h_{1T}^\perp(x, \boldsymbol{p}_T^2)\, \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M}\right]$$

$$= S_T^i\, h_1(x, \boldsymbol{p}_T^2) + \frac{\lambda p_T^i}{M}\, h_{1L}^\perp(x, \boldsymbol{p}_T^2) + \frac{\left(p_T^i p_T^j - \tfrac{1}{2}\boldsymbol{p}_T^2 \delta_{ij}\right) S_T^j}{M^2}\, h_{1T}^\perp(x, \boldsymbol{p}_T^2), \tag{32}$$

with $h_1 = h_{1T} + (\boldsymbol{p}_T^2/2M^2)\, h_{1T}^\perp$. At subleading order $(M/P^+)^1$ (twist-three) one finds

$$\Phi^{[1]}(x, \boldsymbol{p}_T) = \frac{M}{P^+}\, e(x, \boldsymbol{p}_T^2) \tag{33}$$

$$\Phi^{[\gamma^i]}(x, \boldsymbol{p}_T) = \frac{p_T^i}{P^+}\, f^\perp(x, \boldsymbol{p}_T^2), \tag{34}$$

$$\Phi^{[\gamma^i\gamma_5]}(x, \boldsymbol{p}_T) = \frac{M\, S_T^i}{P^+}\, g_T'(x, \boldsymbol{p}_T^2) + \frac{p_T^i}{P^+}\left[\lambda\, g_L^\perp(x, \boldsymbol{p}_T^2) + g_T^\perp(x, \boldsymbol{p}_T^2)\, \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M}\right]$$

$$= \frac{M\, S_T^i}{P^+}\, g_T(x, \boldsymbol{p}_T^2) + \frac{\lambda p_T^i}{P^+}\, g_L^\perp(x, \boldsymbol{p}_T^2) + \frac{\left(p_T^i p_T^j - \tfrac{1}{2}\boldsymbol{p}_T^2 \delta_{ij}\right) S_T^j}{M^2}\, g_T^\perp(x, \boldsymbol{p}_T^2), \tag{35}$$

$$\Phi^{[i\sigma^{ij}\gamma_5]}(x, \boldsymbol{p}_T) = \frac{S_T^i p_T^j - p_T^i S_T^j}{P^+}\, h_T^\perp(x, \boldsymbol{p}_T^2) \tag{36}$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}(x, \boldsymbol{p}_T) = \frac{M}{P^+}\left[\lambda\, h_L(x, \boldsymbol{p}_T^2) + h_T(x, \boldsymbol{p}_T^2)\, \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M}\right], \tag{37}$$

with $g_T = g_T' + (\boldsymbol{p}_T^2/2M^2)\, g_T^\perp$. From here on we will often use the shorthand notation

$$g_{1s}(x, \boldsymbol{p}_T) \equiv \lambda\, g_{1L}(x, \boldsymbol{p}_T^2) + \frac{(\boldsymbol{p}_T \cdot \boldsymbol{S}_T)}{M}\, g_{1T}(x, \boldsymbol{p}_T^2), \tag{38}$$

and similarly shorthand notations $h_{1s}^\perp$, $g_s^\perp$ and h_s. The results after integration over $\boldsymbol{p}_T$ are lightcone correlation functions $\Phi^{[\Gamma]}(x)$ depending only on one lightcone momentum component $p^+ = x\, P^+$, for which the nonlocality is restricted to x^-, i.e., $x^+ = 0$ and $\boldsymbol{x}_T = \boldsymbol{0}_T$. Only the functions f_1, $g_1 = g_{1L}$, h_1, e, g_T and h_L are nonvanishing upon integration over $\boldsymbol{p}_T$. Upon further integration over x a local matrix element is found, e.g., relating the valence distributions to the quark numbers, $\int dx\, (f_1^a(x) - \bar{f}_1^a(x)) = \langle P, S | \overline{\psi}^a(0)\gamma^+\psi^a(0)|P, S\rangle/2P^+ = n_a - \bar{n}_a$.

In naming the functions, we have extended the scheme introduced by Jaffe and Ji. All functions obtained after tracing with a scalar (1) or pseudoscalar ($i\gamma_5$) Dirac matrix are given the name $e_{..}$, those traced with a vector matrix (γ^μ) are given the name $f_{..}$, those traced with an axial vector matrix ($\gamma^\mu\gamma_5$) are given the name $g_{..}$ and finally those traced with the second rank tensor ($i\sigma^{\mu\nu}\gamma_5$) are given the name $h_{..}$. A subscript $'1'$ is given to the twist-two functions, subscripts $'L'$ or $'T'$ refer to the connection with the hadron spin being longitudinal or transverse and a superscript $'\perp'$ signals the explicit presence of transverse momenta with a noncontracted index.

The twist-two distribution functions have a natural interpretation as parton densities. Using the good and bad components of the quark fields, $\psi_\pm = P_\pm \psi$ with $P_\pm = \gamma^-\gamma^+/2$ one finds that the operator combination in $\Phi^{[\gamma^+]}$ is just a density, $\psi_+^\dagger \psi_+$. The other twist-two correlations functions involve in addition chiral projections of the quark field, $\psi_{R/L} = P_{R/L}\psi$ with $P_{R/L} = (1 \pm \gamma_5)/2$ or transverse spin projections $\psi_{\uparrow/\downarrow} = P_{\uparrow/\downarrow}\psi$ with $P_{\uparrow/\downarrow} = (1 \pm \gamma^1\gamma_5)/2$. The correlation functions then become,

$$\Phi^{[\gamma^+]} = f_1 = f_1^{(R)} + f_1^{(L)} = f_1^{(\uparrow)} + f_1^{(\downarrow)}, \tag{39}$$

$$\Phi^{[\gamma^+\gamma_5]} = f_1^{(R)} - f_1^{(L)}, \tag{40}$$

$$\Phi^{[i\sigma^{1+}\gamma_5]} = f_1^{(\uparrow)} - f_1^{(\downarrow)}, \tag{41}$$

where $f_1^{(R)}$ is a matrix element involving the fields $\psi_{+R}^\dagger \psi_{+R}$, etc. Thus, the correlation function $\Phi^{[\gamma^+]}(x, \boldsymbol{p}_T)$ is just the unpolarized quark distribution, which integrated over $\boldsymbol{p}_T$ gives the familiar lightcone momentum distribution $\Phi^{[\gamma^+]}(x) = f_1(x)$. The correlation function $\Phi^{[\gamma^+\gamma_5]}(x, \boldsymbol{p}_T)$ is the chirality distribution (for massless

quarks helicity distribution). It can only be measured in a polarized target, as is evident from the presence of λ and $\boldsymbol{S}_T$ in Eq. 31. In a transversely polarized (spin 1/2) target the chirality distribution is proportional to the transverse momentum of the quark along $\boldsymbol{S}_T$. Integrated over $\boldsymbol{p}_T$ only one distribution survives in the correlation function, $\Phi^{[\gamma^+\gamma_5]}(x) = \lambda\, g_1(x)$, which can only be measured in a longitudinally polarized target. The correlation function $\Phi^{[i\sigma^{1+}\gamma_5]}(x,\boldsymbol{p}_T)$ is the transverse spin distribution and can also only be measured in a polarized target. The possible dependence on target spin and transverse momenta involves three functions, as can be seen in Eq. 32. Using the second expression one sees that one term (involving h_1) is insensitive to the direction of the quark transverse momentum; this part of the correlation function can only be measured in a transversely polarized target. The other two terms in Eq. 32 depend on rank 1 and rank 2 combinations of the transverse momentum of the quark and are accessible in longitudinally and transversely polarized targets, respectively. Integrated over $\boldsymbol{p}_T$ only one transverse spin distribution survives, $\Phi^{[i\sigma^{i+}\gamma_5]}(x) = S_T^i\, h_1(x)$, which can only be measured in a transversely polarized target. It involves a *chirally odd* [5] operator structure, which in a cross section can only appear in combination with another chirally odd operator such as a quark mass term in inclusive scattering, a chirally odd fragmentation function in semi-inclusive scattering or a chirally odd antiquark distribution in Drell-Yan scattering.

The twist-three distributions cannot be expressed as densities or differences of densities. They are related to matrix elements containing one good and one bad quark field. The bad field can in principle be replaced by a good field and a transverse gluon field using the equations of motion. Up to $\mathcal{O}(1/Q)$ one anyway needs to include quark-quark-gluon matrix elements. For this one needs to consider

$$\Phi_{D\,ij}^{\alpha}(P,S;p) = \frac{1}{(2\pi)^4}\int d^4x\; e^{i\,p\cdot x}\langle P,S|\overline{\psi}_j(0)\,\mathcal{L}(0,x;n_-)\,iD^\alpha(x)\psi_i(x)|P,S\rangle. \tag{42}$$

$$\Phi_{D\,ij}^{\alpha\dagger}(P,S;p) = \frac{1}{(2\pi)^4}\int d^4x\; e^{i\,p\cdot x}\langle P,S|\overline{\psi}_j(0)\,iD^\alpha(0)\,\mathcal{L}(0,x;n_-)\psi_i(x)|P,S\rangle, \tag{43}$$

where $iD^\alpha = i\partial^\alpha + gA^\alpha$. The twist analysis is performed by considering the projections

$$\Phi_D^{\alpha[\Gamma]}(x,\boldsymbol{p}_T) = \frac{1}{2}\int dp^-\; Tr\left(\Phi_D^\alpha\Gamma\right)\bigg|_{p^+=xP^+,\;\boldsymbol{p}_T}$$
$$= \int \frac{d\xi^-d^2\xi_\perp}{2\,(2\pi)^3}\; e^{i\,p\cdot\xi}\langle P,S|\overline{\psi}(0)\Gamma\,\mathcal{L}(0,\xi;n_-)\,iD^\alpha(\xi)\psi(\xi)|P,S\rangle\bigg|_{\xi^+=0}, \tag{44}$$

where $\mathcal{L}$ is the same link operator as the one used for the quark-quark correlation functions. Because of the choice of link, which always lies along the minus direction (except for the points $\xi^- = \pm\infty$), one has for the correlation function with a longitudinal gluon field the relation

$$\int \frac{d\xi^-d^2\xi_\perp}{2\,(2\pi)^3}\; e^{i\,p\cdot\xi}\,\langle P,S|\overline{\psi}_j(0)\Gamma\mathcal{L}(0,\xi;n_-)\,iD^+\psi_i(\xi)|P,S\rangle\bigg|_{\xi^+=0} = p^+\,\Phi^{[\Gamma]}(x,\boldsymbol{p}_T). \tag{45}$$

The projections obtained for the quark-quark-gluon correlation functions with transverse gluon fields are not all independent from the ones defined for the quark-quark correlation functions, either. Some can be connected to quark-quark correlation functions with one good and one bad quark field using the QCD equation of motion, $(i\!\!\not{D} - m)\psi(x) = 0$. This gives the relations

$$g_{T\alpha\beta}\,\Phi_D^{\alpha[\sigma^{\beta+}]} = \epsilon_{T\alpha\beta}\,\Phi_D^{\alpha[i\sigma^{\beta+}\gamma_5]} = i\,(Mx\,e - m\,f_1) + \epsilon_{T\,ij}\,p_T^i S_T^j\, xh_T^\perp, \tag{46}$$

$$g_{T\alpha\beta}\,\Phi_D^{\alpha[i\sigma^{\beta+}\gamma_5]} = Mx\,h_s - m\,g_{1s}, \tag{47}$$

$$g_T^{\alpha\beta}\,\Phi_{D\,\beta}^{[\gamma^+]} - i\epsilon_T^{\alpha\beta}\,\Phi_{D\,\beta}^{[\gamma^+\gamma_5]} = p_T^\alpha\,xf^\perp - i\epsilon_T^{\alpha\beta}p_{T\beta}\left(xg_s^\perp - \frac{m}{M}\,h_{1s}^\perp\right) - i\epsilon_T^{\alpha\beta}S_{T\beta}\,(Mx\,g_T' - m\,h_{1T}). \tag{48}$$

A useful quantity to consider is the (color gauge-invariant) correlation function

$$\Phi_A^{\alpha[\Gamma]}(x,\boldsymbol{p}_T) \equiv \Phi_D^{\alpha[\Gamma]}(x,\boldsymbol{p}_T) - p^\alpha\,\Phi^{[\Gamma]}(x,\boldsymbol{p}_T). \tag{49}$$

From Eq. 45 one sees that $\Phi_A^{+[\Gamma]} = 0$, while for the transverse indices $\alpha = 1$ or 2 it reduces after gauge fixing to the pure quark-quark-gluon matrix element (see Fig. 3),

$$\Phi_A^{\alpha[\Gamma]}(x,\boldsymbol{p}_T) \Rightarrow g\int \frac{d\xi^-d^2\xi_\perp}{2\,(2\pi)^3}\; e^{i\,k\cdot\xi}\langle P,S|\overline{\psi}(0)\Gamma\,A^\alpha(\xi)\psi(\xi)|P,S\rangle\bigg|_{\xi^+=0}, \tag{50}$$

needed to calculate the leptoproduction cross sections in subleading order. We use the correlation function Φ_A^α to identify *interaction-dependent* combinations in the distribution functions. As an example, from the first relation above (Eq. 46) we obtain

$$\epsilon_{T\alpha\beta}\,\Phi_A^{\alpha[i\sigma^{\beta+}\gamma_5]} = i\,(Mx\,e - m\,f_1) - \epsilon_{T\,ij}\,p_T^i S_T^j\,\left(h_{1T} - xh_T^\perp\right) \equiv i\,Mx\,\tilde{e} + \epsilon_{T\,ij}\,p_T^i S_T^j\,x\tilde{h}_T^\perp. \tag{51}$$

In this way we can rewrite all twist-three functions in a part containing twist-two distribution functions and an interaction-dependent part, e.g., $e = (m/Mx)\,f_1 + \tilde{e}$. The results for all twist-three functions are explicitly given in Appendix C, including results for the $\boldsymbol{p}_T$-integrated functions.

The antiquark distribution functions in a hadron are obtained from the matrix elements

$$\overline{\Phi}_{ij}(P,S;p) = \frac{1}{(2\pi)^4}\int d^4x\;e^{-i\,p\cdot x}\langle P,S|\mathcal{L}(0,x;n_-)\psi_i(x)\overline{\psi}_j(0)|P,S\rangle. \tag{52}$$

The antiquark distributions should be defined consistent with the replacement $\psi \to \psi^c = C\overline{\psi}^T$, or $\overline{\Phi}^{[\Gamma]} = +\Phi^{c[\Gamma]}$ for $\Gamma = \gamma_\mu$, $i\sigma_{\mu\nu}\gamma_5$ and $i\gamma_5$ and $\overline{\Phi}^{[\Gamma]} = -\Phi^{c[\Gamma]}$ for $\Gamma = 1$ and $\gamma_\mu\gamma_5$. Finally, the anticommutation relations for fermions can be used to obtain, within the (connected) matrix elements, the symmetry relation

$$\overline{\Phi}_{ij}(P,S;k) = -\Phi_{ij}(P,S;-k). \tag{53}$$

For the distribution functions this gives the symmetry relations $\bar{f}_1(x,\boldsymbol{p}_T^2) = -f_1(-x,\boldsymbol{p}_T^2)$ and identically for g_{1T}, h_{1T}, $h_{1T}^\perp$, $g_L^\perp$ and h_L, while $\bar{g}_{1L}(x,\boldsymbol{p}_T^2) = g_{1L}(-x,\boldsymbol{p}_T^2)$ and identically for $h_{1L}^\perp$, e, $f^\perp$, g_T', $g_T^\perp$, $h_T^\perp$ and h_T.

