Can you compute Ramsey Numbers?

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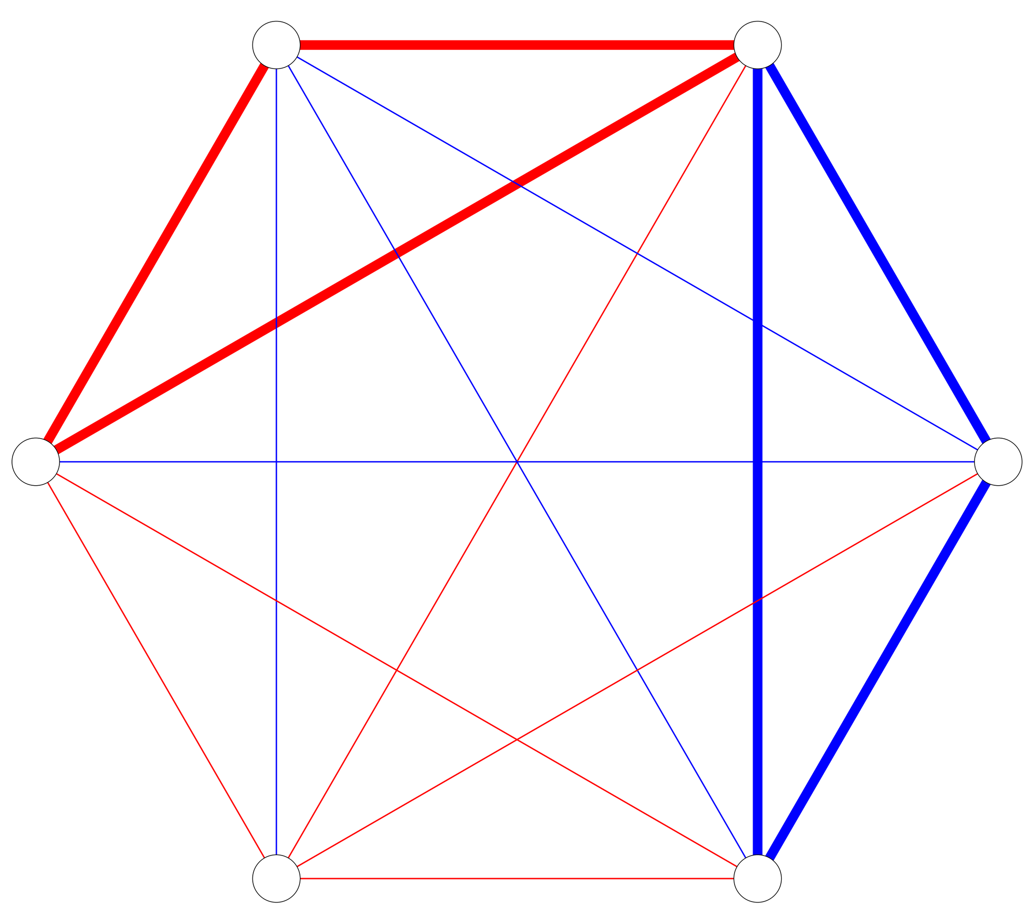
Ramsey Numbers

The Party Problem:
"How many people must be at a party before 3 people all know each other, or have never met?"



The solution to the party problem is 6. But this is only when looking for a relationship between 3 people? How does it change when we have 5 people that know each other, and 2 that do not?

The party problem as a graph

1. Each vertex is a person
2. Each edge is the relationship between two people:
   - Blue - They know each other
   - Red - They do not know each other

A Ramsey Number $R(k,l) = n$ is the number of vertices required before a k-clique or l-independent-set (iset) will always be found in any graph with n vertices.

How difficult are Ramsey Numbers?

There are two sources of difficulty:

1. The number of possible graphs with n vertices is $2^{n(n-1)/2}$
2. The number of checks to find the k-clique (l-iset) is $\binom{n}{k}$ ($\binom{n}{l}$)

To test if $n = R(k,l)$, a computer would need to do

$$O(2^{n^2}\binom{n}{k})$$

work to manually check for a graph that doesn't have the k-clique (l-iset).

$R(k,l)$ as a decision problem

Given three integers – n, k and l – decide if $n = R(k,l)$ by checking if every graph of n vertices has either a k-clique, or l-iset.

