Time-reversal-odd asymmetry in semi-inclusive leptoproduction in quantum chromodynamics

Kaoru Hagiwara
Physics Department, University of Wisconsin—Madison, Madison, Wisconsin 53706

Ken-ichi Hikasa and Naoyuki Kai
Physics Department, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan
(Received 17 June 1982)

The angular distribution of hadrons produced in deep-inelastic lepton-nucleon scattering is studied in perturbative quantum chromodynamics. Time-reversal-odd asymmetry in the distribution is calculated in the one-loop order where quark and gluon final-state interactions at short distances are taken into account. Qualitative features of the asymmetry expected in neutrino, antineutrino, and longitudinally polarized electron or muon scattering are studied. Observation of the asymmetry with the expected sign should provide clear evidence for the existence of the three-gluon coupling.

I. INTRODUCTION

The short-distance behavior of strong interactions has been successfully described by the quantum-chromodynamics (QCD) perturbation theory.[1] Evidence for the existence of gluons (gluon jets) and their small effective coupling to quarks has been qualitatively established. On the other hand, clear evidence for the gluon self-coupling, whose existence is supposed to be essential for both asymptotic freedom and color confinement, is still lacking.

Among various proposed tests to measure the gluon self-coupling, we find that the tests based on observation of time-reversal-odd (T-odd) quantities in hard processes[2–4] are particularly promising. The common features of these tests are that the quantities to be measured are highly sensitive to the gluon self-coupling, and even the qualitative observation of such effects is sufficient to establish its existence. Furthermore, and more importantly, they are fairly insensitive to the poorly known gluon distribution in a nucleon and its fragmentation functions.

Simple T-odd quantities that we can observe without spin measurement are not only T-odd, but also parity-odd (P-odd). Hence observations of T-odd effects require either spin measurements or parity-violating interactions. In e^+e^- annihilation experiments, it is found that we need longitudinally polarized beams at presently available energies.[2,3] In leptoproduction experiments, definite helicity of neutrinos and antineutrinos and their weak-charged-current interaction provides an ideal testing ground for perturbative QCD.

In a recent publication,[4] we reported the results of our one-loop calculation on the T-odd asymmetry in semi-inclusive neutrino and antineutrino charged-current reactions. Two important observations were made: we can define a T-odd observable as an asymmetry in the hadron momentum distribution with respect to the precisely measurable lepton scattering plane, and perturbative QCD gives infrared-finite predictions for the asymmetry. In the one-loop order, which is the leading order for T-odd quantities in perturbative QCD, we found that the asymmetry is quite sensitive to the gluon self-coupling, such that the establishment of its sign will be sufficient to distinguish between QCD and the Abelian gluon model.

In this paper we present the details of our calculation and provide a formalism which can be used in further studies on this subject. We also show predictions for longitudinally-polarized-electron—nucleon scattering.

The paper is organized as follows. In Sec. II we show the kinematics of semi-inclusive leptoproduction and fix our notation. We also explain what kinds of reactions are useful to measure T-odd effects. In Sec. III, we show the parton-model expression for the process and present all the parton scattering cross sections in the leading order, including both T-even and T-odd parts. In Sec. IV, we make rough numerical estimates for the asymmetry in neutrino, antineutrino, and longitudinally-polarized-electron scattering. We find that the results are virtually independent of the gluon distribution in a nucleon and fairly insensitive to the gluon decay functions. Section V is devoted to conclusions.

We include three appendices for completeness. In

Appendix A, some basic facts about T-odd effects are reviewed. In Appendix B, we define the hadronic tensor and the nine independent scalar functions which survive in the massless lepton limit. In Appendix C, we briefly sketch our one-loop calculation of the absorptive part of the parton scattering amplitudes and explain the differences between our leptoproduction calculation and the similar one in e^+e^- annihilation into a quark pair and a gluon.[5,6]

II. KINEMATICS

Consider the semi-inclusive process

$$l(k)+N(P)\to l'(k')+h(P')+\text{ anything }, \qquad (1)$$

where l and l' are leptons, N a target nucleon, and h an observed hadron. k, k', P, and P' are their four-momenta. In the lowest order of the weak and electromagnetic interactions, the nucleon-spin-averaged invariant cross section for the process can be written as

$$\frac{\omega' E' d\sigma}{d^3k' d^3P'}=\frac{1}{4F}K^2 L^{\mu\nu}H_{\mu\nu}\ , \qquad (2)$$

where ω' and E' are the energies of the final lepton and the hadron, respectively, $F=[(k\cdot P)^2-m_l{}^2 m_N{}^2]^{1/2}$ is the invariant flux, and

$$K=\frac{G_F}{\sqrt{2}\pi}$$

for the weak charged-current reaction, and

$$K=\frac{\alpha}{-q^2}$$

with $q=k-k'$ for the electromagnetic interaction. Here G_F denotes the Fermi constant and α the fine-structure constant.

The leptonic tensor reads

$$L_{\mu\nu}=2\left[k_\mu k'_\nu+k_\nu k'_\mu+\frac{q^2}{2}g_{\mu\nu}\mp i\epsilon_{\mu\nu\rho\sigma}k^\rho k'^\sigma\right], \qquad (3)$$

where we take the upper (lower) sign for the left- (right-) handed initial leptons and neglect all lepton masses. The spin-averaged hadronic tensor is defined as

$$H_{\mu\nu}=\frac{1}{(2\pi)^4}\overline{\sum_N}\sum_h\sum_X\int \langle N\,|\,J_\nu^\dagger(0)\,|\,hX\rangle \times\langle hX\,|\,J_\mu(0)\,|\,N\rangle d\Phi(X)\ . \qquad (4)$$

Here the average over N and sum over h are for their helicity states and the summation over X runs every possible final state. J_μ denotes the weak or electromagnetic current and $d\Phi(X)$ is the invariant phase space for the X state (see Appendix B for details).

The hadronic tensor has in general 16 independent scalar functions, 9 of which contribute to the cross sections in the vanishing lepton mass limit. We define these nine functions in Appendix B, which in general depend on $-q^2$ and three scaling variables

$$F_i=F_i(-q^2,x,z,\kappa)\quad (i=L,T,3,\ldots,9) \qquad (5)$$

in the deep-inelastic region where we can neglect the target and hadron masses. The scaling variables are defined as follows:

$$\frac{-q^2}{2q\cdot P}=x\ ,\qquad \frac{P\cdot P'}{q\cdot P}=z\ ,\qquad \frac{q\cdot P'}{P\cdot P'}=-x\left[1-\frac{\kappa^2}{z^2}\right], \qquad (6)$$

where κ represents the transverse momentum of the hadron perpendicular to the current-target axis divided by $(-q^2)^{1/2}$, i.e., $\kappa=P'_T/(-q^2)^{1/2}$. The kinematical limit of this variable reads

$$0\le\kappa\le\left[\frac{1-x}{x}z(1-z)\right]^{1/2}.$$

By contracting with the leptonic tensor we find

$$L^{\mu\nu}H_{\mu\nu}=\frac{2}{y^2}(A+B\cos\phi+C\cos2\phi+D\sin\phi+E\sin2\phi) \qquad (7)$$

with

$$\begin{aligned}A&=2(1-y)F_L+[1+(1-y)^2]F_T\pm y(2-y)F_3\ ,\\ B&=(1-y)^{1/2}[(2-y)F_4\pm yF_5]\ ,\\ C&=(1-y)F_6\ ,\\ D&=(1-y)^{1/2}[\pm yF_7+(2-y)F_8]\ ,\\ E&=(1-y)F_9\ ,\end{aligned} \qquad (8)$$

where we take the upper (lower) sign for the left- (right-) handed lepton beam. We have introduced the scaling variable

$$\frac{q\cdot P}{k\cdot P}=y$$

as usual and ϕ denotes the azimuthal angle between the transverse momentum of the hadron and the scattered lepton with respect to the current-target axis (see Fig. 1). Four of these nine functions (F_3,F_5,F_8,F_9) are P-odd in the hadronic part and three (F_7,F_8,F_9) are T-odd (see Appendix B for details). P-odd functions vanish for the purely electromagnetic current and from Eq. (8) we find that the functions F_3, F_5, and F_7 vanish for the unpolarized beam.

