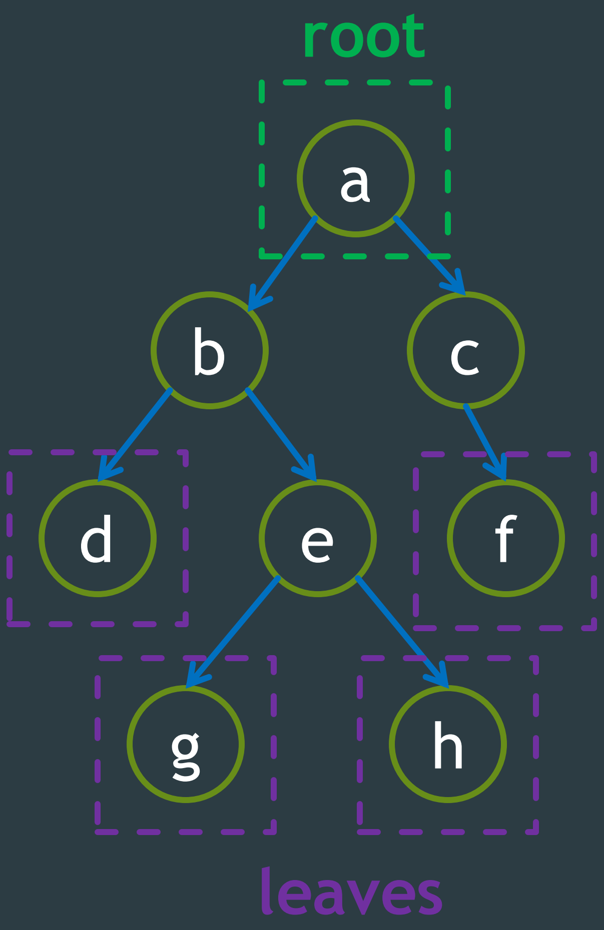
ECE2810J
Data Structures and Algorithms

Trees

- **Learning Objectives:**
  - Know some basic terminology of trees and binary trees
  - Know some basic properties of binary trees
  - Know how to represent a binary tree by an array and a linked list

