086761 - Homework 4

Yuri Feldman, Alexander Shender

December 17, 2017

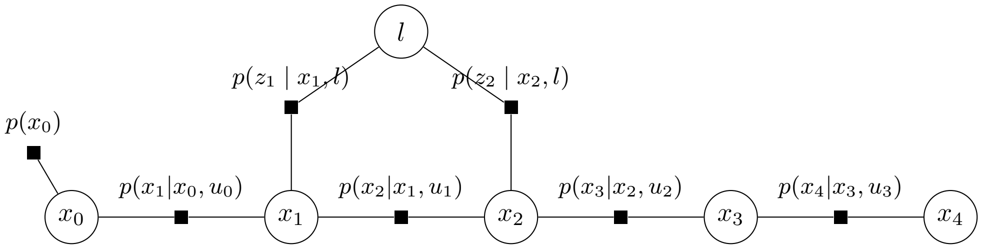
1

1.1

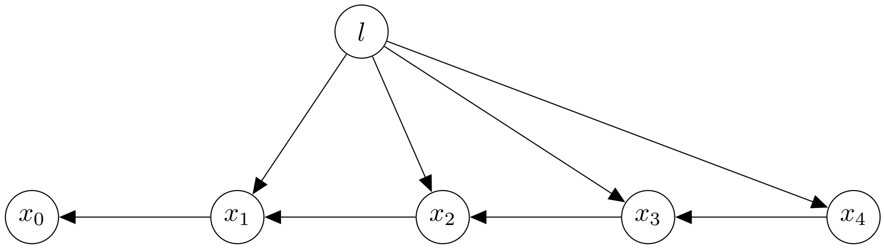
$$p(x_{0:4}, l \mid u_{0:3}, z_1, z_2) = \eta p(x_0)p(x_1|x_0, u_0)p(x_2|x_1, u_1)p(x_3|x_2, u_2)p(x_4|x_3, u_3)p(z_1 \mid x_1, l)p(z_2 \mid x_2, l)$$

1.2

Variable nodes for x_i's and l. A factor node for each one of the 7 factors above, with edges to all involved variables



1.3



Derivation:

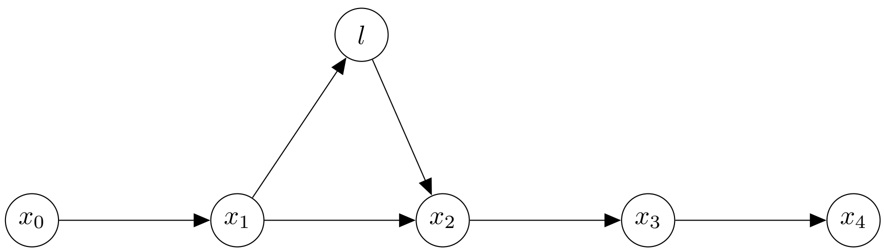
$$
\begin{aligned}
&p(x_0)p(x_1|x_0) = p(x_0|x_1)g_1(x_1)\\
&g_1(x_1)p(x_2|x_1,u_1)p(z_1|x_1,l) = p(x_1|x_2,l)g_2(x_2,l)\\
&g_2(x_2,l)p(x_3|x_2,u_2)p(z_2|x_2,l) = p(x_2|x_3,l)g_3(x_3,l)\\
&g_3(x_3,l)p(x_4|x_3,u_3) = p(x_3|x_4,l)g_4(x_4,l)\\
&g_4(x_4,l) = p(x_4|l)g_5(l)
\end{aligned}
$$

The observed variables z_i's and u_i's are incorporated into the g_i's.

R matrix has 14 nonzero entries:

$$
\begin{array}{cc}
\begin{array}{cccccc} x_0 & x_1 & x_2 & x_3 & x_4 & l \end{array} & \\
\begin{pmatrix}
\times & \times & 0 & 0 & 0 & 0\\
0 & \times & \times & 0 & 0 & \times\\
0 & 0 & \times & \times & 0 & \times\\
0 & 0 & 0 & \times & \times & \times\\
0 & 0 & 0 & 0 & \times & \times\\
0 & 0 & 0 & 0 & 0 & \times
\end{pmatrix} & \begin{array}{c} x_0\\ x_1\\ x_2\\ x_3\\ x_4\\ l \end{array}
\end{array}
\tag{1.1}
$$