The three T-odd functions (F_7,F_8,F_9) are the coefficients of $\sin\phi$ and $\sin 2\phi$ in Eq. (7). This can be easily understood since in the target rest frame,

$$\langle \sin\phi \rangle = \left\langle \frac{\vec{k}\times\vec{k}\,'\cdot\vec{P}\,'_T}{|\vec{k}\times\vec{k}\,'|\;|\vec{P}\,'_T|} \right\rangle ,$$

where $\vec{k}$, $\vec{k}\,'$, and $\vec{P}\,'_T$ are the three vectors of l, l', and the hadron transverse momentum about current direction, respectively. The right-hand side is not only T-odd, but also P-odd. What we can measure in longitudinally-polarized-electron electromagnetic scattering is expressed as

$$\langle \sin\phi \rangle = \mp \left\langle \frac{\vec{s}\times\vec{k}\,'\cdot\vec{P}\,'_T}{|\vec{s}\times\vec{k}\,'|\;|\vec{P}\,'_T|} \right\rangle ,$$

with $\vec{s}$ denoting the spin vector of the electron where we take the upper (lower) sign for left- (right-) handed electrons.

It is phenomenologically important that the asymmetry $\langle \sin\phi \rangle$ is related to the left $(0<\phi<\pi)$ -right $(\pi<\phi<2\pi)$ asymmetry in the hadron momentum distribution with respect to the lepton scattering plane,

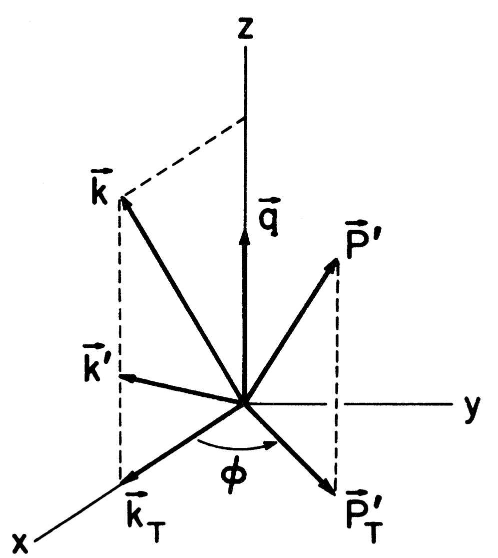

FIG. 1. Three-momenta for the process $l(k)+N(P) \to l'(k')+h(P')+$ anything in the target rest frame.

$$A(-q^2;x,y,z,\kappa) = \frac{\left[\dfrac{d\sigma(\text{left})}{dx\,dy\,dz\,d\kappa^2} - \dfrac{d\sigma(\text{right})}{dx\,dy\,dz\,d\kappa^2}\right]}{\left[\dfrac{d\sigma(\text{left})}{dx\,dy\,dz\,d\kappa^2} + \dfrac{d\sigma(\text{right})}{dx\,dy\,dz\,d\kappa^2}\right]} , \tag{9}$$

which is simply $4/\pi$ times $\langle \sin\phi \rangle$. The lepton scattering plane can be precisely measured even in neutrino and antineutrino charged-current reactions. A measurement of $\langle \sin 2\phi \rangle$ in these reactions may be more involved.

T-odd effects are related to the absorptive part of the scattering amplitude when the interactions preserve time-reversal invariance[7] (see Appendix A for a review on T-odd effects). At short distances, these absorptive parts may be described by QCD perturbation theory. Since the tree amplitudes do not have the absorptive part, the leading contribution to the T-odd functions F_7, F_8, and F_9 will come from the one-loop amplitudes, whose calculation is the subject of the next section.

III. PARTON CROSS SECTIONS

In the parton model, the hadronic tensor can be expressed as a convolution of the parton distribution, $D_{a/N}(\xi)$ in a nucleon, the partonic tensor $\hat{H}^{b/a}_{\mu\nu}$, and the fragmentation function $D_{h/b}(\eta)$:

$$H_{\mu\nu} = \sum_{a,b} \int \frac{d\xi}{\xi} \int \frac{d\eta}{\eta^2} D_{h/b}(\eta) \hat{H}^{b/a}_{\mu\nu} D_{a/N}(\xi) . \tag{10}$$

Here the partonic tensor is defined as

$$\hat{H}^{b/a}_{\mu\nu} = \frac{1}{(2\pi)^4} \overline{\sum_a} \sum_b \sum_X \int \langle a \,|\, J^\dagger_\nu(0) \,|\, bX \rangle \times \langle bX \,|\, J_\mu(0) \,|\, a \rangle d\Phi(X) , \tag{11}$$

where the average over a and sum over b are for their helicity as well as color states and the sum over X runs every possible parton final state. In Eq. (10) we neglect possible effects from the nonperturbative transverse momentum spread in the distribution and the fragmentation functions, which may not invalidate our order-of-magnitude arguments in the following.[8] Within the same approximation, we can identify the azimuthal angle of the produced hadron as that of the parton b, and the nine scalar functions in Eq. (5) can be expressed as

$$F_i(-q^2,x,z,\kappa)=\sum_{a,b}\int \frac{d\xi}{\xi}\int \frac{d\eta}{\eta^2}D_{h/b}(\eta)\hat{F}_i^{b/a}D_{a/N}(\xi) \quad (i=L,T,3,\ldots,9)\ . \tag{12}$$

Here the partonic scalar functions $\hat{F}_i^{b/a}$ are defined in the same manner as we define the hadronic functions (see Appendix B). They can be calculated in QCD perturbation theory and have general forms

$$\hat{F}_i^{b/a}=\hat{F}_i^{b/a}\left[\hat{x},\hat{z},\hat{\kappa},\ln\frac{\mu^2}{-q^2},\frac{p_a{}^2}{-q^2},\frac{p_b{}^2}{-q^2};\frac{\alpha_s}{\pi}(\mu^2)\right] \quad (i=L,T,3,\ldots,9) \tag{13}$$

with

$$\frac{-q^2}{2q\cdot p_a}=\hat{x},\quad \frac{p_a\cdot p_b}{q\cdot p_a}=\hat{z},\quad \frac{q\cdot p_b}{p_a\cdot p_b}=-\hat{x}\left[1-\frac{\hat{\kappa}^2}{\hat{z}^2}\right], \tag{14}$$

and μ being the renormalization point for the QCD coupling constant α_s/π. In our approximation the parton variables [Eq. (14)] are related to the hadronic ones [Eq. (6)] by

$$\hat{x}=\frac{x}{\xi},\quad \hat{z}=\frac{z}{\eta},\quad \hat{\kappa}=\frac{\kappa}{\eta},\quad \hat{\phi}=\phi\ . \tag{15}$$

The partonic scalar functions Eq. (13) in general have mass singularities, powers of $\ln(p_a{}^2/-q^2)$ and $\ln(p_b{}^2/-q^2)$, in the massless parton limit. It has been shown in each order of perturbation theory that these singularities can be universally factorized into the redefinition of the distribution and fragmentation functions.[11] The final formula reads

$$F_i(-q^2,x,z,\kappa)=\sum_{a,b}\int \frac{d\xi}{\xi}\int \frac{d\eta}{\eta^2}\tilde{D}_{h/b}(\eta,M'^2)\tilde{F}_i^{b/a}\left[\hat{x},\hat{z},\hat{\kappa},\ln\frac{\mu^2}{-q^2},\ln\frac{M^2}{-q^2},\ln\frac{M'^2}{-q^2};\frac{\alpha_s}{\pi}(\mu^2)\right]\tilde{D}_{a/N}(\xi,M^2)\ , \tag{16}$$

where $\tilde{D}_{a/N}$ and $\tilde{D}_{h/b}$ are the scale-dependent distribution and fragmentation functions with M and M' their factorization mass scales, respectively. In the leading order, we have the simple expressions

$$F_i(-q^2,x,z,\kappa)=\sum_{a,b}\int \frac{d\xi}{\xi}\int \frac{d\eta}{\eta^2}\tilde{D}_{h/b}(\eta,-q^2)\tilde{F}_i^{b/a}\left[\hat{x},\hat{z},\hat{\kappa};\frac{\alpha_s}{\pi}(-q^2)\right]\tilde{D}_{a/N}(\xi,-q^2) \tag{17}$$

for $i=L,T,3,\ldots,9$.

In the zeroth order in α_s/π, only two of the parton cross sections are nonvanishing, namely $\tilde{F}_T$ and $\tilde{F}_3$. They have the form for the charged-current reaction

$$\tilde{F}_T^{b/a}=|U_{ba}|^2\delta(1-\hat{x})\delta(1-\hat{z})\frac{1}{\pi}\delta(\hat{\kappa}^2),\quad \tilde{F}_3^{b/a}=\pm|U_{ba}|^2\delta(1-\hat{x})\delta(1-\hat{z})\frac{1}{\pi}\delta(\hat{\kappa}^2)\ , \tag{18}$$

where U_{ba} denotes the Kobayashi-Maskawa matrix element[12] and we take the upper sign for quarks and the lower sign for antiquarks. For the purely electromagnetic-current reaction, we have

$$\tilde{F}_T^{b/a}=2\delta_{ba}e_b{}^2\delta(1-\hat{x})\delta(1-\hat{z})\frac{1}{\pi}\delta(\hat{\kappa}^2) \tag{19}$$

with e_b denoting the electromagnetic charge of the parton b in the unit of the proton charge and all other functions vanish.

