Investigations of Azimuthal Asymmetry in Semi-Inclusive Leptoproduction

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Abstract

We consider the azimuthal asymmetries in semi-inclusive deep inelastic leptoproduction arising due to both perturbative and nonperturbative effects at HERMES energies and show that the k_T^2/Q^2 order corrections to $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$ are significant. We also reconsider the results of perturbative effects for $\langle \cos\phi \rangle$ at large momentum transfers [1] using the more recent sets of scale-dependent distribution and fragmentation functions, which bring up to 18% difference in $\langle \cos\phi \rangle$. In the same approach we calculate the $\langle \cos 2\phi \rangle$ as well.

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The semi-inclusive deep inelastic process $l(k_1) + p(P_1) \rightarrow l^{'}(k_2) + h(P_2) + X$, where l and $l^{'}$ are charged leptons and h is a observed hadron, has been recognized [2] as an important testing ground for QCD. In particular, measurement of the azimuthal angle ϕ of the detected hadron around the virtual photon direction (Fig.1) provides information on the production mechanism. Different mechanisms to generate azimuthal asymmetries - $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$ have been discussed in the literature. Georgi and Politzer [2] found a negative contribution to $\langle\cos\phi\rangle$ in the first-order in α_S perturbative theory and proposed the measurement of this quantity as a clean test of QCD. However, partons have nonzero transverse momenta (k_T) as a consequence of being confined by the strong interactions inside hadrons. As Cahn [3] showed, there is a contribution to $\langle\cos\phi\rangle$ from the lowest-order processes due to this intrinsic transverse momentum. Therefore the perturbative QCD alone does not describe the observed azimuthal angular dependence. In connection with this Chay, Ellis and Stirling [1] combined these perturbative and nonperturbative mechanisms and analyzes the quantity $\langle\cos\phi\rangle$ as a function of the detected hadron's transverse momentum cutoff P_C.

In this paper we reconsider the results obtained in Ref. [1] at HERMES energies and show that the k_T^2/Q^2 order corrections to $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$ are significant, whereas at $E665$ energies in Fermilab [4] these contributions are less then 10% [1]. We also recalculate the behavior of $\langle\cos\phi\rangle$ in the kinematic regime at HERA where perturbative QCD effects should dominate, by using the new sets of scale-dependent distribution and fragmentation functions which bring up to 18% difference to quantity $\langle\cos\phi\rangle$ obtained in Ref. [1]. In the same approach the quantity $\langle\cos 2\phi\rangle$ is calculated as well.

The quantities $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$ are defined as

$$\langle\cos\phi\rangle = \frac{\int d\sigma^{(0)}\cos\phi + \int d\sigma^{(1)}\cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}, \tag{1}$$

$$\langle\cos 2\phi\rangle = \frac{\int d\sigma^{(0)}\cos 2\phi + \int d\sigma^{(1)}\cos 2\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}, \tag{2}$$

where $d\sigma^{(0)}$ ($d\sigma^{(1)}$) is the lowest-order (first-order in α_S) hadronic scattering cross section expressed as

$$d\sigma \sim F_i(\xi, Q^2) \otimes d\sigma_{ij} \otimes D_j(\xi^{'}, Q^2),$$

where $F_i(\xi, Q^2)$ is the probability distribution describing an i-type parton with a fraction ξ of the target momentum, $p_1^\mu = \xi P_1^\mu$, $d\sigma_{ij}$ describes the partonic semi-inclusive process (Fig.2) and $D_j(\xi^{'}, Q^2)$ is the probability distribution for a j-type parton to fragment producing a hadron with a fraction $\xi^{'}$ of the partons momentum, $P_2^\mu = \xi^{'} p_2^\mu$. In Eqs.(1, 2) the integrations are over P_{2T}, ϕ, x_H, y and z_H. These usual set of kinematic variables are defined as:

$$x_H = \frac{Q^2}{2(P_1 q)}, \quad y = \frac{(P_1 q)}{(P_1 k_1)}, \quad z_H = \frac{(P_1 P_2)}{(P_1 q)},$$

where q-momentum of the virtual photon ($Q^2 = -q^2$), and the parton variables

$$x = \frac{x_H}{\xi} = \frac{Q^2}{2(p_1 q)}, \quad z = \frac{z_H}{\xi^{'}} = \frac{(p_1 p_2)}{(p_1 q)}.$$

The nonperturbative effects are parameterized by Gaussian distributions for the intrinsic transverse momenta of both the target (proton) and the observed hadron (pion):

$$F_i(\xi, Q^2) \rightarrow d^2k_T \tilde{F}_i(\xi, \vec{k}_T, Q^2) = d^2k_T F_i(\xi, Q^2) f(\vec{k}_T),$$
$$D_j(\xi', Q^2) \rightarrow d^2\rho' \tilde{D}_j(\xi', \vec{\rho'}, Q^2) = d^2\rho' D_j(\xi', Q^2) d(\vec{\rho'}), \quad (3)$$

where

$$f(\vec{k}_T) = \frac{1}{a^2\pi} e^{-k_T^2/a^2}, \quad d(\vec{\rho'}) = \frac{1}{b^2\pi} e^{-\rho'^2/b^2},$$

and $\vec{\rho'}$ is defined to be perpendicular to the direction of motion of the outgoing parton. Then in the center-of-mass frame of the virtual photon and the proton (in the limit $M^2/P_1^2 \ll 1, P_1^2 = Q^2/4x_H(1-x_H)$) the hadron's transverse momentum, perpendicular to $\vec{q}$ is given by (for more details see Ref.[1])

$$\vec{P}_{2T} = \xi' \vec{k}_T + \vec{\rho'} - \frac{(\vec{P}_1 \vec{\rho'})}{P_1^2} \vec{P}_1, \quad (4)$$

and it's magnitude as

$$P_{2T}^2 = (\xi' \vec{k}_T + \vec{\rho'})^2 - \frac{4x_H}{1-x_H} \frac{(\vec{k}_T \vec{\rho'})^2}{Q^2}. \quad (5)$$