Outline

- Trees
- Binary Trees

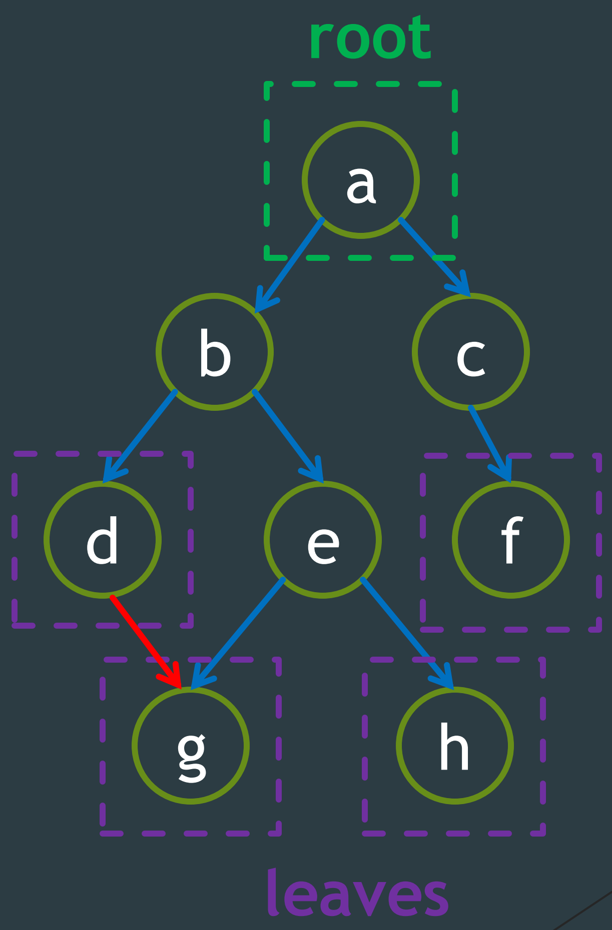


Trees

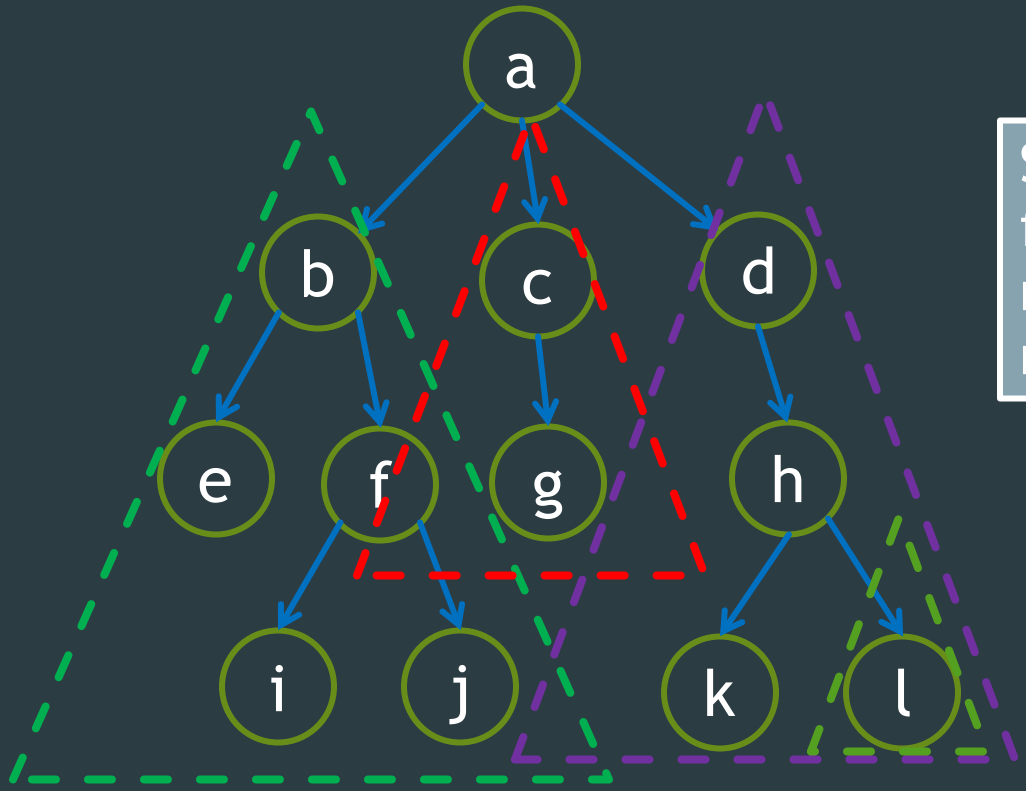
- Tree is an extension of linked list data structure:
  - Each node connects to multiple nodes.
- A tree is a “natural” way to represent hierarchical structure and organization.
- Many problems in computer science can be solved by breaking it down into smaller pieces and arranging the pieces in some form of hierarchical structure.
  - For example: merge sort.

Tree Terminology

- Just like lists, trees are collections of nodes.
- The node at the top of the hierarchy is the root.
- Nodes are connected by edges.
- Edges define parent-child relationship.
  - Root has no parent.
  - All other nodes have **exactly one** parent.
- A node with no children is called a leaf.



