Consider the reconstruction of the position of the vertex of an interaction.

- If the target were just a point, of infinitesimal volume, this position will be reconstructed as a Gaussian function, due to the big number of random variables that are being summed (central limit theorem).

- In reality the target doesn't has differential volume, then it doesn't has differential dimensions.

The reconstructed position will be then a Gaussian convoluted with the spatial function describing the target.

Let's focus on z position. Let be $f_t(z)$ the function that describes the target spatial function, $f_m(z)$ the function that describes the procedure of measurement and $v(z)$ the reconstructed vertex function $v(z) = f_t \star f_m$.

first point:

$$f_t(z) = \delta(z - m) \tag{1}$$

$$f_m(z) = \frac{1}{\sqrt{2\sigma^2}} e^{\frac{-z^2}{2\sigma^2}} \tag{2}$$

$$v(z) = \int_{-\infty}^{\infty} f_t(z - t) f_m(t) dt \tag{3}$$

$$v(z) = \int_{-\infty}^{\infty} \delta(z - t - m) f_m(t) dt \tag{4}$$

$$v(z) = f_m(z - m) = \frac{1}{\sqrt{2\sigma^2}} e^{\frac{-(z-m)^2}{2\sigma^2}} \tag{5}$$

$$\tag{6}$$

Considering the real target as a step function:

$$f_t(z) = \begin{cases} 1, & |z - m| < \frac{w}{2} \\ 0, & o.w. \end{cases} \tag{7}$$

using (3) and (7).

$$v(z) = \int_{-\infty}^{\infty} f_t(z - t) f_m(t) dt \tag{8}$$

$$v(z) = \int_{m-z-w/2}^{m-z+w/2} f_m(t) dt \tag{9}$$

$$v(z) = \int_{m-z-w/2}^{m-z+w/2} \frac{1}{\sqrt{2\sigma^2}} e^{\frac{-t^2}{2\sigma^2}} dt \tag{10}$$

considering the function erf

$$erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx \tag{11}$$

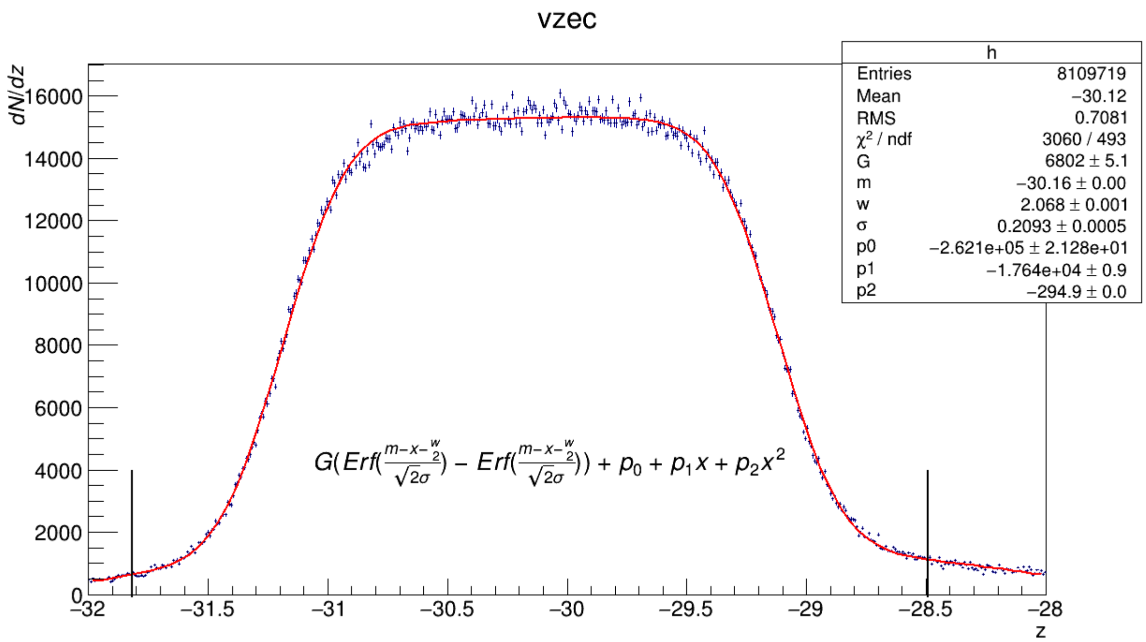

Figure 1: Fitting on Deuterium target using C and D2 data.

using (11) and (10):

$$v(z) = \frac{1}{2}\left(erf(\frac{m-z+w/2}{\sqrt{2}\sigma}) - erf(\frac{m-z-w/2}{\sqrt{2}\sigma}) \right) \tag{12}$$

Adding a second order polynomial and fitting on Carbon and Deuterium events gives: As expected, the dispersion due to vertex measurement (σ on $f_m(z)$) are very close considering the estimations

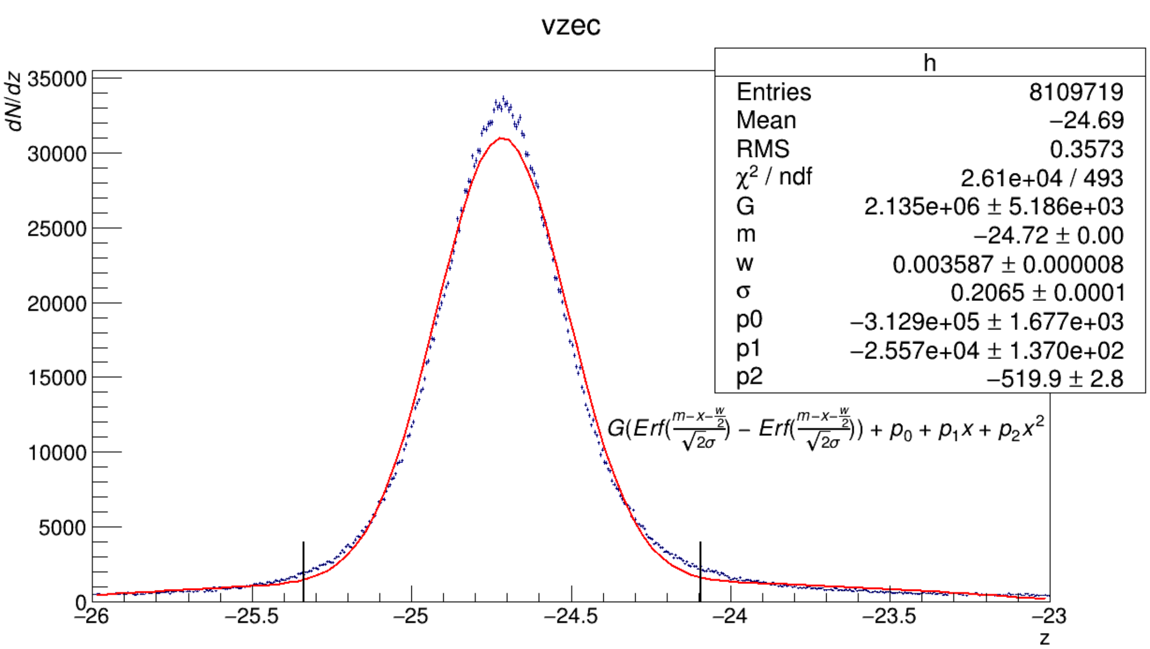

Figure 2: Fitting on Carbon target using C and D2 data.

using each of the targets.

$\boxed{\text{D2: } \sigma = 0.2093 \text{ C: } \sigma = 0.2065}$