In the first order in α_s/π, there are contributions to $\tilde{F}_L$ to $\tilde{F}_6$,[9] while the T-odd terms $\tilde{F}_7$ to $\tilde{F}_9$ remain zero. They have the general form, for the charged-current reaction,

$$\tilde{F}_i^{b/a}=|U_{ba}|^2\frac{\alpha_s}{\pi}f_i^{b/a}(\hat{x},\hat{z})\frac{\hat{z}}{\pi}\delta\left[\hat{\kappa}^2-\frac{1-\hat{x}}{\hat{x}}\hat{z}(1-\hat{z})\right] \quad (i=L,T,3,\ldots,6)\ , \tag{20}$$

where we omit the nonleading terms proportional to the lowest-order cross sections, Eq. (18), in $\tilde{F}_T$ and $\tilde{F}_3$.

The leading contributions to the T-odd functions $\tilde{F}_7$, $\tilde{F}_8$, and $\tilde{F}_9$ come from the absorptive part of the one-loop diagrams shown in Fig. 2. Figure 2(a) shows the amplitudes for current + quark $\rightarrow$ quark + gluon and Fig. 2(b) shows those for current + gluon $\rightarrow$ quark + antiquark. We briefly sketch our calculation of

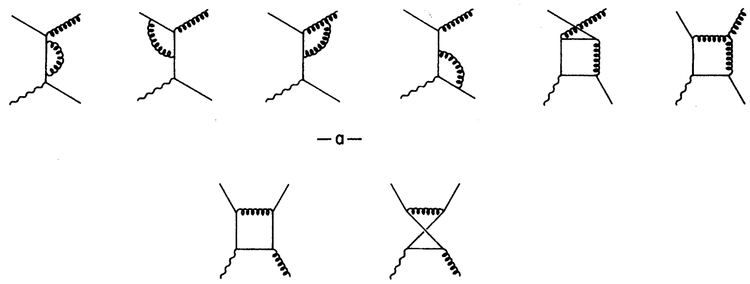

FIG. 2. Diagrams contributing to the absorptive part of the parton scattering amplitude, (a) current + quark →quark + gluon and (b) current + gluon →quark + antiquark, in the one-loop order. Wavy lines denote currents and curly lines are gluons.

the absorptive part of these diagrams in Appendix C. The final results read

$$\hat{F}_i^{b/a}=|U_{ba}|^2\pi\left[\frac{\alpha_s}{\pi}\right]^2 f_i^{b/a}(\hat{x},\hat{z})\frac{\hat{z}}{\pi}\delta\left[\hat{\kappa}^2-\frac{1-\hat{x}}{\hat{x}}\hat{z}(1-\hat{z})\right]\quad(i=7,8,9)\ . \tag{21}$$

For the electromagnetic currents, P-odd functions ($i=3,5,8,9$) vanish and the coupling factor $|U_{ba}|^2$ must be replaced by $2e_q{}^2$ in the remaining functions.

The functions $f_i^{b/a}$ for $i=L,T,3,\ldots,6$ in Eq. (20) are found in Ref. 9 and those for $i=7,8,9$ in Eq. (21) have been reported in our previous publication.[4] We reproduce all these functions here for completeness. The functions $f_i^{q/q}$, $f_i^{G/q}$, $f_i^{\bar{q}/\bar{q}}$, and $f_i^{G/\bar{q}}$ are obtained from the functions $f_i(\hat{x},\hat{z})$ below according to the substitution rule shown in Table I. $f_i^{q/G}$ and $f_i^{\bar{q}/G}$ are obtained from $\bar{f}_i(\hat{x},\hat{z})$ in a similar way. These 18 functions are

$$\begin{aligned}
f_L&=2\hat{x}\hat{z}C_F\ ,\\
f_T&=\frac{1}{2}\left[\frac{1+\hat{x}^2\hat{z}^2}{(1-\hat{x})(1-\hat{z})}+(1-\hat{x})(1-\hat{z})\right]C_F\ ,\\
f_3&=\frac{1}{2}\left[\frac{1+\hat{x}^2\hat{z}^2}{(1-\hat{x})(1-\hat{z})}-(1-\hat{x})(1-\hat{z})\right]C_F\ ,\\
f_4&=2\left[\frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})}\right]^{1/2}[-\hat{x}\hat{z}-(1-\hat{x})(1-\hat{z})]C_F\ ,\\
f_5&=2\left[\frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})}\right]^{1/2}[-\hat{x}\hat{z}+(1-\hat{x})(1-\hat{z})]C_F\ ,\\
f_6&=2\hat{x}\hat{z}C_F\ ,\\
f_7&=\frac{1}{2}\left[\frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})}\right]^{1/2}\{-[\hat{x}\hat{z}-(1-\hat{x})(1-\hat{z})-\hat{z}(1-\hat{z})L(1-\hat{z})]C_AC_F\\
&\qquad -[(1-\hat{x})(1-\hat{z})+2\hat{z}(1-\hat{z})L(1-\hat{z})]C_F{}^2\}\ ,\\
f_8&=\frac{1}{2}\left[\frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})}\right]^{1/2}\{-[\hat{x}\hat{z}+(1-\hat{x})(1-\hat{z})+(1-2\hat{x})\hat{z}(1-\hat{z})L(1-\hat{z})]C_AC_F\\
&\qquad +[(1-\hat{x})(1-\hat{z})+2(1-2\hat{x})\hat{z}(1-\hat{z})L(1-\hat{z})]C_F{}^2\}\ ,\\
f_9&=\tfrac{1}{2}\hat{x}\hat{z}\{[2-(1-2\hat{z})L(1-\hat{z})]C_AC_F-[1-2(1-2\hat{z})L(1-\hat{z})]C_F{}^2\}
\end{aligned}\tag{22}$$

and

TABLE I. The parton functions $f_i^{b/a}$ for various subprocesses expressed in terms of 18 functions f_i and $\bar{f}_i$ shown in Eqs. (22) and (23).

	$f_i^{q/q}$	$f_i^{G/q}$	$f_i^{\bar{q}/\bar{q}}$	$f_i^{G/\bar{q}}$	$f_i^{q/G}$	$f_i^{\bar{q}/G}$
$i=L,T,6$	$f_i(\hat{x},\hat{z})$	$f_i(\hat{x},1-\hat{z})$	$f_i(\hat{x},\hat{z})$	$f_i(\hat{x},1-\hat{z})$	$\bar{f}_i(\hat{x},\hat{z})$	$\bar{f}_i(\hat{x},1-\hat{z})$
$i=3,9$	$f_i(\hat{x},\hat{z})$	$f_i(\hat{x},1-\hat{z})$	$-f_i(\hat{x},\hat{z})$	$-f_i(\hat{x},1-\hat{z})$	$\bar{f}_i(\hat{x},\hat{z})$	$\bar{f}_i(\hat{x},1-\hat{z})$
$i=4,7$	$f_i(\hat{x},\hat{z})$	$-f_i(\hat{x},1-\hat{z})$	$f_i(\hat{x},\hat{z})$	$-f_i(\hat{x},1-\hat{z})$	$\bar{f}_i(\hat{x},\hat{z})$	$-\bar{f}_i(\hat{x},1-\hat{z})$
$i=5,8$	$f_i(\hat{x},\hat{z})$	$-f_i(\hat{x},1-\hat{z})$	$-f_i(\hat{x},\hat{z})$	$f_i(\hat{x},1-\hat{z})$	$\bar{f}_i(\hat{x},\hat{z})$	$-\bar{f}_i(\hat{x},1-\hat{z})$

$$
\begin{aligned}
\bar{f}_L &= 4\hat{x}(1-\hat{x})T_F\,,\\
\bar{f}_T &= [\hat{x}^2+(1-\hat{x})^2]\frac{\hat{z}^2+(1-\hat{z})^2}{2\hat{z}(1-\hat{z})}T_F\,,\\
\bar{f}_3 &= -[\hat{x}^2+(1-\hat{x})^2]\frac{1-2\hat{z}}{2\hat{z}(1-\hat{z})}T_F\,,\\
\bar{f}_4 &= -2\left[\frac{\hat{x}(1-\hat{x})}{\hat{z}(1-\hat{z})}\right]^{1/2}(1-2\hat{x})(1-2\hat{z})T_F\,,\\
\bar{f}_5 &= 2\left[\frac{\hat{x}(1-\hat{x})}{\hat{z}(1-\hat{z})}\right]^{1/2}(1-2\hat{x})T_F\,,\\
\bar{f}_6 &= 4\hat{x}(1-\hat{x})T_F\,,\\
\bar{f}_7 &= \left[\frac{\hat{x}(1-\hat{x})}{\hat{z}(1-\hat{z})}\right]^{1/2}\{(1-\hat{x})[\hat{z}L(\hat{z})-(1-\hat{z})L(1-\hat{z})]+\hat{x}(1-2\hat{z})\}\left[C_F-\frac{C_A}{2}\right]T_F\,,\\
\bar{f}_8 &= \left[\frac{\hat{x}(1-\hat{x})}{\hat{z}(1-\hat{z})}\right]^{1/2}\{(1-\hat{x})[\hat{z}L(\hat{z})+(1-\hat{z})L(1-\hat{z})]-\hat{x}\}\left[C_F-\frac{C_A}{2}\right]T_F\,,\\
\bar{f}_9 &= \hat{x}(1-\hat{x})[L(\hat{z})-L(1-\hat{z})]\left[C_F-\frac{C_A}{2}\right]T_F\,,
\end{aligned}
\tag{23}
$$

where $C_F=\frac{4}{3}$, $C_A=3$, and $T_F=\frac{1}{2}$ are the color factors and

$$L(t)=\frac{1}{t}\left[1+\frac{1}{t}\ln(1-t)\right].$$

These functions govern completely the leading-order behavior of the three-jet events in leptoproduction.