The procedure for the fragmentation functions obtained from the matrix element $\Delta(P_h,S_h;k)$ is similar. One again starts with the general decomposition in twelve amplitudes $A_i(k^2,k\cdot P_h)$, replacing $P \to P_h$ and $S \to S_h$. In this case time reversal invariance cannot be applied, as the matrix elements Δ_{ij} involve out-states containing a hadron h, in contrast to the plane wave states in the case of the matrix elements Φ_{ij}. A consequence is that there are more fragmentation functions. The functions that appear in hard scattering processes can again be expressed as specific Dirac projections of the correlation functions, integrated over k^+,

$$\Delta^{[\Gamma]}(z,\boldsymbol{k}_T') = \frac{1}{4z}\int dk^+\;Tr(\Delta\,\Gamma)\bigg|_{k^- = P_h^-/z,\;\boldsymbol{k}_T}$$
$$= \int \frac{d\xi^+ d^2\xi_T}{4z\,(2\pi)^3}\;e^{ik\cdot\xi}\,Tr\langle 0|\mathcal{L}(0,\xi;n_+)\,\psi(\xi)a_h^\dagger a_h\overline{\psi}(0)\Gamma|0\rangle\bigg|_{\xi^-=0}. \tag{54}$$

The arguments of the functions are the lightcone fraction $z = P_h^-/k^-$ and the transverse momentum $\boldsymbol{k}_T' = -z\boldsymbol{k}_T$, which is the perpendicular momentum of the hadron h with respect to the quark momentum. In the definition of the correlation functions Δ the path is chosen in the plane $x^- = 0$, in essence along the x^+-direction, indicated with 'path' $= n_+$ (see Appendix B). The spin vector is now parametrized as

$$S_h = \left[\frac{\lambda_h P_h^-}{M_h},-\frac{\lambda_h M}{2P_h^-},\boldsymbol{S}_{hT}\right] = \frac{\lambda_h zQ}{M_h\sqrt{2}}\,n_- + \frac{\lambda_h M_h}{zQ\sqrt{2}}\,n_+ + S_{hT} \approx \lambda_h\,\frac{P_h}{M_h} + S_{hT}. \tag{55}$$

In the last expression the contribution proportional to n_+, which is irrelevant up to subleading order is omitted. The role of the final state spin vector in determining the polarization of the produced hadrons is explained in Appendix A. The following fragmentation functions appear at leading order $(M_h/P_h^-)^0$ (twist-two),

$$\Delta^{[\gamma^-]}(z,\boldsymbol{k}_T') = D_1(z,\boldsymbol{k}_T'^2) + \frac{\epsilon_{T\,ij}\,k_T^i S_{hT}^j}{M_h}\,D_{1T}^\perp(z,\boldsymbol{k}_T'^2), \tag{56}$$

$$\Delta^{[\gamma^-\gamma_5]}(z,\boldsymbol{k}_T') = G_{1s}(z,\boldsymbol{k}_T') \tag{57}$$

$$\Delta^{[i\sigma^{i-}\gamma_5]}(z,\boldsymbol{k}_T') = S_{hT}^i\,H_{1T}(z,\boldsymbol{k}_T'^2) + \frac{k_T^i}{M_h}\,H_{1s}^\perp(z,\boldsymbol{k}_T') + \frac{\epsilon_T^{ij}k_{Tj}}{M_h}\,H_1^\perp(z,\boldsymbol{k}_T'^2). \tag{58}$$

At subleading order $(M_h/P_h^-)^1$ (twist-three) one finds

$$\Delta^{[1]}(z,\boldsymbol{k}_T') = \frac{M_h}{P_h^-}\,E(z,\boldsymbol{k}_T'^2), \tag{59}$$

$$\Delta^{[\gamma^i]}(z, \boldsymbol{k}'_T) = \frac{k_T^i}{P_h^-}\, D^\perp(z, \boldsymbol{k}'^2_T), \tag{60}$$

$$\Delta^{[i\gamma_5]}(z, \boldsymbol{k}'_T) = \frac{M_h}{P_h^-}\, E_s(z, \boldsymbol{k}'_T), \tag{61}$$

$$\Delta^{[\gamma^i\gamma_5]}(z, \boldsymbol{k}'_T) = \frac{M_h\, S_{hT}^i}{P_h^-}\, G'_T(z, \boldsymbol{k}'^2_T) + \frac{k_T^i}{P_h^-}\, G_s^\perp(z, \boldsymbol{k}'_T), \tag{62}$$

$$\Delta^{[i\sigma^{ij}\gamma_5]}(z, \boldsymbol{k}'_T) = \frac{S_{hT}^i k_T^j - k_T^i S_{hT}^j}{P_h^-}\, H_T^\perp(z, \boldsymbol{k}'^2_T) + \frac{M_h\, \epsilon_T^{ij}}{P_h^-}\, H(z, \boldsymbol{k}'^2_T), \tag{63}$$

$$\Delta^{[i\sigma^{-+}\gamma_5]}(z, \boldsymbol{k}'_T) = \frac{M_h}{P_h^-}\, H_s(z, \boldsymbol{k}'_T). \tag{64}$$

In the above expressions we have used the shorthand notations G_{1s} etc. standing for

$$G_{1s}(z, \boldsymbol{k}'_T) = \lambda_h\, G_{1L}(z, \boldsymbol{k}'^2_T) + \frac{(\boldsymbol{k}_T \cdot \boldsymbol{S}_{hT})}{M_h}\, G_{1T}(z, \boldsymbol{k}'^2_T). \tag{65}$$

There are more functions allowed in this case because of the non-applicability of time-reversal invariance, allowing more amplitudes in the expansion for Δ_{ij} as compared to that for Φ_{ij} (see discussion following Eq. 25). The fragmentation functions which have no equivalent in the distribution functions, $D_{1T}^\perp$, $H_1^\perp$, E_L, E_T and H, will be referred to as *time-reversal odd*. The results after integration over $\boldsymbol{k}'_T$ define the lightcone correlation functions $\Delta^{[\Gamma]}(z)$. Nonvanishing upon $\boldsymbol{k}'_T$-integration are the functions D_1, $G_1 = G_{1L}$, $H_1 = H_{1T} + (\boldsymbol{k}_T^2/2M_h^2)\, H_{1T}^\perp$, E, E_L, $G_T = G'_T + (\boldsymbol{k}_T^2/2M_h^2)\, G_T^\perp$, H and H_L and $D_{1T}^{\perp(1)} = (\boldsymbol{k}_T^2/2M_h^2)\, D_{1T}^\perp$. Summing over all produced hadrons h and integrating over z one can use completeness in the final state to see e.g. that one has (for any flavor) the momentum sum rule $\sum_h \int dz\, zD_1(z) = 1$.

A note on the names of the fragmentation functions. Because of the analogy with the distribution functions in essence the same names are used but using the capital letter, except for the functions corresponding to the $f_{..}$ distribution functions, for which we have used the more familiar name $D_{...}$. The connection with the naming scheme used by Jaffe and Ji, namely using the same letter as the distribution functions with a hat is straightforward. We have used capital letters in order to avoid double symbols over the function names.

The twist-two functions have a natural interpretation as decay functions, $\Delta^{[\gamma^-]}$ being the probability of a quark to produce a hadron in a specific spin state (characterized by the spin vector S_h). Note that the decay probability for an unpolarized quark with nonzero transverse momentum, can lead to a transverse polarization in the production of spin 1/2 particles. This transverse polarization is orthogonal to the quark momentum. Such an effect violates time reversal invariance. For an out-state this is induced by nonvanishing interactions in the final state. Indeed, as we will see below, the function $D_{1T}^\perp$ is a purely interaction-dependent function. It has no equivalent in the distribution functions. The correlation functions $\Delta^{[\gamma^-\gamma_5]}$ and $\Delta^{[i\sigma^{i-}\gamma_5]}$ are differences of probabilities for quarks with different chiralities or transverse spins, respectively, to produce a hadron in a specific spin state. For the latter also a new decay function appears because of the non-applicability of time reversal invariance. A transversely polarized quark with nonzero transverse momentum can produce unpolarized hadrons, in particular it can produce spinless particles, such as pions. The relevant function $H_1^\perp$ is the one appearing in the single spin Collins effect [24]. Note that neutral pions cannot be produced, since CP-invariance of the strong interactions enforces T-invariance.

A useful fragmentation function is the one for a quark (with momentum k) producing a quark with momentum p ($p^- = z\, k^-$), for which one has $\Delta^{[\gamma^-]} = \frac{1}{2}\, \delta(1-z)\, \delta^2(\boldsymbol{p}_T - \boldsymbol{k}_T)$. This result is useful to find cross sections for production of a current jet with (measured) transverse momentum $\boldsymbol{p}_T$ in lepto-production.

The twist analysis for the projections of the quark-quark-gluon matrix elements,

$$\Delta_D^{\alpha[\Gamma]}(z, \boldsymbol{k}'_T) = \int \frac{d\xi^+ d^2\xi_\perp}{4z\,(2\pi)^3}\, e^{i\,k\cdot\xi}\, Tr\, \langle 0|\mathcal{L}(0, \xi; n_+)\, \psi(\xi)\, iD^\alpha(\xi)\, a_h^\dagger a_h\, \overline{\psi}(0)\Gamma|0\rangle\bigg|_{\xi^-=0}, \tag{66}$$

is analogous to the distribution part. The correlation function $\Delta_D^{-[\Gamma]}$ is because of the choice of the path in the link operator trivially related to $k^-\, \Delta^{[\Gamma]}$. Using the equations of motion one obtains the relations

$$g_{T\alpha\beta}\, \Delta_D^{\alpha[\sigma^{\beta-}]} = -\epsilon_{T\alpha\beta}\, \Delta_D^{\alpha[i\sigma^{\beta-}\gamma_5]} = i\left(\frac{M_h}{z}E - m\, D_1 - i\frac{M_h}{z}H + i\frac{k_T^2}{M_h}H_1^\perp\right)$$

$$-\epsilon_{Tij}\, k_T^i S_{hT}^j\, (\frac{1}{z}H_T^\perp + i\,\frac{m}{M_h}\, D_{1T}^\perp), \tag{67}$$

$$g_{T\alpha\beta}\, \Delta_D^{\alpha[i\sigma^{\beta-}\gamma_5]} = \frac{M_h}{z}H_s - m\, G_{1s} + i\,\frac{M_h}{z}E_s, \tag{68}$$

$$g_T^{\alpha\beta}\Delta_{D\,\beta}^{[\gamma^-]} + i\epsilon_T^{\alpha\beta}\,\Delta_{D\,\beta}^{[\gamma^-\gamma_5]} = k_T^\alpha\left(\frac{1}{z}D^\perp + i\,\frac{m}{M_h}\,H_1^\perp\right) + i\epsilon_T^{\alpha\beta}k_{T\beta}\left(\frac{1}{z}G_s^\perp - \frac{m}{M_h}\,H_{1s}^\perp\right)$$
$$+ i\epsilon_T^{\alpha\beta}S_{hT\beta}\left(\frac{M_h}{z}G_T' - m\,H_{1T}\right) \tag{69}$$

Again it is useful to consider the quantity

$$\Delta_A^{\alpha[\Gamma]}(z,\boldsymbol{k}_T') = \Delta_D^{\alpha[\Gamma]}(z,\boldsymbol{k}_T') - k^\alpha\,\Delta^{[\Gamma]}(z,\boldsymbol{k}_T'), \tag{70}$$

which vanishes for $\alpha = -$ and which for transverse indices reduces after gauge-fixing to the quark-quark-gluon correlation function (Fig. 3) needed in the calculation of the leptoproduction amplitude. As for the distribution functions, it is used to define *interaction-dependent* parts and to express the twist-three functions into twist-two fragmentation functions and interaction-dependent parts, explicitly given in Appendix C.

A consistent definition of antiquark fragmentation functions implies

$$\overline{\Delta}_{ij}(P_h, S_h; k) = -\Delta_{ij}(P_h, S_h; -k), \tag{71}$$

with the symmetry relations $\overline{D}_1(z,\boldsymbol{k}_T'^2) = D_1(-z,\boldsymbol{k}_T'^2)$ and identically for G_{1T}, H_{1T}, $H_{1T}^\perp$, E_T, $G_L^\perp$, H and H_L, while $\overline{G}_{1L}(z,\boldsymbol{k}_T'^2) = -G_{1L}(-z,\boldsymbol{k}_T'^2)$ and identically for $D_{1T}^\perp$, $H_{1L}^\perp$, $H_1^\perp$, E, $D^\perp$, E_L, G_T', $G_T^\perp$, $H_T^\perp$ and H_T.

IV. THE COMPLETE TREE-LEVEL RESULT

Up to $\mathcal{O}(1/Q)$ one needs to include the contributions of the handbag diagram (Fig. 4), now calculated up to this order with in addition irreducible diagrams with one gluon coupling either to the soft part involving hadron H or the soft part involving hadron h (see Fig. 5). The expressions thus involve the quark-quark-gluon correlation functions. The momentum-conserving delta-function at the photon vertex is written (neglecting $1/Q^2$ contributions) as

$$\delta^4(p+q-k) = \delta(p^+ + q^+)\,\delta(q^- - k^-)\,\delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T), \tag{72}$$

fixing $p^+ = -q^+ = x_B P^+$ and $k^- = q^- = P_h^-/z_h$. The full result, neglecting $1/Q^2$ contributions and still omitting flavor indices, is then given by

$$2M\,\mathcal{W}_{\mu\nu} = e^2 \int dp^-\, dk^+\, d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\left\{\mathrm{Tr}\,(\Phi(p)\gamma_\mu\Delta(k)\gamma_\nu)\right.$$
$$-\mathrm{Tr}\left(\gamma_\alpha\frac{\slashed{n}_+}{Q\sqrt{2}}\gamma_\nu\Phi_A^\alpha(p)\gamma_\mu\Delta(k)\right) - \mathrm{Tr}\left(\gamma_\mu\frac{\slashed{n}_+}{Q\sqrt{2}}\gamma_\alpha\Delta(k)\gamma_\nu\Phi_A^{\alpha\dagger}(p)\right)$$
$$\left. -\mathrm{Tr}\left(\gamma_\nu\frac{\slashed{n}_-}{Q\sqrt{2}}\gamma_\alpha\Phi(p)\gamma_\mu\Delta_A^{\alpha\dagger}(k)\right) - \mathrm{Tr}\left(\gamma_\alpha\frac{\slashed{n}_-}{Q\sqrt{2}}\gamma_\mu\Delta_A^\alpha(k)\gamma_\nu\Phi(p)\right)\right\}. \tag{73}$$

In this expression the terms with $\slashed{n}_\pm$ arise from fermion propagators in the hard part neglecting contributions that will appear suppressed by powers of Q^2, i.e.,

$$\frac{\slashed{p}_1 + \slashed{q} + m}{(p_1+q)^2} \approx \frac{(p_1^+ + q^+)\gamma^-}{2(p_1^+ + q^+)q^-} = \frac{\gamma^-}{2q^-} = \frac{\slashed{n}_+}{Q\sqrt{2}} \approx \frac{x_B\,\slashed{P}}{Q^2}, \tag{74}$$

$$\frac{\slashed{k}_1 - \slashed{q} + m}{(k_1-q)^2} \approx \frac{(k_1^- - q^-)\gamma^+}{-2(k_1^- - q^-)q^+} = \frac{\gamma^+}{(-2q^+)} = \frac{\slashed{n}_-}{Q\sqrt{2}} \approx \frac{\slashed{P}_h}{z_h Q^2}. \tag{75}$$

Furthermore, the only relevant quark-quark-gluon correlation functions are those in which the argument of the gluon field is equal to that of one of the quark fields, discussed in the previous section. These correlation functions can be related to the quark-quark correlation functions using the QCD equations of motion. Such relations are

essential to ensure that at order $1/Q$ the sum of diagrams in Figs 4 and 5 yields an electromagnetically gauge-invariant result.

The result can be expressed in terms of the twist-two and twist-three distribution and fragmentation functions and perpendicular tensors and vectors. These vectors, $k_\perp^\mu$, $p_\perp^\mu$, $S_\perp^\mu$ and $S_{h\perp}^\mu$, are obtained from the expansion into the Cartesian directions of k_T^μ, p_T^μ, S_T^μ and S_{hT}^μ, respectively. For that one can use the result

$$g_T^{\mu\nu} = g_\perp^{\mu\nu} - \frac{Q_T}{Q}\,\hat{q}^{\{\mu}\hat{x}^{\nu\}} + \frac{Q_T}{Q}\,\hat{t}^{\{\mu}\hat{x}^{\nu\}}, \tag{76}$$

obtained from Eqs 20 and 21. The full expression for the symmetric and antisymmetric parts of the hadronic tensor are,

$$2M\,\mathcal{W}_S^{\mu\nu} = 2z_h \int d^2\mathbf{k}_T\, d^2\mathbf{p}_T\, \delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

$$\times \Bigg\{ -g_\perp^{\mu\nu}\left[f_1 D_1 + g_{1s} G_{1s} + \frac{\epsilon_\perp^{\rho\sigma} k_{\perp\rho} S_{h\perp\sigma}}{M_h}\, f_1 D_{1T}^\perp \right]$$

$$- \frac{k_\perp^{\{\mu} p_\perp^{\nu\}} + (\mathbf{k}_\perp \cdot \mathbf{p}_\perp)\, g_\perp^{\mu\nu}}{M M_h}\, h_{1s}^\perp H_{1s}^\perp - \frac{k_\perp^{\{\mu} S_\perp^{\nu\}} + (\mathbf{k}_\perp \cdot \mathbf{S}_\perp)\, g_\perp^{\mu\nu}}{M_h}\, h_{1T} H_{1s}^\perp$$

$$- \frac{p_\perp^{\{\mu} S_{h\perp}^{\nu\}} + (\mathbf{p}_\perp \cdot \mathbf{S}_{h\perp})\, g_\perp^{\mu\nu}}{M}\, h_{1s}^\perp H_{1T} - \left(S_\perp^{\{\mu} S_{h\perp}^{\nu\}} + (\mathbf{S}_\perp \cdot \mathbf{S}_{h\perp})\, g_\perp^{\mu\nu} \right) h_{1T} H_{1T}$$

$$- \frac{\left(k_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} p_{\perp\rho} + p_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} k_{\perp\rho} \right)}{2 M M_h}\, h_{1s}^\perp H_1^\perp - \frac{\left(k_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{\perp\rho} + S_\perp^{\{\mu} \epsilon_\perp^{\nu\}\rho} k_{\perp\rho} \right)}{2 M_h}\, h_{1T} H_1^\perp$$

$$+ \frac{2\,\hat{t}^{\{\mu} k_\perp^{\nu\}}}{Q}\Bigg[-f_1 D_1 + f_1 \frac{D^\perp}{z_h} - g_{1s} G_{1s} + g_{1s} \frac{G_s^\perp}{z_h}$$