Subtrees

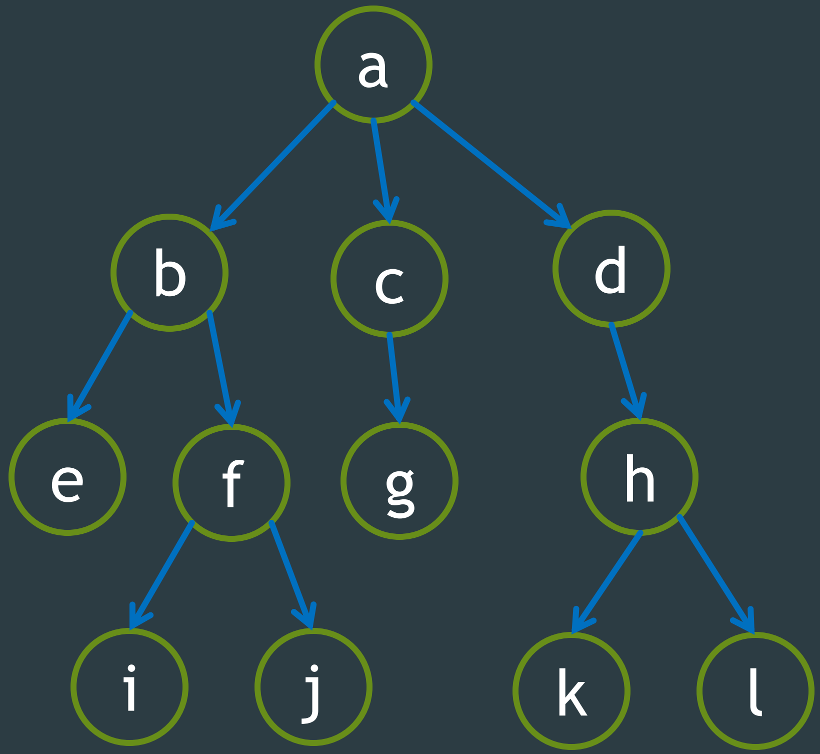


Subtree can be defined for any node in general, not just for the root node.

More Tree Terminology

- f is the child of b.
- b is the parent of f.

- Nodes that share the same parent are siblings.
  - b and c are the siblings of d.
  - e is the sibling of f.



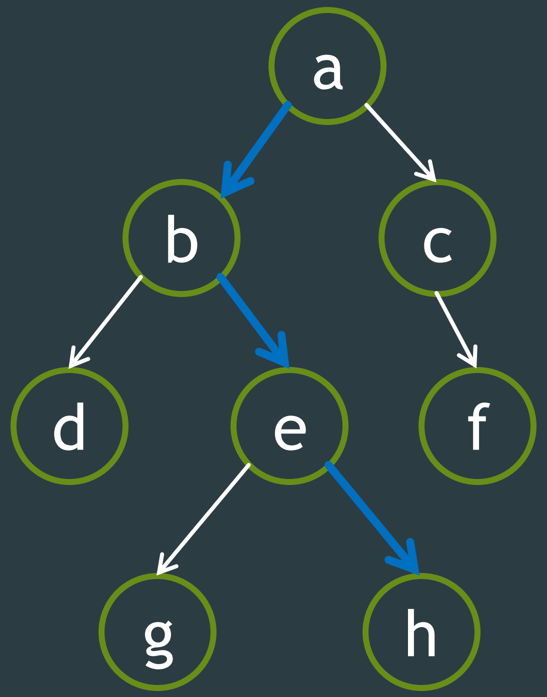
Path

- A path is a sequence of nodes such that the next node in the sequence is a child of the previous.
  - E.g., a → b → e → h is a path.
  - The path length is 3.

- Path length may be 0

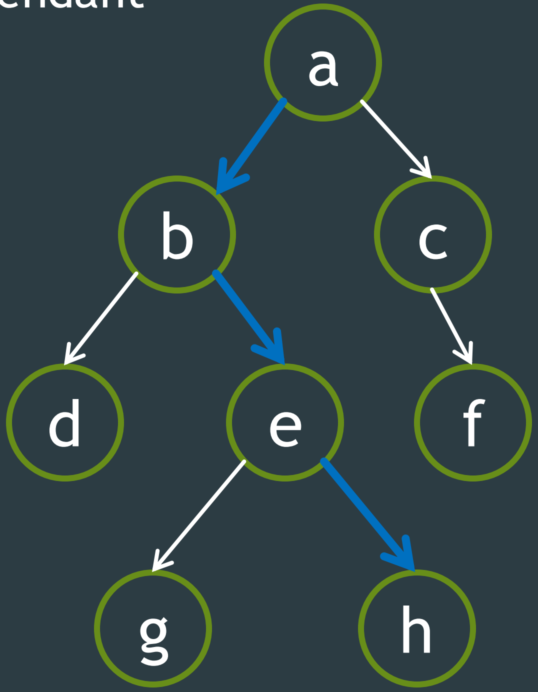
  e.g., b going to itself is a path and its length is 0.

- <u>**Claim**</u>: If there exists a path between two nodes, then this path is the unique path between these two nodes.