Let us examine the qualitative properties of the T-odd functions f_7 to f_9 and $\bar{f}_7$ to $\bar{f}_9$. The functions $\bar{f}_i$ come from the current-gluon-fusion mechanism. We should add the contributions to the asymmetry both from the quark and antiquark in the final state unless we identify the jet species. According to the substitution rule in Table I and the properties of the functions $\bar{f}_i$ under exchange of $\hat{z}$ and $1-\hat{z}$, the functions $\bar{f}_8$ and $\bar{f}_9$ cannot contribute and only $\bar{f}_7$ remains effective. Hence there is no contribution from the current-gluon-fusion process to $\langle\sin 2\phi\rangle$. Even for the asymmetry $\langle\sin\phi\rangle$, the contribution from the fusion mechanism is suppressed by the color factor $T_F(C_F-C_A/2)$ in $\bar{f}_7$, which is about $\frac{1}{50}$ times smaller in magnitude than the color factor C_FC_A. Therefore, we can expect that the T-odd asymmetries are insensitive to (or sometimes independent of) the gluon distribution in a nucleon.[13]

The asymmetries will thus be mainly governed by current-quark scattering processes expressed by f_7, f_8, and f_9. Because of the fragmentation mechanism, the asymmetries in the hadron momentum distribution are governed by the moderately large-$\hat{z}$ behavior of these parton cross sections. Let us show the $\hat{z}\to 1$ behavior of these functions:

$$f_7=\frac{1}{2}\left[\frac{\hat{x}}{1-\hat{x}}\right]^{1/2}\left[-\left[\frac{\hat{x}}{\sqrt{1-\hat{z}}}-\frac{1+\hat{x}}{2}\sqrt{1-\hat{z}}\right]C_A C_F+\hat{x}\sqrt{1-\hat{z}}C_F{}^2+O((1-\hat{z})^{3/2})\right],$$

$$f_8=\frac{1}{2}\left[\frac{\hat{x}}{1-\hat{x}}\right]^{1/2}\left[-\left[\frac{\hat{x}}{\sqrt{1-\hat{z}}}+\frac{1-3\hat{x}}{2}\sqrt{1-\hat{z}}\right]C_A C_F+\hat{x}\sqrt{1-\hat{z}}C_F{}^2+O((1-\hat{z})^{3/2})\right], \tag{24}$$

$$f_9=\frac{\hat{x}}{2}\{[\tfrac{3}{2}-\tfrac{5}{6}(1-\hat{z})]C_A C_F-\tfrac{4}{3}(1-\hat{z})C_F{}^2+O((1-\hat{z})^{3/2})\}\ .$$

These functions describe contributions from the quark jet to the asymmetries. From Table I we see that the contribution from the gluon jet can be obtained from them by exchanging $\hat{z}$ with $1-\hat{z}$. We find that the gluon-jet contributions are suppressed by powers of $\sqrt{1-\hat{z}}$ for the large-$\hat{z}$ region.

The above investigations show that we can expect the qualitative behavior of the T-odd asymmetries at moderate values of hadron momentum fraction z from the expression in Eqs. (24). f_7 and f_8 determine $\langle \sin\phi \rangle$ and f_9 determines $\langle \sin 2\phi \rangle$. For all these functions, we see that the coefficients of $C_A C_F$ are larger than those of $C_F{}^2$ and have opposite signs. This reflects the fact that the final-state interactions between quark and gluon states shown in Fig. 2(a) are the t-channel gluon exchange for the diagrams with the three-gluon coupling and they are the s- or u-channel quark exchange for the diagrams with only the quark-quark-gluon coupling. Therefore, we can expect the sign of the asymmetry $\langle \sin\phi \rangle$ to be negative and that of $\langle \sin 2\phi \rangle$ to be positive in both neutrino and antineutrino scattering. The sign of the asymmetry $\langle \sin\phi \rangle$ will be negative (positive) for the left- (right-) handed electron scattering. All these signs of the asymmetries reflect strongly the existence of the gluon self-coupling because they are essentially determined by the $C_A C_F$ terms. In the next section we make a crude estimate for the magnitude of these asymmetries.

IV. NUMERICAL ESTIMATES

In order to see the observable effects of the T-odd functions F_7, F_8, and F_9, we show in this section crude parton-model estimates for the z dependence of the asymmetries $\langle \sin\phi \rangle$ and $\langle \sin 2\phi \rangle$ in semi-inclusive leptoproduction.

By integrating over the azimuthal angle of the scattered lepton (l') and the transverse momentum of the hadron (h) in Eqs. (2) and (7), we find

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi}=K^2\frac{sx}{2}\frac{\pi}{z}\int (A+B\cos\phi+C\cos 2\phi+D\sin\phi+E\sin 2\phi)d\kappa^2\ . \tag{25}$$

The coefficients $A,B,\ldots,E$ are expressed in terms of the nine hadron functions $F_L,F_T,F_3,\ldots,F_9$ [Eq. (8)], which are the convolution integrals of the effective parton distribution functions $\widetilde{D}_{a/N}$, the hard-scattering part $\widetilde{F}_i^{b/a}$, and the fragmentation functions $\widetilde{D}_{h/b}$ [Eq. (17)]. The coefficient A in Eq. (25) is order 1, B and C are order α_s/π, and D and E are order $\pi(\alpha_s/\pi)^2$.

We need the effective quark and gluon distribution functions, their effective fragmentation functions, and the value of the running coupling constant in the leading-order analysis. In the following, we make a bold approximation in which we neglect all scaling violation effects. In view of this and the other approximation implicit in the relations between the parton variables and the hadronic ones [Eq. (15)] where we neglect possible smearing effects due to the nonperturbative transverse-momentum fluctuation (see Ref. 8), as well as the target and hadron masses, our predictions should be regarded as order-of-magnitude estimates.

We define the z-dependent average values of the asymmetries by

$$\langle \sin(m\phi) \rangle=\int\int\int dx\,dy\,d\phi\,\sin(m\phi)\frac{d\sigma}{dx\,dy\,dz\,d\phi}\Big/\int\int\int dx\,dy\,d\phi\frac{d\sigma}{dx\,dy\,dz\,d\phi}\ , \tag{26}$$

for $m=1$ and $m=2$, where we restrict the integration domain of the lepton variables x and y to the deep-inelastic region; $-q^2>5$ GeV^2 and $(q+P)^2>5$ GeV^2. We use the quark distribution function parametrized by Barger and Phillips[14] and the quark fragmentation functions by Sehgal.[15] We take only charged pions as detected hadrons and set

$$\begin{aligned}\widetilde{D}_{h/q}(\eta)&=\widetilde{D}_{\pi^+/u}(\eta)+\widetilde{D}_{\pi^-/u}(\eta)\\&=[1.05(1-\eta)^2+0.05]/\eta\ ,\end{aligned} \tag{27}$$

where u denotes an up quark. For the gluon distribution in a nucleon we take the form

$$\tilde{D}_{G/N}(\xi)=\frac{n+1}{2}(1-\xi)^n/\xi \tag{28}$$

with $3\leq n\leq 7$. For the gluon fragmentation function we examine the two cases

$$\tilde{D}^{\rm hard}_{h/G}(\eta)=\tilde{D}_{h/q}(\eta) \tag{29}$$

or

$$\tilde{D}^{\rm soft}_{h/G}(\eta)=2\int_\eta^1 \frac{d\eta'}{\eta'}\tilde{D}_{h/q}(\eta')\ . \tag{30}$$

We take $\alpha_s/\pi=0.1$ for definiteness.