$$+ \frac{M}{M_h} x_B\, h_s H_{1s}^\perp - \frac{m}{M_h}\, g_{1s} H_{1s}^\perp - \frac{\mathbf{p}_\perp \cdot \mathbf{S}_{h\perp}}{M}\, h_{1s}^\perp H_{1T}$$

$$+ \frac{\mathbf{p}_\perp \cdot \mathbf{S}_{h\perp}}{M}\, h_{1s}^\perp \frac{H_T^\perp}{z_h} - \mathbf{S}_\perp \cdot \mathbf{S}_{h\perp}\, h_{1T} H_{1T} + \mathbf{S}_\perp \cdot \mathbf{S}_{h\perp}\, h_{1T} \frac{H_T^\perp}{z_h} \Bigg]$$

$$+ \frac{2\,\hat{t}^{\{\mu} p_\perp^{\nu\}}}{Q}\Bigg[x_B\, f^\perp D_1 + x_B\, g_s^\perp G_{1s} + \frac{M_h}{M}\, h_{1s}^\perp \frac{H_s}{z_h} - \frac{m}{M}\, h_{1s}^\perp G_{1s} + \frac{k_\perp^2}{M M_h}\, h_{1s}^\perp H_{1s}^\perp$$

$$+ \frac{\mathbf{k}_\perp \cdot \mathbf{S}_{h\perp}}{M}\, h_{1s}^\perp H_{1T} + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M_h}\, x_B\, h_T^\perp H_{1s}^\perp + \mathbf{S}_\perp \cdot \mathbf{S}_{h\perp}\, x_B\, h_T^\perp H_{1T} \Bigg]$$

$$+ \frac{2M\,\hat{t}^{\{\mu} S_\perp^{\nu\}}}{Q}\Bigg[x_B\, g_T' G_{1s} + \frac{M_h}{M}\, h_{1T} \frac{H_s}{z_h} - \frac{m}{M}\, h_{1T} G_{1s} + \frac{k_\perp^2}{M M_h}\, h_{1T} H_{1s}^\perp$$

$$- \frac{\mathbf{k}_\perp \cdot \mathbf{p}_\perp}{M M_h}\, x_B\, h_T^\perp H_{1s}^\perp + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_{h\perp}}{M}\, h_{1T} H_{1T} - \frac{\mathbf{p}_\perp \cdot \mathbf{S}_{h\perp}}{M}\, x_B\, h_T^\perp H_{1T} \Bigg]$$

$$+ \frac{2M_h\,\hat{t}^{\{\mu} S_{h\perp}^{\nu\}}}{Q}\Bigg[\frac{M}{M_h}\, x_B\, h_s H_{1T} + g_{1s} \frac{G_T'}{z_h} - \frac{m}{M_h}\, g_{1s} H_{1T} + \frac{\mathbf{k}_\perp \cdot \mathbf{p}_\perp}{M M_h}\, h_{1s}^\perp H_{1T}$$

$$- \frac{\mathbf{k}_\perp \cdot \mathbf{p}_\perp}{M M_h}\, h_{1s}^\perp \frac{H_T^\perp}{z_h} + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M_h}\, h_{1T} H_{1T} - \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M_h}\, h_{1T} \frac{H_T^\perp}{z_h} \Bigg]$$

$$+ \frac{2\,\hat{t}^{\{\mu} \epsilon_\perp^{\nu\}\rho} k_{\perp\rho}}{Q}\Bigg[\frac{M}{M_h}\, x_B\, h_s H_1^\perp - \frac{m}{M_h}\, g_{1s} H_1^\perp - \frac{\mathbf{k}_\perp \cdot \mathbf{S}_{h\perp}}{M_h}\, f_1 D_{1T}^\perp + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_{h\perp}}{M_h}\, x_B\, f^\perp D_{1T}^\perp \Bigg]$$

$$+ \frac{2\,\hat{t}^{\{\mu} \epsilon_\perp^{\nu\}\rho} p_{\perp\rho}}{Q}\Bigg[\frac{M_h}{M}\, h_{1s}^\perp \frac{H}{z_h} + \frac{k_\perp^2}{M M_h}\, h_{1s}^\perp H_1^\perp + \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M_h}\, x_B\, h_T^\perp H_1^\perp \Bigg]$$

$$+ \frac{2M\,\hat{t}^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{\perp\rho}}{Q}\Bigg[\frac{M_h}{M}\, h_{1T} \frac{H}{z_h} + \frac{k_\perp^2}{M M_h}\, h_{1T} H_1^\perp - \frac{\mathbf{k}_\perp \cdot \mathbf{p}_\perp}{M M_h}\, x_B\, h_T^\perp H_1^\perp \Bigg]$$

$$+\frac{2M_h\,\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{h\perp\rho}}{Q}\left[\frac{\boldsymbol{k}_\perp^2}{M_h^2}\,f_1D_{1T}^\perp-\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{M_h^2}\,x_{\scriptscriptstyle B}\,f^\perp D_{1T}^\perp\right]\Bigg\}\tag{77}$$

and

$$2M\,\mathcal{W}_A^{\mu\nu}=2z_h\int d^2\boldsymbol{k}_{\scriptscriptstyle T}\,d^2\boldsymbol{p}_{\scriptscriptstyle T}\,\delta^2(\boldsymbol{p}_{\scriptscriptstyle T}+\boldsymbol{q}_{\scriptscriptstyle T}-\boldsymbol{k}_{\scriptscriptstyle T})$$

$$\times\left\{i\,\epsilon_\perp^{\mu\nu}\left[f_1G_{1s}+g_{1s}D_1\right]+i\frac{k_\perp^{[\mu}S_{h\perp}^{\nu]}}{M_h}\,g_{1s}D_{1T}^\perp\right.$$

$$+i\frac{2\,\hat{t}^{[\mu}k_\perp^{\nu]}}{Q}\left[-\frac{M}{M_h}\,x_{\scriptscriptstyle B}\,eH_1^\perp+\frac{m}{M_h}\,f_1H_1^\perp+\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_{h\perp}}{M_h}\,g_{1s}D_{1T}^\perp\right.$$

$$+\frac{\boldsymbol{p}_\perp\cdot\boldsymbol{S}_{h\perp}}{M_h}\,\frac{m}{M}\,h_{1s}^\perp D_{1T}^\perp-\frac{\boldsymbol{p}_\perp\cdot\boldsymbol{S}_{h\perp}}{M_h}\,x_{\scriptscriptstyle B}\,g_s^\perp D_{1T}^\perp$$

$$\left.-\boldsymbol{S}_\perp\cdot\boldsymbol{S}_{h\perp}\,\frac{M}{M_h}\,x_{\scriptscriptstyle B}\,g_T'D_{1T}^\perp+\boldsymbol{S}_\perp\cdot\boldsymbol{S}_{h\perp}\,\frac{m}{M_h}\,h_{1T}D_{1T}^\perp\right]$$

$$+i\frac{2\,\hat{t}^{[\mu}p_\perp^{\nu]}}{Q}\,\frac{M_h}{M}\,h_{1s}^\perp\frac{E_s}{z_h}+i\frac{2M\,\hat{t}^{[\mu}S_\perp^{\nu]}}{Q}\,\frac{M_h}{M}\,h_{1T}\frac{E_s}{z_h}$$

$$+i\frac{2M_h\,\hat{t}^{[\mu}S_{h\perp}^{\nu]}}{Q}\left[-\frac{\boldsymbol{k}_\perp^2}{M_h^2}\,g_{1s}D_{1T}^\perp-\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{MM_h}\,\frac{m}{M_h}\,h_{1s}^\perp D_{1T}^\perp+\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{p}_\perp}{M_h^2}\,x_{\scriptscriptstyle B}\,g_s^\perp D_{1T}^\perp\right.$$

$$\left.+\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M_h}\,\frac{M}{M_h}\,x_{\scriptscriptstyle B}\,g_T'D_{1T}^\perp-\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M_h}\,\frac{m}{M_h}\,h_{1T}D_{1T}^\perp\right]$$

$$+i\frac{2\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}k_{\perp\rho}}{Q}\left[f_1\frac{G_s^\perp}{z_h}-f_1G_{1s}+g_{1s}\frac{D^\perp}{z_h}-g_{1s}D_1+\frac{M}{M_h}\,x_{\scriptscriptstyle B}\,eH_{1s}^\perp-\frac{m}{M_h}\,f_1H_{1s}^\perp\right]$$

$$+i\frac{2\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}p_{\perp\rho}}{Q}\left[x_{\scriptscriptstyle B}\,f^\perp G_{1s}+x_{\scriptscriptstyle B}\,g_s^\perp D_1+\frac{M_h}{M}\,h_{1s}^\perp\frac{E}{z_h}-\frac{m}{M}\,h_{1s}^\perp D_1\right]$$

$$+i\frac{2M\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}}{Q}\left[x_{\scriptscriptstyle B}\,g_T'D_1+\frac{M_h}{M}\,h_{1T}\frac{E}{z_h}-\frac{m}{M}\,h_{1T}D_1\right]$$

$$+i\frac{2M_h\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{h\perp\rho}}{Q}\left[f_1\frac{G_T'}{z_h}+\frac{M}{M_h}\,x_{\scriptscriptstyle B}\,eH_{1T}-\frac{m}{M_h}\,f_1H_{1T}\right]\Bigg\},\tag{78}$$

where $\{\mu\nu\}$ indicates symmetrization of indices and $[\mu\nu]$ indicates antisymmetrization. Note that one can make the replacement $\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}a_{\perp\rho}=\epsilon^{\mu\nu\rho\sigma}a_{\perp\rho}\hat{q}_\sigma$. The quark distribution functions in hadron H depend on $x_{\scriptscriptstyle B}$ and $\boldsymbol{p}_T^2$, while the quark fragmentation functions into hadron h depend on z_h and $|\boldsymbol{P}_{h\perp}-z\boldsymbol{p}_T|^2$, the squared perpendicular momentum of hadron h with respect to the quark.

The cross sections are obtained from the hadronic tensor after contraction with the lepton tensor (Eq. 4). These contractions, given in Table I use azimuthal angles in the perpendicular plane (see section 2) defined with respect to the (lepton) scattering plane and the (spacelike) virtual photon momentum,

$$\hat{x}\cdot a_\perp=-\hat{x}\cdot\boldsymbol{a}_\perp=-a_x=-|\boldsymbol{a}_\perp|\cos\phi_a,\tag{79}$$

$$\epsilon_\perp^{\mu\nu}\hat{x}_\mu a_{\perp\nu}\equiv\hat{x}\wedge\boldsymbol{a}_\perp=a_y=|\boldsymbol{a}_\perp|\sin\phi_a,\tag{80}$$

where we have used $\boldsymbol{a}_\perp\wedge\boldsymbol{b}_\perp\equiv\epsilon_\perp^{\rho\sigma}a_{\perp\rho}b_{\perp\sigma}$.

V. RESULTS FOR LEPTOPRODUCTION INTEGRATED OVER TRANSVERSE MOMENTA

After integration over the transverse momenta ($\boldsymbol{P}_{h\perp}=-z_h\boldsymbol{q}_{\scriptscriptstyle T}$) the integrations over $\boldsymbol{k}_{\scriptscriptstyle T}$ and $\boldsymbol{p}_{\scriptscriptstyle T}$ in Eqs 77 and 78 can be performed leading to

$$2M \int d^2 P_{h\perp} \mathcal{W}^{\mu\nu} = 2z_h \left\{ -g_\perp^{\mu\nu} \Big[f_1 D_1 + \lambda \lambda_h \, g_1 G_1 \Big] - \left(S_\perp^{\{\mu} S_{h\perp}^{\nu\}} + (\boldsymbol{S}_\perp \cdot \boldsymbol{S}_{h\perp}) \, g_\perp^{\mu\nu} \right) h_1 H_1 \right.$$

$$+ i \, \epsilon_\perp^{\mu\nu} \Big[f_1 \lambda_h \, G_1 + \lambda \, g_1 D_1 \Big] + \lambda_h \, \frac{2M \, \hat{t}^{\{\mu} S_\perp^{\nu\}}}{Q} \left[x_B \, g_T G_1 + \frac{M_h}{M} \, h_1 \frac{\tilde{H}_L}{z_h} \right]$$

$$+ \lambda \, \frac{2M_h \, \hat{t}^{\{\mu} S_{h\perp}^{\nu\}}}{Q} \left[\frac{M}{M_h} \, x_B \, h_L H_1 + g_1 \frac{\tilde{G}_T}{z_h} \right] + \frac{2M_h \, \hat{t}^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{\perp\rho}}{Q} \, h_1 \frac{\tilde{H}}{z_h}$$

$$+ \frac{2M_h \, \hat{t}^{\{\mu} \epsilon_\perp^{\nu\}\rho} S_{h\perp\rho}}{Q} \, f_1 D_{1T}^{\perp(1)} + i\lambda_h \, \frac{2M\hat{t}^{[\mu} S_\perp^{\nu]}}{Q} \, \frac{M_h}{M} \, h_1 \frac{\tilde{E}_L}{z_h}$$

$$- i\lambda \, \frac{2M_h \, \hat{t}^{[\mu} S_{h\perp}^{\nu]}}{Q} \, g_1 D_{1T}^{\perp(1)} + i \, \frac{2M_h \, \hat{t}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{h\perp\rho}}{Q} \left[\frac{M}{M_h} \, x_B \, eH_1 + f_1 \frac{\tilde{G}_T}{z_h} \right]$$

$$\left. + i \, \frac{2M \, \hat{t}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{\perp\rho}}{Q} \left[x_B \, g_T D_1 + \frac{M_h}{M} \, h_1 \frac{\tilde{E}}{z_h} \right] \right\}, \tag{81}$$

where the quark distribution functions in hadron H depend on x_B, while the quark fragmentation functions into hadron h depend on z_h. The functions indicated with a tilde appear in the quark-quark-gluon correlation functions Φ_A and Δ_A discussed in section 3. These interaction-dependent correlation functions are explicitly given in Appendix C, just as their $\boldsymbol{k}'_T$-integrated results. The function $D_{1T}^{\perp(1)}(z)$ is the $(\boldsymbol{k}_T^2/2M_h^2)$-weighted and $\boldsymbol{k}'_T$-integrated result of $D_{1T}^\perp(z, \boldsymbol{k}'_T)$ (see also Appendix C).

Separating the cross sections into parts for unpolarized (O) and longitudinally polarized (L) leptons and unpolarized (O), longitudinally polarized (L) or transversely polarized (T) hadrons in the initial state one obtains the lepto-production cross section

$$\frac{d\sigma(\vec{\ell}\vec{H} \to \ell'\vec{h}X)}{dx_B \, dy \, dz_h} = \frac{d\sigma_{OO}}{dx_B \, dy \, dz_h} + \frac{d\sigma_{OL}}{dx_B \, dy \, dz_h} + \frac{d\sigma_{OT}}{dx_B \, dy \, dz_h}$$
$$+ \frac{d\sigma_{LO}}{dx_B \, dy \, dz_h} + \frac{d\sigma_{LL}}{dx_B \, dy \, dz_h} + \frac{d\sigma_{LT}}{dx_B \, dy \, dz_h} \tag{82}$$

with (including again the flavor indices)

$$\frac{d\sigma_{OO}}{dx_B \, dy \, dz_h} = \frac{4\pi\alpha^2 s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left\{ \left(\frac{y^2}{2} + 1 - y \right) x_B f_1^a(x_B) \, D_1^a(z_h) \right.$$

$$\left. + 2|\boldsymbol{S}_{h\perp}| \, (2-y)\sqrt{1-y} \, \sin(\phi_s^h) \, \frac{M_h}{Q} \, x_B \, f_1^a(x_B) D_{1T}^{\perp(1) \, a}(z_h) \right\}, \tag{83}$$

$$\frac{d\sigma_{OL}}{dx_B \, dy \, dz_h} = \frac{4\pi\alpha^2 s}{Q^4} \lambda \sum_{a,\bar{a}} e_a^2 \left\{ \lambda_h \left(\frac{y^2}{2} + 1 - y \right) x_B \, g_1^a(x_B) \, G_1^a(z_h) \right.$$

$$\left. - 2|\boldsymbol{S}_{h\perp}| \, (2-y)\sqrt{1-y} \, \cos(\phi_s^h) \left[\frac{M}{Q} \, x_B^2 \, h_L^a(x_B) H_1^a(z_h) + \frac{M_h}{Q} \, x_B \, g_1^a(x_B) \frac{\tilde{G}_T^a(z_h)}{z_h} \right] \right\}, \tag{84}$$

$$\frac{d\sigma_{OT}}{dx_B \, dy \, dz_h} = \frac{4\pi\alpha^2 s}{Q^4} |\boldsymbol{S}_\perp| \sum_{a,\bar{a}} e_a^2 \left\{ 2 \, (2-y)\sqrt{1-y} \, \sin(\phi_s) \, \frac{M_h}{Q} \, x_B \, h_1^a(x_B) \frac{\tilde{H}^a(z_h)}{z_h} \right.$$

$$- 2\lambda_h \, (2-y)\sqrt{1-y} \, \cos(\phi_s) \left[\frac{M}{Q} \, x_B^2 \, g_T^a(x_B) G_1^a(z_h) + \frac{M_h}{Q} \, x_B \, h_1^a(x_B) \frac{\tilde{H}_L^a(z_h)}{z_h} \right]$$

$$\left. - |\boldsymbol{S}_{h\perp}| \, (1-y) \, \cos(\phi_s + \phi_s^h) \, x_B \, h_1^a(x_B) H_1^a(z_h) \right\}, \tag{85}$$

$$\frac{d\sigma_{LO}}{dx_B \, dy \, dz_h} = \frac{4\pi\alpha^2 s}{Q^4} \lambda_e \sum_{a,\bar{a}} e_a^2 \left\{ \lambda_h \, y \left(1 - \frac{y}{2} \right) x_B \, f_1^a(x_B) \, G_1^a(z_h) \right.$$

$$-2\left|\boldsymbol{S}_{h\perp}\right|y\sqrt{1-y}\,\cos(\phi_s^h)\left[\frac{M}{Q}\,x_B^2\,e^a(x_B)H_1^a(z_h)+\frac{M_h}{Q}\,x_B\,f_1^a(x_B)\frac{\tilde{G}_T^a(z_h)}{z_h}\right]\Bigg\},\qquad(86)$$

$$\frac{d\sigma_{LL}}{dx_B\,dy\,dz_h}=\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,\lambda\sum_{a,\bar{a}}e_a^2\left\{y\left(1-\frac{y}{2}\right)x_B g_1^a(x_B)\,D_1^a(z_h)\right.$$

$$\left.+2\left|\boldsymbol{S}_{h\perp}\right|y\sqrt{1-y}\,\sin(\phi_s^h)\,\frac{M_h}{Q}\,x_B\,g_1^a(x_B)D_{1T}^{\perp(1)\,a}(z_h)\right\},\qquad(87)$$

$$\frac{d\sigma_{LT}}{dx_B\,dy\,dz_h}=\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\left|\boldsymbol{S}_\perp\right|\sum_{a,\bar{a}}e_a^2\left\{-2\,y\sqrt{1-y}\,\cos(\phi_s)\left[\frac{M}{Q}\,x_B^2\,g_T^a(x_B)D_1^a(z_h)+\frac{M_h}{Q}\,x_B\,h_1^a(x_B)\frac{\tilde{E}^a(z_h)}{z_h}\right]\right.$$

$$\left.-2\,\lambda_h\,y\sqrt{1-y}\,\sin(\phi_s)\,\frac{M_h}{Q}\,x_B\,h_1^a(x_B)\frac{\tilde{E}_L^a(z_h)}{z_h}\right\}.\qquad(88)$$