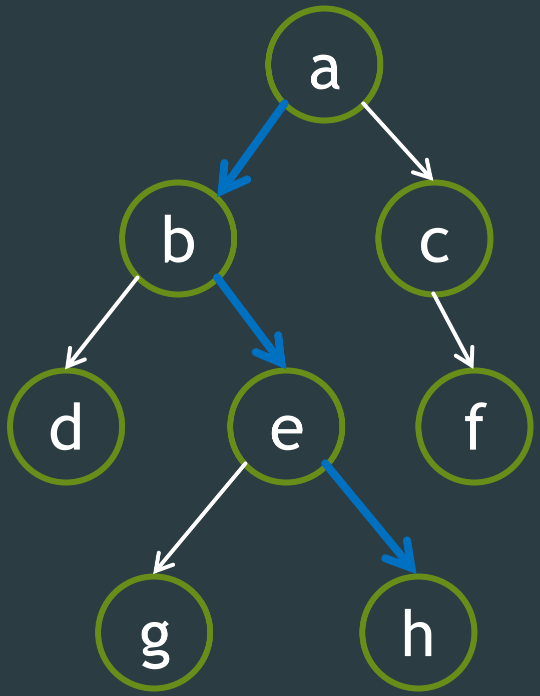
Ancestors and Descendants

- If there exists a path from a node A to a node B, then A is an ancestor of B and B is a descendant of A.
  - E.g., a is an ancestor of h and h is a descendant of a.



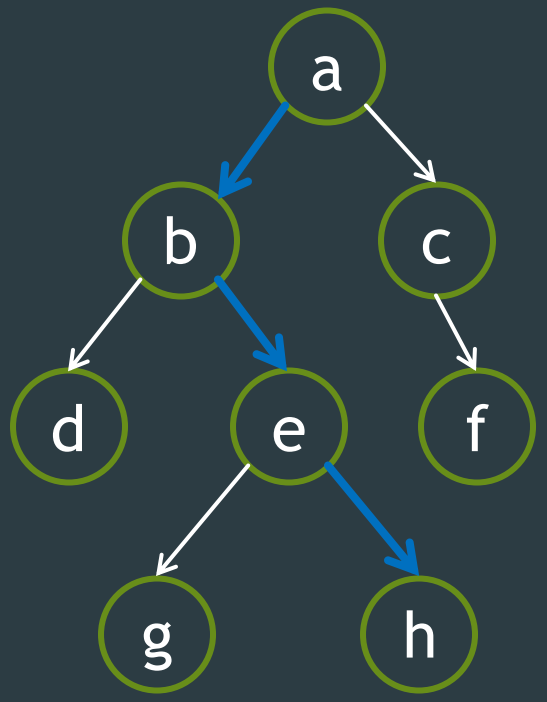
Depth, Level, and Height of a Node

- The depth or level of a node is the length of the unique path from the **root** to the node.
  - E.g., depth(b)=1, depth(a)=0.
- The height of a node is the length of the **longest** path from the node to a **leaf**.
  - E.g., height(b)=2, height(a)=3.
  - All leaves have height zero.



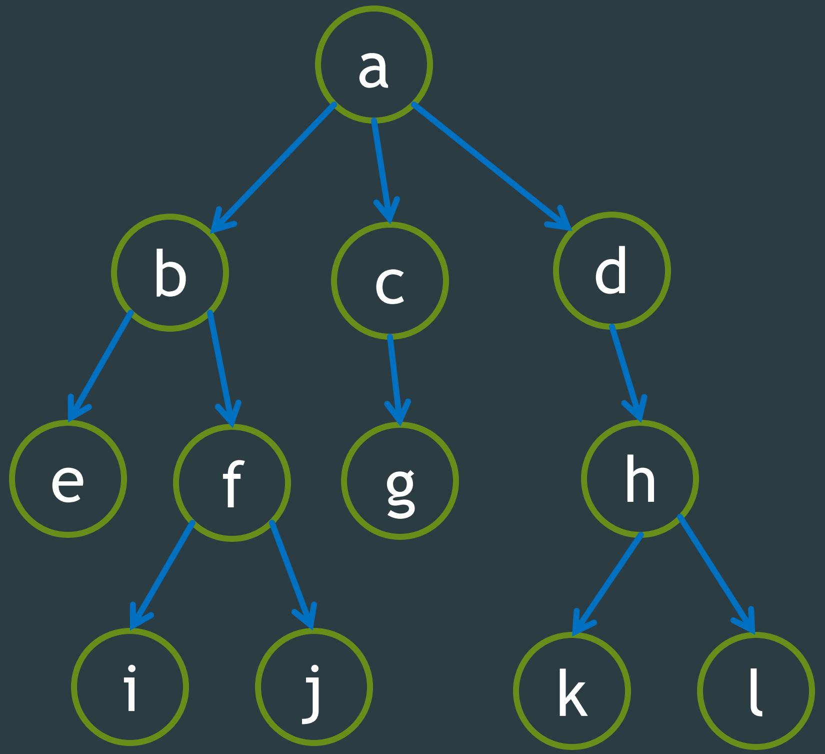
Depth, Level, and Height of a Tree

- The height of a tree is the height of its root.
  - This is also known as the depth of a tree.
  - The depth of the tree on the right is 3.
- The number of levels of a tree is the height of the tree plus one.
  - The number of levels of the tree on the right is 4.



