## Notes on the Spruit-Tayler Dynamo (February 2015)

These notes extend the notes I sent following the Sedgewick meeting. I have skipped the summary of Spruit 2002 and how the Spruit-Tayler dynamo is implemented in stellar structure codes (see Heger et al. 2005 for this).

### Systems to analyze

The first step is to define a family of models in which the dynamo could operate. The simplest family I have found are the 1D cylindrical equilibria in which gravity is provided by a line mass along the rotation axis. We can then write

$$\mathbf{g} = -\hat{\varpi}\frac{V^2}{\varpi} \tag{1}$$

Let the ratio of centrifugal to gravitational acceleration be a constant such that the rotational velocity $v = V_r$ is constant. Assume the gas is isothermal; $P = \rho a^2$, and the plasma $\beta \equiv 8\pi P/B^2$ is a constant. Then equilibrium can be achieved for

$$\rho = \rho_0 \left(\frac{\varpi}{\varpi_0}\right)^{-\alpha}, \tag{2}$$

as long as the following relationship holds

$$\alpha\left(1+\frac{1}{\beta}\right) - \frac{V^2}{a^2}\left(1 - \frac{V_0^2}{V^2} + \frac{2a^2}{\beta V^2}\right) = 0. \tag{3}$$

Note that these models are very different from those envisaged by Tayler, who discusses instability near the axis of rotation where $g_\varpi \propto \varpi$. In the absence of rotation, we can test this configuration for stability using the criteria in Tayler 1973. First, we consider axisymmetric stability ($m - 0$ modes). For our 1D models, stability to axisymmetric disturbances is determined from his eqn. (2.13), and requires

$$\left[\alpha\left(x+\frac{2}{\beta}\right)+\frac{4}{\beta}\right]\left(\gamma+\frac{2}{\beta}\right) - \left(x+\frac{4}{\beta}\right)^2 > 0, \tag{4}$$

where $x \equiv V^2/a^2$. In the limit of no magnetic field, eqn. (4) reduces to $\gamma - 1 > 0$. I have explored this equation with Mathematica and not found any regimes of instability.

The ST dynamo is based on an $m = 1$ instability. For our models, the relevant criterion is eqn. (2.20) of Tayler (eqn. 2.21 is irrelevant in 1D and eqn. 2.22 is equivalent to 2.21).

For our models it is necessary and sufficient that

$$\alpha\left(x+\frac{2}{\beta}\right)-\frac{x^2}{\gamma}-\frac{2}{\beta}>0. \tag{5}$$

The figure shows that instability is possible for this type of system. Cases with lower $\gamma$ are

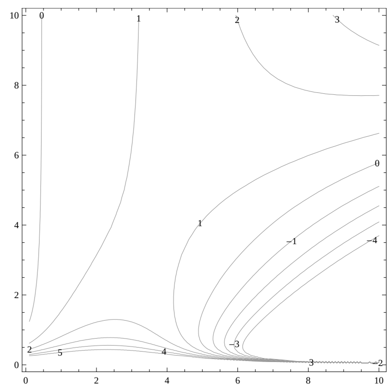

Fig. 1.— Contour plot in the $(x,\beta)$ plane of the criterion given in eqn. (5) for $\gamma = 1.1$. The contour labels indicate the left hand side, so the contours labeled zero are stability boundaries.

more unstable. This can be achieved with efficient thermal diffusion, as discussed in Spruit 2002. There are two regimes of instability, one with high $\beta$ and low $V/a$, which has a very shallow density profile, and one with low $\beta$, high $V/a$, and a steeper profile.

These models are not realistic, but might be a good place to start because they are analytically tractable. The next step would be a full linear stability analysis which includes rotation and diffusion (thermal and magnetic). GYRE doesn't seem to have MHD implemented - could we do it with Dedalus?