The familiar inclusive cross section is easily obtained by using the results for the fragmentation function of a quark into a quark and summing over the quark spins,

$$\frac{d\sigma(\vec{\ell}\vec{H}\to\ell'X)}{dx_B\,dy}=\frac{d\sigma_{OO}}{dx_B\,dy}+\frac{d\sigma_{LL}}{dx_B\,dy}+\frac{d\sigma_{LT}}{dx_B\,dy}\qquad(89)$$

with

$$\frac{d\sigma_{OO}}{dx_B\,dy}=\frac{4\pi\alpha^2\,s}{Q^4}\sum_{a,\bar{a}}e_a^2\left[\frac{y^2}{2}+1-y\right]x_B f_1^a(x_B)\qquad(90)$$

$$\frac{d\sigma_{LL}}{dx_B\,dy}=\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,\lambda\sum_{a,\bar{a}}e_a^2\,y\left(1-\frac{y}{2}\right)x_B g_1^a(x_B)\qquad(91)$$

$$\frac{d\sigma_{LT}}{dx_B\,dy}=-\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\left|\boldsymbol{S}_\perp\right|\sum_{a,\bar{a}}e_a^2\,2\,y\sqrt{1-y}\,\cos(\phi_s)\,\frac{M}{Q}\,x_B^2\,g_T^a(x_B).\qquad(92)$$

These inclusive results lead to the tree-level results for the structure functions $F_1(x_B,Q^2)$, $F_2(x_B,Q^2)$, $g_1(x_B,Q^2)$ and $g_2(x_B,Q^2)$ which were given in the Introduction. Comparing semi-inclusive and inclusive results one finds the familiar result for the number of produced particles

$$N_h(x_B,z_h)=\frac{\frac{d\sigma_{OO}}{dx_B\,dy\,dz_h}}{\frac{d\sigma_{OO}}{dx_B\,dy}}=\frac{\sum_{a,\bar{a}}e_a^2\,f_1^a(x_B)\,D_1^a(z_h)}{\sum_{a,\bar{a}}e_a^2\,f_1^a(x_B)}.\qquad(93)$$

Asymmetries in the number of produced particles for polarized leptons and/or polarized target hadrons are obtained from the cross sections by putting S_h to zero,

$$N_h(x_B,z_h;\lambda_e;\lambda,\boldsymbol{S}_\perp)=N_h(x_B,z_h)\left[1+\left|\boldsymbol{S}_\perp\right|\sin(\phi_s)\,\Delta N_{h,OT}+\lambda_e\lambda\,\Delta N_{h,LL}+\lambda_e\left|\boldsymbol{S}_\perp\right|\cos(\phi_s)\,\Delta N_{h,LT}\right],\qquad(94)$$

involving the nonvanishing ones of the following asymmetries,

$$\Delta N_{h,OL}(x_B,z_h)=\frac{\sigma(\ell\,\vec{H})-\sigma(\ell\,\overleftarrow{H})}{\sigma(\ell\,\vec{H})+\sigma(\ell\,\overleftarrow{H})}=\frac{\sigma_{OL}}{\sigma_{OO}}=0,\qquad(95)$$

$$\Delta N_{h,OT}(x_B,z_h)=\frac{\sigma(\ell\,\vec{H})-\sigma(\ell\,\overleftarrow{H})}{\sigma(\ell\,\vec{H})+\sigma(\ell\,\overleftarrow{H})}=\frac{\sigma_{OT}}{\sigma_{OO}}=\frac{2(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2}\,\frac{M_h}{Q}\,\frac{\sum_{a,\bar{a}}e_a^2\,h_1^a(x_B)\,\tilde{H}^a(z_h)/z_h}{\sum_{a,\bar{a}}e_a^2\,f_1^a(x_B)\,D_1^a(z_h)},\qquad(96)$$

$$\Delta N_{h,LO}(x_B,z_h)=\frac{\sigma(\vec{\ell}\,H)-\sigma(\overleftarrow{\ell}\,H)}{\sigma(\vec{\ell}\,H)+\sigma(\overleftarrow{\ell}\,H)}=\frac{\sigma_{LO}}{\sigma_{OO}}=0,\qquad(97)$$

$$\Delta N_{h,LL}(x_B,z_h)=\frac{\sigma(\vec{\ell}\,\vec{H})-\sigma(\overleftarrow{\ell}\,\vec{H})-\sigma(\vec{\ell}\,\overleftarrow{H})+\sigma(\overleftarrow{\ell}\,\overleftarrow{H})}{\sigma(\vec{\ell}\,\vec{H})+\sigma(\overleftarrow{\ell}\,\vec{H})+\sigma(\vec{\ell}\,\overleftarrow{H})+\sigma(\overleftarrow{\ell}\,\overleftarrow{H})}=\frac{\sigma_{LL}}{\sigma_{OO}}=\frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2}\,\frac{\sum_{a,\bar{a}}e_a^2\,g_1^a(x_B)\,D_1^a(z_h)}{\sum_{a,\bar{a}}e_a^2\,f_1^a(x_B)\,D_1^a(z_h)},\qquad(98)$$

$$\Delta N_{h,LT}(x_B, z_h) = \frac{\sigma(\overrightarrow{\ell}\,\overrightarrow{H}) - \sigma(\overleftarrow{\ell}\,\overrightarrow{H}) - \sigma(\overrightarrow{\ell}\,\overleftarrow{H}) + \sigma(\overleftarrow{\ell}\,\overleftarrow{H})}{\sigma(\overrightarrow{\ell}\,\overrightarrow{H}) + \sigma(\overleftarrow{\ell}\,\overrightarrow{H}) + \sigma(\overrightarrow{\ell}\,\overleftarrow{H}) + \sigma(\overleftarrow{\ell}\,\overleftarrow{H})} = \frac{\sigma_{LT}}{\sigma_{OO}}$$

$$= \frac{2(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \left[\frac{M}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, x_B\, g_T^a(x_B)\, D_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)\, D_1^a(z_h)} + \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, x_B\, h_1^a(x_B)\, \tilde{E}^a(z_h)/z_h}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)\, D_1^a(z_h)} \right]. \quad (99)$$

For the double asymmetries, $\Delta N_{h,LL}$ and $\Delta N_{h,LT}$ it is (at least in principle) sufficient to consider either the asymmetry for the electron spins or for the target spins since $\Delta N_{h,OL} = \Delta N_{h,LO} = 0$. Although $\Delta N_{h,OT} \neq 0$, the asymmetries in $\Delta N_{h,OT}$ and $\Delta N_{h,LT}$ involve different transverse directions. Of the asymmetries in the number of produced hadrons only $\Delta N_{h,LL}$ is leading. The other asymmetries are proportional to $1/Q$. For a target with transverse polarization in the scattering plane the result is proportional to the transverse spin distribution h_1^a and the twist-three fragmentation function $\tilde{H}^a$. For a target with transverse polarization orthogonal to the scattering plane the result contains the product of the twist-three distribution g_T^a with the ordinary fragmentation function D_1^a and the product of the transverse spin distribution h_1^a with the twist-three fragmentation function $\tilde{E}^a$ [19]. We note that in the naive parton model approach the tilde functions do not appear at tree level.

Measuring polarization in the final state, gives several new possibilities to study quark-quark and quark-quark-gluon correlation functions. As discussed in Appendix A the coefficient of S_h in $d\sigma/dx_B dy dz_h$ determines the polarization. It is convenient to parametrize the polarization vector $\mathcal{P}_h^\mu$ in the same way as S_h^μ (Eq. 55) with lightcone helicity Λ_h and transverse polarization $\mathcal{P}_{h\perp}$. The polarization induced in the final state is

$$\Lambda_h N_h(x_B, z_h; \lambda_e; \lambda, \boldsymbol{S}_\perp) = \lambda \frac{\sum_{a,\bar{a}} e_a^2\, g_1^a(x_B)\, G_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} + \lambda_e \frac{y(1-\frac{1}{2}y)}{1-y+\frac{1}{2}y^2} \frac{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)\, G_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)}$$

$$- \boldsymbol{S}_x \frac{2(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \left[\frac{M}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, x_B\, g_T^a(x_B) G_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} + \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, h_1^a(x_B)\tilde{H}_L^a(z_h)/z_h}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} \right]$$

$$- \boldsymbol{S}_y \frac{2y\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, h_1^a(x_B)\, \tilde{E}_L^a(z_h)/z_h}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)}, \quad (100)$$

$$\mathcal{P}_{hx} N_h(x_B, z_h; \lambda_e; \lambda, \boldsymbol{S}_\perp) = -\boldsymbol{S}_x \frac{1-y}{1-y+\frac{1}{2}y^2} \frac{\sum_{a,\bar{a}} e_a^2\, h_1^a(x_B)\, H_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)}$$

$$- \lambda \frac{2(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \left[\frac{M}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, x_B\, h_L^a(x_B)H_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} + \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, g_1^a(x_B)\tilde{G}_T^a(z_h)/z_h}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} \right]$$

$$- \lambda_e \frac{2y\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \left[\frac{M}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, x_B\, e^a(x_B)H_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} + \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)\tilde{G}_T^a(z_h)/z_h}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)} \right], \quad (101)$$

$$\mathcal{P}_{hy} N_h(x_B, z_h; \lambda_e; \lambda, \boldsymbol{S}_\perp) = \boldsymbol{S}_y \frac{1-y}{1-y+\frac{1}{2}y^2} \frac{\sum_{a,\bar{a}} e_a^2\, h_1^a(x_B)\, H_1^a(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)}$$

$$+ \frac{2(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)\, D_{1T}^{\perp(1)\,a}(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)}$$

$$+ \lambda_e \lambda \frac{2y\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \frac{M_h}{Q} \frac{\sum_{a,\bar{a}} e_a^2\, g_1^a(x_B)\, D_{1T}^{\perp(1)\,a}(z_h)}{\sum_{a,\bar{a}} e_a^2\, f_1^a(x_B)}. \quad (102)$$

In leading order the final state polarization is proportional to the corresponding polarization (longitudinal or transverse) of the target hadron and the polarization (longitudinal) of the lepton. The transfer parameters involve the leading distribution functions f_1^a, g_1^a and h_1^a and the leading fragmentation functions G_1^a and H_1^a. For instance, the measurement of transversely polarized Λ's has been proposed [10] as a way to obtain the transverse spin distribution h_1^a.

In the complete result up to order $1/Q$ final state polarization arises from several other polarizations in the initial state. Especially noteworthy is the (transverse) polarization in the production of a spin $1/2$ particle (e.g. a Λ) for an unpolarized initial state, the term involving $f_1^a(x_B)\, D_{1T}^{\perp(1)\,a}(z_h)$ in Eq. 102. This polarization is purely transverse, orthogonal to the lepton scattering plane. It involves the (interaction-dependent) time-reversal odd fragmentation function $D_{1T}^{\perp a}$.

At order $1/Q$ one finds also transverse polarization in the production of hadrons, coming from longitudinal polarization in the initial state or vice versa. This offers new possibilities to obtain twist-three distribution functions such as $g_T^a(x)$ and also $h_L^a(x)$. For the latter this semi-inclusive polarization transfer measurement is the equivalent of a double-spin asymmetry measurement in Drell-Yan scattering [19,20]. We note the presence of several interaction-dependent functions which complicate the naive parton picture for these observables.

VI. AZIMUTHAL ASYMMETRIES

The transverse momentum dependence in Eqs 77 and 78 lead after integration over $\boldsymbol{k}_T$ and $\boldsymbol{p}_T$ to a dependence on $\boldsymbol{q}_T = -\boldsymbol{P}_{h\perp}/z$. Furthermore transverse directions appear in the spin vectors. It is useful to project the momenta $\boldsymbol{p}_\perp$ and $\boldsymbol{k}_\perp$ onto the direction $\hat{\boldsymbol{h}}$,

$$a_\perp^\mu = (\hat{\boldsymbol{h}} \cdot \boldsymbol{a}_\perp)\,\hat{h}^\mu + (\hat{\boldsymbol{h}} \wedge \boldsymbol{a}_\perp)\,\epsilon_\perp^{\mu\rho}\,\hat{h}_\rho, \tag{103}$$

$$\epsilon_\perp^{\mu\rho} a_{\perp\rho} = -(\hat{\boldsymbol{h}} \wedge \boldsymbol{a}_\perp)\,\hat{h}^\mu + (\hat{\boldsymbol{h}} \cdot \boldsymbol{a}_\perp)\,\epsilon_\perp^{\mu\rho}\,\hat{h}_\rho. \tag{104}$$

