<u>Lab 4 – Image Filtering</u>

Goal: Introduction to image filtering in spatial and in frequency domains.

I. Introduction

In the early days of image processing the use of Discrete Fourier Transform (DFT) was very restricted because of its high computational complexity. With the introduction of the FFT algorithm the complexity of DFT was reduced and DFT became an extremely important practical tool of image processing.

In this lab we'll study the properties of DFT and two practical applications:

- Computation of convolution by two methods – direct method (in spatial domain) and indirect method (in frequency domain).
- Computation of edge enhancement by unsharp masking.

II. Preliminary report

1. Prove the following properties of the Continuous 2-D Fourier Transform:
   - Linearity property
   - Scaling property
   - Rotation property
2. Explain the Periodicity property of DFT. What difficulties it imposes on image processing? Explain your answer.
3. Explain the Conjugate Symmetry property of DFT. How it can be utilized in algorithms to reduce the number of computational operations?
4. The magnitude of spectrum usually has a large dynamic range. How do you usually display the magnitude of spectrum?
5. Explain the purpose of the functions **fft.fft2** and **fft.fftshift** from **numpy** library.
6. Which values are returned by the function **cv2.getOptimalDFTSize**? Explain why it’s useful?

7. Recall the 1D rectangular function: $\prod(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & else \end{cases}$ and its Fourier Transform (FT).

   a) Write the expression of the 2D rectangular function $\prod(x, y)$ and derive its 2D FT (in Cartesian coordinates).

   b) Generate a 256×256 px binary image (Grayscale) of a **centered square window** of dimensions 64×64.

      Use **fft.fft2** and **fft.fftshift** to derive its 2D FT. Display both images and explain your results.

8. We now consider a circular rectangular function defined using *polar coordinates*:

$$\text{circ}(r, \theta; R) = \begin{cases} 1, & r \leq R \\ 0, & r > R \end{cases}$$

   a) Show that the 2D FT of $\text{circ}(r, \theta; 1)$, i.e., setting $R = 1$, in *polar coordinates* is the "Sombrero" function:

$$\mathcal{F}_{2D}\{\text{circ}(r; 1)\} = \frac{J_1(2\pi\rho)}{\rho} \triangleq \pi \cdot \text{somb}(\rho)$$

      where:

      - $J_1(\alpha)$ is *order 1 Bessel function* defined as: $\alpha J_1(\alpha) = \int_0^\alpha x \cdot J_0(x) dx$.
      - Reminder: $J_0(\alpha) = \frac{1}{2\pi} \int_0^{2\pi} e^{-j\alpha \cos\theta} d\theta$ is the *order 0 Bessel function*.

   b) Create a 256×256 px binary image (Grayscale) of a **centered circular window** with $R = 32$.

      Use **fft.fft2** and **fft.fftshift** to derive its 2D FT. Display both images and explain your results.

   c) Why is it called the "Sombrero" function?

   d) Is there resemblance between the FTs of the square and circular rectangular functions?

9. Study the pythons module **scipy.ndimage**. Give a short explanation about the following filters: **uniform_filter**, **median_filter**, **gaussian_filter**, **laplace**, **prewitt** and **sobel**. What is an important parameter of these operations?

10. Explain the purpose of Unsharp Masking (USM) (**skimage.filters.unsharp_mask**). Give a short explanation of this process.

III. Description of the Experiment

Run Lab4.ipynb using Google Colab and follow the instructions in the 3 sections:

i. Part 1 - DFT Properties
ii. Part 2 – Convolution and Image Filtering
iii. Part 3 – Unsharp Masking (USM)

Once done, save your notebook in a PDF and make sure you add it to your final report.

IV. Final report

Submit the results of testing and demonstrations from the 3 parts of the experiment, with the image of your choice. Make sure you explain your results in each step, either in the provided spaces within the notebook or in an attached document.

Good Luck!