

Vision-Aided Navigation (086761)

Homework #4

Submission in pairs by: 18 December 2016, 14:30.

1. Factor graph, variable elimination and Bayes net. Consider a smoothing and mapping (SAM) problem where a robot travels through an unknown environment and captures observations using its onboard sensors (e.g. camera). Assume the robot starts at time t_0 , with a known prior $p(x_0)$ and consider motion and observation models $p(x_k|x_{k-1}, u_{k-1})$ and $p(z_{k,i}|x_k, l_i)$, respectively, where l_i denotes the i th landmark. The robot moves according to given controls and observes a single landmark at time instances t_1 and t_2 .
 - (a) Write the joint pdf corresponding to the above scenario until time t_4 : $p(x_{0:4}, l|u_{0:3}, z_1, z_2)$.
 - (b) Draw the corresponding factor graph. Explain in detail the relation to the joint pdf and indicate to what each node and edge in graph corresponds.
 - (c) Eliminate the factor graph into a Bayes net, assuming elimination order $x_0, x_1, x_2, x_3, x_4, l$. Explain in detail to what each node and edge in the Bayes net correspond. Show the corresponding square root information matrix R (you can only indicate what entries are non-zero in that matrix).
 - (d) Repeat the previous clause using a different variable elimination order: $x_4, x_3, x_2, l, x_1, x_0$.
 - (e) Which of the two elimination orders you would prefer in terms of estimation accuracy and computational aspects? Please explain in detail.
2. Variable ordering. Consider a Jacobian matrix A obtained by linearizing all the terms in a SAM problem (e.g. as in question 1). Access the matrix A from the piazza course website¹.
 - (a) Calculate the square root information matrix R from A (e.g. via QR factorization). Plot its sparsity pattern and indicate the number of non-zero entries (e.g. use `spy` command in Matlab).
 - (b) Calculate a better variable ordering, e.g. using the COLAMD algorithm (`colamd` in Matlab). Recalculate the square root information matrix R using the new variable ordering. Plot its sparsity pattern, indicate the number of non-zero entries, and compare to the previous case (before re-ordering).

¹<https://piazza.com/technion.ac.il/spring2015/086761/resources>.