We will not consider polarization in the final state. The result for the symmetric and antisymmetric parts in the hadronic tensor are,

$$
\begin{aligned}
2M\,\mathcal{W}_S^{\mu\nu} = 2z_h \Bigg\{ &-g_\perp^{\mu\nu}\,I[f_1 D_1] \\
&+\frac{2\hat{t}^{\{\mu}\hat{h}_\perp^{\nu\}}}{Q}\left(I\left[(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp)\,f_1\left(\frac{D^\perp}{z_h} - D_1\right)\right] + x_{\scriptscriptstyle B}\,I\left[(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)\,f^\perp D_1\right]\right) \\
&-\lambda\Bigg\{\hat{h}^{\{\mu}\epsilon_\perp^{\nu\}\rho}\hat{h}_\rho\,I\left[\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp)(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp) - \boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp}{MM_h}\,h_{1L}^\perp H_1^\perp\right] \\
&\qquad -2\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho}\hat{h}_\rho\left(\frac{M}{Q}\,I\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp}{M_h}\left(x_{\scriptscriptstyle B}\,h_L - \frac{m}{M}\,g_{1L}\right)H_1^\perp\right]\right. \\
&\qquad\qquad \left.+\frac{M_h}{Q}\,I\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp}{M}\,h_{1L}^\perp\left(\frac{H}{z_h} + \frac{\boldsymbol{k}_\perp^2}{M_h^2}\,H_1^\perp\right)\right]\right)\Bigg\} \\
&-\frac{1}{2}\left(\hat{h}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{\perp\rho} + S_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}\hat{h}_\rho\right)I\left[\frac{(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp)}{M_h}\,h_1 H_1^\perp\right] \\
&-\hat{h}^{\{\mu}\epsilon_\perp^{\nu\}\rho}\hat{h}_\rho\,(\hat{\boldsymbol{h}} \cdot \boldsymbol{S}_\perp)\,I\left[\frac{4(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)^2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp) - \boldsymbol{p}_\perp^2\,(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp) - 2(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)(\boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp)}{2\,M^2\,M_h}\,h_{1T}^\perp H_1^\perp\right] \\
&+\left(2\,\hat{h}^\mu\hat{h}^\nu + g_\perp^{\mu\nu}\right)(\hat{\boldsymbol{h}} \wedge \boldsymbol{S}_\perp) \\
&\qquad\times I\left[\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)(\boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp) + \boldsymbol{p}_\perp^2\,(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp) - 4(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)^2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp)}{2\,M^2\,M_h}\,h_{1T}^\perp H_1^\perp\right] \\
&+2\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{\perp\rho}\left(\frac{M}{Q}\,I\left[\frac{\boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp}{2MM_h}\left(x_{\scriptscriptstyle B}h_T - \frac{m}{M}\,g_{1T} - x_{\scriptscriptstyle B}h_T^\perp\right)H_1^\perp\right]\right. \\
&\qquad\qquad \left.+\frac{M_h}{Q}\,I\left[h_1\left(\frac{H}{z_h} + \frac{\boldsymbol{k}_\perp^2}{M_h^2}\,H_1^\perp\right)\right]\right) \\
&+2\hat{t}^{\{\mu}\epsilon_\perp^{\nu\}\rho}\hat{h}_\rho\,(\hat{\boldsymbol{h}} \cdot \boldsymbol{S}_\perp)\left(\frac{M_h}{Q}\,I\left[\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)^2 - \boldsymbol{p}_\perp^2}{2M^2}\,h_{1T}^\perp\left(\frac{H}{z_h} + \frac{\boldsymbol{k}_\perp^2}{M_h^2}\,H_1^\perp\right)\right]\right. \\
&\qquad\qquad \left.+\frac{M}{Q}\,I\left[\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp)(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp) - \boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp}{2MM_h}\left(x_{\scriptscriptstyle B}h_T - \frac{m}{M}\,g_{1T} + x_{\scriptscriptstyle B}h_T^\perp\right)H_1^\perp\right]\right) \\
&+2\hat{t}^{\{\mu}\hat{h}^{\nu\}}(\hat{\boldsymbol{h}} \wedge \boldsymbol{S}_\perp)\left(\frac{M_h}{Q}\,I\left[\frac{2(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)^2 - \boldsymbol{p}_\perp^2}{2M^2}\,h_{1T}^\perp\left(\frac{H}{z_h} + \frac{\boldsymbol{k}_\perp^2}{M_h^2}\,H_1^\perp\right)\right]\right)
\end{aligned}
$$

$$- \frac{M}{Q} I\Big[\frac{\boldsymbol{k}_\perp \cdot \boldsymbol{p}_\perp - 2\,(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_\perp)(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp)}{2MM_h}\,\Big(x_B h_T - \frac{m}{M}\,g_{1T} + x_B h_T^\perp\Big)\,H_1^\perp\Big]\Big)\Big\} \qquad (105)$$

and

$$\begin{aligned}
2M\,\mathcal{W}_A^{\mu\nu} = 2\,z_h\,\Big\{\;& i\,\lambda\,\epsilon_\perp^{\mu\nu}\,I\Big[g_{1L}D_1\Big] + i\,\epsilon_\perp^{\mu\nu}\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{S}_\perp)\,I\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{M}\,g_{1T}D_1\Big]\\[4pt]
& -2i\,\hat{t}^{[\mu}\hat{h}^{\nu]}\,\frac{M}{Q}\,I\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M_h}\,\Big(x_B\,e - \frac{m}{M}\,f_1\Big)\,H_1^\perp\Big]\\[4pt]
& +2i\,\lambda\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}\hat{h}_\rho\,\Big(\frac{M_h}{Q}\,I\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M_h}\,g_{1L}\,\Big(\frac{D^\perp}{z_h} - D_1\Big)\Big] + \frac{M}{Q}\,I\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{M}\,x_B g_L^\perp D_1\Big]\\[4pt]
& \qquad\qquad + \frac{M_h}{Q}\,I\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{M}\,h_{1L}^\perp\,\Big(\frac{E}{z_h} - \frac{m}{M}D_1\Big)\Big]\Big)\\[4pt]
& +2i\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}\hat{h}_\rho\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{S}_\perp)\,\Big(\frac{M_h}{Q}\,I\Big[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp) - \boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp}{2MM_h}\,g_{1T}\,\Big(\frac{D^\perp}{z_h} - D_1\Big)\Big]\\[4pt]
& \qquad\qquad + \frac{M}{Q}\,I\Big[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)^2 - \boldsymbol{p}_\perp^2}{2M^2}\,x_B g_T^\perp D_1\Big]\\[4pt]
& \qquad\qquad + \frac{M_h}{Q}\,I\Big[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)^2 - \boldsymbol{p}_\perp^2}{2M^2}\,h_{1T}^\perp\,\Big(\frac{E}{z_h} - \frac{m}{M}D_1\Big)\Big]\Big)\\[4pt]
& +2i\,\hat{t}^{[\mu}\hat{h}^{\nu]}\,(\hat{\boldsymbol{h}}\wedge\boldsymbol{S}_\perp)\,\Big(\frac{M_h}{Q}\,I\Big[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp) - \boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp}{2MM_h}\,g_{1T}\,\Big(\frac{D^\perp}{z_h} - D_1\Big)\Big]\\[4pt]
& \qquad\qquad + \frac{M}{Q}\,I\Big[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)^2 - \boldsymbol{p}_\perp^2}{2M^2}\,x_B g_T^\perp D_1\Big]\\[4pt]
& \qquad\qquad + \frac{M_h}{Q}\,I\Big[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)^2 - \boldsymbol{p}_\perp^2}{2M^2}\,h_{1T}^\perp\,\Big(\frac{E}{z_h} - \frac{m}{M}D_1\Big)\Big]\Big)\\[4pt]
& +2i\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\,\Big(\frac{M}{Q}\,I\Big[x_B g_T D_1\Big] + \frac{M_h}{Q}\,I\Big[h_1\,\Big(\frac{E}{z_h} - \frac{m}{M}D_1\Big)\Big]\\[4pt]
& \qquad\qquad + \frac{M_h}{Q}\,I\Big[\frac{\boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp}{2MM_h}\,g_{1T}\,\Big(\frac{D^\perp}{z_h} - D_1\Big)\Big]\Big)\Big\},
\end{aligned} \qquad (106)$$

with (including again flavor indices) convolution integrals of the type

$$I\Big[(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp)\,f\,D\Big]\,(x_B, z_h, Q_T) = \sum_{a,\bar{a}} e_a^2 \int d^2\boldsymbol{p}_T\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,f^a(x_B, \boldsymbol{p}_T^2)D^a(z_h, |\boldsymbol{P}_{h\perp} - z_h\boldsymbol{p}_T|^2). \qquad (107)$$

For a gaussian transverse momentum dependence the relevant integrals are evaluated in Appendix D. In principle one can now identify structure functions by looking at the most general expression for the hadronic tensor [21]. We will omit this step. Rather, we immediately give the cross section following from the above tensor. We will use the results for gaussian transverse momentum dependence. The complete result for the cross section is again written as a sum over terms depending on lepton (O and L) and target hadron (O, L and T) polarizations. The contributions that are nonzero in leading order in $1/Q$ are

$$\frac{d\sigma_{OO}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2\,s}{Q^4} \sum_{a,\bar{a}} e_a^2\,\Big(\frac{y^2}{2} + 1 - y\Big)\,x_B f_1^a(x_B)\,D_1^a(z_h)\,\frac{\mathcal{G}(Q_T;R)}{z_h^2}, \qquad (108)$$

$$\frac{d\sigma_{OL}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha^2\,s}{Q^4}\,\lambda \sum_{a,\bar{a}} e_a^2\,(1-y)\,\sin(2\phi_h)\,\frac{Q_T^2\,R^4}{MM_h\,R_H^2\,R_h^2}\,x_B h_{1L}^{\perp\,a}(x_B)H_1^{\perp\,a}(z_h)\,\frac{\mathcal{G}(Q_T;R)}{z_h^2}, \qquad (109)$$

$$\frac{d\sigma_{OT}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha^2 \, s}{Q^4} \, |\boldsymbol{S}_\perp| \sum_{a,\bar{a}} e_a^2 \left\{ (1-y) \sin(\phi_h + \phi_s) \frac{Q_T \, R^2}{M_h \, R_h^2} \, x_B h_1^a(x_B) H_1^{\perp \, a}(z_h) \right.$$

$$\left. + (1-y) \sin(3\phi_h - \phi_s) \frac{Q_T^3 \, R^6}{2M^2 M_h \, R_H^4 \, R_h^2} \, x_B h_{1T}^{\perp a}(x_B) H_1^{\perp \, a}(z_h) \right\} \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \qquad (110)$$

$$\frac{d\sigma_{LL}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2 \, s}{Q^4} \, \lambda_e \lambda \sum_{a,\bar{a}} e_a^2 \, y \left(1 - \frac{y}{2}\right) \, x_B \, g_{1L}^a(x_B) \, D_1^a(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \qquad (111)$$

$$\frac{d\sigma_{LT}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2 \, s}{Q^4} \, \lambda_e \, |\boldsymbol{S}_\perp| \sum_{a,\bar{a}} e_a^2 \, y \left(1 - \frac{y}{2}\right) \, \cos(\phi_h - \phi_s) \frac{Q_T \, R^2}{M \, R_H^2} \, x_B \, g_{1T}^a(x_B) D_1^a(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \quad (112)$$

where $\mathcal{G}(Q_T; R) = (R^2/\pi) \exp(-Q_T^2 R^2)$, i.e., a gaussian of which the fall-off is determined by a radius R. This radius is related to the radii R_H and R_h governing the fall-off of $f(x, \boldsymbol{k}_T^2)$ and $D(z, \boldsymbol{k}_T'^2)$ as $R^2 = R_H^2 R_h^2 / (R_H^2 + R_h^2)$. These radii again may depend on the longitudinal momentum and on the specific function, i.e., $R_H = R_H^f(x_B)$ and $R_h = R_h^D(z_h)$. Note that in leading order $\sigma_{LO} = 0$. These results have been discussed in Refs [22] and [23]. The possibility to measure h_1^a by considering asymmetry in the production of hadrons (e.g. pions), the first term in Eq. 110, was first discussed by Collins [24]. The complete results show that the measurements of specific azimuthal asymmetries in deep-inelastic lepton-hadron scattering with polarized beam and target allow the measurement of all six (x- and k_T-dependent) quark distributions, of course keeping in mind that also a separation of the different flavors is needed. For obtaining a first indication for the behavior of the results, the dominance of u-quarks in the proton, especially at moderate x-values will be helpful. In all the expressions only two fragmentation functions play a role. The first is the 'ordinary' fragmentation function D_1^a, the second is the time-reversal odd function $H_1^{\perp a}$.

In principle the general expression for the hadronic tensor in the beginning of this section allows the calculation of all azimuthal asymmetries in the cross section that contribute up to order $1/Q$. Below, we give first the results for unpolarized spin 1/2 targets, which for unpolarized leptons is the extension of Eq. 108,

$$\frac{d\sigma_{OO}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2 \, s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left\{ \left(\frac{y^2}{2} + 1 - y \right) \, x_B f_1^a(x_B) \, D_1^a(z_h) \right.$$

$$\left. - 2(2-y)\sqrt{1-y} \cos(\phi_h) \frac{Q_T}{Q} \left(\frac{R^2}{R_H^2} x_B^2 f^{\perp \, a}(x_B) D_1^a(z_h) - \frac{R^2}{R_h^2} x_B f_1^a(x_B) \frac{\tilde{D}^{\perp \, a}(z_h)}{z_h} \right) \right\} \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \quad (113)$$

$$\frac{d\sigma_{LO}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha^2 \, s}{Q^4} \, \lambda_e \sum_{a,\bar{a}} e_a^2 \, 2y\sqrt{1-y} \, \sin\phi_h \frac{Q_T}{Q} \frac{M \, R^2}{M_h \, R_h^2} \, x_B^2 \, \tilde{e}^a(x_B) \, H_1^{\perp \, a}(z_h) \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \quad (114)$$

The $\langle \cos(\phi_h) \rangle$ asymmetry in unpolarized leptoproduction, unfortunately is rather complicated, involving one twist-three distribution function ($f^{\perp a}$) and one twist-three fragmentation function ($D^{\perp a}$) [18]. It is important to point out, however, that the $\langle \cos(\phi_h) \rangle$ asymmetry is not only a kinematical effect. It reduces to a kinematical factor only depending on y and Q^2 when the interaction-dependent pieces in the twist-three functions are set to zero, $\tilde{f}^{\perp a} = 0$ and $\tilde{D}^{\perp a} = 0$, implying $f^{\perp a} = f_1^a/x_B$ and $D^{\perp a} = z_h D_1^{\perp a}$. At order $1/Q$ there is no $\langle \cos(2\phi_h) \rangle$ asymmetry in the deep-inelastic leptoproduction cross section. For polarized leptons and unpolarized targets a $\langle \sin(\phi_h) \rangle$ asymmetry is found [25], involving the interaction-dependent part of the distribution function e^a and the time-reversal odd fragmentation function $H_1^{\perp a}$. Noteworthy is that it is the same fragmentation function that appears in several of the leading azimuthal asymmetries for polarized targets.

As indicated, the above asymmetries have been discussed in various papers. The remaining asymmetries at order $1/Q$, i.e. those for polarized targets, can also be extracted from the general result at the beginning of this section. They are

$$\frac{d\sigma_{OL}}{dx_B \, dy \, dz_h \, d^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha^2 \, s}{Q^4} \, \lambda \sum_{a,\bar{a}} e_a^2 \left\{ (1-y) \sin(2\phi_h) \frac{Q_T^2 \, R^4}{M M_h \, R_H^2 R_h^2} \, x_B h_{1L}^{\perp a}(x_B) H_1^{\perp \, a}(z_h) \right.$$

$$- 2(2-y)\sqrt{1-y} \sin(\phi_h) \frac{Q_T}{Q} \left(\frac{R^6}{M M_h \, R_H^4 R_h^4} \left(\frac{R_h^2 - R_H^2}{R^2} - Q_T^2 R_h^2 \right) x_B h_{1L}^{\perp a}(x_B) \, H_1^{\perp \, a}(z_h) \right.$$

$$\left. \left. + \frac{M \, R^2}{M_h \, R_h^2} x_B^2 \tilde{h}_L^a(x_B) \, H_1^{\perp \, a}(z_h) - \frac{M_h \, R^2}{M \, R_H^2} x_B h_{1L}^{\perp a}(x_B) \frac{\tilde{H}^a(z_h)}{z} \right) \right\} \frac{\mathcal{G}(Q_T; R)}{z_h^2}, \qquad (115)$$

$$\frac{d\sigma_{OT}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha^2\,s}{Q^4}\,|\boldsymbol{S}_\perp|\sum_{a,\bar{a}}e_a^2\left\{(1-y)\,\sin(\phi_h+\phi_s)\frac{Q_T\,R^2}{M_h\,R_h^2}\,x_B h_1^a(x_B)H_1^{\perp\,a}(z_h)\right.$$

$$+(1-y)\,\sin(3\phi_h-\phi_s)\frac{Q_T^3\,R^6}{2M^2M_h\,R_H^4\,R_h^2}\,x_B h_{1T}^{\perp\,a}(x_B)H_1^{\perp\,a}(z_h)$$

$$-2(2-y)\sqrt{1-y}\,\sin(\phi_s)\left(\frac{Q_T}{Q}\,\frac{Q_T\,R^8}{2M^2M_h\,R_H^6R_h^4}\left[\frac{2R_H^2-R_h^2}{R^2}+Q_T^2R_h^2\right]x_B h_{1T}^{\perp\,a}(x_B)\,H_1^{\perp\,a}(z_h)\right.$$

$$\left.+\frac{M_h}{Q}\,x_B h_1^a(x_B)\frac{\tilde{H}^a(z_h)}{z_h}-\frac{Q_T}{Q}\,\frac{Q_T\,R^4}{2M_h\,R_H^2R_h^2}x_B^2\left(\tilde{h}_T^a(x_B)+\tilde{h}_T^{\perp\,a}(x_B)\right)H_1^{\perp\,a}(z_h)\right)$$

$$+2(2-y)\sqrt{1-y}\,\sin(2\phi_h-\phi_s)\left(\frac{Q_T}{Q}\,\frac{Q_T\,R^4}{2M_h\,R_H^2R_h^2}\,x_B\left(2h_1^a(x_B)-x_B\tilde{h}_{1L}^{\perp\,a}(x_B)\right)H_1^{\perp\,a}(z_h)\right.$$

$$\left.\left.-\frac{M_h}{Q}\,\frac{Q_T^2\,R^4}{2M^2\,R_H^4}\,x_B h_{1T}^{\perp\,a}(x_B)\frac{\tilde{H}^a(z_h)}{z_h}\right)\right\}\frac{\mathcal{G}(Q_T;R)}{z_h^2},\qquad (116)$$

$$\frac{d\sigma_{LL}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\lambda\sum_{a,\bar{a}}e_a^2\left\{y\left(1-\frac{y}{2}\right)x_B\,g_{1L}^a(x_B)\,D_1^a(z_h)\right.$$

$$-2y\sqrt{1-y}\,\cos(\phi_h)\frac{Q_T}{Q}\left(\frac{R^2}{R_H^2}\,x_B^2 g_L^{\perp\,a}(x_B)D_1^a(z_h)-\frac{R^2}{R_h^2}\,x_B\,g_{1L}^a(x_B)\frac{\tilde{D}^{\perp\,a}(z_h)}{z_h}\right.$$

$$\left.\left.+\frac{M_h\,R^2}{M\,R_H^2}\,x_B h_{1L}^{\perp\,a}(x_B)\frac{\tilde{E}^a(z_h)}{z_h}\right)\right\}\frac{\mathcal{G}(Q_T;R)}{z_h^2},\qquad (117)$$

$$\frac{d\sigma_{LT}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_{h\perp}} = \frac{4\pi\alpha^2\,s}{Q^4}\,\lambda_e\,|\boldsymbol{S}_\perp|\sum_{a,\bar{a}}e_a^2\left\{y\left(1-\frac{y}{2}\right)\cos(\phi_h-\phi_s)\frac{Q_T\,R^2}{M\,R_H^2}\,x_B\,g_{1T}^a(x_B)D_1^a(z_h)\right.$$

$$-2y\sqrt{1-y}\,\cos(\phi_s)\left(\frac{M}{Q}\,x_B^2 g_T^a(x_B)D_1^a(z_h)+\frac{M_h}{Q}\,x_B h_1^a(x_B)\frac{\tilde{E}^a(z_h)}{z_h}\right.$$

$$\left.+\frac{M_h}{Q}\,\frac{R^2}{2MM_h\,R_H^2R_h^2}\left(1-Q_T^2\,R^2\right)x_B g_{1T}^a(x_B)\frac{\tilde{D}^{\perp\,a}(z_h)}{z_h}\right)$$

$$-2y\sqrt{1-y}\,\cos(2\phi_h-\phi_s)\frac{Q_T}{Q}\left(\frac{Q_T\,R^4}{2M\,R_H^4}\,x_B^2 g_T^{\perp\,a}(x_B)D_1^a(z_h)+\frac{Q_T\,R^4}{2M\,R_H^4}\,\frac{M_h}{M}\,x_B h_{1T}^{\perp\,a}(x_B)\frac{\tilde{E}^a(z_h)}{z_h}\right.$$

$$\left.\left.-\frac{Q_T\,R^4}{2M\,R_H^2R_h^2}\,x_B g_{1T}^a\frac{\tilde{D}^{\perp\,a}(z_h)}{z_h}\right)\right\}\frac{\mathcal{G}(Q_T;R)}{z_h^2}.\qquad (118)$$