If we allow the initial parton to have intrinsic transverse momentum $\vec{p_1} = \xi \vec{P}_1 + \vec{k}_T$, the parton cross section at lowest order (Fig.2(a)) is modified [3] to

$$\frac{d\sigma_{ij}}{dxdydzdp_{2T}^2 d\phi} = \frac{2\pi\alpha^2}{yQ^2} Q_i^2 \delta_{ij} \delta(1-x)\delta(1-z)\delta^2(\vec{p_{2T}} - \vec{k}_T)$$
$$\left\{1 + (1-y)^2 + \frac{4p_{2T}^2}{Q^2}(1-y) - \frac{4p_{2T}}{Q}\cos\phi(2-y)(1-y)^{1/2} + \frac{8p_{2T}^2}{Q^2}(1-y)\cos 2\phi\right\}, \quad (6)$$

where Q_i is a charge of i-type parton.

Using this parton cross section along with the distribution and fragmentation functions of Eq.(3) and observed hadron's transverse momentum defined in Eq.(4), one may obtain an explicit expression for the hadronic cross section at lowest-order in α_S

$$\int d\sigma^{(0)} \cos\phi d\phi = -\frac{8\pi^2\alpha^2}{Q^2} \int dy dx_H dz_H dP_{2T}^2 d^2k_T d^2\rho' \frac{(2-y)(1-y)^{1/2}}{y}$$
$$\sum_j Q_j^2 F_j(x_H, Q^2) D_j(z_H, Q^2) \frac{k_T}{Q} f(k_T) d(\rho') \delta^2(\vec{P}_T - \xi' \vec{k}_T - \vec{\rho'} + \frac{(\vec{P}_1 \vec{\rho'})}{P_1^2} \vec{P}_1), \quad (7)$$

$$\int d\sigma^{(0)} \cos 2\phi d\phi = \frac{8\pi^2\alpha^2}{Q^2} \int dy dx_H dz_H dP_{2T}^2 d^2k_T d^2\rho' \frac{k_T^2}{Q^2}(1-y)$$
$$\sum_j Q_j^2 F_j(x_H, Q^2) D_j(z_H, Q^2) f(k_T) d(\rho') \delta^2(\vec{P}_T - \xi' \vec{k}_T - \vec{\rho'} + \frac{(\vec{P}_1 \vec{\rho'})}{P_1^2} \vec{P}_1), \quad (8)$$

$$\int d\sigma^{(0)} d\phi = \frac{4\pi^2\alpha^2}{Q^2}\int dydx_H dz_H dP_{2T}^2 d^2k_T d^2\rho' \left\{1 + (1-y)^2 + \frac{4k_T^2}{Q^2}(1-y)\right\}$$

$$\sum_j Q_j^2 F_j(x_H, Q^2) D_j(z_H, Q^2) f(k_T) d(\rho') \delta^2(\vec{P}_T - \xi' \vec{k}_T - \vec{\rho'} + \frac{(\vec{P}_1\vec{\rho'})}{P_1^2}\vec{P}_1), \tag{9}$$

where the lower limit of the integrating over P_{2T} is P_C (observed hadron's transverse momentum cutoff).

At large momentum transfers, the intrinsic transverse momenta of the partons of a few hundred MeV cannot produce hadrons with larger transverse momenta and the nonperturbative effects from $\sigma^{(0)}$ are suppressed. Therefore, $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$ are, to a good approximations,

$$\langle \cos\phi \rangle \approx \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}. \tag{10}$$

$$\langle \cos 2\phi \rangle \approx \frac{\int d\sigma^{(1)} \cos 2\phi}{\int d\sigma^{(1)}}. \tag{11}$$

The numerators and denominator of these equations can be written in following form

$$\int d\sigma^{(1)} \cos\phi d\phi = \frac{8\alpha_S\alpha^2}{3Q^2}\frac{(2-y)(1-y)^{1/2}}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\times$$

$$\sum_j Q_j^2 \left\{F_j(\xi, Q^2) A D_j(\xi', Q^2) + F_j(\xi, Q^2) B D_G(\xi', Q^2) + F_G(\xi, Q^2) C D_j(\xi', Q^2)\right\}, \tag{12}$$

$$\int d\sigma^{(1)} \cos 2\phi d\phi = \frac{8\alpha_S\alpha^2}{3Q^2}\frac{1-y}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\times$$

$$\sum_j Q_j^2 \left\{F_j(\xi, Q^2) A' D_j(\xi', Q^2) + F_j(\xi, Q^2) B' D_G(\xi', Q^2) + F_G(\xi, Q^2) C' D_j(\xi', Q^2)\right\}, \tag{13}$$

$$\int d\sigma^{(1)} d\phi = \frac{4\alpha_S\alpha^2}{3Q^2}\frac{1}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\times$$

$$\sum_j Q_j^2 \left\{F_j(\xi, Q^2) A'' D_j(\xi', Q^2) + F_j(\xi, Q^2) B'' D_G(\xi', Q^2) + F_G(\xi, Q^2) C'' D_j(\xi', Q^2)\right\}, \tag{14}$$