In Fig. 3 we show our predictions for $\langle \sin\phi \rangle$ in neutrino (ν) and antineutrino ($\bar{\nu}$) charged-current scattering off an isoscalar target at $E_{\rm beam}=100$ GeV.[16] We neglect Cabibbo (Kobayashi-Maskawa) mixing for simplicity. The solid lines are for the hard-gluon fragmentation functions and the dashed lines are for the soft ones. The contribution from the gluon-fusion mechanism is negligible and the differences from varying the gluon distribution function are virtually indistinguishable. As expected in the previous section, the sign of the asymmetry is negative for both reactions and their magnitude is at the few-percent level. Although we cannot determine the precise value of the hadron momentum fraction z in these reactions, the mild z dependences of the asymmetry may help us smearing or integrating over a certain region of z values. The lepton scattering plane is defined by the directions of the beam and the scattered leptons which are both well determined. The left-right asymmetry with respect to this plane [Eq. (9)] is simply $4/\pi$ times $\langle \sin\phi \rangle$ and may be precisely measured.

Shown in Fig. 4 are the predictions for $\langle \sin 2\phi \rangle$ in the same reactions at the same energy. The solid and dashed lines denote the predictions for the hard- and soft-gluon fragmentation, respectively. These predictions are independent of the gluon distributions in a nucleon. The asymmetries are positive while their magnitudes tend to be smaller than a half-percent level. Measurements of $\langle \sin 2\phi \rangle$ require careful final-state analyses in these reactions.

We show in Fig. 5 our predictions for $\langle \sin\phi \rangle$ in the purely electromagnetic scattering of the left-handed electron off an isoscalar target at $E_{\rm beam}=20$ GeV. The symbols are the same as before and the predictions are very insensitive to the gluon distribution. The asymmetry for the right-handed electron scattering is obtained by just reversing the sign in the figure. Hence the asymmetry $\langle \sin\phi \rangle$ in the longitudinally-polarized-electron scattering is about a half-percent in magnitude and negative (positive) for the left- (right-) handed polarization. The difference in magnitude between the opposite helicity experiments and the asymmetry $\langle \sin 2\phi \rangle$ is proportional to the parity-violating effects in these reactions.[17]

V. CONCLUSION

In this paper we have presented details of our one-loop calculations on T-odd asymmetries in semi-inclusive leptoproduction processes. At short distances, formation of the quark and gluon "final states" is supposed to be responsible to the hadron-

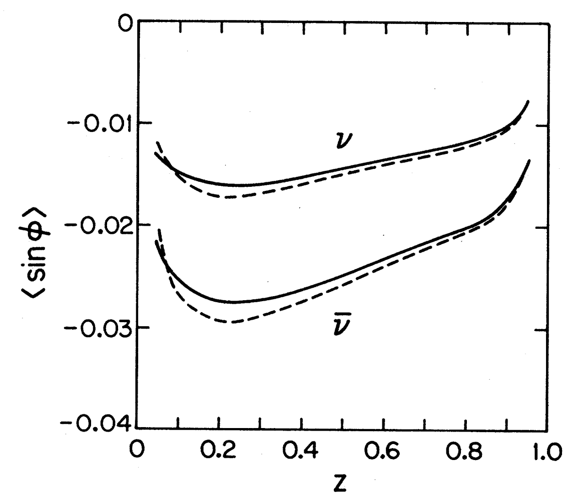

FIG. 3. Predictions for $\langle \sin\phi \rangle$ in semi-inclusive neutrino and antineutrino scattering off isoscalar target at $E_{\rm beam}=100$ GeV. Solid lines show our predictions by using the hard form [Eq. (29)] for the gluon fragmentation function and dashed lines are obtained by using the soft form [Eq. (30)].

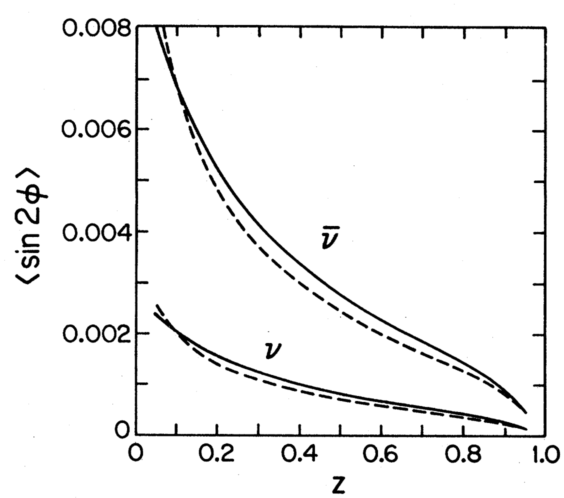

FIG. 4. Predictions for $\langle \sin 2\phi \rangle$ in semi-inclusive neutrino and antineutrino scattering off isoscalar target at $E_{\rm beam}=100$ GeV. Symbols are the same as those in Fig. 3.

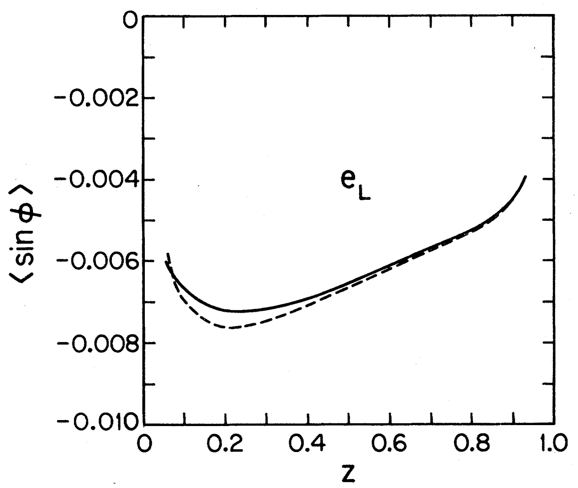

FIG. 5. Predictions for $\langle \sin\phi \rangle$ in semi-inclusive polarized (left-handed) electron scattering off isoscalar target at $E_{\rm beam}=20$ GeV. Symbols are the same as those in Fig. 3. Predictions for the right-handed electroproduction can be obtained by reversing the sign.

jet structures. T-odd asymmetries in the hadron-jet system may therefore be determined by the absorptive part of the parton scattering amplitudes. These quantities can be calculated in perturbation theory as far as they are free from mass (soft and collinear) singularities. Since the quark and gluon "final-state" interactions are sensitive to the existence of the gluon self-coupling, observation of T-odd asymmetries can be an ideal place to test the non-Abelian nature of the color gauge symmetry.[2-4]

We find that in the leptoproduction processes we can define an infrared-finite T-odd quantity as the left-right asymmetry in the hadron momentum distribution with respect to the lepton scattering plane. The magnitude of the asymmetries is predicted to be of the order of a few percent in neutrino and antineutrino charged-current reactions and about a half-percent in longitudinally-polarized-electron electromagnetic scattering. The signs of these asymmetries strongly reflect the existence of the three-gluon coupling and their measurements provide a clear evidence for its existence.

In our numerical estimates in Sec. IV, we have calculated the asymmetries in the P_T'-integrated inclusive spectrum for simplicity.[18] If we probe the high-P_T' region ($\kappa \sim 1$) where the lowest-order contribution to the cross section becomes negligible at high energies, the T-odd asymmetry would be of the order of α_s instead of $\alpha_s{}^2/\pi$. Another possible approach may be to measure the asymmetries within the clean three-jet events, where again we can expect their magnitudes to be of the order of α_s. The overall cross sections, however, become much smaller. Further phenomenological studies may be important.

ACKNOWLEDGMENTS

One of the authors (N.K.) wishes to thank Professor T. Muta for valuable discussions. The authors would like to thank F. Herzog and M. Olsson for reading the manuscript. Numerical computations are supported by the Institute for Nuclear Study, University of Tokyo. This research was also supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the Department of Energy under Contract No. DE-AC02-76ER00881.

APPENDIX A: T-ODD EFFECTS

In this appendix we briefly review the well-known facts about T-odd effects[7] for completeness.

The scattering amplitude from the state i to the state f can be written as

$$S_{fi}=\delta_{fi}+i(2\pi)^4\delta^4(P_f-P_i)T_{fi}$$

in general. Unitarity of the S matrix reads

$$T_{fi}-T_{if}^{*}=iA_{fi} \tag{A1}$$

with

$$A_{fi}=\sum_n T_{nf}^{*}T_{ni}(2\pi)^4\delta^4(P_n-P_i)\ , \tag{A2}$$

where the sum over n runs over every possible state. We call A_{fi} the absorptive part of the amplitude T_{fi}.