Because the results involve mostly higher harmonics in the ϕ_h-dependence and involve most of the allowed combinations of twist-two and twist-three distribution and fragmentation functions discussed in section 3, we do not expect that these results soon will be used to extract the new information they contain on the structure of the nucleon, specifically the quark-quark-gluon correlations contained in the interaction dependent pieces. In first instance, the result have been given because they can be used to estimate azimuthal asymmetries. Such estimates may be necessary in the absence of full azimuthal coverage. We note that the estimate of the asymmetries in the naive parton model is obtained by setting all interaction-dependent pieces (functions with a tilde) to zero. In that case the twist-three functions can be expressed in the twist-two ones. The explicit relations are given in Appendix C. Note, however, that although the functions appear in the $1/Q$ parts of the cross section, there is no reason to expect that the interaction-dependent functions themselves are smaller than the twist-two functions. They are just different matrix elements involving gluon fields.

Finally, we want to discuss the case in which only the azimuthal direction of the current jet is detected, without an analysis of this jet. The measurement of the (small) transverse momentum of the current jet allows (in principle) a direct study of the transverse momentum of the distribution functions. The starting point are Eqs 77 and 78 in which one uses the quark $\rightarrow$ quark fragmentation function. The nonvanishing contributions in the cross section (with $\boldsymbol{p}_\perp$ being the jet transverse momentum involving azimuthal angle ϕ_j) are

$$\frac{d\sigma_{OO}}{dx_B \, dy \, d^2\boldsymbol{p}_\perp} = \frac{4\pi\alpha^2 \, s}{Q^4} \sum_{a,\bar{a}} e_a^2 \left\{ \left(\frac{y^2}{2} + 1 - y \right) \, x_B f_1^a(x_B, \boldsymbol{p}_\perp^2) \right.$$

$$\left. -2(2-y)\sqrt{1-y} \, \cos(\phi_j) \frac{|\boldsymbol{p}_\perp|}{Q} \, x_B^2 f^{\perp \, a}(x_B, \boldsymbol{p}_\perp^2) \right\}, \tag{119}$$

$$\frac{d\sigma_{LL}}{dx_B \, dy \, d^2\boldsymbol{p}_\perp} = \frac{4\pi\alpha^2 \, s}{Q^4} \lambda_e \lambda \sum_{a,\bar{a}} e_a^2 \left\{ y \left(1 - \frac{y}{2} \right) \, x_B \, g_{1L}^a(x_B, \boldsymbol{p}_\perp^2) \right.$$

$$\left. -2y\sqrt{1-y} \, \cos(\phi_j) \frac{|\boldsymbol{p}_\perp|}{Q} \, x_B^2 g_L^{\perp \, a}(x_B, \boldsymbol{p}_\perp^2) \right\}. \tag{120}$$

$$\frac{d\sigma_{LT}}{dx_B \, dy \, d^2\boldsymbol{p}_\perp} = \frac{4\pi\alpha^2 \, s}{Q^4} \lambda_e |\boldsymbol{S}_\perp| \sum_{a,\bar{a}} e_a^2 \left\{ y \left(1 - \frac{y}{2} \right) \frac{|\boldsymbol{p}_\perp|}{M} \, \cos(\phi_j - \phi_s) \, x_B \, g_{1T}^a(x_B, \boldsymbol{p}_\perp^2) \right.$$

$$-2y\sqrt{1-y} \, \cos(\phi_s) \frac{M}{Q} \, x_B^2 g_T^a(x_B, \boldsymbol{p}_\perp^2)$$

$$\left. -2y\sqrt{1-y} \, \cos(2\phi_j - \phi_s) \frac{M}{Q} \frac{|\boldsymbol{p}_\perp|^2}{2M^2} \, x_B^2 g_T^{\perp \, a}(x_B, \boldsymbol{p}_\perp^2) \right\}. \tag{121}$$

Again decomposing the twist-three functions in these jet asymmetries into their twist-two parts and the interaction-dependent piece, one obtains the naive parton model results, if the interaction-dependent pieces are set to zero. For the unpolarized case, the cross section then becomes proportional to f_1^a, reducing the $\langle\cos(\phi_j)\rangle$ asymmetry to a purely kinematical effect depending only on y and Q as discussed in Ref. [26]. Similarly, in the case of longitudinal polarization setting the interaction-dependent pieces to zero, produces a result proportional to the quark helicity distribution g_1^a plus a quark mass term. Except for this mass term the $\langle\cos(\phi_j)\rangle$ asymmetry is purely kinematical in that case. We would like to turn things around, however, and state the importance of finding deviations of the naive parton results for unpolarized and polarized jet production in order to get an idea of the quark-quark-gluon correlation functions $\tilde{f}^{\perp a}$ and $\tilde{g}_L^{\perp a}$ in a nucleon. For transversely polarized hadrons one can study $g_T^a(x_B, \boldsymbol{p}_\perp^2)$, which integrated over $\boldsymbol{p}_\perp$ can also be measured in inclusive leptoproduction.

VII. SUMMARY

In this paper the complete tree-level result up to order $1/Q$ for polarized deep-inelastic leptoproduction has been presented. The formalism is the diagrammatic approach in which soft hadronic parts are represented by expectation values of nonlocal - here bilocal - combinations of quark and gluon fields, referred to as correlation functions. Both quark-quark and quark-quark-gluon correlation functions need to be included in order $1/Q$. The quark-quark-gluon correlation functions can be related to the quark-quark correlation functions using the QCD equations of motion. Such relations are essential to ensure an electromagnetically gauge-invariant result. Essential in our treatment is also the explicit treatment of quark transverse momenta in order to study azimuthal asymmetries in semi-inclusive lepto-production processes.

The full result for the hadronic tensor contains a large number of terms. They are some of the allowed terms in the most general expansion of the hadronic tensor, which can be expressed in terms of 'standard' tensor structures built from the momenta and spin vectors of the hadrons involved multiplied with structure functions. Having such an expansion one could read off the result for the structure functions. In view of the large number of structure functions that are allowed in principle, we found it more practical to split up the cross section in the parts involving the lepton polarizations (unpolarized or longitudinal polarization) and hadron polarizations (unpolarized, longitudinal and transverse polarization).

In the leptoproduction cross section in which one integrates over all transverse momenta the results have been presented as the numbers of produced hadrons $N_h(x_B, z_h)$ and the asymmetries in these numbers arising from the polarization of initial state lepton or hadron. Finally the polarization of a produced (spin 1/2) hadron has been discussed. The results include among others the following features

- An asymmetry in produced particles (e.g. pions) is obtained at order $1/Q$ for unpolarized leptons and transversely polarized hadrons (Eq. 96) proportional to the transverse spin distribution h_1^a and a twist-three fragmentation function.

- A transverse polarization orthogonal to the scattering plane is induced in spin 1/2 hadrons in the final state (e.g. Λ-baryons) starting with an unpolarized lepton and unpolarized target hadron. This polarization is proportional to the unpolarized quark distributions f_1^a and the fragmentation function $D_{1T}^{\perp(1)\,a}$ (Eq. 102). For longitudinally polarized lepton and target hadron a similar polarization emerges, now proportional to the quark helicity distribution g_1^a and the same fragmentation function $D_{1T}^{\perp(1)\,a}$. Although the fragmentation function is a new one, allowed because of the non-applicability of time-reversal invariance in the production, the comparison of induced polarization for polarized and unpolarized initial state may give another handle on determination of quark helicity distributions, especially because the production of Λ's produces different flavor weighting compared with inclusive processes.

- For completeness also all leading asymmetries in number of particles and in induced polarization have been included in Eqs 95-99 and Eqs 100-102. These are the ones that appear in the naive parton model.

The dependence on quark transverse momenta becomes explicit in semi-inclusive lepto-production in the transverse momentum of the produced hadrons. Here the following features emerge,

- All six twist-two x- and $\boldsymbol{p}_T$-dependent quark distribution functions for a spin 1/2 hadron can be accessed in leading order asymmetries if one considers lepton and hadron polarizations. One of the asymmetries involves the transverse spin distribution h_1^a. On the production side, only two different fragmentation functions are involved, the familiar unpolarized fragmentation function D_1^a and the (interaction-dependent) fragmentation function $H_1^{\perp a}$.

- At order $1/Q$ a $\langle \sin(\phi_h) \rangle$ asymmetry is found for polarized leptons and unpolarized hadrons. This probes the interaction-dependent distribution functions $\tilde{e}^a$ in combination with the same fragmentation function $H_1^{\perp a}$ mentioned in the previous item.

Several more azimuthal asymmetries are found for polarized targets. The results have been given explicitly because they indicate the type of asymmetries to be expected in deep-inelastic leptoproduction. At this point we also want to point out once more that our results represent tree-level calculations. The inclusion of gluon ladder-graphs give corrections proportional to α_s and $\alpha_s \ln Q^2$. For the cross sections integrated over transverse momenta, they lead to scale-dependence of the distribution functions. They will also enter as additional contributions to several of the observables, including the ones that are zero at tree-level. A well-known example of the latter is the longitudinal structure function $F_L = F_2 - 2x_{_B}F_1$, which is proportional to α_s. Work along these lines with explicit treatment of transverse momenta of quarks is under investigation.

APPENDIX A: SPIN VECTORS

In this appendix we would like to remind the reader of the role of the spin vectors S and S_h for initial state and final state hadrons, respectively. A more extended discussion of aspects related to the role of the spin vector of the produced hadron (applied in e^+e^- annihilation) can be found in Ref. [27]. Starting with the quantities

$$\tilde{\mathcal{W}}_{ij,kl}^{\mu\nu}(q,P,P_h) = \langle h;k|T|H;i\rangle \langle H;j|T^\dagger|h;l\rangle$$
$$- \quad \text{generalized tensor in } 2\times2 \text{ H-spin-space (ij) and in } 2\times2 \text{ h-spin-space (kl)},$$

$$\rho_{ij}(P,S) = \langle H;i|\sum_\alpha |\alpha\rangle p(\alpha)\langle\alpha|H;j\rangle \quad \overset{\text{H rest frame}}{\equiv} \quad \frac{1}{2}\left(\mathbf{1}_{ij} + \boldsymbol{\sigma}_{ij}\cdot\boldsymbol{S}\right)$$
$$- \quad \text{initial state density matrix , defining } S,$$

$$R_{kl}^*(P_h,f) = \langle h;k|f\rangle \langle f|h;l\rangle \quad \overset{\text{h rest frame}}{\equiv} \quad w(f)\left(\mathbf{1}_{kl} + \boldsymbol{\sigma}_{kl}^*\cdot\boldsymbol{S}_h\right)$$
$$- \quad \text{decay matrix into final states f in } 2\times2 \text{ h-spin-space (kl), defining } S_h,$$

where the latter is normalized to $\sum_f R_{kl}^*(f) = \delta_{kl}$. Note that the summation over f is just symbolic for all kinematic variables appearing in the decay. An example of $w(f)$ is the center-of-mass distribution $w(\theta_{\pi N})$

in the decay of the Λ baryon. Starting with some (general) initial state, one has the following result for the semi-inclusive hadronic tensor in which the decay products of hadron h are detected,

$$\mathcal{W}^{\mu\nu}(i \to f) = \rho_{ji}(P,S)\,\tilde{\mathcal{W}}^{\mu\nu}_{ij,kl}(q,P,P_h)\,R_{lk}(P_h,f) \equiv \tilde{\mathcal{W}}^{\mu\nu}_{kl}(q;P,S;P_h)\,R_{lk}(P_h,f). \tag{A1}$$

This shows for instance why the spin vector, originating from $\rho(P,S)$ can only appear linearly in the parametrization of the tensor. Summing (or integrating) over the decay products of h one is not able to measure the polarization of h,

$$\sum_f \mathcal{W}^{\mu\nu}(i \to h \to f) = \tilde{\mathcal{W}}^{\mu\nu}_{kk}(q;P,S;P_h) \equiv \mathcal{W}^{\mu\nu}(q;P,S;P_h,S_h=0). \tag{A2}$$

The general result in the h-spin-space can (in the rest-frame of hadron h) be written as the unpolarized result times the production density matrix parametrized in terms of the polarization $\boldsymbol{\mathcal{P}}_h$,

$$\tilde{\mathcal{W}}^{\mu\nu}_{kl}(q;P,S;P_h) \equiv \mathcal{W}^{\mu\nu}(q;P,S;P_h,0)\,\frac{1}{2}\Big(\mathbf{1}_{kl} + \boldsymbol{\sigma}_{kl}\cdot\boldsymbol{\mathcal{P}}_h(q;P,S;P_h)\Big). \tag{A3}$$

Thus one can write in the h-rest frame,

$$\mathcal{W}^{\mu\nu}(i \to f) = \mathcal{W}^{\mu\nu}(q;P,S;P_h,0)\,w(f)\Big(1 + \boldsymbol{\mathcal{P}}(q;P,S;P_h)\cdot\boldsymbol{S}_h(P_h;f)\Big)$$
$$\equiv \mathcal{W}^{\mu\nu}(q;P,S;P_h,S_h)\,w(f). \tag{A4}$$

This shows that also the spin vector S_h appears only linearly, while it multiplies the polarization vector as appearing in the production matrix. Parametrizing the polarization vector $\boldsymbol{\mathcal{P}}_h$ similar as S_h in a longitudinal polarization Λ_h and a transverse polarization $\boldsymbol{\mathcal{P}}_{hT}$, which because the polarization vector parametrizes a density matrix satisfy $\Lambda_h^2 + \boldsymbol{\mathcal{P}}_{hT}^2 \le 1$, one can in a general frame write for $\boldsymbol{\mathcal{P}}\cdot\boldsymbol{S}_h = \Lambda_h\,\lambda_h + \boldsymbol{\mathcal{P}}_{h\perp}\cdot\boldsymbol{S}_{h\perp}$.