Degree

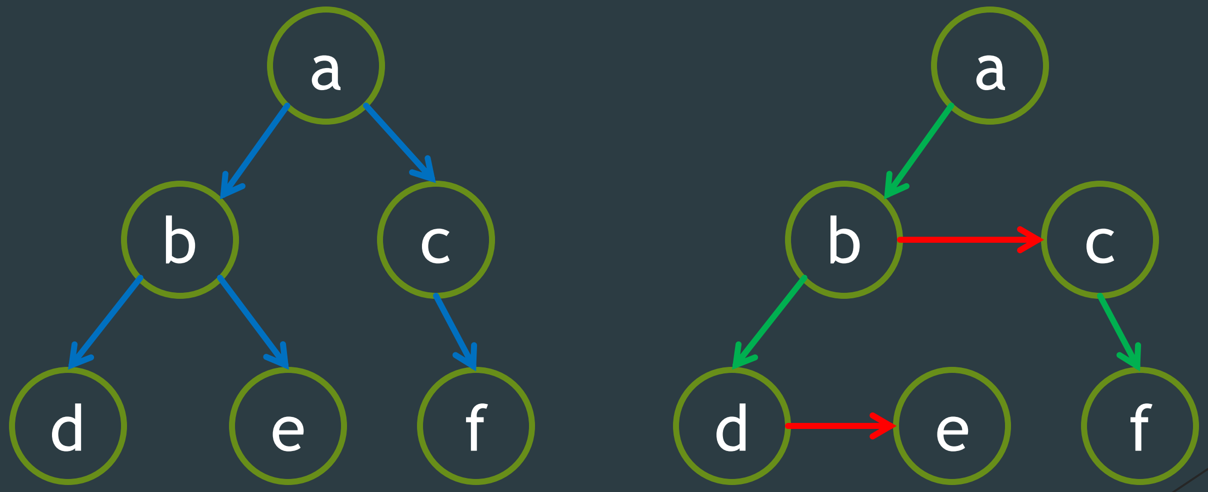
- The degree of a node is the number of children of a node.
  - E.g., degree(a) = 3, degree(c) = 1.
- The degree of a tree is the maximum degree of a node in the tree.
  - The degree of the tree on the right is 3.