It is straightforward to see that the unitarity relation can be rewritten as

$$|T_{fi}|^2=|T_{if}|^2-2\,\mathrm{Im}(T_{fi}^{*}A_{fi})-|A_{fi}|^2\ . \tag{A3}$$

Let the states $\tilde{i}$ and $\tilde{f}$ be the states made from i and f, respectively, by reversing the directions of both three-momenta and spins. Subtracting $|T_{\tilde{f}\tilde{i}}|^2$ from both sides of Eq. (A3) we find

$$|T_{fi}|^2-|T_{\tilde{f}\tilde{i}}|^2=(|T_{if}|^2-|T_{\tilde{f}\tilde{i}}|^2)-2\,\mathrm{Im}(T_{fi}^{*}A_{fi})-|A_{fi}|^2\ . \tag{A4}$$

Here T-odd effects are defined by the left-hand side of this equation. The first term in the right-hand side (RHS) vanishes if the interactions preserve time-reversal invariance (detailed balance). The remaining terms in the RHS include the absorptive part. In perturbation theory, the absorptive part of the amplitude appears in the one- or higher-loop order. Hence the second term in the RHS of Eq. (A4) gives the leading contribution to the T-odd effects in the perturbation theory.

APPENDIX B: HADRONIC TENSOR

The spin-averaged hadronic tensor in semi-inclusive lepton scattering processes off a nucleon is defined in Eq. (4) as

$$H_{\mu\nu}=\frac{1}{(2\pi)^4}\overline{\sum_N}\sum_h\sum_X\int\langle N|J_\nu^\dagger(0)|hX\rangle\langle hX|J_\mu(0)|N\rangle d\Phi(X) \tag{B1}$$

with

$$d\Phi(X)=(2\pi)^4\delta^4\left[q+P-P'-\sum_{i\in X}P_i\right]\prod_{i\in X}\frac{d^3P_i}{(2\pi)^3 2E_i}\ , \tag{B2}$$

the invariant phase space for the X state.

The hadronic tensor $H_{\mu\nu}$ has the most general form

$$\begin{aligned}(-q^2)^2H_{\mu\nu}=&-q^2g_{\mu\nu}H_1+q_\mu q_\nu H_2+P_\mu P_\nu H_3+P'_\mu P'_\nu H_4\\&+q_{\{\mu}P_{\nu\}}H_5+q_{\{\mu}P'_{\nu\}}H_6+P_{\{\mu}P'_{\nu\}}H_7+iq_{[\mu}v_{\nu]}H_8+iP_{[\mu}v_{\nu]}H_9+iP'_{[\mu}v_{\nu]}H_{10}\\&+iq_{[\mu}P_{\nu]}H_{11}+iq_{[\mu}P'_{\nu]}H_{12}+iP_{[\mu}P'_{\nu]}H_{13}+q_{\{\mu}v_{\nu\}}H_{14}+P_{\{\mu}v_{\nu\}}H_{15}+P'_{\{\mu}v_{\nu\}}H_{16}\ .\end{aligned} \tag{B3}$$

Here the symbols $a_{\{\mu}b_{\nu\}}$ represent $a_\mu b_\nu+a_\nu b_\mu$ and $a_{[\mu}b_{\nu]}$ denote $a_\mu b_\nu-a_\nu b_\mu$ and

$$v^\mu=\frac{1}{-q^2}\epsilon^{\mu\nu\rho\sigma}q_\nu P_\rho P'_\sigma$$

with $\epsilon^{\mu\nu\rho\sigma}$ the completely antisymmetric tensor ($\epsilon^{0123}=-\epsilon_{0123}=-1$). The 16 real scalar functions H_i ($i=1$ to 16) are dimensionless functions of the five variables

$$H_i=H_i\left[\frac{q\cdot P}{-q^2},\frac{q\cdot P'}{-q^2},\frac{P\cdot P'}{-q^2},\frac{m_N{}^2}{-q^2},\frac{m_h{}^2}{-q^2}\right],$$

while we are interested in the deep-inelastic limit of these functions where only the first three of these variables remain finite [i.e., x, z, κ in Eq. (6)].

From the definition (B3) we see that the first ten terms are T-even and the remaining six are T-odd. The terms with H_8 to H_{10} and H_{14} to H_{16} are P-odd and they are absent for currents with definite parity.[19] In the case of the electromagnetic current, the tensor satisfies the current conservation condition

$$q^\mu H_{\mu\nu}=q^\nu H_{\mu\nu}=0\ . \tag{B4}$$

Even in the case of the weak currents, the above equations hold when we can neglect quark masses. Nine of these sixteen functions are independent in this limit and five are nonvanishing for the currents with definite parity. Expressions in terms of the nine independent functions are easily obtained from Eq. (B3) by simply omitting terms with H_2, H_5, H_6, H_8, H_{11}, H_{12}, H_{14} and by making the following replacements in the remaining terms:

$$g_{\mu\nu}\to g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\ ,$$

$$P_\mu\to P_\mu-\frac{q\cdot P}{q^2}q_\mu\ ,$$

and

$$P'_\mu\to P'_\mu-\frac{q\cdot P'}{q^2}q_\mu\ .$$

By contracting the hadronic tensor (B3) with the leptonic one (3) we find the expressions shown in Eqs. (7) and (8). The nine functions F_L, F_T, $F_3,\ldots,F_9$ in Eq. (8) are related to the nine independent scalar functions H_i ($i=1,3,4,7,9,10,13,15,16$) in Eq. (B3) as follows:

$$
\begin{aligned}
F_L &= H_1 + \frac{1}{4x^2} H_3 \\
&\quad + \frac{1}{4}\left[z + \frac{\kappa^2}{z}\right]^2 H_4 + \frac{1}{2x}\left[z + \frac{\kappa^2}{z}\right] H_7 , \\
F_T &= -H_1 + \frac{\kappa^2}{2} H_4 , \\
F_3 &= \frac{\kappa^2}{2x} H_{10} , \\
F_4 &= -\kappa\left[\left[z + \frac{\kappa^2}{z}\right] H_4 + \frac{1}{x} H_7\right] , \\
F_5 &= -\frac{\kappa}{2x}\left[\frac{1}{x} H_9 + \left[z + \frac{\kappa^2}{z}\right] H_{10}\right] , \\
F_6 &= \kappa^2 H_4 , \\
F_7 &= \frac{\kappa}{x} H_{13} , \\
F_8 &= -\frac{\kappa}{2x}\left[\frac{1}{x} H_{15} + \left[z + \frac{\kappa^2}{z}\right] H_{16}\right] , \\
F_9 &= \frac{\kappa^2}{x} H_{16} .
\end{aligned}
\tag{B5}
$$

APPENDIX C: PARTON SCATTERING AMPLITUDES

In this appendix we briefly sketch our one-loop calculation of the absorptive parts of the parton scattering amplitudes shown in Fig. 2. We also show differences between our calculation and the crossly related one in e^+e^- annihilation into a quark pair and a gluon.[6]

Let us consider the process current (q)+quark$(p_1)$$\rightarrow$quark$(p_2)$+gluon$(p_3)$. Let the current spin index be μ, the gluon spin index be α, and its color index a. Quark helicity is preserved in the massless limit. The amplitude for definite helicity quarks has the general form

$$T^a_{\mu\alpha} = g T^a T_{\mu\alpha}$$

with g the strong coupling and T^a the generator of the color gauge group. In the covariant perturbation theory with massless quarks, $T_{\mu\alpha}$ can in general be expanded as

$$T_{\mu\alpha} = \sum_{i=1}^{13} a_i M^{(i)}_{\mu\alpha} \tag{C1}$$

with

$$
\begin{aligned}
&M^{(1)}_{\mu\alpha} = \gamma_\mu \gamma_\alpha \not{q}/(-q^2), \quad M^{(2)}_{\mu\alpha} = g_{\mu\alpha} \not{q}/(-q^2) , \\
&M^{(3)}_{\mu\alpha} = p_{1\alpha} \gamma_\mu/(-q^2), \quad M^{(4)}_{\mu\alpha} = p_{2\alpha} \gamma_\mu/(-q^2) , \\
&M^{(5)}_{\mu\alpha} = q_\mu \gamma_\alpha/(-q^2), \quad M^{(6)}_{\mu\alpha} = p_{1\mu} \gamma_\alpha/(-q^2) , \\
&M^{(7)}_{\mu\alpha} = p_{2\mu} \gamma_\alpha/(-q^2), \quad M^{(8)}_{\mu\alpha} = p_{1\mu} p_{2\alpha} \not{q}/(-q^2)^2 , \\
&M^{(9)}_{\mu\alpha} = p_{2\mu} p_{1\alpha} \not{q}(-q^2)^2, \quad M^{(10)}_{\mu\alpha} = q_\mu p_{1\alpha} \not{q}(-q^2)^2 , \\
&M^{(11)}_{\mu\alpha} = p_{1\mu} p_{1\alpha} \not{q}/(-q^2)^2, \quad M^{(12)}_{\mu\alpha} = q_\mu p_{2\alpha} \not{q}/(-q^2)^2 \\
&M^{(13)}_{\mu\alpha} = p_{2\mu} p_{2\alpha} \not{q}/(-q^2)^2 .
\end{aligned}
\tag{C2}
$$