APPENDIX B: THE LINK OPERATOR

In this appendix we discuss the path structure of the link operator used in the definition of the various correlation functions. In hard processes one encounters the correlation functions integrated over at least one lightcone component of the parton momentum. The actual component which one needs is actually fixed by the large vector q. This vector can be used to fix a lightlike vector for each given (small, i.e. $p^2 \ll Q^2$) vector p,

$$n_p \propto \frac{1}{Q}\left(p + \frac{2\,p\cdot q}{Q^2}\,q\right). \tag{B1}$$

For the correlation function $\Phi(P,S;p)$ one obtains as a consequence of the fact that $p^2 \sim P^2 \sim p\cdot P \ll Q^2$

$$n_p \approx n_P = \left(\frac{Q}{A},\,0,\,\mathbf{0}_T\right) \;=\; n_-. \tag{B2}$$

The lightlike vector is (approximately, i.e., up to corrections of $\mathcal{O}(1/Q^2)$) equal for all momenta in a 'connected' soft part. These momenta themselves define the 'other' lightlike vector, $p/Q \propto P/Q \propto n_+$. Similarly one finds for $\Delta(P_h,S_h;k)$ that

$$n_k \approx n_{P_h} = \left(0,\,\frac{A}{Q},\,\mathbf{0}_T\right) \;=\; n_+, \tag{B3}$$

while $k/Q \propto P_h/Q \propto n_-$. The lightlike vector determines the integration used, for instance in $\int dp^-\,\Phi(P,S;p)$ the component $p^- = (Q/A)\,p\cdot n_+$. It also fixes the path structure in the link operator

$$\mathcal{L}(0,x;n_-) = \mathcal{P}\exp\left(-ig\int_0^x ds^\mu\,A_\mu(s)\right), \tag{B4}$$

The link operator $\mathcal{L}(0,x;n_-)$ is actually constructed as the average of two links with paths P_1 and P_2. For Φ both lie in the $x^+ = 0$ plane and run along the n_- direction except for endpoints at $x^- = \pm\infty$. The path P_1 runs from $[0,0,\mathbf{0}_T]$ to $[x^-,0,\boldsymbol{x}_T]$ via $[+\infty,0,\mathbf{0}_T]$ and $[+\infty,0,\boldsymbol{x}_T]$, while the path P_2 runs from $[0,0,\mathbf{0}_T]$ to $[x^-,0,\boldsymbol{x}_T]$

via $[-\infty, 0, \mathbf{0}_T]$ and $[-\infty, 0, \mathbf{x}_T]$. In the gauge $A^+ = n_- \cdot A = 0$, the link operator in the projections $\Phi^{[\Gamma]}$ becomes unity, except for the parts at $x^- = \pm\infty$. However, it is well-known that $A^+ = 0$ does not completely fix the gauge. The residual gauge freedom is for instance fixed by imposing an antisymmetric boundary condition on A_T [28]. Fixing in this way the residual gauge freedom on A_T, $A_T[-\infty, x^+, \mathbf{x}_T] = -A_T[\infty, x^+, \mathbf{x}_T]$, allows a unique inversion of the relation $G^{+i} = \partial^+ A_T^i$,

$$A_T^i[x^-, x^+, \mathbf{x}_T] = \frac{1}{2} \int dy^- \, \epsilon(x^- - y^-) \, G^{+i}[y^-, x^+, \mathbf{x}_T] \equiv \frac{1}{\partial^+} G^{+i}[x^-, x^+, \mathbf{x}_T]. \tag{B5}$$

The choice $\mathcal{L}(0, x; n_-) = (\mathcal{L}_1(0, x; P_1) + \mathcal{L}_2(0, x; P_2))/2$ then reduces to unity.

For the link operator in the correlation functions Δ all choices are based on the n_+-direction. It becomes simply unity in the gauge fixed by $A^- = 0$ and an antisymmetric boundary condition for A_T.

APPENDIX C: EXPLICIT QUARK-QUARK-GLUON CORRELATION FUNCTIONS

For the correlation function,

$$\Phi_A^{\alpha[\Gamma]}(x, \mathbf{p}_T) = \Phi_D^{\alpha[\Gamma]}(x, \mathbf{p}_T) - p^\alpha \, \Phi^{[\Gamma]}(x, \mathbf{p}_T), \tag{C1}$$

one has $\Phi_A^{+[\Gamma]} = 0$ while the functions with transverse indices reduce after gauge fixing ($A^+ = 0$) to a quark-quark-gluon matrix element (Eq. 50). It is convenient to parametrize the following combinations (in which all indices α and β are transverse only) as

$$\Phi_A^{\alpha[i\sigma^{\beta+}\gamma_5]} - \Phi_A^{\beta[i\sigma^{\alpha+}\gamma_5]} = i\epsilon_T^{\alpha\beta} \, Mx \, \tilde{e} - \left(S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta \right) x \, \tilde{h}_T^\perp, \tag{C2}$$

$$g_{T\alpha\beta} \, \Phi_A^{\alpha[i\sigma^{\beta+}\gamma_5]} = Mx \, \tilde{h}_s, \tag{C3}$$

$$\Phi_A^{\alpha[\gamma^+]} - i\epsilon_T^{\alpha\beta} \, \Phi_{A\,\beta}^{[\gamma^+\gamma_5]} = p_T^\alpha \, x \, \tilde{f}^\perp - i\epsilon_T^{\alpha\beta} p_{T\beta} \, x\tilde{g}_s^\perp - i\epsilon_T^{\alpha\beta} S_{T\beta} \, Mx \, \tilde{g}_T', \tag{C4}$$

in terms of (interaction-dependent) functions indicated with a tilde. Using the results for Φ_D as following from the equations of motion (see section 3), one can split the twist-three distribution functions into the twist-two part and an interaction-dependent part. The first part is the one which is obtained in the naive parton model. The complete list is

$$e(x, \mathbf{p}_T^2) = \frac{m}{M} \frac{f_1(x, \mathbf{p}_T^2)}{x} + \tilde{e}(x, \mathbf{p}_T^2), \tag{C5}$$

$$f^\perp(x, \mathbf{p}_T^2) = \frac{f_1(x, \mathbf{p}_T^2)}{x} + \tilde{f}^\perp(x, \mathbf{p}_T^2), \tag{C6}$$

$$g_T'(x, \mathbf{p}_T^2) = \frac{m}{M} \frac{h_{1T}(x, \mathbf{p}_T^2)}{x} + \tilde{g}_T'(x, \mathbf{p}_T^2), \tag{C7}$$

$$g_L^\perp(x, \mathbf{p}_T^2) = \frac{g_{1L}(x, \mathbf{p}_T^2)}{x} + \frac{m}{M} \frac{h_{1L}^\perp(x, \mathbf{p}_T^2)}{x} + \tilde{g}_L^\perp(x, \mathbf{p}_T^2), \tag{C8}$$

$$g_T^\perp(x, \mathbf{p}_T^2) = \frac{g_{1T}(x, \mathbf{p}_T^2)}{x} + \frac{m}{M} \frac{h_{1T}^\perp(x, \mathbf{p}_T^2)}{x} + \tilde{g}_T^\perp(x, \mathbf{p}_T^2), \tag{C9}$$

$$h_T^\perp(x, \mathbf{p}_T^2) = \frac{h_{1T}(x, \mathbf{p}_T^2)}{x} + \tilde{h}_T^\perp(x, \mathbf{p}_T^2), \tag{C10}$$

$$h_L(x, \mathbf{p}_T^2) = \frac{m}{M} \frac{g_{1L}(x, \mathbf{p}_T^2)}{x} - \frac{\mathbf{p}_T^2}{M^2} \frac{h_{1L}^\perp(x, \mathbf{p}_T^2)}{x} + \tilde{h}_L(x, \mathbf{p}_T^2), \tag{C11}$$

$$h_T(x, \mathbf{p}_T^2) = \frac{m}{M} \frac{g_{1T}(x, \mathbf{p}_T^2)}{x} - \frac{h_{1T}(x, \mathbf{p}_T^2)}{x} - \frac{\mathbf{p}_T^2}{M^2} \frac{h_{1T}^\perp(x, \mathbf{p}_T^2)}{x} + \tilde{h}_T(x, \mathbf{p}_T^2). \tag{C12}$$

The $\mathbf{p}_T$-integrated results involve the functions

$$e(x) \equiv \int d^2\mathbf{p}_T \, e(x, \mathbf{p}_T^2) = \frac{m}{M} \frac{f_1(x)}{x} + \tilde{e}(x), \tag{C13}$$

$$g_T(x) \equiv \int d^2\boldsymbol{p}_T \left[g_T'(x, \boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T^2}{2M^2} g_T^\perp(x, \boldsymbol{p}_T^2) \right] = \frac{g_{1T}^{(1)}(x)}{x} + \frac{m}{M} \frac{h_1(x)}{x} + \tilde{g}_T(x), \tag{C14}$$

$$h_L(x) \equiv \int d^2\boldsymbol{p}_T \, h_L(x, \boldsymbol{p}_T^2) = -2 \frac{h_{1L}^{\perp(1)}(x)}{x} + \frac{m}{M} \frac{g_1(x)}{x} + \tilde{h}_L(x), \tag{C15}$$

where the functions $\tilde{e}(x)$, $\tilde{g}_T(x)$ and $\tilde{h}_L(x)$ are the $\boldsymbol{p}_T$-integrated results of the interaction-dependent parts. The functions with upper index (1) indicate $\boldsymbol{p}_T^2/2M^2$-weighted functions,

$$g_{1T}^{(1)}(x) \equiv \int d^2\boldsymbol{p}_T \, \frac{\boldsymbol{p}_T^2}{2M^2} g_{1T}(x, \boldsymbol{p}_T^2), \tag{C16}$$

$$h_{1L}^{\perp(1)}(x) \equiv \int d^2\boldsymbol{p}_T \, \frac{\boldsymbol{p}_T^2}{2M^2} h_{1L}^\perp(x, \boldsymbol{p}_T^2). \tag{C17}$$

Using the explicit expansion into amplitudes one can derive the following relations for the p_T integrated distribution functions [20], which clearly exhibit the role of transverse momenta and interaction terms,

$$g_T(x) = \int_x^1 dy \, \frac{g_1(y)}{y} + \frac{m}{M} \left[\frac{h_1(x)}{x} - \int_x^1 dy \, \frac{h_1(y)}{y^2} \right] + \tilde{g}_T(x) - \int_x^1 dy \, \frac{\tilde{g}_T(y)}{y},$$

$$= g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) \tag{C18}$$

$$h_L(x) = 2x \int_x^1 dy \, \frac{h_1(y)}{y^2} + \frac{m}{M} \left[\frac{g_1(x)}{x} - 2x \int_x^1 dy \, \frac{g_1(y)}{y^3} \right] + \tilde{h}_L(x) - 2x \int_x^1 dy \, \frac{\tilde{h}_L(y)}{y^2}$$

$$= h_1(x) - 2 \frac{d}{dx} h_{1L}^{\perp(1)}(x). \tag{C19}$$

The decomposition of g_T contains the Wandzura-Wilczek part [29], a quark mass term and interaction-dependent parts. Noteworthy is the appearance of the $(\boldsymbol{p}_T^2/2M^2)$-weighted twist-two functions, e.g. the function $g_{1T}^{(1)}(x)$ [In Ref. [7] referred to as $B^A(x)$].

For the fragmentation part one has for the projections

$$\Delta_A^{\alpha[\Gamma]}(z, \boldsymbol{k}_T) = \Delta_D^{\alpha[\Gamma]}(z, \boldsymbol{k}_T) - k^\alpha \, \Delta^{[\Gamma]}(z, \boldsymbol{k}_T), \tag{C20}$$

the result $\Delta_A^{-[\Gamma]} = 0$, while for transverse indices

$$\Delta_A^{\alpha[\Gamma]}(z, \boldsymbol{k}_T) = \int \frac{d\xi^+ d^2\xi_\perp}{4z \, (2\pi)^3} \, e^{i k \cdot \xi} \, Tr \, \langle 0 | \psi(\xi) \, A^\alpha(x) \, a_h^\dagger a_h \, \overline{\psi}(0) \Gamma | 0 \rangle \Big|_{x^- = 0}. \tag{C21}$$

It is convenient to parametrize the following combinations (in which all indices α and β are transverse only) as

$$\Delta_A^{\alpha[i\sigma^{\beta-}\gamma_5]} - \Delta_A^{\beta[i\sigma^{\alpha-}\gamma_5]} = -i\epsilon_T^{\alpha\beta} \left(\frac{M_h}{z} \tilde{E} - i \frac{M_h}{z} \tilde{H} \right)$$
$$- \left(S_{hT}^\alpha k_T^\beta - k_T^\alpha S_{hT}^\beta \right) \left(\frac{1}{z} \tilde{H}_T^\perp + i \frac{m}{M_h} \tilde{D}_{1T}^\perp \right), \tag{C22}$$

$$g_{T\alpha\beta} \, \Delta_A^{\alpha[i\sigma^{\beta-}\gamma_5]} = \frac{M_h}{z} \tilde{H}_s + i \frac{M_h}{z} \tilde{E}_s, \tag{C23}$$

$$\Delta_A^{\alpha[\gamma^-]} + i\epsilon_T^{\alpha\beta} \, \Delta_{A\beta}^{[\gamma^-\gamma_5]} = k_T^\alpha \left(\frac{1}{z} \tilde{D}^\perp + i \frac{m}{M_h} \tilde{H}_1^\perp \right) - \frac{k_T^\alpha}{M_h} \epsilon_{T\,ij} k_T^i S_{hT}^j \tilde{D}_{1T}^\perp$$
$$+ i\epsilon_T^{\alpha\beta} k_{T\beta} \frac{1}{z} \tilde{G}_s^\perp + i\epsilon_T^{\alpha\beta} S_{hT\beta} \frac{M_h}{z} \tilde{G}_T', \tag{C24}$$

in terms of (interaction-dependent) functions indicated with a tilde and depending on z and $\boldsymbol{k}_T' = -z\boldsymbol{k}_T$. One again can use them to split the fragmentation functions into a 'parton' piece and interaction dependent functions,

$$D_{1T}^\perp(z, \boldsymbol{k}_T'^2) = \tilde{D}_{1T}^\perp(z, \boldsymbol{k}_T'^2), \tag{C25}$$

$$H_1^\perp(z, \boldsymbol{k}_T'^2) = \tilde{H}_1^\perp(z, \boldsymbol{k}_T'^2), \tag{C26}$$

$$E(z, \boldsymbol{k}_T'^2) = \frac{m}{M_h} z D_1(z, \boldsymbol{k}_T'^2) + \tilde{E}(z, \boldsymbol{k}_T'^2), \tag{C27}$$

$$D^\perp(z, \boldsymbol{k}_T'^2) = z D_1(z, \boldsymbol{k}_T'^2) + \tilde{D}^\perp(z, \boldsymbol{k}_T'^2), \tag{C28}$$

$$E_s(z, \boldsymbol{k}_T') = \tilde{E}_s(z, \boldsymbol{k}_T'), \tag{C29}$$

$$G_T'(z, \boldsymbol{k}_T'^2) = \frac{m}{M_h} z H_{1T}(z, \boldsymbol{k}_T'^2) + \tilde{G}_T'(z, \boldsymbol{k}_T'^2), \tag{C30}$$

$$G_s^\perp(z, \boldsymbol{k}_T') = z G_{1s}(z, \boldsymbol{k}_T') + \frac{m}{M_h} z H_{1s}^\perp(z, \boldsymbol{k}_T') + \tilde{G}_s^\perp(z, \boldsymbol{k}_T'), \tag{C31}$$

$$H_T^\perp(z, \boldsymbol{k}_T'^2) = z H_{1T}(z, \boldsymbol{k}_T'^2) + \tilde{H}_T^\perp(z, \boldsymbol{k}_T'^2), \tag{C32}$$

$$H(z, \boldsymbol{k}_T'^2) = -\frac{\boldsymbol{k}_T^2}{M_h^2} z H_1^\perp(z, \boldsymbol{k}_T'^2) + \tilde{H}(z, \boldsymbol{k}_T'^2), \tag{C33}$$

$$H_s(z, \boldsymbol{k}_T') = \frac{m}{M_h} z G_{1s}(z, \boldsymbol{k}_T') - \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_{hT}}{M_h} z H_{1T}(z, \boldsymbol{k}_T'^2) - \frac{\boldsymbol{k}_T^2}{M_h^2} z H_{1s}^\perp(z, \boldsymbol{k}_T') + \tilde{H}_s(z, \boldsymbol{k}_T'). \tag{C34}$$

Note that the twist-two time-reversal odd functions $D_{1T}^\perp$ and $H_1^\perp$ are interaction-dependent. Below we give some of the $\boldsymbol{k}_T'$-integrated functions that turn out to be relevant in the integrated cross sections. They are

$$E(z) \equiv \int d^2\boldsymbol{k}_T' \, E(z, \boldsymbol{k}_T'^2) = z^2 \int d^2\boldsymbol{k}_T \, E(z, z^2\boldsymbol{k}_T^2) = \frac{m}{M_h} z D_1(z) + \tilde{E}(z), \tag{C35}$$

$$G_T(z) \equiv \int d^2\boldsymbol{k}_T' \left[G_T'(z, \boldsymbol{k}_T'^2) + \frac{\boldsymbol{k}_T^2}{2M_h^2} G_T^\perp(z, \boldsymbol{k}_T'^2) \right] = z G_{1T}^{(1)}(z) + \frac{m}{M_h} z H_1(z) + \tilde{G}_T(z), \tag{C36}$$

$$H_L(z) \equiv \int d^2\boldsymbol{k}_T' \, H_L(z, \boldsymbol{k}_T'^2) = -2z H_{1L}^{\perp(1)}(z) + \frac{m}{M_h} z G_1(z) + \tilde{H}_L(z), \tag{C37}$$

while the $\boldsymbol{k}_T^2/2M_h^2$-weighted functions (indicated with index (1)) are,

$$D_{1T}^{\perp(1)}(z) \equiv \int d^2\boldsymbol{k}_T' \, \frac{\boldsymbol{k}_T^2}{2M_h^2} \, D_{1T}^\perp(z, \boldsymbol{k}_T'^2), \tag{C38}$$

$$G_{1T}^{(1)}(z) \equiv \int d^2\boldsymbol{k}_T' \, \frac{\boldsymbol{k}_T^2}{2M_h^2} \, G_{1T}(z, \boldsymbol{k}_T'^2), \tag{C39}$$

$$H_{1L}^{\perp(1)}(z) \equiv \int d^2\boldsymbol{k}_T' \, \frac{\boldsymbol{k}_T^2}{2M_h^2} \, H_{1L}^\perp(z, \boldsymbol{k}_T'^2). \tag{C40}$$

The above decomposition leads in combination with the amplitude expansion (based on Lorentz invariance) to the following relations,

$$G_T(z) = z \int_z^1 dy \, \frac{G_1(y)}{y^2} + \frac{m}{M_h} \left[z H_1(z) - z \int_z^1 dy \, \frac{H_1(y)}{y} \right] + \tilde{G}_T(z) - z \int_z^1 dy \, \frac{\tilde{G}_T(y)}{y^2},$$
$$= G_1(z) + z \frac{d}{dz} \left(z G_{1T}^{(1)}(z) \right), \tag{C41}$$

$$H_L(z) = 2 \int_z^1 dy \, \frac{H_1(y)}{y} + \frac{m}{M_h} \left[z G_1(z) - 2 \int_z^1 dy \, G_1(y) \right] + \tilde{H}_L(z) - 2 \int_z^1 dy \, \frac{\tilde{H}_L(y)}{y}$$
$$= H_1(z) - 2z \frac{d}{dz} \left(z H_{1L}^{\perp(1)}(z) \right). \tag{C42}$$

Analogous to the sum rules $\int_0^1 dx \, g_2(x) = 0$ [30], where $g_2 = g_T - g_1$, and $\int_0^1 dx \, h_2(x) = 0$ [31,20], where $h_2 = 2(h_L - h_1)$, one obtains sum rules

$$\int_0^1 dz \, \frac{G_2(z)}{z} = 0, \tag{C43}$$

where $G_2 = G_T - G_1$ and

$$\int_0^1 dz \, \frac{H_2(z)}{z} = 0, \tag{C44}$$

where $H_2 = 2(H_L - H_1)$. As for the distribution functions, one must be aware of convergence problems in the integrals and of the fact that in the cross sections other contributions beyond tree-level may be present. We consider these sum rules at this point as merely academic.

