Vision-Aided Navigation (086761)
Homework #4

Submission in pairs by: 17 December 2017, 13:30. Electronic submission is preferred.

1. Factor graph, variable elimination and Bayes net. Consider a smoothing and mapping (SAM) problem where a robot travels through an unknown environment and captures observations using its onboard sensors (e.g. camera). Assume the robot starts at time t_0, with a known prior $p(x_0)$ and consider motion and observation models $p(x_k|x_{k-1}, u_{k-1})$ and $p(z_{k,i}|x_k, l_i)$, respectively, where l_i denotes the ith landmark. The robot moves according to given controls and observes a single landmark at time instances t_1 and t_2.

   (a) Write the joint pdf corresponding to the above scenario until time t_4: $p(x_{0:4}, l|u_{0:3}, z_1, z_2)$.

   (b) Draw the corresponding factor graph. Explain in detail the relation to the joint pdf and indicate to what each node and edge in graph corresponds.

   (c) Eliminate the factor graph into a Bayes net, assuming elimination order $x_0, x_1, x_2, x_3, x_4, l$. Explain in detail to what each node and edge in the Bayes net correspond. Show the corresponding square root information matrix R (you can only indicate what entires are non-zero in that matrix).

   (d) Repeat the previous clause using a different variable elimination order: $x_4, x_3, x_2, l, x_1, x_0$.

   (e) Which of the two elimination orders you would prefer in terms of estimation accuracy and computational aspects? Please explain in detail.

2. Incremental factorization. Consider now the robot, from question 1, executes command u_4 and moves to a new location; denote its new pose by x_5. Assume the robot observes again the landmark l from the new location.

   (a) Draw the factor graph of the problem and indicate the new factors and variable nodes.

   (b) Consider the Bayes net from question 1(c) with elimination order $x_0, x_1, x_2, x_3, x_4, l$. Perform incremental factorization by updating this Bayes net with the new information using the elimination order $x_0, x_1, x_2, x_3, x_4, l, x_5$. Indicate what entries in the Bayes net have been changed in this process.

   (c) Show the corresponding updated square root information matrix R (as earlier, you can only indicate what entires are non-zero in that matrix), and indicate what entries in that matrix have been changed with respect to the same matrix from a previous time.

3. Variable ordering. Consider a Jacobian matrix A obtained by linearizing all the terms in a SAM problem (e.g. as in question 1). Access the matrix A from the piazza course website[1].

   (a) Calculate the square root information matrix R from A (e.g. via QR factorization). Plot its sparsity pattern and indicate the number of non-zero entries (e.g. use spy command in Matlab).

   (b) Calculate a better variable ordering, e.g. using the COLAMD algorithm (colamd in Matlab). Recalculate the square root information matrix R using the new variable ordering. Plot its sparsity pattern, indicate the number of non-zero entries, and compare to the previous case (before re-ordering).

[1] https://piazza.com/technion.ac.il/fall2017/086761/resources.