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VAN - Homework #1

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Basic probability:
1.
(a)
$p(x;\mu_x,\Sigma_x) = \frac{1}{\sqrt{|2\pi\Sigma_x|}}\exp\left(\frac{-1}{2}(x-\mu_x)^T\Sigma_x^{-1}(x-\mu_x)\right)$
(b)
$\mu_y=$

$$\mathbf{E}(y) = \mathbf{E}(Ax+b) = A\mathbf{E}(x)+b =$$

$A\mu_x + b$
$\Sigma_y =$

$$\mathbf{E}((y-\mu_y)(y-\mu_y)^T) = \mathbf{E}((Ax+b-A\mu_x-b)(Ax+b-A\mu_x-b)^T) = \mathbf{E}((A(x-\mu_x)(A(x-\mu_x))^T) =$$

$$\mathbf{E}(A(x-\mu_x)(x-\mu_x)^TA^T) = A\mathbf{E}((x-\mu_x)(x-\mu_x)^T)A^T =$$

$= A\Sigma_x A^T$
y ~ N(Aμ_x+b, AΣ_xA^T)

[-4]
its not proving y is gaussian distributed...

2.
(a)
$p(x|z) = \frac{p(z|x)p(x)}{p(z)} = \eta p(z|x)p(x) = \eta\frac{1}{\sqrt{|2\pi R|}}\exp\left(\frac{-1}{2}\|z-Hx\|_R^2\right)\frac{1}{\sqrt{|2\pi\Sigma_0|}}\exp\left(\frac{-1}{2}\|x-\hat{x}_0\|_{\Sigma_0}^2\right)$
$= \eta\frac{1}{\sqrt{|2\pi R|}\sqrt{|2\pi\Sigma_0|}}\exp\left(\frac{-1}{2}\left(\|z-Hx\|_R^2 + \|x-\hat{x}_0\|_{\Sigma_0}^2\right)\right)$
(b)
$x^* =$

$$\underset{x}{\operatorname{argmax}}[p(x|z)] = \underset{x}{\operatorname{argmin}}[-logp(x|z)] = \underset{x}{\operatorname{argmin}}[-logp(z|x)-logp(x)] = \underset{x}{\operatorname{argmin}}\,J(x)$$

Where:

$$J(x) =$$

$$\|x-\hat{x}_0\|_{\Sigma_0}^2 + \|Hx-z\|_R^2 = \left\|\Sigma_0^{\frac{-1}{2}}(x-\hat{x}_0)\right\|^2 + \left\|R^{\frac{-1}{2}}(Hx-z)\right\|^2 =$$

$$\left\|(\Sigma_0^{\frac{-1}{2}}\ R^{\frac{-1}{2}}H)^T x - (\Sigma_0^{\frac{-1}{2}}\hat{x}_0\ R^{\frac{-1}{2}}z)^T\right\|^2$$

$$A = (\Sigma_0^{\frac{-1}{2}}\ R^{\frac{-1}{2}}H)^T$$

$$B = (\Sigma_0^{\frac{-1}{2}}\hat{x}_0\ R^{\frac{-1}{2}}z)^T$$

$$x* = \underset{x}{\operatorname{argmin}}\ \|Ax - B\|^2$$

$Ax^* - B = 0$
$x^* = (A^T A)^{-1} A^T B$
$p(x|z) = N((A^T A)^{-1} A^T B,\ (A^T A)^{-1})$

Wet Part:

1.

(a)

```
function R = calculateRotationMatrixFromAngles(roll, pitch ,yaw)
Rx = [1 0 0; 0 cosd(roll) -sind(roll); 0 sind(roll) cosd(roll)];
Ry = [cosd(pitch) 0 sind(pitch); 0 1 0; -sind(pitch) 0 cosd(pitch)];
Rz = [cosd(yaw) -sind(yaw) 0; sind(yaw) cosd(yaw) 0; 0 0 1];
R = Rz * Ry * Rx;
end
```

(b)

calculateRotationMatrixFromAngles(pi / 4, pi / 5, pi / 7)

$$\begin{bmatrix} 0.9999 & -0.0077 & 0.0111 \\ 0.0078 & 0.9999 & -0.0136 \\ -0.0110 & 0.0137 & 0.9998 \end{bmatrix}$$

[-5]
incorrect

(c)

```
function angles = rotationMatrixToAngles(rotationMatrix)
yaw = atan(rotationMatrix(2,1) / rotationMatrix(1,1));
pitch = asin(-rotationMatrix(3,1));
roll = atan(rotationMatrix(3,2) / rotationMatrix(3,3));
angles = [roll*180/pi pitch*180/pi yaw*180/pi];
end
```

(d)

rotationMatrixToAngles([0.813797681 -0.440969611 0.378522306; 0.46984631 0.882564119 0.0180283112; -0.342020143 0.163175911 0.925416578])

$\begin{bmatrix} 30 & 20 & 10 \end{bmatrix}$

[-3]
incorrect

2.

function v = rigidTransformation(R, t, vector)

v = ((R * vector) + t);

end

incorrect
[-5]

$$l^C = R_G^C l^G + t_{C\to G}^C = \begin{bmatrix} 0.5363 & -0.8440 & 0 \\ 0.8440 & 0.5363 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 450 \\ 400 \\ 50 \end{bmatrix} + \begin{bmatrix} -451.2459 \\ 257.0322 \\ 400 \end{bmatrix} = \begin{bmatrix} -547.5109 \\ 881.3522 \\ 450 \end{bmatrix}$$

3.

(a)

$$l^C = R_G^C l^G + t_{C\to G}^G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{l^G} \\ y_{l^G} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_{l^G} \\ y_{l^G} + 1 \end{bmatrix}$$

$$l^C = E_G^C R_G^C l^G + t_{C\to G}^G + e_{C\to G}^G = \begin{bmatrix} 0.9998 & -0.0174 \\ 0.0174 & 0.9998 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{l^G} \\ y_{l^G} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} = \begin{bmatrix} 0.9998x_{l^G} - 0.0174y_{l^G} \\ 0.0174x_{l^G} + 0.9998y_{l^G} + 1.01 \end{bmatrix}$$

(b)

x1$\bigoplus$x2 = $\{R_1R_2, R_1t_2 + t_1\}$

commanded:

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$t = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

actual:

```
function [R t] = poseComposition(R1, t1, R2, t2)
R = (R1 * R2);
t = (R1 * t2 + t1);
end
R = [cosd(1) -sind(1); sind(1) cosd(1)];
t = [0; 1.01];
R1 = R;
t1 = t;
R2 = R;
t2 = t;
for m = 1:9
 [R2 t2] = poseComposition(R1, t1, R2, t2);
end
```


[-10]
where is the comm and actual path drawing?

R2

t2

R2 =
$$\begin{bmatrix} 0.9848 & -0.1736 \\ 0.1736 & 0.9848 \end{bmatrix}$$
t2 =
$$\begin{bmatrix} -0.7914 \\ 10.0562 \end{bmatrix}$$
The Dead Reckoning Error:

Distance:

norm(t2 - [0;10]) = $\sqrt{(0-(-0.7914))^2 + (10-10.0562)^2} = \sqrt{0.7914^2 + (-0.0562)^2}$
= 0.7934 [-3]

Angle: incorrect

atan2(R2(2,1), R2(1,1)) = 10.0 $\rightarrow \theta = 0.0$