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Class: CSCE 350
Homework Assignment #2

1. For each of the following functions, indicate the class Θ(g(n) the function belongs to.(Use the simplest g(n) possible in your answer).Prove your assertions using limits. Show your work

- $(n^3 + n)^7$
  - t(n) = $(n^3 + n)^7$, guess of g(n)= $(n^3 + n)^7 \approx (n^3)^7 \approx n^{21}$
  - $\lim_{n \to inf} \frac{t(n)}{g(n)} = \frac{(n^3+n)^7}{n^{21}} = \lim_{n \to inf} \frac{7n^9+n^7+n^{21}+7n^{19}+21n^{17}+35n^{15}+35n^{13}+21n^{11}}{n^{21}}$

    $= \lim_{n \to inf} \frac{\frac{d^{21}(7n^9+n^7+n^{21}+7n^{19}+21n^{17}+35n^{15}+35n^{13}+21n^{11})}{dn^{21}}}{\frac{d^{21}(n^{21})}{dn^{21}}} = 1$
  - Since the highest "x" exponent value of "n" in both numerator and denominator is same value and they both have a form of $a_n * x^n + a_{n-1} * x^{n-1} + a_{n-2} * x^{n-2} ... + a_0 * x^0$, it is known that L'Hospital rule could be applied "x" number of times to find the limit of the function.
  - t (n) has the same order of growth as g(n), $(n^3 + n)^7 \in \Theta(n^{21})$
- $n * log(n^2) + (n-2)^2 * log(n/2)$
  - t(n) = $n * log(n^2) + (n-2)^2 * log(n/2)$; Since $(n-2)^2 * log(n/2)$ has a larger time complexity than $n * log(n^2)$, $(n-2)^2 * log(n/2) \approx (n^2) * log(n)$, guess of g(n)=$(n^2) * log(n)$
  - $\lim_{n \to inf} \frac{n*log(n^2) + (n-2)^2*log(n/2)}{(n^2)*log(n)} = \lim_{n \to inf} \frac{n*log(n^2)}{n^2*log(n)} + \lim_{n \to inf} \frac{(n-2)^2*log(n/2)}{n^2*log(n)}$
  - $\lim_{n \to inf} \frac{n*log(n^2)}{n^2*log(n)} = \lim_{n \to inf} \frac{(log(n^2))'}{(n*log(n))'} = \lim_{n \to inf} \frac{\frac{2*log(e)}{n}}{log(n)+log(e)}$

    $= \lim_{n \to inf} \frac{(log(n^2)*n)'}{(2log(e))'} + (n)' = \lim_{n \to inf} (log(n^2) + 2log(e) + 1)$

    $= \lim_{n \to inf} (log(x^2) + 2log(e) + 1)' = \lim_{n \to inf} (\frac{2}{x*ln(10)}) \approx 0$
  - $\lim_{n \to inf} \frac{(n-2)^2*log(n/2)}{n^2*log(n)} = \lim_{n \to inf} \frac{log(n/2)}{log(n)} - 4 \lim_{n \to inf} \frac{n*log(n/2)}{n^2*log(n)} + 4 \lim_{n \to inf} \frac{log(n/2)}{n^2*log(n)}$
  - $\lim_{n \to inf} \frac{(log(n/2))'}{(log(n))'} = \lim_{n \to inf} \frac{\frac{1}{x*ln(10)}}{\frac{1}{x*log(10)}} = 1$
  - $\lim_{n \to inf} \frac{(n*log(n/2))'}{(n^2*log(n))'} =$

    $\lim_{n \to inf} \frac{(\frac{log(e)}{n})'}{(log(n)+log(e)-log(2))'} = \lim_{n \to inf} \frac{\frac{-log(e)}{n^2}}{\frac{log(e)}{n}} = \lim_{n \to inf} \frac{-1}{n} \approx 0$
  - $\lim_{n \to inf} \frac{(log(n/2))'}{(n^2*log(n))'} = \lim_{n \to inf} \frac{(\frac{log(e)}{n})'}{(n*(2log(n)+2log(e)-2log(2)))'}$

$$= \lim_{n \to inf} \frac{(\frac{-log(e)}{n})}{((2log(n)+3log(e)-2log(2)))} \approx 0$$, same form as before.