A Simple Implementation of Tree

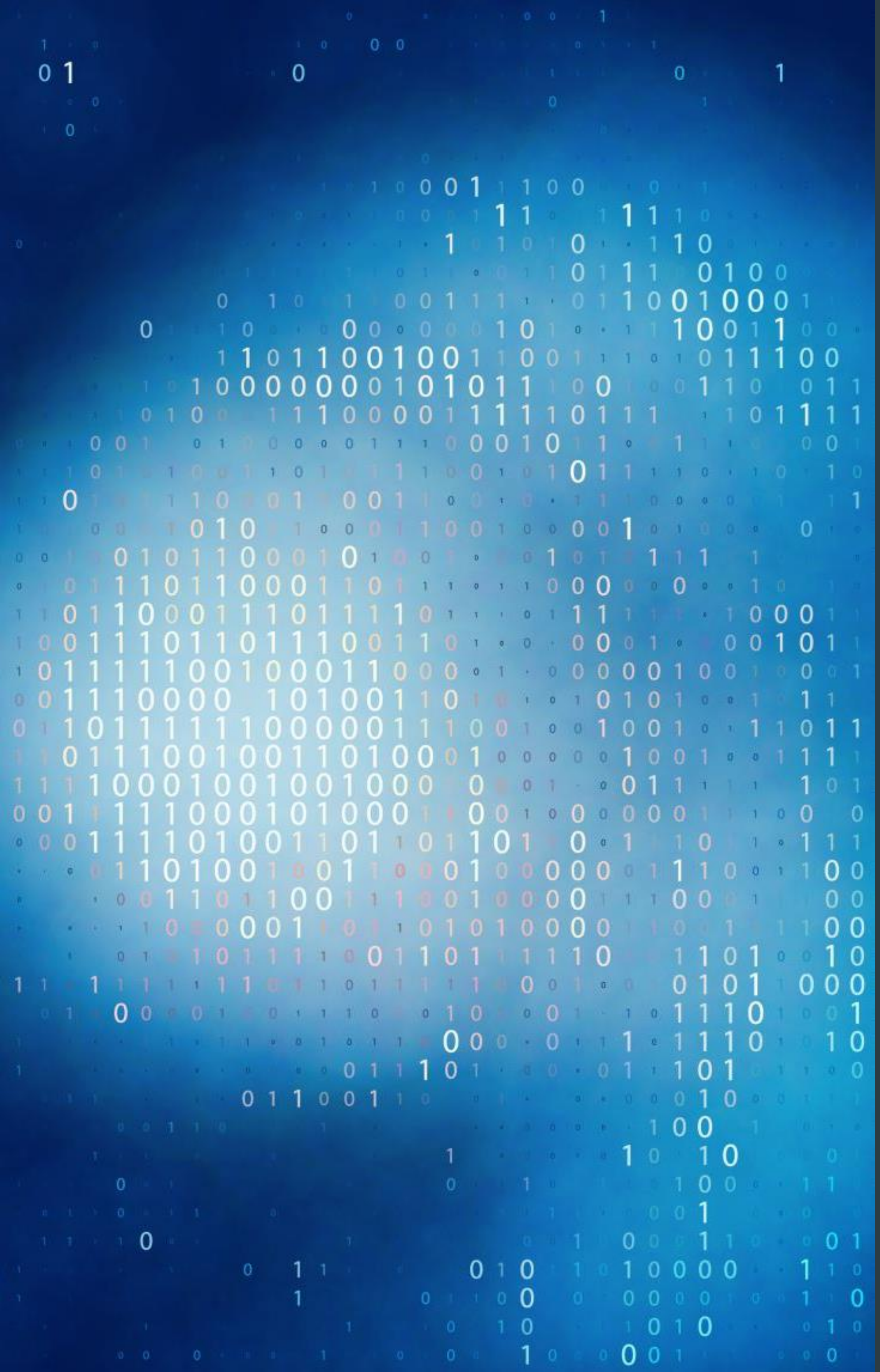
- Each node is part of a linked list of **siblings**.
- Additionally, each node stores a pointer to its first child.

```
struct node {
  Item item;
  node *firstChild;
  node *nextSibling;
};
```



Code Exercise: Trees (~15 mins)

- 1.1 Complete the implementation:
  - Canvas -> Code Exercise -> trees.cpp
  - Construct the tree
  - Function to insert new elements
  - Function to find elements in the tree

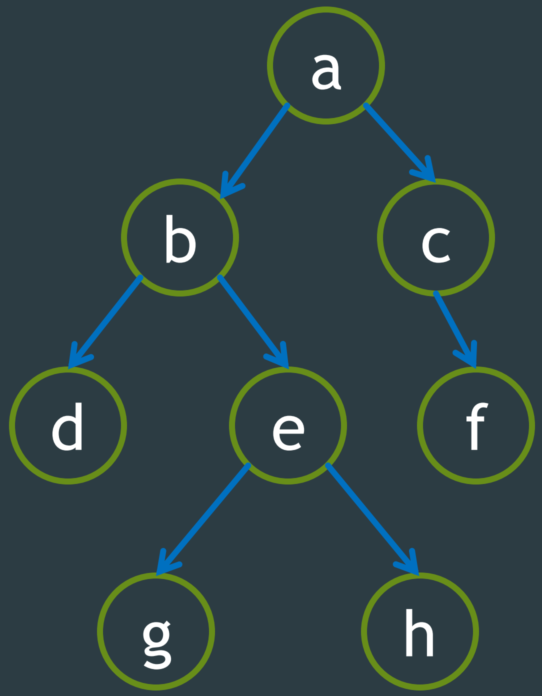


Outline

- Trees
- Binary Trees

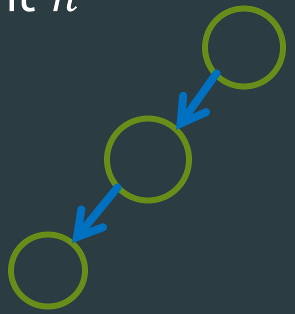
Binary Tree

- Every node can only have at most two children.
- An empty tree is a special binary tree.