Using Oracles and Non-Deterministic Classical Turing Machines

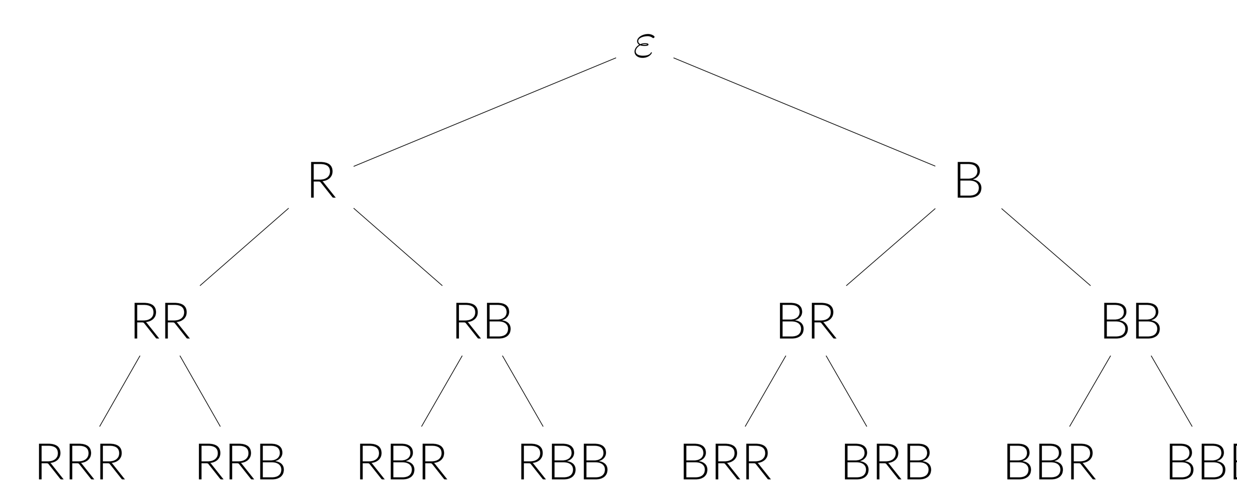
$$n \stackrel{?}{=} R(k,l) \in \text{coNP}^{\text{NP}}$$

Using just Non-Deterministic Quantum Turing Machines

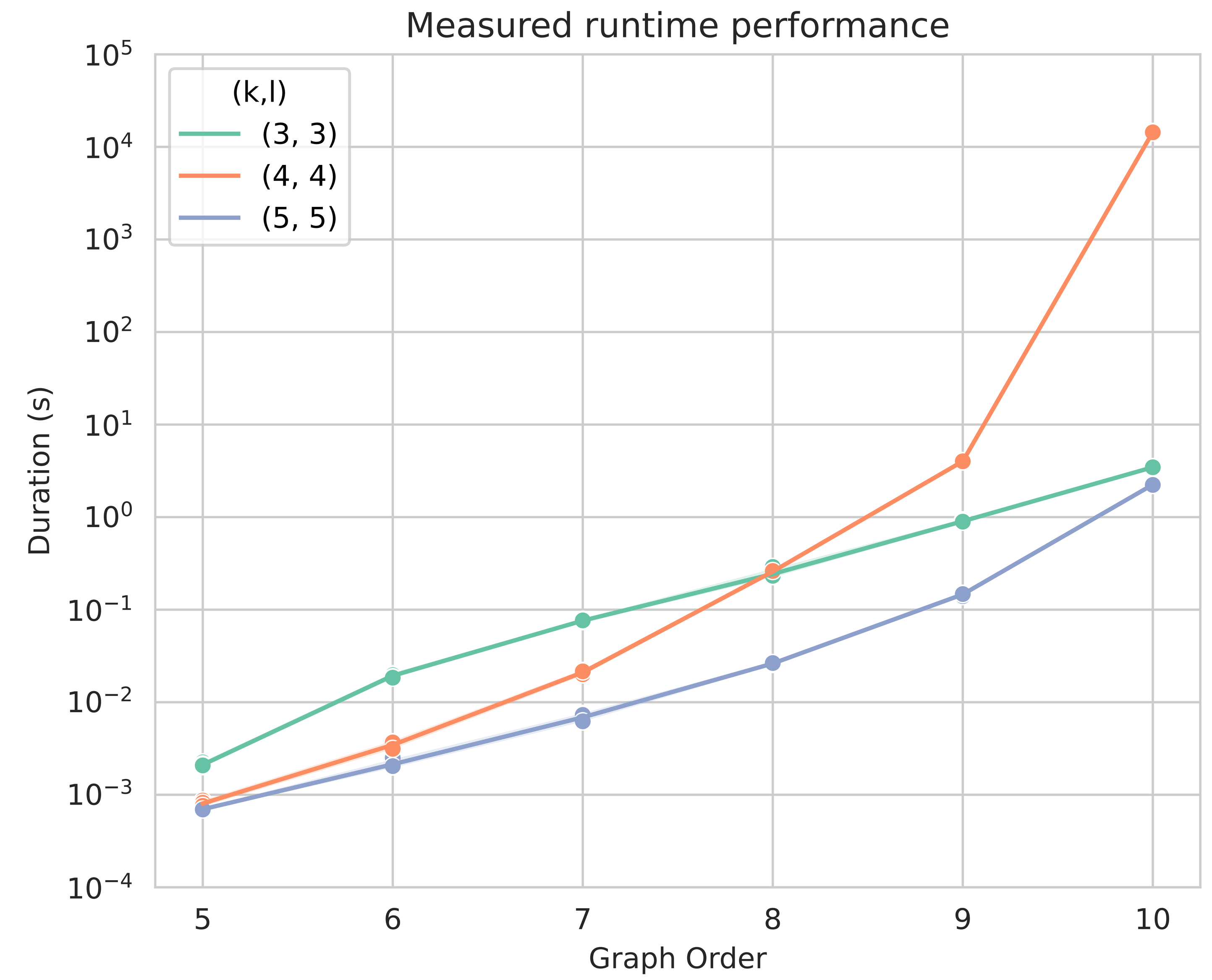
$$n \stackrel{?}{=} R(k,l) \in \text{QMA}$$