where

$$A = -\left\{\frac{xz}{(1-x)(1-z)}\right\}^{1/2} [xz + (1-x)(1-z)]$$

$$B = \left\{\frac{x(1-z)}{(1-x)z}\right\}^{1/2} [x(1-z) + (1-x)z]$$

$$C = -\frac{3}{8}\left\{\frac{x(1-x)}{z(1-z}\right\}^{1/2} (1-2x)(1-2z)$$

$$A' = xz$$

$$B^{'} = x(1-z)$$

$$C^{'} = \frac{3}{4}x(1-x)$$

$$A^{''} = [1+(1-y)^2]\frac{x^2+z^2}{(1-x)(1-z)} + 2y^2(1+xz) + 4(1-y)(1+3xz)$$

$$B^{''} = [1+(1-y)^2]\frac{x^2+(1-z)^2}{z(1-x)} + 2y^2(1+x-xz) + 4(1-y)(1+3x(1-z))$$

$$C^{''} = \frac{3}{8}\left\{[1+(1-y)^2][x^2+(1-x)^2]\frac{z^2+(1-z)^2}{z(1-z)} + 16x(1-y)(1-x)\right\}$$

These expressions are identical with previous perturbative results in Ref.[5] and the quantities A, B and C and those with primes and two primes arise from diagrams Figs.2(b)-2(d) respectively.

Let us consider how $\langle\cos\phi\rangle$ as defined in Eq.(1) with P_{2T} cutoff P_C, behaves numerically with including both leading-order QCD (Eqs.(12,14)) and intrinsic transverse momentum (Eqs.(7,9)). We use the Glück, et. al. (GRV) parton distribution functions [6] for $F_j(\xi, Q^2)$ in Eq.(3) and the scale-dependent Binnewies, et.al. (BKK) parameterizations [8] for the quark and gluon fragmentation functions to charged pions. Our numerical results at HERMES energies: $E_l = 27.5GeV$, $Q^2 > 1GeV^2$, $0.1 < y < 0.85$, $0.02 < x_H < 0.4$ and $0.2 < z_H < 1$, presented in Fig. 3 (at these ranges the difference in $\langle\cos\phi\rangle$ with distribution functions of Ref. [7] (HMSR) is less then a few percents). In order to make an average over the range of Q^2, we also (as in Ref. [1]) use the relation $Q^2 = 2ME_l x_H y$, where M is the proton mass. The curves correspond to integrating over the same ranges with keeping the k_T^2/Q^2 term in Eq.(5) (Fig.3 (a)) and with neglecting the term of the order k_T^2/Q^2 so that $\vec{P}_{2T} = \xi^{'}\vec{k}_T + \vec{\rho}^{'}$ approximation (Fig.3 (b)). In both cases we take $a = b = 0.3GeV$, which corresponds an average intrinsic transverse momenta of $\langle k_T\rangle = \langle p_T\rangle = 0.27GeV$. Note, that this choice is arbitrary. The numerical magnitude of $\langle k_T\rangle$ is at present rather uncertain and there are not measurements of the unpolarized azimuthal asymmetries at HERMES yet. In this respect our aim is only to show the role of the k_T^2/Q^2 corrections and their quantitative contributions in azimuthal asymmetries at HERMES kinematics (small Q^2 and relatively large x) for some reasonable numerical magnitude of $\langle k_T\rangle$. The reason of choice of a small value of mean k_T comes from the fact that in the covariant parton model the k_T depends on kinematical variables: the small and moderate Q^2 (and relatively large x) requiring a small mean k_T. In the same approach we calculate also the angular moment $\langle\cos 2\phi\rangle$ as defined in Eq.(2) using the Eqs.(13, 14) and Eqs.(7,9). The numerical results are illustrated in Fig.4. One can see from Figs.3,4 that the contribution of the term k_T^2/Q^2 to $\langle\cos\phi\rangle$ and to $\langle\cos 2\phi\rangle$ is significant. Thus, one can conclude that in kinematic regime of HERMES the error of $\vec{P}_{2T} = \xi^{'}\vec{k}_T + \vec{\rho}^{'}$ approximation (valid to order k_T/Q) is rather big. The reason of this is mainly conditioned by small Q^2 and relatively large x_H. Note, that for relatively large values of the $\langle k_T\rangle$, $\langle p_T\rangle$, the magnitudes of the nonperturbative $|\langle\cos\phi\rangle|$ and $\langle\cos 2\phi\rangle$ increase and the k_T^2/Q^2 order corrections become more essential. The contributions of perturbative effects in this regime are not exceed a few percents.

Moreover, it is important to mention here that the complete behavior of azimuthal distributions may be predicted only after inclusion of higher-twist mechanisms, as suggested by Berger [9]. He considered the case of single pion production taking into account pion bound-state effects, which generates azimuthal asymmetries with opposite sign respect to perturbative QCD and intrinsic transverse momenta effects. More recently Brandenburg, et.al. [10] reconsidered Berger's mechanism and discussed the way of disentangle the effects from those considered above.

