Hands-on exercises

Question 1.

(a) Based on the given information, the projection matrix is given by:

Assumption: skew = 0

$$P = K_{3\times3}\,[\,R \mid t\,]_{(3\times4)} = \begin{bmatrix} f_x & s & u_0 \\ 0 & f_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} [\,R_{3\times3} \mid t_{3\times1}]$$

$$P = \begin{bmatrix} 480 & 0 & 320 \\ 0 & 480 & 270 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5363 & -0.8440 & 0 & -451.2459 \\ 0.8440 & 0.5363 & 0 & 257.0322 \\ 0 & 0 & 1 & 400 \end{bmatrix}$$

$$P = \begin{bmatrix} 257.42 & -405.12 & 320 & -88598.032 \\ 405.12 & 257.42 & 270 & 231375.456 \\ 0 & 0 & 1 & 400 \end{bmatrix}$$

(b) In total, the projection of a 3D point onto the screen is performed in 3 stages:

1. The points is translated and rotated into the camera coordinates using Rotation + translation matrix
2. The point is translated into the homogeneous coordinates
3. The homogenous coordinates are being normalized into the regular pixel position coordinates

Solving each step separately:

1. Convert the 3D point into Camera coordinates

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = [R \mid t\,] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5363 & -0.8440 & 0 & -451.2459 \\ 0.8440 & 0.5363 & 0 & 257.0322 \\ 0 & 0 & 1 & 400 \end{bmatrix} \begin{bmatrix} 350 \\ -250 \\ -35 \\ 1 \end{bmatrix} = \begin{bmatrix} -52.54 \\ 418.357 \\ 365 \end{bmatrix}$$

2. Get the homogenous coordinates:

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = K_{3\times3} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} 91580.368 \\ 299361.456 \\ 365 \end{bmatrix}$$

3. Obtaining the normalized (real) coordinates:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{\tilde{u}}{\tilde{w}} \\ \frac{\tilde{v}}{\tilde{w}} \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 250.9 \\ 820.17 \end{bmatrix}$$

(c) Re-Projection error – is defined as:

$$v = z - \pi(x, l)$$

In our case:

$$z = \begin{bmatrix} 241.5 \\ 169 \end{bmatrix}$$

$$\pi(x, l) = \begin{bmatrix} 250.9 \\ 820.17 \end{bmatrix}$$

Meaning:

$$v = \begin{bmatrix} 241.5 \\ 169 \end{bmatrix} - \begin{bmatrix} 250.9 \\ 820.17 \end{bmatrix} = \begin{bmatrix} -9.405 \\ -651.168 \end{bmatrix}$$