1.4



Derivation:

$$
\begin{aligned}
&p(x_4|x_3)\\
&p(x_3|x_2)\\
&p(x_2|x_1)p(z_2|x_2,l) = p(x_2|x_1,l)g_1(x_1,l)\\
&g_1(x_1,l)p(z_1|x_1,l) = p(l|x_1)g_2(x_1)\\
&g_2(x_1)p(x_1|x_0) = p(x_1|x_0)g_3(x_0)
\end{aligned}
$$

The observed variables z_i's and u_i's are incorporated into the g_i's. R matrix has 12 nonzero entries:

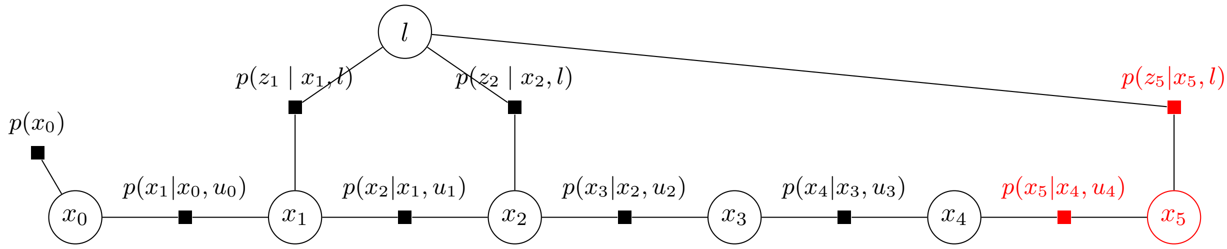
$$
\begin{array}{c c}
\begin{array}{cccccc} x_4 & x_3 & x_2 & l & x_1 & x_0 \end{array} & \\
\begin{pmatrix}
\times & \times & 0 & 0 & 0 & 0 \\
0 & \times & \times & 0 & 0 & 0 \\
0 & 0 & \times & \times & \times & 0 \\
0 & 0 & 0 & \times & \times & 0 \\
0 & 0 & 0 & 0 & \times & \times \\
0 & 0 & 0 & 0 & 0 & \times
\end{pmatrix} & \begin{array}{c} x_4 \\ x_3 \\ x_2 \\ l \\ x_1 \\ x_0 \end{array}
\end{array}
\tag{1.2}
$$

1.5

Both orders are equivalent in terms of estimation accuracy - they represent the same posterior. In terms of computations, in the latter case, the R matrix will contain less nonzero entries (corresponding to edges), so back-substitution will be more efficient.

2

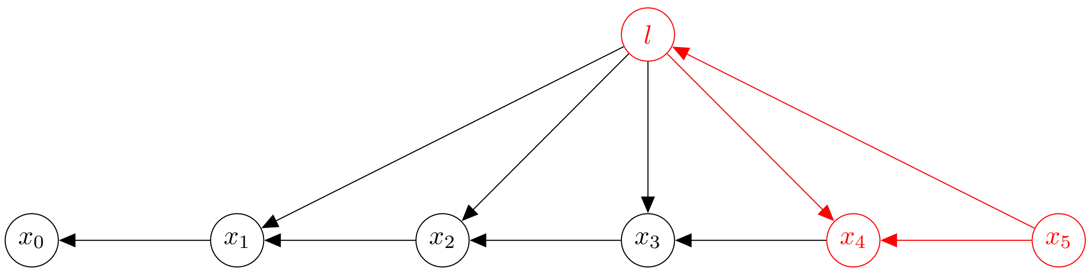
2.1



New variable (x_5) and two factors (motion model, observation model) in red.

2.2

Updated nodes appear in red (all nodes that lie from the previous root l to nodes of variables involved in new factors)



We resume derivation, adding the new factors and re-eliminating x_4 and l:

$$\mathbb{P}(x_{0:5}, l \mid u_{0:4}, z_1, z_2, z_5) \propto \tag{2.1}$$

$$\mathbb{P}(x_0 \mid x_1)\ \mathbb{P}(x_1 \mid x_2, l)\ \mathbb{P}(x_2 \mid x_3, l)\ \mathbb{P}(x_3 \mid x_4, l)\ g_4(x_4, l) \mathbb{P}(x_5 \mid x_4, u_4) \mathbb{P}(z_5 \mid l, x_5) \tag{2.2}$$

$$\propto \mathbb{P}(x_0 \mid x_1)\ \mathbb{P}(x_1 \mid x_2, l)\ \mathbb{P}(x_2 \mid x_3, l)\ \mathbb{P}(x_3 \mid x_4, l)\ \mathbb{P}(x_4 \mid x_5, l)\ \mathbb{P}(l \mid x_5)\ \mathbb{P}(x_5) \tag{2.3}$$