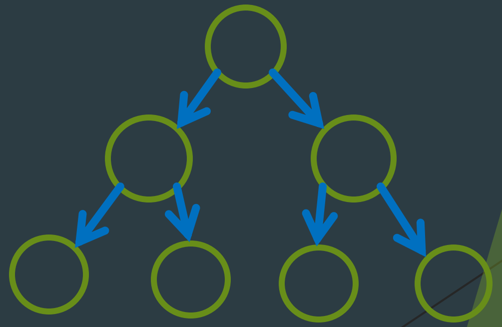


Binary Tree Properties

- What is the minimum number of nodes in a binary tree of height h (i.e., has $h+1$ levels)?
  - Answer: At least one node at each level.
  - $h+1$ levels means at least $h+1$ nodes.



- What is the maximum number of nodes in a binary tree of height h (i.e., has $h+1$ levels)?
  - Answer: At most 2^k nodes at level k.
  - Maximum number of nodes is
$$1+2+2^2+\cdots+2^h=2^{h+1}-1$$



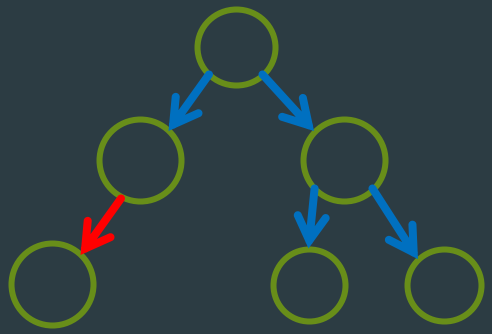
Number Of Nodes and Height

- **Claim** (from the previous slide): Let n be the number of nodes in a binary tree whose height is h (i.e., has $h + 1$ levels).
  - We have $h + 1 \leq n \leq 2^{h+1} - 1$.

- **Question**: given n nodes, what is the height h of the tree?
  - $\log_2(n + 1) - 1 \leq h \leq n - 1$

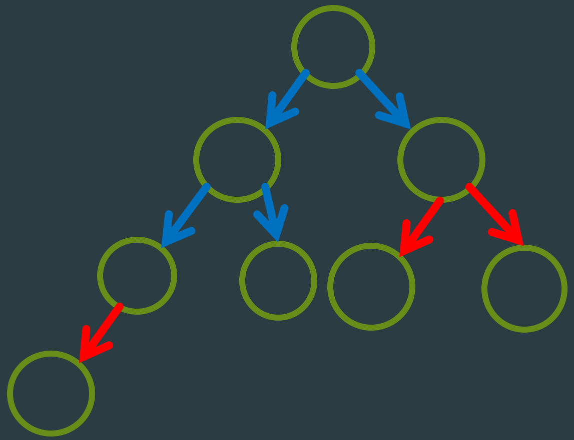
Types of Binary Trees

- A binary tree is proper if every node has 0 or 2 children.

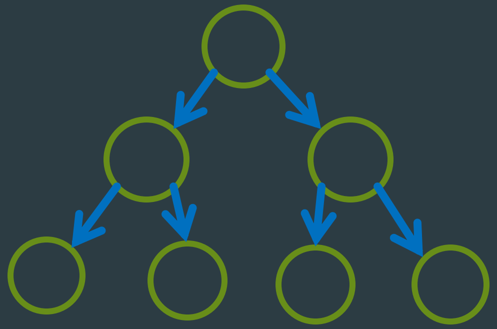


- A binary tree is complete if:

1. every level except the lowest is fully populated, and
2. the lowest level is populated **from left to right.**



- A binary tree is perfect if every level is fully populated.