These 13 amplitudes are of course not independent and they are subject to the current conservation conditions

$$q^\mu T_{\mu\alpha} = p_3^\alpha T_{\mu\alpha} = 0 \tag{C3}$$

and to a γ-matrix identity.[20] We define the six independent amplitudes b_i $(i=1,2,\ldots,6)$ by a calculational convenience as follows:

$$T_{\mu\alpha} = \sum_{i=1}^{6} b_i \mathscr{M}^{(i)}_{\mu\alpha} \tag{C4}$$

with

$$
\begin{aligned}
\mathscr{M}^{(1)}_{\mu\alpha} &= \tfrac{1}{2}(\hat{z}-\hat{x}) M^{(1)}_{\mu\alpha} + (1-\hat{z})(M^{(2)}_{\mu\alpha} - M^{(5)}_{\mu\alpha} - M^{(6)}_{\mu\alpha} + (1-\hat{x})(M^{(3)}_{\mu\alpha} - M^{(4)}_{\mu\alpha} + M^{(7)}_{\mu\alpha}) , \\
\mathscr{M}^{(2)}_{\mu\alpha} &= M^{(9)}_{\mu\alpha} + \frac{1-\hat{x}-\hat{z}}{4\hat{x}}(M^{(1)}_{\mu\alpha} - 2M^{(4)}_{\mu\alpha}) - \tfrac{1}{2} M^{(7)}_{\mu\alpha} , \\
\mathscr{M}^{(3)}_{\mu\alpha} &= M^{(10)}_{\mu\alpha} - \tfrac{1}{2} M^{(1)}_{\mu\alpha} + M^{(4)}_{\mu\alpha} - \frac{1-\hat{z}}{2\hat{x}} M^{(5)}_{\mu\alpha} - M^{(7)}_{\mu\alpha} , \\
\mathscr{M}^{(4)}_{\mu\alpha} &= M^{(11)}_{\mu\alpha} + \frac{1}{2\hat{x}}[\tfrac{1}{2} M^{(1)}_{\mu\alpha} - M^{(4)}_{\mu\alpha} - (1-\hat{z}) M^{(6)}_{\mu\alpha} + M^{(7)}_{\mu\alpha}] , \\
\mathscr{M}^{(5)}_{\mu\alpha} &= M^{(12)}_{\mu\alpha} - \tfrac{1}{2} M^{(1)}_{\mu\alpha} + M^{(2)}_{\mu\alpha} + M^{(4)}_{\mu\alpha} - \frac{1+\hat{x}}{2\hat{x}} M^{(5)}_{\mu\alpha} - M^{(6)}_{\mu\alpha} , \\
\mathscr{M}^{(6)}_{\mu\alpha} &= M^{(13)}_{\mu\alpha} + \frac{1-\hat{x}-\hat{z}}{2\hat{x}}\left[\frac{1}{2} M^{(1)}_{\mu\alpha} - M^{(2)}_{\mu\alpha} - M^{(4)}_{\mu\alpha} + M^{(5)}_{\mu\alpha} + M^{(6)}_{\mu\alpha} - \frac{1-\hat{x}}{1-\hat{x}-\hat{z}} M^{(7)}_{\mu\alpha}\right] ,
\end{aligned}
\tag{C5}
$$

where $\hat{x}=-q^2/2qp_1$, and $\hat{z}=p_1p_2/qp_1$.

We find in the tree order only one of these six amplitudes is nonvanishing, namely

$$b_1=\frac{2\hat{x}}{(1-\hat{x})(1-\hat{z})},\quad b_i=0 \text{ for } i\neq 1 . \tag{C6}$$

The tree amplitude is thus determined by only one amplitude.

The absorptive part of the amplitude (C4) can be written as

$$A_{\mu\alpha}=2\sum_{i=1}^{6}(\mathrm{Im}b_i)\mathcal{M}_{\mu\alpha}^{(i)} \tag{C7}$$

and the leading contributions come from the one-loop diagrams in Fig. 2(a). We calculate these diagrams by dimensionally regularizing the IR singularities. We find

$$\begin{aligned}
\mathrm{Im}b_1&=\left\{\frac{\hat{x}}{2(1-\hat{x})(1-\hat{z})}\left[-\frac{f(\epsilon)}{\epsilon}+1-2\hat{z}+\hat{z}^2L(\hat{z})-(1-\hat{z})^2L(1-\hat{z})\right]C_A\right.\\
&\quad\left.+\frac{\hat{x}}{1-\hat{x}}[-1+(1-\hat{z})L(1-\hat{z})]C_F\right\}\alpha_s ,\\
\mathrm{Im}b_2&=\frac{\hat{x}^2}{1-\hat{z}}\{[-2+(1-2\hat{z})L(1-\hat{z})]C_A+[1-2(1-2\hat{z})L(1-\hat{z})]C_F\}\alpha_s ,\\
\mathrm{Im}b_3&=\frac{\hat{x}^2}{1-\hat{z}}\{[1+\hat{z}L(1-\hat{z})]C_A-[1+2\hat{z}L(1-\hat{z})]C_F\}\alpha_s ,\\
\mathrm{Im}b_4&=\frac{\hat{x}^3}{1-\hat{z}}\{L(1-\hat{z})C_A-[1+2L(1-\hat{z})]C_F\}\alpha_s ,\\
\mathrm{Im}b_5&=\mathrm{Im}b_6=0
\end{aligned} \tag{C8}$$

in this order. Here $D=4-2\epsilon$ is the continuous dimension and

$$f(\epsilon)=\Gamma(1+\epsilon)\left[\frac{4\pi\mu^2\hat{x}}{(-q^2)(1-\hat{x})}\right]^{\epsilon}$$

with μ the unit of mass and

$$L(t)=\frac{1}{t}\left[1+\frac{1}{t}\ln(1-t)\right] .$$

From the general argument in Appendix A [see Eq. (A4)] we find that the lowest-order contribution to the T-odd quantity comes from the interference term between the one-loop absorptive part (C7) and the tree amplitude. Since the tree amplitude has only one nonvanishing component (C6), we find by explicit calculation that only four of the six amplitudes in Eq. (C7), $\mathrm{Im}b_3$ to $\mathrm{Im}b_6$, can contribute to the T-odd effects in the lowest order. $\mathrm{Im}b_1$ and $\mathrm{Im}b_2$ shown in Eq. (C8) can contribute only as radiative corrections to them. Hence only one amplitude b_1 in Eq. (C6) determines all the T-even functions f_L to f_6 in Eq. (22) and two absorptive amplitudes $\mathrm{Im}b_3$ and $\mathrm{Im}b_4$ in Eq. (C8) are relevant to the remaining three T-odd functions, f_7 to f_9.

The amplitudes for the process

current(q)+gluon(p_1)$\to$quark(p_2)+antiquark(p_3)

are obtained from the previous amplitudes by the substitution

$$(q,p_1,p_2)\to(q,-p_3,p_2) ,$$

and accordingly,

$$\hat{x}\to-\frac{\hat{x}}{(\hat{z}-\hat{x})},\quad \hat{z}\to\frac{(1-\hat{x})}{(\hat{z}-\hat{x})} .$$

We can define the independent amplitudes for this process by

$$\overline{T}_{\mu\alpha}=\sum_{i=1}^{6}\overline{b}_i\overline{\mathcal{M}}_{\mu\alpha}^{(i)} , \tag{C9}$$

where $\overline{\mathcal{M}}_{\mu\alpha}^{(i)}$ are made from $\mathcal{M}_{\mu\alpha}^{(i)}$ in Eqs. (C5) by the above substitution. In the tree order, we have

$$\overline{b}_1=\frac{2\hat{x}(\hat{z}-\hat{x})}{\hat{z}(1-\hat{z})} \tag{C10}$$

and all other amplitudes vanish. The relevant absorptive amplitudes are again four from the six and they read

$$\begin{aligned}
\mathrm{Im}\bar{b}_3 &= -\frac{2\hat{x}^2}{1-\hat{z}}[\hat{x}-(1-\hat{x})L(1-\hat{z})] \\
&\quad \times \left[C_F-\frac{C_A}{2}\right]\alpha_s\ , \\
\mathrm{Im}\bar{b}_4 &= -\frac{2\hat{x}^3}{1-\hat{z}}[1+L(1-\hat{z})]\left[C_F-\frac{C_A}{2}\right]\alpha_s\ , \\
\mathrm{Im}\bar{b}_5 &= -\frac{2\hat{x}^2}{\hat{z}}[\hat{x}-(1-\hat{x})L(\hat{z})] \\
&\quad \times \left[C_F-\frac{C_A}{2}\right]\alpha_s\ , \\
\mathrm{Im}\bar{b}_6 &= \frac{2\hat{x}^3}{\hat{z}}[1+L(\hat{z})]\left[C_F-\frac{C_A}{2}\right]\alpha_s\ .
\end{aligned} \tag{C11}$$

These functions completely determine the nine functions $\bar{f}_L$ to $\bar{f}_9$ in Eq. (23).

