Vision-Aided Navigation (086761)
Homework #3

Submission in pairs by: 26 November 2017, 13:30. Electronic submission is preferred.

1. Consider a prior $p(x) = N(\hat{x}_0, \Sigma_0)$ on random variable x and two measurements from different observation models (e.g. representing different sensors):

$$z_1 = h_1(x) + v_1 \quad , \quad z_2 = h_2(x) + v_2,$$

with $v_1 \sim N(0, \Sigma_{v1})$ and $v_2 \sim N(0, \Sigma_{v2})$.

   (a) Develop an expression for the a posteriori information matrix $I = \Sigma^{-1}$, such that $p(x|z_1, z_2) = N(\mu, \Sigma)$.

   (b) Consider now a pinhole camera sensor with the corresponding measurement model:

$$z = \pi(x, l) + v \quad , \quad v \sim N(0, \Sigma_v)$$

   An image observation z of a landmark l is obtained. Additionally, a prior $p(l) = N\left(\hat{l}_0, \Sigma_{l0}\right)$ on l is available. Indicate what is the *initial* re-projection error, i.e. using information from $p(x|z_1, z_2)$ and $p(l)$.

   (c) Write an expression for the joint pdf $p(x, l|z_1, z_2, z)$ in terms of prior and measurement likelihood terms.

   (d) Develop an expression for the joint information matrix I' over x and l as in $p(x, l|z_1, z_2, z) = N\left(\times, I'^{-1}\right)$. Consider $p(x|z_1, z_2) = N(\mu, \Sigma)$ is given.

2. Consider two camera poses $x_1 \doteq (R_1, t_1)$ and $x_2 \doteq (R_2, t_2)$, where the following convention is assumed

$$R_i \equiv R^G_{C_i} \quad , \quad t_i \equiv t^G_{C_i \to G} \quad i = 1, 2.$$

Here, the superscript G denotes some global reference frame. Assume each camera captures an image, and let $z_1 = (u_1, v_1)^T$ and $z_2 = (u_2, v_2)^T$ be two corresponding image observations from these two images.

   (a) Develop the epipolar constraint, expressing all quantities in the *second* camera frame. Express the constraint in the form

$$h(x_1, x_2, z_1, z_2) = 0.$$

   (b) Assume the true values of the camera poses x_1 and x_2 are not actually known, and instead we have a prior on each camera (e.g. from GPS):

$$p(x_1) = N(\mu_{01}, \Sigma_{01}) \quad , \quad p(x_2) = N(\mu_{02}, \Sigma_{02}).$$

   Assume the residual error in the epipolar constraint from the previous clause can be modeled as zero-mean Gaussian with covariance Σ_{ep}, derive a probabilistic expression for $MAP\ p(x_1, x_2|z_1, z_2)$.

3. Prove the fundamental matrix is singular (please explain each step in the proof).