086761 - Homework 4

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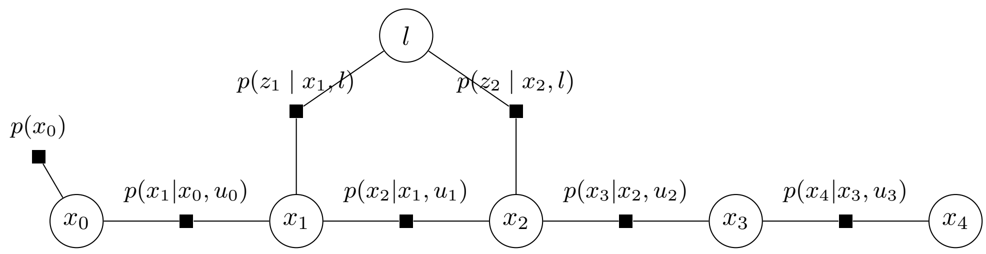
1

1.1

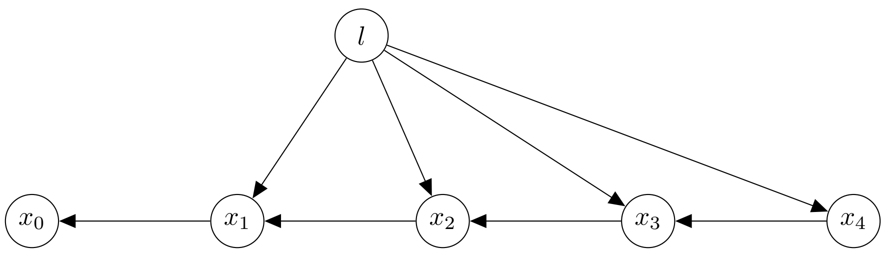
$$p(x_{0:4}, l \mid u_{0:3}, z_1, z_2) = \eta p(x_0)p(x_1|x_0, u_0)p(x_2|x_1, u_1)p(x_3|x_2, u_2)p(x_4|x_3, u_3)p(z_1 \mid x_1, l)p(z_2 \mid x_2, l)$$

1.2

Variable nodes for x_i's and l. A factor node for each one of the 7 factors above, with edges to all involved variables



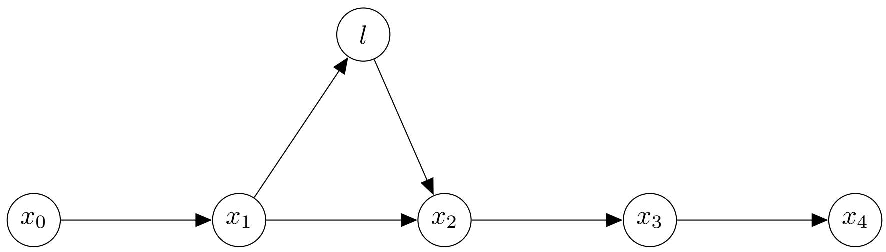
1.3



Derivation:

$$\begin{aligned}
&p(x_0)p(x_1|x_0) = p(x_0|x_1)g_1(x_1)\\
&g_1(x_1)p(x_2|x_1, u_1)p(z_1|x_1, l) = p(x_1|x_2, l)g_2(x_2, l)\\
&g_2(x_2, l)p(x_3|x_2, u_2)p(z_2|x_2, l) = p(x_2|x_3, l)g_3(x_3, l)\\
&g_3(x_3, l)p(x_4|x_3, u_3) = p(x_3|x_4, l)g_4(x_4, l)\\
&g_4(x_4, l) = p(x_4|l)g_5(l)
\end{aligned}$$

The observed variables z_i's and u_i's are incorporated into the g_i's.



1.4

Derivation:

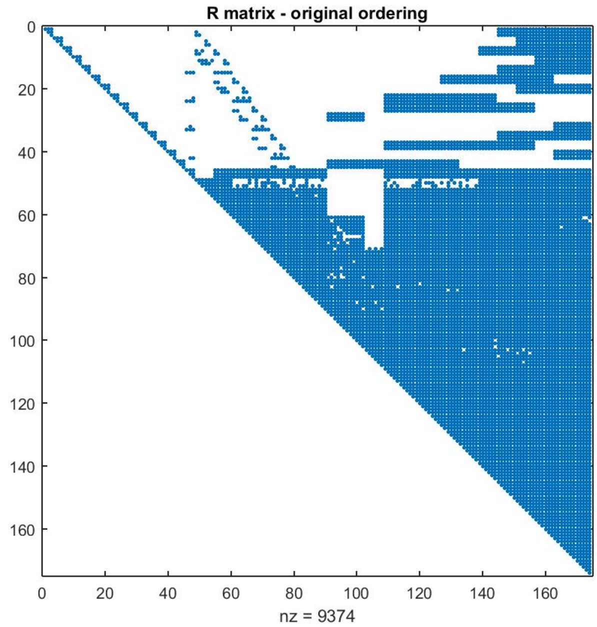
$$
\begin{aligned}
&p(x_4|x_3)\\
&p(x_3|x_2)\\
&p(x_2|x_1)p(z_2|x_2,l) = p(x_2|x_1,l)g_1(x_1,l)\\
&g_1(x_1,l)p(z_1|x_1,l) = p(l|x_1)g_2(x_1)\\
&g_2(x_1)p(x_1|x_0) = p(x_1|x_0)g_3(x_0)
\end{aligned}
$$

The observed variables z_i's and u_i's are incorporated into the g_i's.

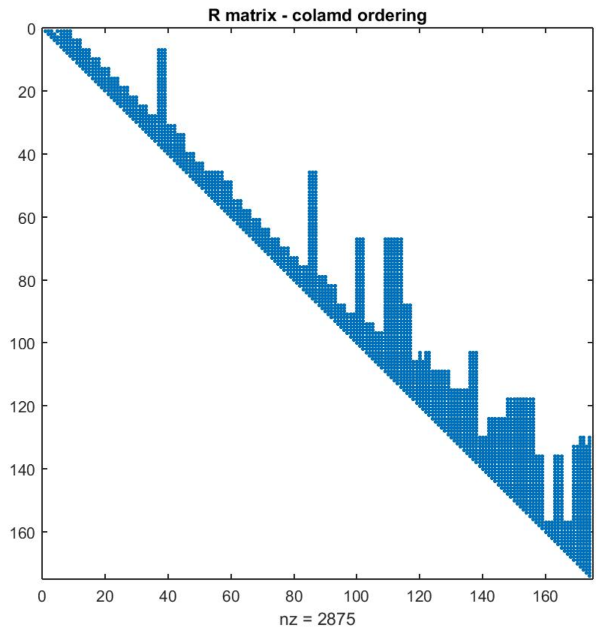
1.5

Both orders are equivalent in terms of estimation accuracy - they represent the same posterior. In terms of computations, in the latter case, the R matrix will contain less nonzero entries (corresponding to edges), so back-substitution will be more efficient.

2



(a)



(b)

Information matrix after rearranging variables with colamd has less than a third of the entries of the original matrix, likely allowing the back-substitution in $R\Delta = d$ to run 3 times as fast.

3 Code

```
%% Load data
close all; clear all;
load hw4_A

%% Original ordering
[~, R] = qr(A);
R = R(1:size(A, 2), :);
spy(R); title('R matrix - original ordering');
print('-djpeg', 'original_R.jpg');

%% New ordering
[~, R] = qr(A(:, colamd(A)));
R = R(1:size(A, 2), :);
spy(R); title('R matrix - colamd ordering');
print('-djpeg', 'new_R.jpg');
```