- $\lim_{n \to inf} \frac{(n-2)^2 * log(n/2)}{n^2 * log(n)} = 1 - 4(0) + 4(0) = 1$
- $\lim_{n \to inf} \frac{n * log(n^2) + (n-2)^2 * log(n/2)}{(n^2) * log(n)} = 0 + 1 = 1$
- t (n) has the same order of growth as g(n), $\left(n * log(n^2) + (n-2)^2 * log(n/2)\right)^7 \in \Theta(n^2 * log(n))$.

2. Solve the following sums. The final answer must be listed in terms of 'n'. Show your work. Find their orders of growth using the simplest g(g(n) the function belongs to. (Use then) possible.

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- (12 points) $\sum_{i=0}^{n-1} (i^2 - 2)^2 = \sum_{i=0}^{n-1} i^4 - 4i^2 + 4 = \sum_{i=0}^{n-1} i^4 - 4 * \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 4$
  - Forumlas: $\sum_{i=1}^{n} i^4 = \frac{n*(2n+1)*(n+1)*(3n^2+3n-1)}{30}$, $\sum_{i=1}^{n} i^2 = \frac{n*(n+1)*(2n+1)}{6}$
  - $\sum_{i=1}^{n-1} i^4 = \frac{(n-1)*(2(n-1)+1)*((n-1)+1)*(3(n-1)^2+3(n-1)-1)}{30} = \frac{n^5}{5} - \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30} \approx (\frac{n^5}{5}) \in \Theta(n^5)$
  - $4\sum_{i=0}^{n-1} i^2 = 4\sum_{i=1}^{n-1} i^2 = \frac{4(n-1)*((n-1)+1)*(2(n-1)+1)}{6} = 4 * (\frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6}) \approx (\frac{4n^3}{3}) \in \Theta(n^3)$
  - $\sum_{i=0}^{n-1} 4 = (4n) \in \Theta(n)$
  - $\sum_{i=0}^{n-1} i^4 - 4 * \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} 4$

    $\approx (\frac{n^5}{5}) \in \Theta(n^5) + (\frac{4n^3}{3}) \in \Theta(n^3) + (4n) \in \Theta(n) \approx (\frac{n^5}{5}) \in \Theta(n^5)$
- (12 points)
  - $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1} (i + j) = \sum_{i=0}^{n-1} [0 + \sum_{j=1}^{i-1} i + \sum_{j=1}^{i-1} j]$
  - $\sum_{i=0}^{n-1} [\sum_{j=1}^{i-1} i + \sum_{j=1}^{i-1} j] = \sum_{i=0}^{n-1} (i^2 + \frac{(i-1)((i-1)+1)}{2}) = \sum_{i=0}^{n-1} (i^2 + \frac{i^2}{2} - \frac{i}{2}) = \frac{1}{2}\sum_{i=0}^{n-1} 3i^2 - i$

    $= \frac{1}{2}\sum_{i=0}^{n-1} 3i^2 - i = 0 + \frac{1}{2}\sum_{i=1}^{n-1} 3i^2 - i = \frac{1}{2}\sum_{i=1}^{n-1} 3i^2 - \frac{1}{2}\sum_{i=1}^{n-1} i = \frac{n*(n-1)}{2} =$
  - $\frac{1}{2}\sum_{i=1}^{n-1} 3i^2 = \frac{3}{2}(\frac{(n-1)(n-1+1)(2(2n-1)+1)}{6}) = n^3 - \frac{5n^2}{4} + \frac{n}{4} \approx (n^3) \in \Theta(n^3)$

- $-\frac{1}{2}\sum_{i=1}^{n-1} i = \frac{-1}{2} * (\frac{n*(n-1)}{2}) = \frac{-n^2}{4} + \frac{n}{4} \approx -(\frac{n^2}{4}) \in \Theta(n^2)$
- $\sum_{i=0}^{n-1}\sum_{j=0}^{i-1}(i + j) = \frac{1}{2}\sum_{i=0}^{n-1} 3i^2 - i$

  $\approx (n^3) \in \Theta(n^3) - (\frac{n^2}{4}) \in \Theta(n^2) \approx (n^3) \in \Theta(n^3)$

3. Solve the following recurrence relations using the method of backward substitutions. Give the solution to the problem in terms of 'n'. Show your work.