If we now focus to large Q^2 values and larger transverse momenta for the observed hadrons, the nonperturbative contributions are much less important (the contributions of $\sigma^{(0)}$ are negligible). In Ref. [1] the authors estimated 10% theoretical uncertainty, due to the indetermination of the distribution and fragmentation functions. We recalculate the quantity $\langle \cos\phi \rangle$ by formulae of Eq.(10) for the same ranges as in Ref.[1] using the new sets of scale-dependent parton distribution [6] and fragmentation functions [8]. In Fig.5 we also exhibit the result of Ref. [1], where parton distribution [7] and scale-independent Segal's fragmentation functions [11] have been used. From Fig.5 one can conclude that those new distribution and fragmentation functions bring up to 18% difference to quantity $\langle \cos\phi \rangle$. This difference arises most probably due to the discrepancy between GRV and HMSR distribution functions at small x ($x < 0.1$).

Fig.6 displays the result for quantity $\langle \cos 2\phi \rangle$ calculated by formulae of Eq.(11) in the same range using the recent sets of Q^2 depending distribution functions.

In summary, we have investigated the azimuthal asymmetries in semi-inclusive deep inelastic leptoproduction arising due to both perturbative and nonperturbative effects at HERMES energies. We have showed that due to small Q^2 and relatively large x_H in that kinematical regime, the k_T^2/Q^2 order corrections to $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$ are significant. At small Q^2 (at moderate Q^2 as well) these quantities are somewhat sensitive to the intrinsic transverse momentum, and consequently, the measurement of the azimuthal asymmetries may provide a good way to obtain $\langle k_T \rangle$.

Moreover, we have reconsidered the results of perturbative effects for $\langle \cos\phi \rangle$ [1] in the kinematic regime at HERA using the more recent Q^2 depending parton distribution and fragmentation functions, which bring up to 18% difference in $\langle \cos\phi \rangle$. In the same approach we have calculated the $\langle \cos 2\phi \rangle$ as well.

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Figure Captions

Fig.1 The definition of the azimuthal angle ϕ.

Fig.2 Diagrams contributed to quantity $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$ in the zeroth-order in α_S (a), and the first-order in α_S: (b - d). The dashed line is a gluon.

Fig.3 $\langle \cos\phi \rangle$ at HERMES energies in the (a) - k_T^2/Q^2 order, (b) - k_T/Q order.

Fig.4 $\langle \cos 2\phi \rangle$ at HERMES energies in the (a) - k_T^2/Q^2 order, (b) - k_T/Q order.

Fig.5 $\langle \cos\phi \rangle$ at large transfer momentum with using the (a) - parton distribution functions of Ref. [7] and fragmentation functions of Ref. [11], (b) - recent scale-dependent parton distribution [6] and fragmentation [8] functions. The kinematical cuts are $Q^2 = 100 GeV^2$, $0.2 < y < 0.8$, $0.05 < x_H < 0.15$ and $0.3 < z_H < 1$.

Fig.6 $\langle \cos 2\phi \rangle$ at large transfer momentum with using the recent scale-dependent parton distribution [6] and fragmentation [8] functions. The kinematical cuts are the same as in Fig.5.

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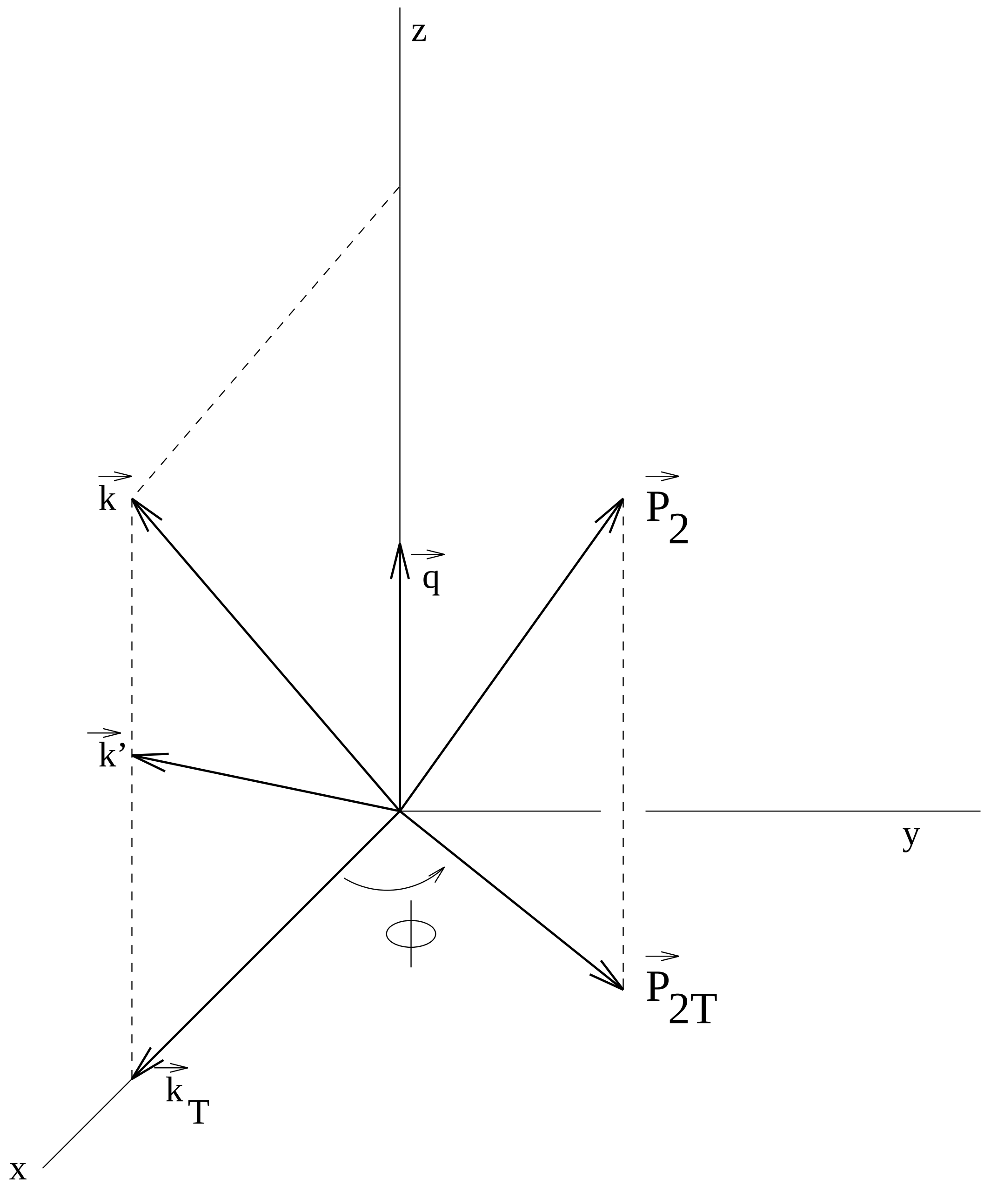


