Relations for ideal gas

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1 Ideal gas equation

$$P\nu = RT$$

2 Isentropic process with constant specific heat capacities

$$s = \text{constant}$$

2.1 Constant specific heats

$$\frac{P_2}{P_1} = \left(\frac{\nu_1}{\nu_2}\right)^k$$

$$\frac{T_2}{T_1} = \left(\frac{\nu_1}{\nu_2}\right)^{k-1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

2.2 Variable specific heats

2.2.1 Given specific volume

If specific volume of the state of interest is given then get temperature of the state of interest numerically. Use Newton-Raphson to find roots of function

$$f(T) = (C_0 - R)\ln\frac{T}{T_1} + \frac{C_1}{1000}(T - T_1) + \frac{C_2}{2\cdot 10^6}(T^2 - T_1^2) + \frac{C_2}{3\cdot 10^9}(T^3 - T_1^3) + R\ln\frac{\nu_2}{\nu_1}$$

and it's derivative

$$f'(T) = \frac{C_0 - R}{T} + \frac{C_1}{1000} + \frac{C_2}{10^6}T + \frac{C_2}{10^9}T^2$$

2.2.2 Given pressure

If pressure of the state of interest is given then get temperature of the state of interest numerically. Use Newton-Raphson to find roots of function

$$f(T) = C_0\ln\frac{T_2}{T_1} + \frac{C_1}{1000}(T_2 - T_1) + \frac{C_2}{2\cdot 10^6}(T_2^2 - T_1^2) + \frac{C_2}{3\cdot 10^9}(T_2^3 - T_1^3) - R\ln\frac{P_2}{P_1}$$

and it's derivative

$$f'(T) = \frac{C_0}{T} + \frac{C_1}{1000} + \frac{C_2}{10^6}T + \frac{C_2}{10^9}T^2$$

2.2.3 Given temperature

If temperature T is given no iteration is required.

$$\nu_2 = \exp\left(\frac{R - C_0}{R}\ln\frac{T}{T_1} - \frac{C_1}{1000R}(T - T_1) - \frac{C_2}{2 \cdot 10^6 R}(T^2 - T_1^2) - \frac{C_2}{3 \cdot 10^9 R}(T^3 - T_1^3) - \ln\nu_1\right)$$

3 Isobaric process

$$P = \text{constant}$$

$$s_2 = s_1 + c_\nu \ln\frac{T_2}{T_1} + \mathrm{R}\ln\frac{\nu_2}{\nu_1}$$

4 Isochoric process

$$\nu = \text{constant}$$

$$s_2 = s_1 + c_\nu \ln\frac{T_2}{T_1} + \mathrm{R}\ln\frac{\nu_2}{\nu_1}$$

5 Isothermal process

$$T = \text{constant}$$

$$P\nu = \text{constant}$$

$$s_2 = s_1 + c_\nu \ln\frac{T_2}{T_1} + \mathrm{R}\ln\frac{\nu_2}{\nu_1}$$

6 var specific heat

$$s_2 - s_1 = \int_1^2 c_{v0}\frac{dT}{T} + R\ln\frac{\nu_2}{\nu_1}$$

$$c_{p0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3$$

where, $\theta = T/1000$ and T is in Kelvin.

$$c_{p0} = C_0 + C_1\frac{T}{1000} + C_2\left(\frac{T}{1000}\right)^2 + C_3\left(\frac{T}{1000}\right)^3$$

Note theta

$$c_{v0} = c_{p0} - R$$

$$\int_1^2 c_{v0}\frac{dT}{T} = \int_1^2 (c_{p0}-R)\frac{dT}{T} = \int_1^2 c_{p0}\frac{dT}{T} - \int_1^2 R\frac{dT}{T} = \int_1^2 c_{p0}\frac{dT}{T} - R\ln\frac{T_2}{T_1}$$

$$\begin{aligned}
\int_1^2 c_{p0}\frac{dT}{T} &= \int_1^2 \left(C_0 + C_1\frac{T}{1000} + C_2\left(\frac{T}{1000}\right)^2 + C_3\left(\frac{T}{1000}\right)^3\right)\frac{dT}{T} \\
&= C_0\int_1^2\frac{dT}{T} + \frac{C_1}{1000}\int_1^2 dT + \frac{C_2}{10^6}\int_1^2 TdT + \frac{C_3}{10^9}\int_1^2 T^2dT \\
&= C_0\ln\frac{T_2}{T_1} + \frac{C_1}{1000}(T_2-T_1) + \frac{C_2}{2\cdot 10^6}(T_2^2-T_1^2) + \frac{C_2}{3\cdot 10^9}(T_2^3-T_1^3)
\end{aligned}$$