A. (11 points) X(n) = X(n-1)+5 for n>0, X(0) = 1
   - X(n-1) = X(n-1-1)+5 = X(n-2)+5
   - X(n-2) = X(n-2-1)+5=X(n-3)+5
   - X(n) = X(n-1)+5 = X(n-2)+ 5 +5 = X(n-3)+5+5+5 = X(n-3)+15
   - X(n) = X(n-i)+i*5
   - n-i=0,n=i
   - X(n) = X(0)+n*5 = **<u>1+5*n</u>**

B. (15 points) X(n) = X(n-1)+5n for n>0, X(1) = 5
   - X(n-1) = X(n-1-1)+5(n-1) = X(n-2)+5(n-1)
   - X(n-2) = X(n-2-1)+5(n-2)=X(n-3)+5(n-2)
   - X(n) = X(n-1)+5n = X(n-2)+5(n-1)+5n = X(n-3)+5(n-2)+5(n-1)+5n
   - X(n) = X(n-i)+5(n-i+1)+5(n-i+2)+5n
   - n-i=1,i=n-1
   - X(n) = X(1)+5(1+1)+5(1+2)+5n =
     X(1)+5(2)+5(3)+5n=5+10+15+...+5n=**<u>5n*(n+1)/2</u>**

C. (15 points) X(n) = X(n/3)+n for n>1, X(1) = 1, n = 3^k
   - $X(3^k) = X(3^{k-1}) + 3^k$
   - $X(3^0) = 1$
   - $X(3^{k-1}) = X(3^{k-2}) + 3^{k-1}$
   - $X(3^{k-2}) = X(3^{k-3}) + 3^{k-2}$
   - $X(3^k) = X(3^{k-1}) + 3^k =$
     $X(3^{k-2}) + 3^{k-1} + 3^k = X(3^{k-3}) + 3^{k-2} + 3^{k-1} + 3^k$
   - $X(3^k) = X(3^{k-3}) + 3^{k-2} + 3^{k-1} + 3^k = X(3^{k-i}) + 3^{k-i+1} + 3^{k-i+2} + 3^k$
   - k-i=0,i=-k
   - $X(3^k) = X(3^0) + 3^{0+1} + 3^{0+2} + 3^k = 1 + 3^1 + 3^2 ... + 3^k = (3^{k+1} - 1)/2 =$
     <u>3(n-1)/2</u>

4. (Use the20 points) Solve the following linear second-order recurrence relation:

X(n) -3X(n-1)+X(n-2)=3   for n>1,

Preconditions: X(0)=0, X(1)=1

- $r^2 - 3r + 1 = 0$
- $r_{1,2} = \frac{3 \pm \sqrt{5}}{2}$
- **General Solution =** $X(n) = \alpha * (\frac{3+\sqrt{5}}{2})^n + \beta * (\frac{3-\sqrt{5}}{2})^n$
- 1c-3c+1c=3 => -c=3 => c=-3
- $X(0) = 0 = \alpha * (\frac{3+\sqrt{5}}{2})^0 + \beta * (\frac{3-\sqrt{5}}{2})^0 = \alpha * 1 + \beta * 1 <=> \alpha =- \beta$
- $X(1) = 1 = \alpha * (\frac{3+\sqrt{5}}{2})^1 + \beta * (\frac{3-\sqrt{5}}{2})^1 =- \beta * (\frac{3+\sqrt{5}}{2})^1 + \beta * (\frac{3-\sqrt{5}}{2})^1$
- $X(1) =- \beta * \sqrt{5} <=> \beta = -\frac{1}{\sqrt{5}}$
- **Particular Solution =** $X(n) = \frac{1}{\sqrt{5}} * (\frac{3+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}} * (\frac{3-\sqrt{5}}{2})^n - 3$