Fig. 1

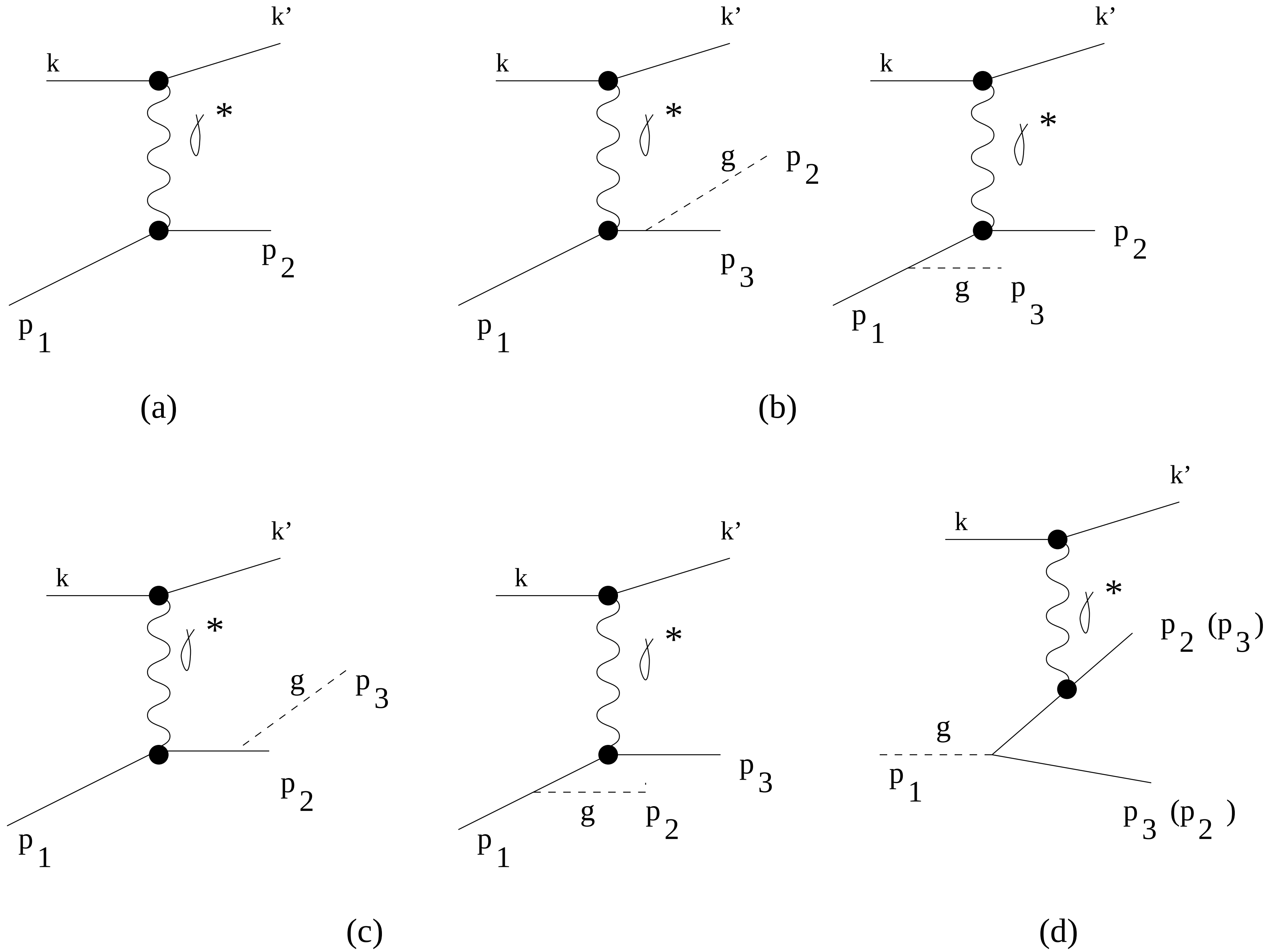