The process $e^+e^-\to\gamma^*(q)\to\bar{q}(p_1)+q(p_2)+G(p_3)$ is related to the process $\gamma^*(q)+q(p_1)\to q(p_2)+G(p_3)$ by the substitution

$$(q,p_1,p_2)\to(q,-p_1,p_2)$$

and the analytic continuation in q^2 from $q^2<0$ to $q^2>0$. We can again define the independent amplitudes in this process by

$$\widetilde{T}_{\mu\alpha}=\sum_{i=1}^{6}\widetilde{b}_i\widetilde{\mathscr{M}}^{(i)}_{\mu\alpha}\ , \tag{C12}$$

where $\widetilde{\mathscr{M}}^{(i)}_{\mu\alpha}$ are made from $\mathscr{M}^{(i)}_{\mu\alpha}$ [Eq. (C5)] by the above substitution. Once again, we find only four of the six absorptive amplitudes, $\mathrm{Im}\widetilde{b}_3$ to $\mathrm{Im}\widetilde{b}_6$, can contribute to the T-odd effects in the lowest order. These four amplitudes are found to be zero in the one-loop order[6,21] and the T-odd effects cannot be measured in e^+e^- annihilation in this order when quark mass effects are neglected.

This can be understood as follows. The amplitude for the process $e^+e^-\to q\bar{q}G$ with massless quarks (C12) has a particular property in the one-loop order that its absorptive part is proportional to the singular terms (with soft and/or collinear singularities) of its real part; more precisely,

$$\mathrm{Im}\widetilde{T}_{\mu\alpha}(\text{one-loop})=\pi\lim_{\epsilon\to 0}\epsilon\,\mathrm{Re}\widetilde{T}_{\mu\alpha}(\text{one-loop}) \tag{C13}$$

in the dimensional regularization scheme.[22] On the other hand, it is not too difficult to see in the one-loop order that the singular part of the amplitude is proportional to the tree amplitude

$$\lim_{\epsilon\to 0}\epsilon\,\mathrm{Re}\widetilde{T}_{\mu\alpha}(\text{one-loop})\propto\widetilde{T}_{\mu\alpha}(\text{tree})\ . \tag{C14}$$

From Eqs. (C13) and (C14), we find that the absorptive part of the one-loop amplitude is proportional to the tree amplitude. Since the tree amplitude has only one component $\widetilde{b}_1$, we find

$$\mathrm{Im}\widetilde{b}_i(\text{one-loop})=0\quad\text{for } i\neq 1\ , \tag{C15}$$

which has been the condition for the vanishing T-odd effects. Details will appear elsewhere.

---

[1]For a recent review, see, e.g., A. J. Buras, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn, 1981), p. 666, and references therein.

[2]A. De Rújula, R. Petronzio, and B. Lautrup, Nucl. Phys. B146, 50 (1978).

[3]K. Fabricius, I. Schmitt, G. Kramer, and G. Schierholz, Phys. Rev. Lett. 45, 867 (1980).

[4]K. Hagiwara, K. Hikasa, and N. Kai, Phys. Rev. Lett. 47, 983 (1981).

[5]T-odd asymmetry is found to vanish in the one-loop order for e^+e^- annihilation into a quark pair and a gluon in massless-quark theory. See Ref. 6.

[6]J. G. Körner, G. Kramer, G. Schierholz, K. Fabricius, and I. Schmitt, Phys. Lett. 94B, 207 (1980).

[7]A. De Rújula, J. M. Kaplan, and E. de Rafael, Nucl. Phys. B35, 365 (1971).

[8]It is known that the finite transverse-momentum (p_T) spread of parton distribution in a nucleon plays a significant role in the angular asymmetry like $\langle\cos\phi\rangle$ or $\langle\cos 2\phi\rangle$. It is because the finite p_T of the parton contributes to the asymmetries by the order of $\langle p_T\rangle/(-q^2)^{1/2}$ or $\langle p_T{}^2\rangle/(-q^2)$, which may be comparable to the order-α_s/π perturbation effects at moderate energies (see Refs. 9 and 10 for these arguments). On the other hand, the axially symmetric p_T distribution cannot contribute to the T-odd asymmetries as $\langle\sin\phi\rangle$ and $\langle\sin 2\phi\rangle$. Those effects are expected to be of the order of $\pi(\alpha_s/\pi)^2\langle p_T{}^2\rangle/(-q^2)$, and may be described as the small smearing correction to our results even at moderate energies.

[9]H. Georgi and H. D. Politzer, Phys. Rev. Lett. 40, 3 (1978); J. Cleymans, Phys. Rev. D 18, 954 (1978); A. Méndez, Nucl. Phys. B145, 199 (1978); G. Köpp, R. Maciejko, and P. M. Zerwas, *ibid.* B144, 123 (1978).

[10]F. Ravndal, Phys. Lett. 43B, 301 (1973); R. N. Cahn, *ibid.* 78B, 269 (1978); P. Mazzanti, R. Odorico, and V. Roberto, *ibid.* 80B, 111 (1978); A. Méndez, A. Raychaudhari, and V. J. Stenger, Nucl. Phys. B148, 499 (1979).

[11]D. Amati, R. Petronzio, and G. Veneziano, Nucl. Phys. B140, 54 (1978); B146, 29 (1978); R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, Nucl.

Phys. B152, 285 (1979); A. H. Mueller, Phys. Rev. D 18, 3705 (1978).

[12]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[13]In the fake theory with $T_F=C_F=1$ and $C_A=0$ which we call the Abelian gluon model, there is no such suppression. Both T_FC_F and C_F^2 become one. Indeed, we find substantial contributions to the asymmetry from the gluon fusion mechanism in this model, which makes the predictions qualitatively different from those of QCD. See Ref. 4.

[14]V. Barger and R. J. N. Phillips, Nucl. Phys. B73, 269 (1974).

[15]L. M. Sehgal, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977,* edited by F. Gutbrod (DESY, Hamburg, 1977), p. 837.

[16]If we reduce the values of the beam energy but retain all our parameters and cuts the same, the sign and shape of the curves in Fig. 3 remain roughly unchanged and their magnitudes decrease (increase) by about 7% (4%) for $E_{\text{beam}}=50$ GeV and by about 25% (13%) for $E_{\text{beam}}=20$ GeV in the neutrino (antineutrino) scattering.

[17]Neutral-current contributions to the T-odd asymmetries may be important in the electron-proton collider experiments being planned.

[18]Although our P_T' integrated asymmetries, $\langle \sin\phi \rangle$ and $\langle \sin 2\phi \rangle$, are formally free from mass singularities, they may not be free from ambiguities due to the nonperturbative hadronic final-state interactions which could be significant in small-P_T' regions where the bulk of the cross section stays at foreseeable high energies. In this respect, introduction of a cutoff in P_T' or in some kind of a hadronic jet measure discussed below may be desirable not only from the experimental, but also from the theoretical point of view.

[19]Time-reversal invariance does not imply the absence of the T-odd terms since the T operation is antiunitary. See Appendix A.

[20]See Eq. (6) with $M=0$ in M. Perrottet, Lett. Nuovo Cimento 7, 915 (1973).

[21]G. Schierholz, in *Current Topics in Elementary Particle Physics,* edited by K. H. Mütter and K. Schilling (Plenum, New York, 1981), p. 77.

[22]The similar identity between the "imaginary part" and the "real part" of the partonic tensor $\hat{H}_{\mu\nu}$ quoted in Ref. 21 is misleading. This is because the imaginary part of the amplitude contributes to the antisymmetric part of the tensor, while the real part contributes to the symmetric part (or vice versa for parity-odd currents) in the one-loop order, which is simply a consequence of the Hermiticity. Furthermore, the relation (C13) is specific to this process; no similar relation holds in the leptoproduction amplitudes (Ref. 4) nor the heavy-quarkonium decay amplitude (Ref. 2), nor even in the $e^+e^-\rightarrow q\bar{q}G$ amplitude with finite quark masses (Ref. 3).