APPENDIX D: CONVOLUTIONS FOR GAUSSIAN DISTRIBUTIONS

In order to study the behavior of the convolutions of distribution and fragmentation functions it is useful to consider gaussian distributions,

$$
\begin{aligned}
f(x, \boldsymbol{p}_T^2) &= f(x, 0)\,\exp(-R_H^2 \boldsymbol{p}_T^2) \\
&= f(x)\,\frac{R_H^2}{\pi}\,\exp(-R_H^2 \boldsymbol{p}_T^2) \equiv f(x)\,\mathcal{G}(|\boldsymbol{p}_T|; R_H),
\end{aligned} \tag{D1}
$$

$$
\begin{aligned}
D(z, \boldsymbol{k}_T'^2) &= D(z, 0)\,\exp(-R_h^2 \boldsymbol{k}_T^2). \\
&= D(z)\,\frac{R_h^2}{\pi z^2}\,\exp(-R_h^2 \boldsymbol{k}_T^2) = \frac{D(z)}{z^2}\,\mathcal{G}(|\boldsymbol{k}_T|; R_h) = D(z)\,\mathcal{G}\left(z|\boldsymbol{k}_T|; \frac{R_h}{z}\right).
\end{aligned} \tag{D2}
$$

In that case the convolution becomes

$$
\begin{aligned}
I[f\,D](x, z, Q_T) &= \int d^2\boldsymbol{p}_T\, f(x, \boldsymbol{p}_T^2)\, D(z, |\boldsymbol{P}_{h\perp} - z\boldsymbol{p}_T|^2) \\
&= \frac{\pi}{R_H^2 + R_h^2}\,\exp\left(-\frac{Q_T^2\,R_H^2 R_h^2}{R_H^2 + R_h^2}\right)\,f(x, \boldsymbol{0}_T)\,D(z, \boldsymbol{0}_T) \\
&= f(x)\,D(z)\,\frac{\mathcal{G}(Q_T; R)}{z^2},
\end{aligned} \tag{D3}
$$

where $R^2 = R_H^2\,R_h^2/(R_H^2 + R_h^2)$. The other convolutions that appear in the cross sections are of the form

$$
I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f\,D\right] = \frac{Q_T\,R^2}{M\,R_H^2}\,I[f\,D], \tag{D4}
$$

$$
I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\,f\,D\right] = -\frac{Q_T\,R^2}{M_h\,R_h^2}\,I[f\,D], \tag{D5}
$$

$$
I\left[\frac{\boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}\,f\,D\right] = \frac{R^2}{MM_h\,R_H^2\,R_h^2}\left(1 - Q_T^2\,R^2\right)\,I[f\,D], \tag{D6}
$$

$$
I\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\,\frac{\boldsymbol{p}_T^2}{M^2}\,f\,D\right] = \frac{Q_T\,R^6}{M_h\,M^2\,R_h^4\,R_H^4}\left(\frac{R_h^2 - R_H^2}{R^2} - Q_T^2\,R_h^2\right)\,I[f\,D], \tag{D7}
$$

$$
I\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{M^2}\,f\,D\right] = \frac{Q_T^2\,R^4}{M^2\,R_H^4}\,I[f\,D], \tag{D8}
$$

$$
I\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2 - \boldsymbol{k}_T^2}{M_h^2}\,f\,D\right] = \frac{Q_T^2\,R^4}{M_h^2\,R_h^4}\,I[f\,D], \tag{D9}
$$

$$
I\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}\,f\,D\right] = -\frac{Q_T^2\,R^4}{MM_h\,R_H^2\,R_h^2}\,I[f\,D], \tag{D10}
$$

$$
I\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}\,\frac{\boldsymbol{p}_T^2}{M^2}\,f\,D\right] = -\frac{Q_T^2\,R^8}{M^3 M_h\,R_H^6\,R_h^4}\left(\frac{2R_H^2 - R_h^2}{R^2} + Q_T^2\,R_h^2\right)\,I[f\,D], \tag{D11}
$$

$$
I\left[\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - 2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\boldsymbol{p}_T\cdot\boldsymbol{k}_T) - \boldsymbol{p}_\perp^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{M^2 M_h}\,f\,D\right] = -\frac{Q_T^3\,R^6}{M^2 M_h\,R_H^4\,R_h^2}\,I[f\,D]. \tag{D12}
$$

Note that we have suppressed possible dependence of the radii on the specific function or the kinematic variables irrelevant in the integration over transverse momenta. We can trivially generalize the results by taking into account such dependence, $R_H \to R_H^f(x)$ and $R_h \to R_h^D(z)$.

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TABLE I. Contractions of the lepton tensor $L_{\mu\nu}$ with tensor structures appearing in the hadron tensor.

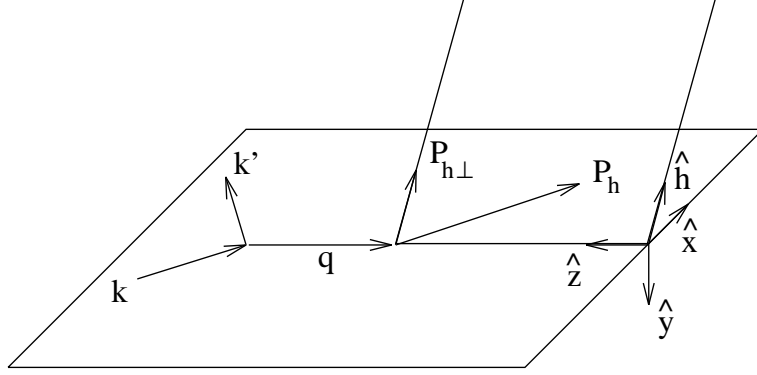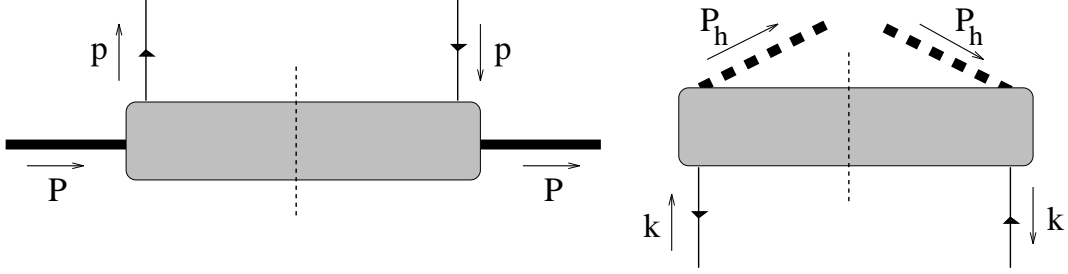| $w^{\mu\nu}$	$L_{\mu\nu} w^{\mu\nu}$
$-g_\perp^{\mu\nu}$	$\frac{4Q^2}{y^2}\left(1 - y + \frac{1}{2}y^2\right)$
$a_\perp^{\{\mu} b_\perp^{\nu\}} - (a_\perp \cdot b_\perp)\, g_\perp^{\mu\nu}$	$\frac{4Q^2}{y^2}(1-y)\,\|a_\perp\|\,\|b_\perp\|\,\cos(\phi_a + \phi_b)$
$\frac{1}{2}\left(a_\perp^{\{\mu}\,\epsilon_\perp^{\nu\}\rho} b_{\perp\rho} + b_\perp^{\{\mu}\,\epsilon_\perp^{\nu\}\rho} a_{\perp\rho}\right)$ $= a_\perp^{\{\mu}\,\epsilon_\perp^{\nu\}\rho} b_{\perp\rho} - (\epsilon_\perp^{\rho\sigma} a_{\perp\rho} b_{\perp\sigma})\, g_\perp^{\mu\nu}$	$-\frac{4Q^2}{y^2}(1-y)\,\|a_\perp\|\,\|b_\perp\|\,\sin(\phi_a + \phi_b)$
$\hat{t}^{\{\mu} a_\perp^{\nu\}}$	$-\frac{4Q^2}{y^2}(2-y)\sqrt{1-y}\,\|a_\perp\|\,\cos\phi_a$
$\hat{t}^{\{\mu}\,\epsilon_\perp^{\nu\}\rho}\, a_{\perp\rho}$	$\frac{4Q^2}{y^2}(2-y)\sqrt{1-y}\,\|a_\perp\|\,\sin\phi_a$
$i\,\epsilon_\perp^{\mu\nu}$	$\lambda_e\,\frac{4Q^2}{y^2}\,y\left(1 - \frac{y}{2}\right)$
$i\,a_\perp^{[\mu} b_\perp^{\nu]}$	$\lambda_e\,\frac{4Q^2}{y^2}\,y\left(1 - \frac{y}{2}\right)\,\|a_\perp\|\,\|b_\perp\|\,\sin(\phi_b - \phi_a)$
$i\,\hat{t}^{[\mu} a_\perp^{\nu]}$	$-\lambda_e\,\frac{4Q^2}{y^2}\,y\sqrt{1-y}\,\|a_\perp\|\,\sin\phi_a$
$i\,\hat{t}^{[\mu}\,\epsilon_\perp^{\nu]\rho} a_{\perp\rho}$	$-\lambda_e\,\frac{4Q^2}{y^2}\,y\sqrt{1-y}\,\|a_\perp\|\,\cos\phi_a$

FIG. 1. Kinematics for ℓH scattering.

FIG. 2. Quark-quark correlation functions $\Phi(P, S; p)$ (left) and $\Delta(P_h, S_h; k)$ (right) giving quark distributions and fragmentation functions, respectively.
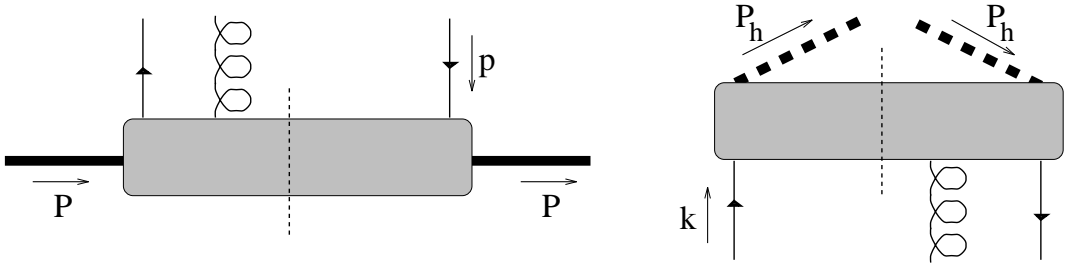

FIG. 3. Quark-quark-gluon correlation functions Φ_A (left) and Δ_A (right) contributing in hard scattering processes at subleading order.
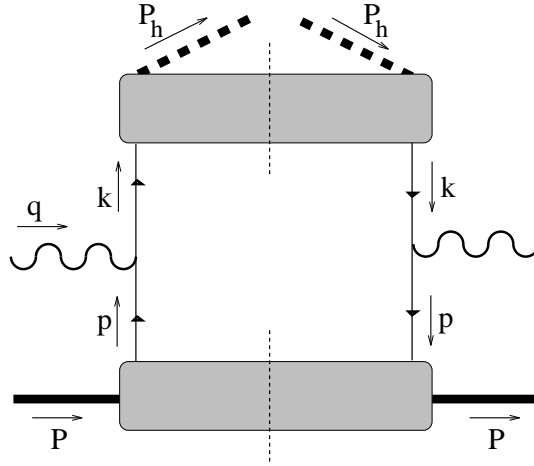
FIG. 4. Quark diagram contributing to ℓH scattering in leading order. There is a similar antiquark contribution.
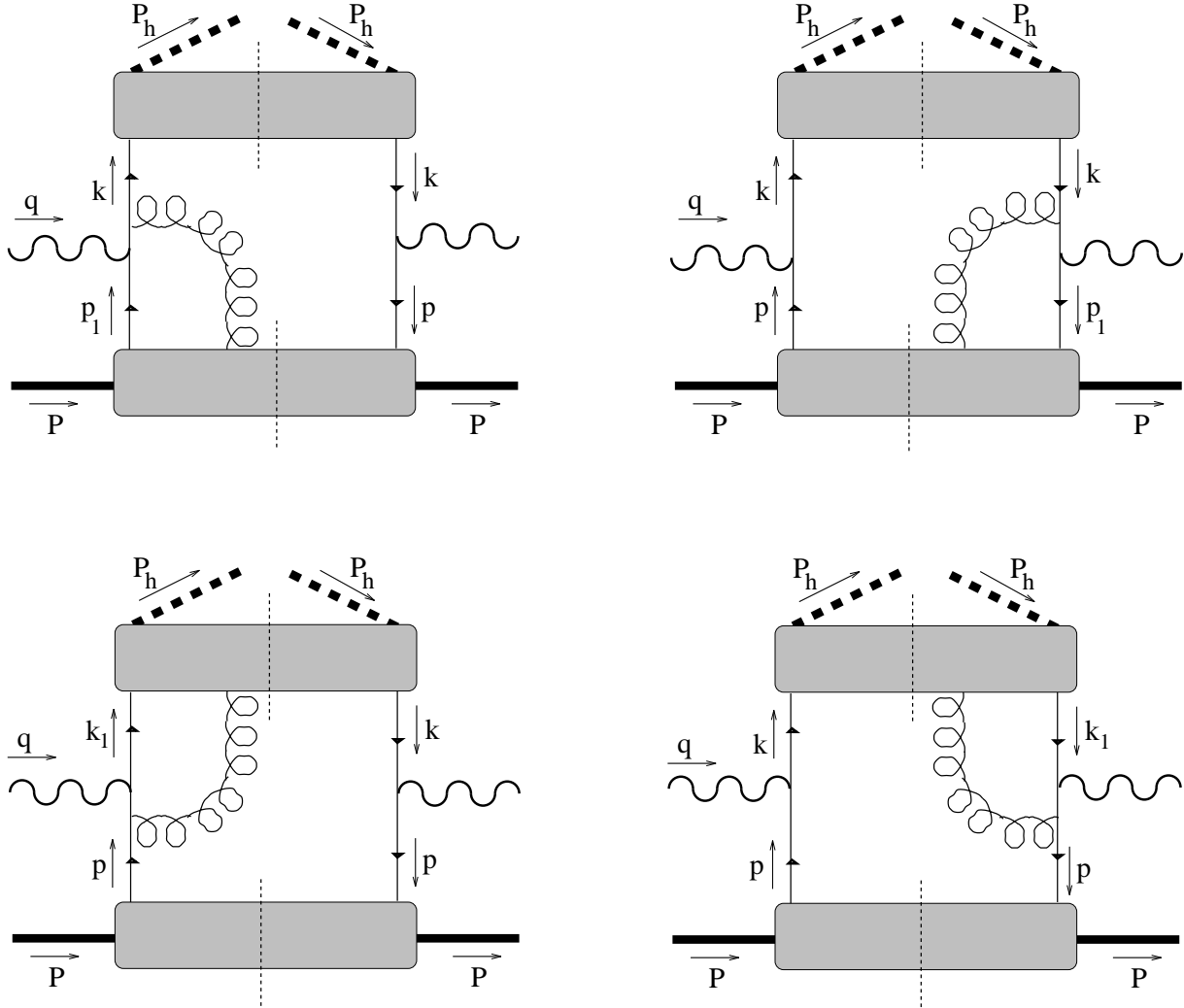

FIG. 5. Quark diagrams contributing to ℓH scattering at order $1/Q$.