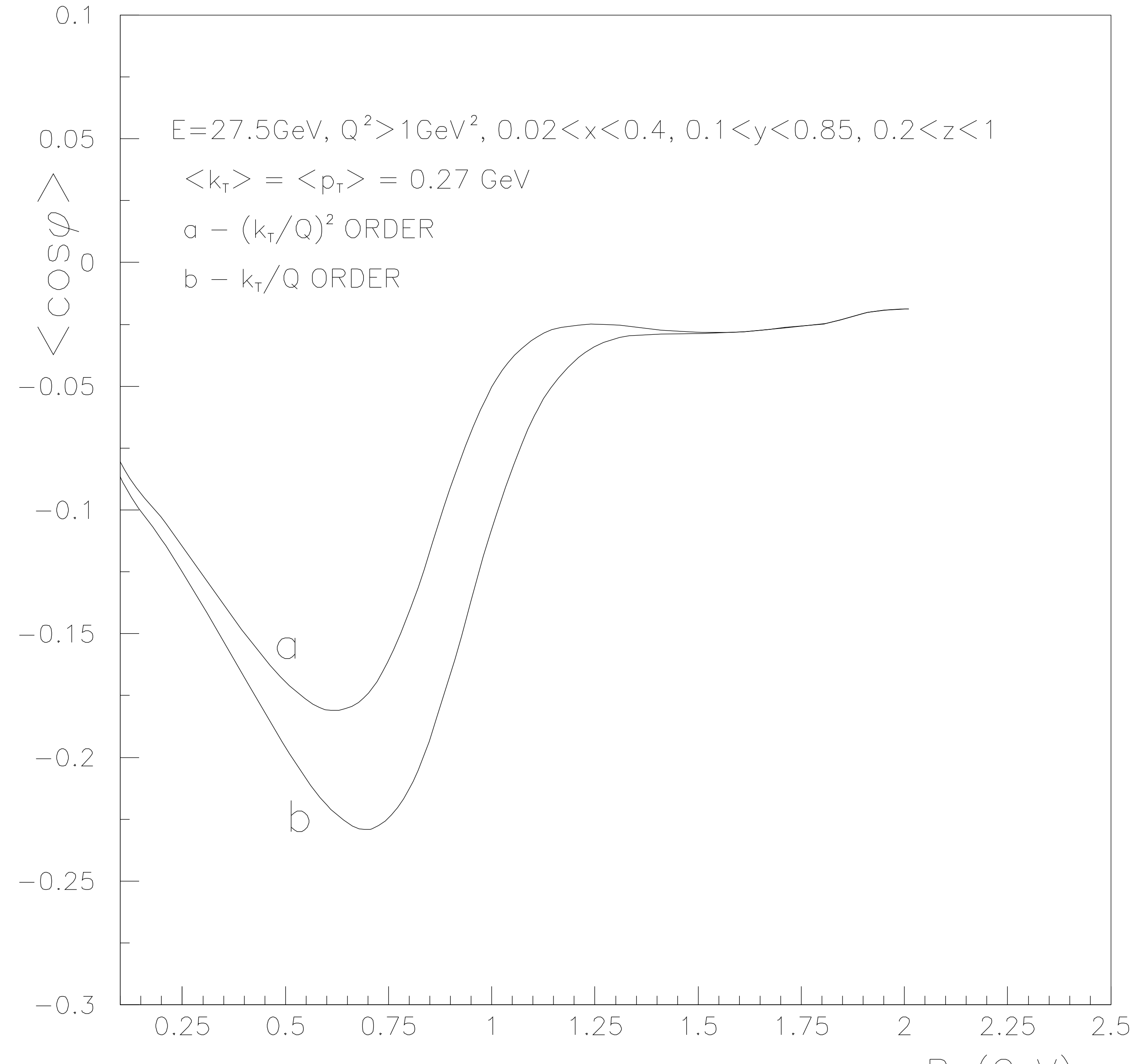