Using a classical computer

Search over the tree construction of all the graphs.

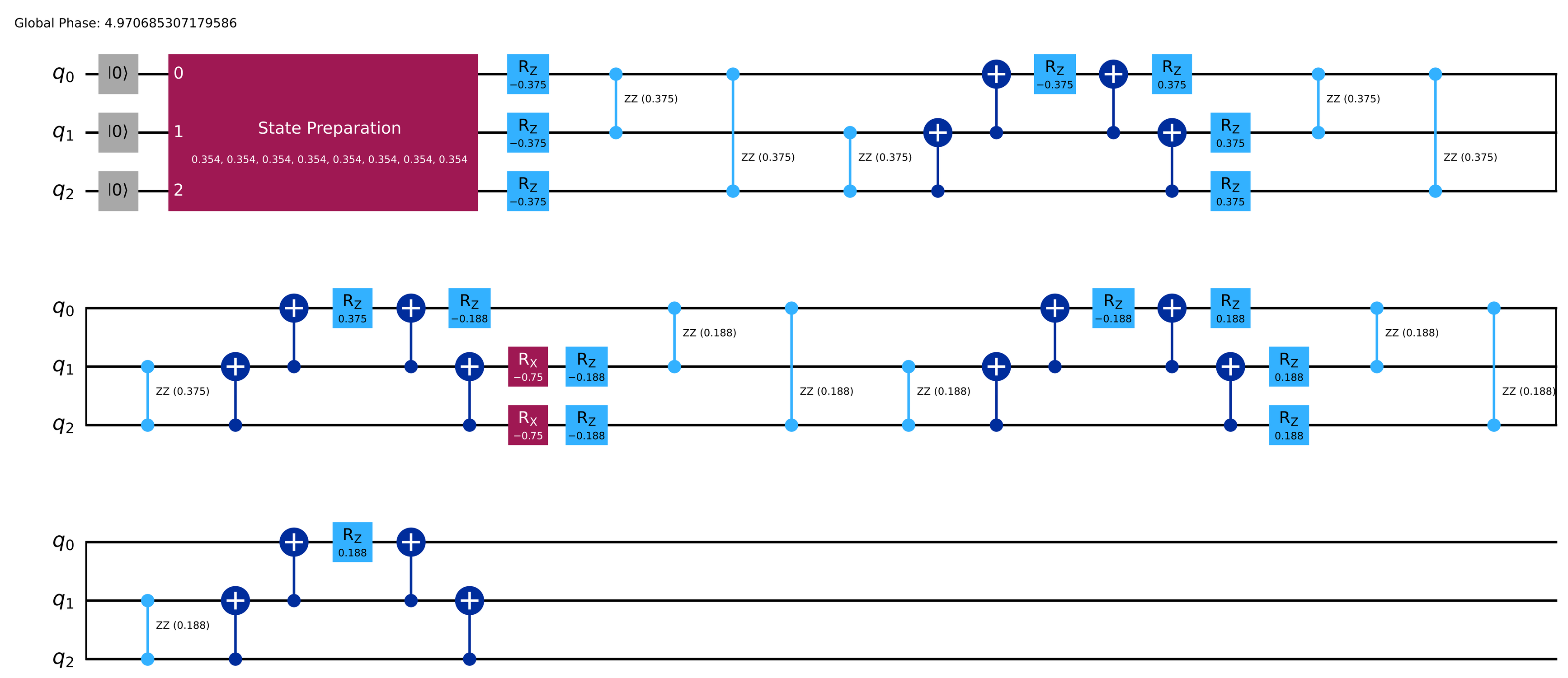


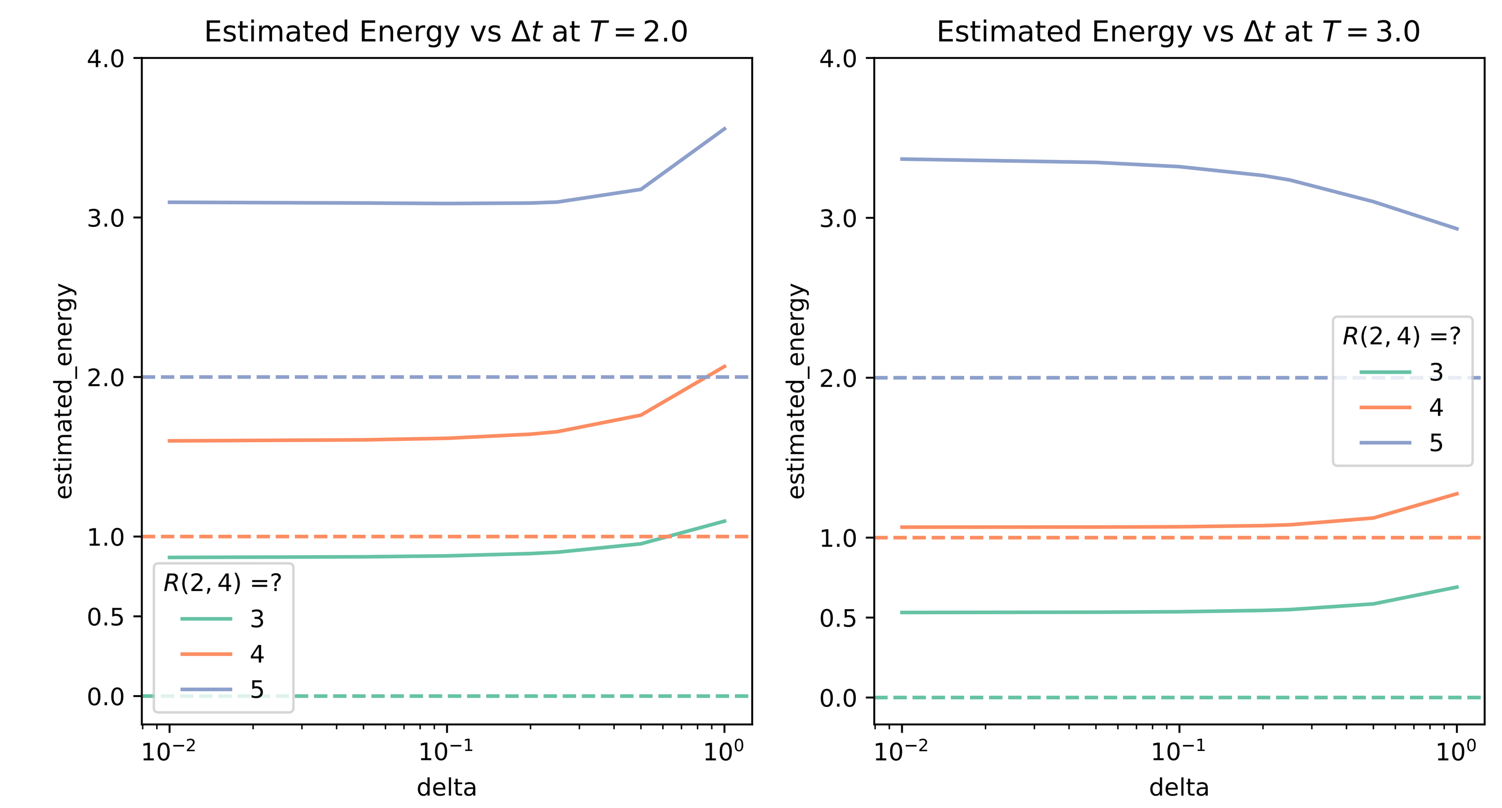
Check each node in the tree, and prune the branch if either the k-clique or l-iset is found. Return the graph at a leaf node to show $n \neq R(k,l)$.



Using a quantum computer

Encode a Ramsey Number decision $n \stackrel{?}{=} R(k,l)$ into a Hamiltonian H^{kl}, and then extract the minimum eigenvalue (energy) using Adiabatic Quantum Optimisation.





In the above figure, a dashed line is the expected energy, and the solid line is the measured energy. The Adiabatic Quantum Optimisation Algorithm was not able to achieve correct results under any configuration of the problem.

Can you compute a Ramsey Number?

NO

The next Ramsey Number $R(5,5)$ will take 10^{134} years. $R(6,6)$ overflows a double precision float with an underestimating projection.

More information is available here