As a result,

$$\int_1^2 c_{v0}\frac{dT}{T} = (C_0-R)\ln\frac{T_2}{T_1} + \frac{C_1}{1000}(T_2-T_1) + \frac{C_2}{2\cdot 10^6}(T_2^2-T_1^2) + \frac{C_2}{3\cdot 10^9}(T_2^3-T_1^3)$$

Finally,

$$s_2 - s_1 = (C_0-R)\ln\frac{T_2}{T_1} + \frac{C_1}{1000}(T_2-T_1) + \frac{C_2}{2\cdot 10^6}(T_2^2-T_1^2) + \frac{C_2}{3\cdot 10^9}(T_2^3-T_1^3) + R\ln\frac{\nu_2}{\nu_1}$$

For isentropic process, $s_2 = s_1$.

$$0 = (C_0-R)\ln\frac{T_2}{T_1} + \frac{C_1}{1000}(T_2-T_1) + \frac{C_2}{2\cdot 10^6}(T_2^2-T_1^2) + \frac{C_2}{3\cdot 10^9}(T_2^3-T_1^3) + R\ln\frac{\nu_2}{\nu_1}$$

If specific volume ν_2 is given, the unknown is T_2. Writing the function in terms of T,

$$f(T) = (C_0-R)\ln\frac{T}{T_1} + \frac{C_1}{1000}(T-T_1) + \frac{C_2}{2\cdot 10^6}(T^2-T_1^2) + \frac{C_2}{3\cdot 10^9}(T^3-T_1^3) + R\ln\frac{\nu_2}{\nu_1}$$

7 Newton Raphson

For NR, derivative of $f(T)$ with respect to T is needed.

$$f'(T) = \frac{C_0-R}{T} + \frac{C_1}{1000} + \frac{C_2}{10^6}T + \frac{C_2}{10^9}T^2$$

Iterate

$$T_{\text{new}} = T - \frac{f(T)}{f'(T)}$$

until $T_{\text{new}} - T < \epsilon$

8 var specific heat 2

$$s_2 - s_1 = \int_1^2 c_{P0}\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

$$c_{p0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3$$

where, $\theta = T/1000$ and T is in Kelvin.

$$c_{p0} = C_0 + C_1 \frac{T}{1000} + C_2 \left(\frac{T}{1000}\right)^2 + C_3 \left(\frac{T}{1000}\right)^3$$

$$\begin{aligned}
\int_1^2 c_{p0} \frac{dT}{T} &= \int_1^2 \left(C_0 + C_1 \frac{T}{1000} + C_2 \left(\frac{T}{1000}\right)^2 + C_3 \left(\frac{T}{1000}\right)^3\right) \frac{dT}{T} \\
&= C_0 \int_1^2 \frac{dT}{T} + \frac{C_1}{1000} \int_1^2 dT + \frac{C_2}{10^6} \int_1^2 T dT + \frac{C_3}{10^9} \int_1^2 T^2 dT \\
&= C_0 \ln \frac{T_2}{T_1} + \frac{C_1}{1000}(T_2 - T_1) + \frac{C_2}{2 \cdot 10^6}(T_2^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T_2^3 - T_1^3)
\end{aligned}$$

Finally,

$$s_2 - s_1 = C_0 \ln \frac{T_2}{T_1} + \frac{C_1}{1000}(T_2 - T_1) + \frac{C_2}{2 \cdot 10^6}(T_2^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T_2^3 - T_1^3) - R \ln \frac{P_2}{P_1}$$

For isentropic process, $s_2 = s_1$.

$$0 = C_0 \ln \frac{T_2}{T_1} + \frac{C_1}{1000}(T_2 - T_1) + \frac{C_2}{2 \cdot 10^6}(T_2^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T_2^3 - T_1^3) - R \ln \frac{P_2}{P_1}$$

If pressure P_2 is given, the unknown is T_2. Writing the function in terms of T,

$$f(T) = C_0 \ln \frac{T_2}{T_1} + \frac{C_1}{1000}(T_2 - T_1) + \frac{C_2}{2 \cdot 10^6}(T_2^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T_2^3 - T_1^3) - R \ln \frac{P_2}{P_1}$$