Fig. 2



E=27.5GeV, Q²>1GeV², 0.02<x<0.4, 0.1<y<0.85, 0.2<z<1
<k_T> = <p_T> = 0.27 GeV
a – (k_T/Q)² ORDER
b – k_T/Q ORDER
<cosφ>
a
b
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−0.3
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
P_c (GeV)

Fig.3

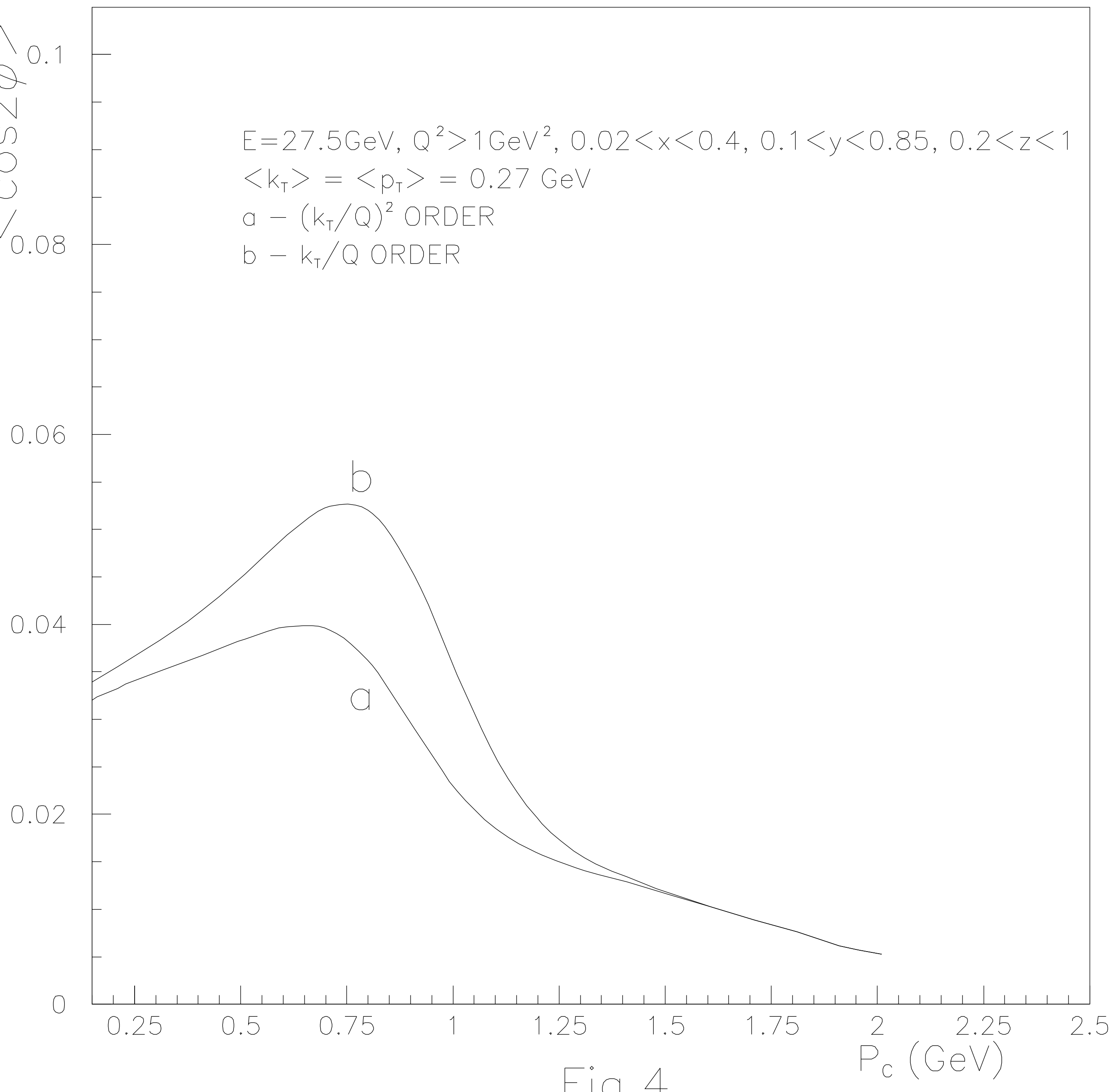
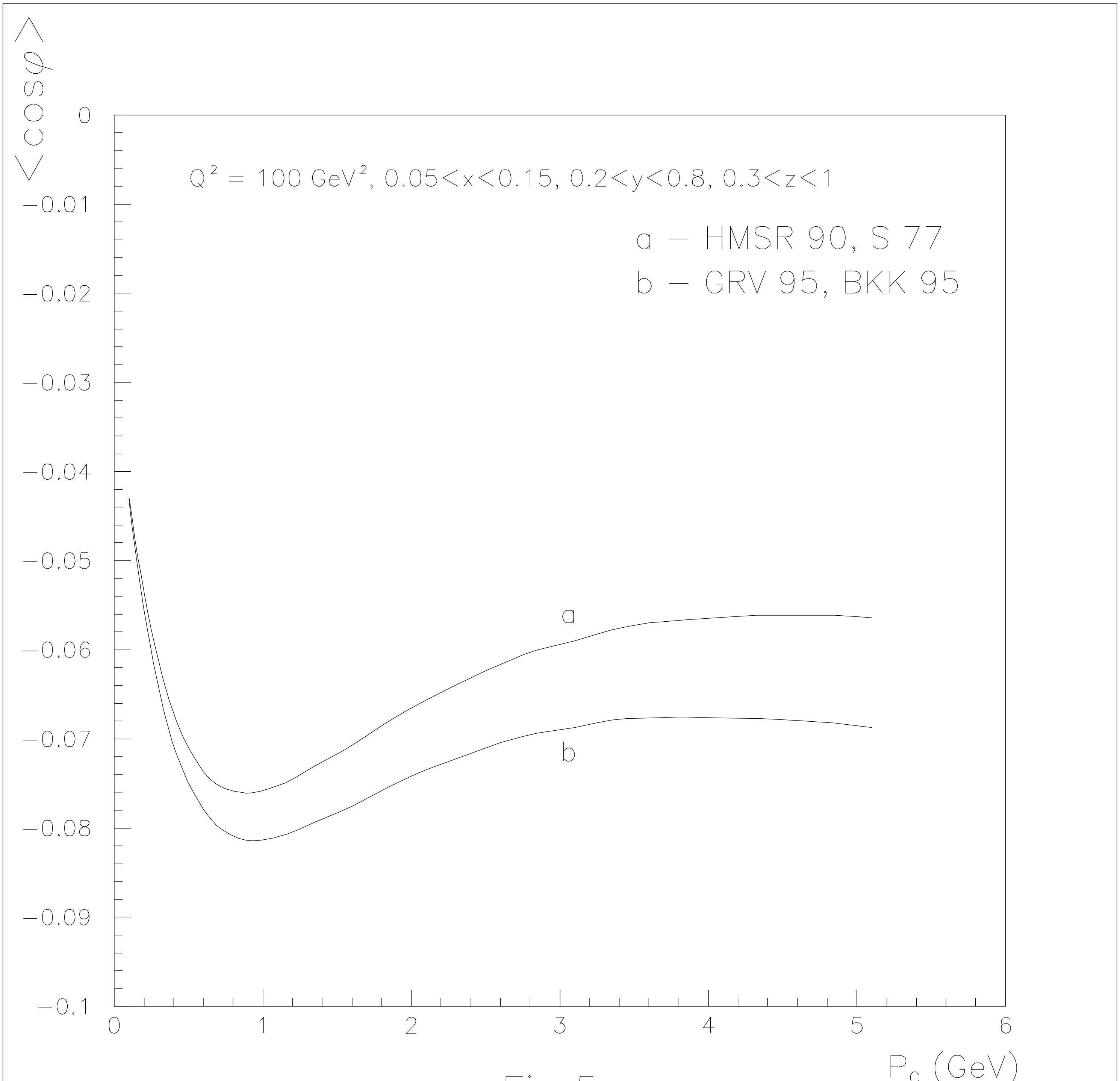
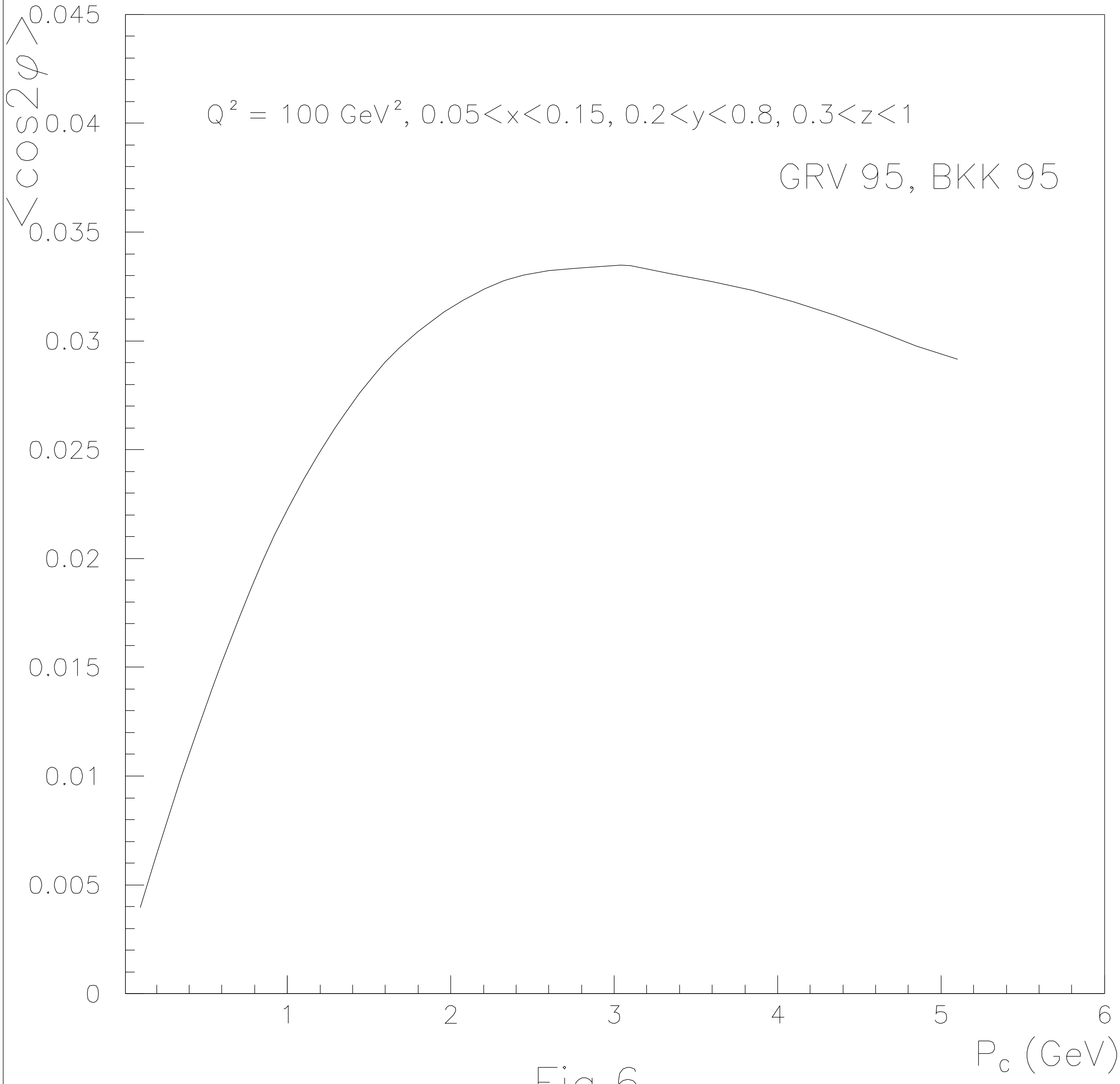

E=27.5GeV, Q²>1GeV², 0.02<x<0.4, 0.1<y<0.85, 0.2<z<1
<kT> = <pT> = 0.27 GeV
a − (kT/Q)² ORDER
b − kT/Q ORDER
<cos2φ>
0.1
0.08
0.06
0.04
0.02
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
b
a
Pc (GeV)

Fig.4



$\langle \cos\varphi \rangle$
$Q^2 = 100$ GeV^2, $0.05<x<0.15$, $0.2<y<0.8$, $0.3<z<1$
a – HMSR 90, S 77
b – GRV 95, BKK 95
a
b
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
−0.07
−0.08
−0.09
−0.1
0
1
2
3
4
5
6
P_c (GeV)

Fig.5



$\langle \cos 2\varphi \rangle$
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Q^2 = 100 GeV^2, 0.05<x<0.15, 0.2<y<0.8, 0.3<z<1
GRV 95, BKK 95
1
2
3
4
5
6
P_c (GeV)

Fig.6