2.3

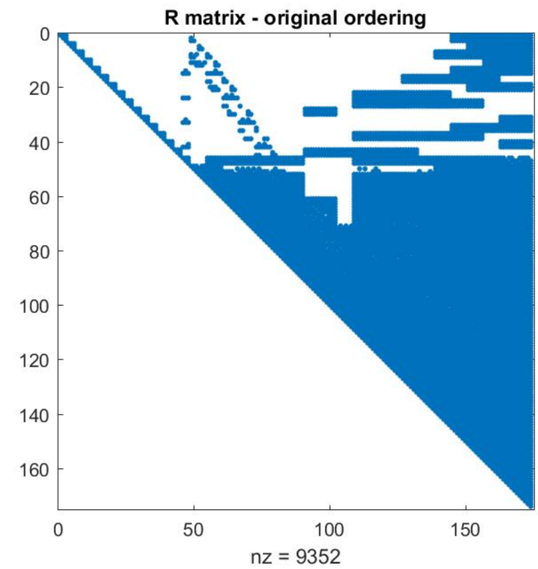
After Givens' rotations, updated R:

$$
\begin{array}{c c}
\begin{array}{ccccccc} x_0 & x_1 & x_2 & x_3 & x_4 & l & x_5 \end{array} & \\
\begin{pmatrix}
\color{blue}{\times} & \color{blue}{\times} & 0 & 0 & 0 & 0 & 0 \\
0 & \color{blue}{\times} & \color{blue}{\times} & 0 & 0 & \color{blue}{\times} & 0 \\
0 & 0 & \color{blue}{\times} & \color{blue}{\times} & 0 & \color{blue}{\times} & 0 \\
0 & 0 & 0 & \color{blue}{\times} & \color{blue}{\times} & \color{blue}{\times} & 0 \\
0 & 0 & 0 & 0 & \color{red}{\times} & \color{red}{\times} & \color{red}{\times} \\
0 & 0 & 0 & 0 & 0 & \color{red}{\times} & \color{red}{\times} \\
0 & 0 & 0 & 0 & 0 & 0 & \color{red}{\times}
\end{pmatrix} & \begin{array}{c} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ l \\ x_5 \end{array}
\end{array}
\tag{2.4}
$$

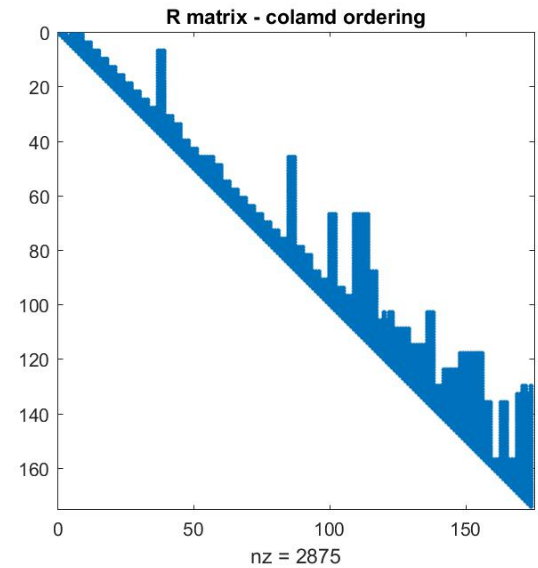
Red color indicates (generally nonzero) entries updated w.r.t. previous R matrix.

3

Information matrix after rearranging variables with colamd has less than a third of the entries of the original matrix, likely allowing the back-substitution in $R\Delta = d$ to run 3 times as fast.


R matrix - original ordering
0
20
40
60
80
100
120
140
160
0
50
100
150
nz = 9352

(a)


R matrix - colamd ordering
0
20
40
60
80
100
120
140
160
0
50
100
150
nz = 2875

(b)

4 Code

```
%% Load data
close all; clear all;
load hw4_A

%% Original ordering
[~, R] = qr(A);
R = R(1:size(A, 2), :);
spy(R); title('R matrix - original ordering');
print('-djpeg', 'original_R.jpg');

%% New ordering
[~, R] = qr(A(:, colamd(A)));
R = R(1:size(A, 2), :);
spy(R); title('R matrix - colamd ordering');
print('-djpeg', 'new_R.jpg');
```