9 Newton Raphson

For NR, derivative of $f(T)$ with respect to T is needed.

$$f'(T) = \frac{C_0}{T} + \frac{C_1}{1000} + \frac{C_2}{10^6} T + \frac{C_2}{10^9} T^2$$

Iterate

$$T_{\text{new}} = T - \frac{f(T)}{f'(T)}$$

until $T_{\text{new}} - T < \epsilon$

10 T is given find ν

Transpose ν_2 in the following equation

$$0 = (C_0 - R) \ln \frac{T}{T_1} + \frac{C_1}{1000}(T - T_1) + \frac{C_2}{2 \cdot 10^6}(T^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T^3 - T_1^3) + R \ln \frac{\nu_2}{\nu_1}$$

such that

$$-R(\ln \nu_2 - \ln \nu_1) = (C_0 - R) \ln \frac{T}{T_1} + \frac{C_1}{1000}(T - T_1) + \frac{C_2}{2 \cdot 10^6}(T^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T^3 - T_1^3)$$

$$-R \ln \nu_2 + R \ln \nu_1 = (C_0 - R) \ln \frac{T}{T_1} + \frac{C_1}{1000}(T - T_1) + \frac{C_2}{2 \cdot 10^6}(T^2 - T_1^2) + \frac{C_2}{3 \cdot 10^9}(T^3 - T_1^3)$$

$$-R\ln\nu_2 = (C_0 - R)\ln\frac{T}{T_1} + \frac{C_1}{1000}(T - T_1) + \frac{C_2}{2\cdot 10^6}(T^2 - T_1^2) + \frac{C_2}{3\cdot 10^9}(T^3 - T_1^3) - R\ln\nu_1$$

$$\ln\nu_2 = \frac{R - C_0}{R}\ln\frac{T}{T_1} - \frac{C_1}{1000R}(T - T_1) - \frac{C_2}{2\cdot 10^6 R}(T^2 - T_1^2) - \frac{C_2}{3\cdot 10^9 R}(T^3 - T_1^3) - \ln\nu_1$$

$$\nu_2 = \exp\left(\frac{R - C_0}{R}\ln\frac{T}{T_1} - \frac{C_1}{1000R}(T - T_1) - \frac{C_2}{2\cdot 10^6 R}(T^2 - T_1^2) - \frac{C_2}{3\cdot 10^9 R}(T^3 - T_1^3) - \ln\nu_1\right)$$

11 qs and qr isochoric

$$q_s = \int_1^2 c_{\nu 0} dT$$

$$c_{p0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3$$

$$c_{\nu 0} = c_{p0} - R$$

$$c_{\nu 0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3 - R$$

$$c_{\nu 0} = C_0 + \frac{C_1}{10^3}T + \frac{C_2}{10^6}T^2 + \frac{C_3}{10^9}T^3 - R$$

$$q_s = \int_1^2 \left(C_0 + \frac{C_1}{10^3}T + \frac{C_2}{10^6}T^2 + \frac{C_3}{10^9}T^3 - R\right) dT$$

$$q_s = (C_0 - R)\int_1^2 dT + \frac{C_1}{10^3}\int_1^2 T dT + \frac{C_2}{10^6}\int_1^2 T^2 dT + \frac{C_3}{10^9}\int_1^2 T^3 dT$$

$$q_s = (C_0 - R)(T_2 - T_1) + \frac{C_1}{2\cdot 10^3}(T_2^2 - T_1^2) + \frac{C_2}{3\cdot 10^6}(T_2^3 - T_1^3) + \frac{C_3}{4\cdot 10^9}(T_2^4 - T_1^4)$$

Unknown is T_2.

$$q_s = (C_0 - R)(T - T_1) + \frac{C_1}{2\cdot 10^3}(T^2 - T_1^2) + \frac{C_2}{3\cdot 10^6}(T^3 - T_1^3) + \frac{C_3}{4\cdot 10^9}(T^4 - T_1^4)$$

$$f(T) = (C_0 - R)(T - T_1) + \frac{C_1}{2\cdot 10^3}(T^2 - T_1^2) + \frac{C_2}{3\cdot 10^6}(T^3 - T_1^3) + \frac{C_3}{4\cdot 10^9}(T^4 - T_1^4) - q_s$$

Derivative

$$f'(T) = (C_0 - R) + \frac{C_1}{10^3}T + \frac{C_2}{10^6}T^2 + \frac{C_3}{10^9}T^3$$