Vision-Aided Navigation (086761)
Homework #1

Submission in pairs by: 20 November 2016, 14:30.

Basic probability

1. Consider a random vector x with a Gaussian distribution:

$$x \sim N\left(\mu_x, \Sigma_x\right).$$

   (a) Write an explicit expression for $p\left(x\right)$.

   (b) Consider a linear transformation $y = Ax + b$. Show y has a Gaussian distribution, $y \sim N\left(\mu_y, \Sigma_y\right)$, and find expressions of μ_y and Σ_y in terms of μ_x and Σ_x.

2. Let $p\left(x\right) = N\left(\hat{x}_0, \Sigma_0\right)$ be a prior distribution over $x \in \mathbb{R}^n$ with known mean $\hat{x}_0 \in \mathbb{R}^n$ and covariance $\Sigma_0 \in \mathbb{R}^{n\times n}$. Consider a given measurement $z \in \mathbb{R}^m$ with a corresponding linear measurement model $z = Hx + v$, where H is a measurement matrix and v is Gaussian noise $v \sim N\left(0, R\right)$ with covariance R. The matrices $H \in \mathbb{R}^{m\times n}$ and $R \in \mathbb{R}^{m\times m}$ are known.

   (a) Write an expression for the a posteriori probability function (pdf) over x, $p\left(x|z\right)$, in terms of the prior $p\left(x\right)$ and measurement likelihood $p\left(z|x\right)$.

   (b) Derive analytically an expression for the maximum a posteriori (MAP) estimate $x^\star$ and the associated covariance Σ or information matrix $I = \Sigma^{-1}$ such that $p\left(x|z\right) = N\left(x^\star, \Sigma\right)$.
   Useful relation: $\left\|a\right\|_\Sigma^2 = \left\|\Sigma^{-1/2}a\right\|^2$ where $\Sigma^{-1} = \Sigma^{-T/2}\Sigma^{-1/2}$.

Hands-on Exercises - Please print and submit your code.

1. Rotations. Implement transformation from rotation matrix to Euler angles and vice versa

   (a) Implement a function that receives as input Euler angles (roll angle ϕ, pitch angle θ,and yaw angle ψ) and calculates the corresponding rotation matrix assuming roll-pitch-yaw order: $R = R_Z\left(\psi\right) R_Y\left(\theta\right) R_X\left(\phi\right)$.

   (b) What is the rotation matrix for $\psi = \pi/7$, $\theta = \pi/5$, and $\phi = \pi/4$?

   (c) Implement a function that receives as input a rotation matrix and calculates the corresponding Euler angles assuming roll-pitch-yaw order.

   (d) What are the Euler angles <u>in degrees</u> for the following rotation matrix (Body to Global, assuming roll-pitch-yaw order):

$$R_B^G = \begin{bmatrix} 0.813797681 & -0.440969611 & 0.378522306 \\ 0.46984631 & 0.882564119 & 0.0180283112 \\ -0.342020143 & 0.163175911 & 0.925416578 \end{bmatrix}$$

2. 3D rigid transformation. The coordinates of a 3D point in a global frame are

$$l^G = \left(450, 400, 50\right)^T.$$

   This 3D point is observed by a camera whose pose is described by the following rotation and translation with respect to the global frame:

$$R_G^C = \begin{bmatrix} 0.5363 & -0.8440 & 0 \\ 0.8440 & 0.5363 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$t^G_{C\to G} = (-451.2459, 257.0322, 400)^T$$

Calculate the 3D point coordinates in a camera frame ($l^C =?$). Write an explicit expression for the appropriate 3D transformation.

3. Pose composition. An autonomous *ground* vehicle (robot) is commanded to move forward by 1 meter each time. Due to imperfect control system, the robot instead moves forward by 1.01 meter and also rotates by 1 degree.
   Remark: In this exercise we consider a 2D scenario, where pose is defined in terms of x-y coordinates and an orientation (heading) angle.

   (a) Write expressions for the corresponding commanded and actual transformations - note these are relative to the robot frame.

   (b) Assuming robot starts moving from the origin, calculate evolution of robot pose (in terms of x-y position and orientation angle) for 10 steps using pose composition. Draw the commanded and actual robot pose for 10 steps. What is the dead reckoning error at the end?

4. Shniki & Noam are building their own UAV (Unmanned Aerial Vehicle), all they have left is to calculate the rotation matrix between the UAV body reference system to the Global reference system.
   Shniki insisted they use only quaterniones in order to create the required rotation quaternion. Help them find the rotation quaternion.

   (a) Calculate the quaternion for a 10^o Roll.

   (b) Calculate the quaternion for a 28^o Pitch.

   (c) Calculate the quaternion for a 5^o Yaw.

   (d) Using 4(a) - 4(c) calculate the required rotation quaternion (roll => pitch => yaw)

   (e) ## BONUS ## - Noam insists there is only one way to create a DCM given a quaternion, while Shniki insists that there are at least two different ways.
   Who is right, Shniki or Noam? proove your point by providing ways to calculate the equivalent DCM matrix using only the quaternion from 4(d)
   If you agree with Noam provide one way, if you agree with Shniki provide at least two different ways.