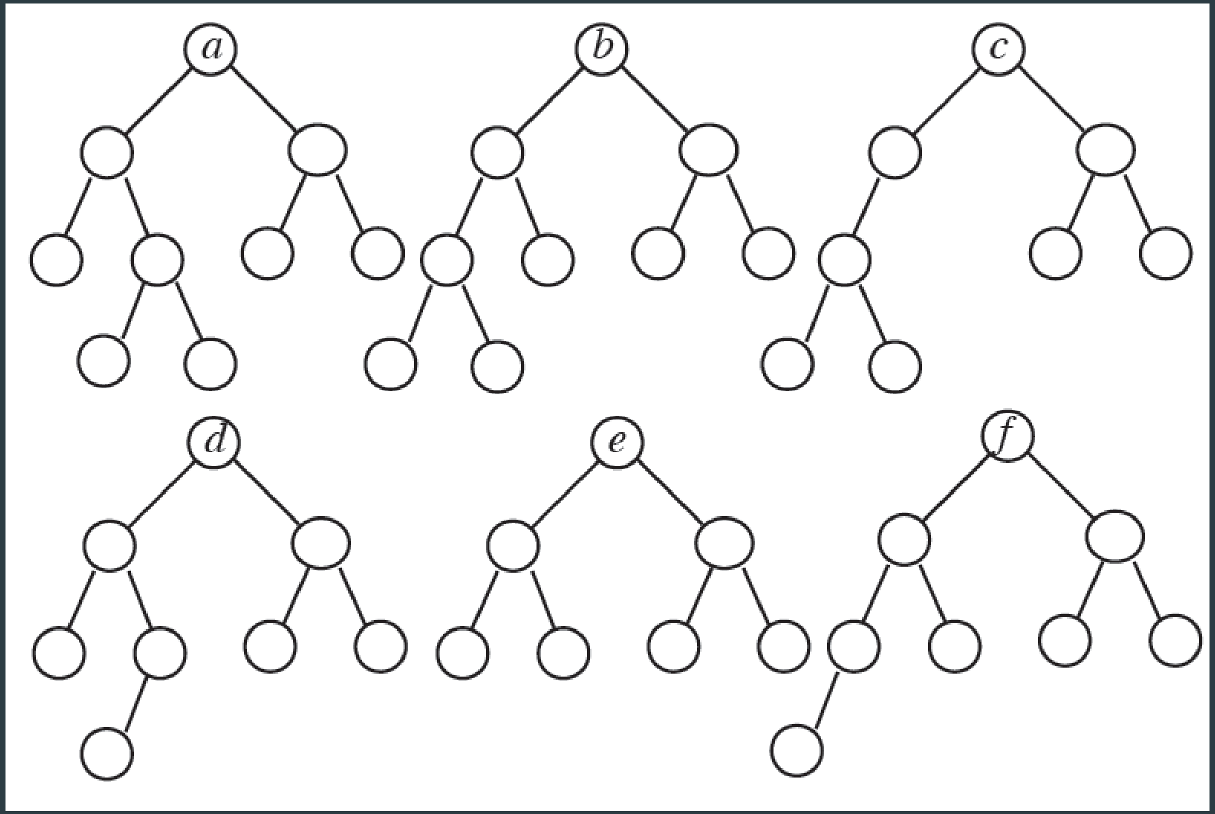
Which Statements Are Correct?

A. Trees a and d are proper.

B. Tree c is complete.

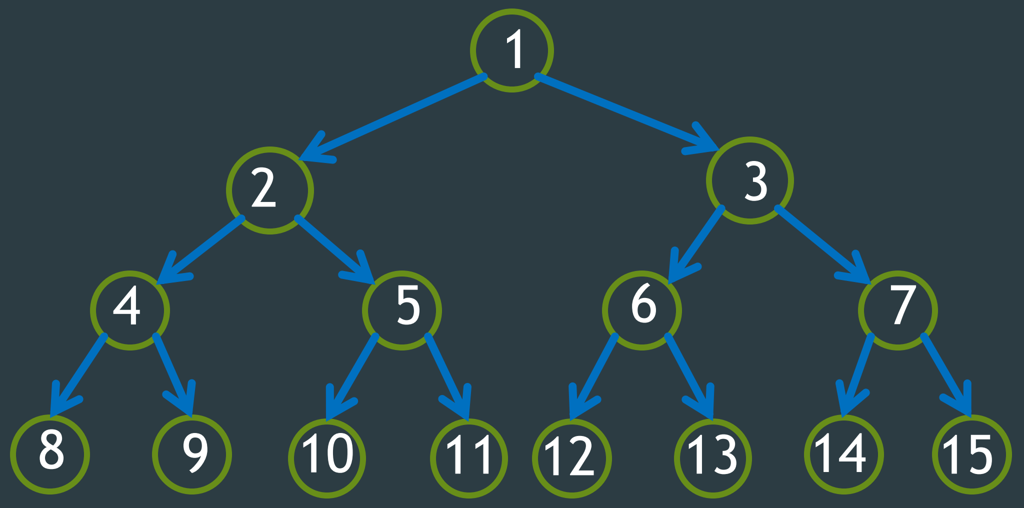
C. Trees b and f are complete.

D. Tree e is perfect.

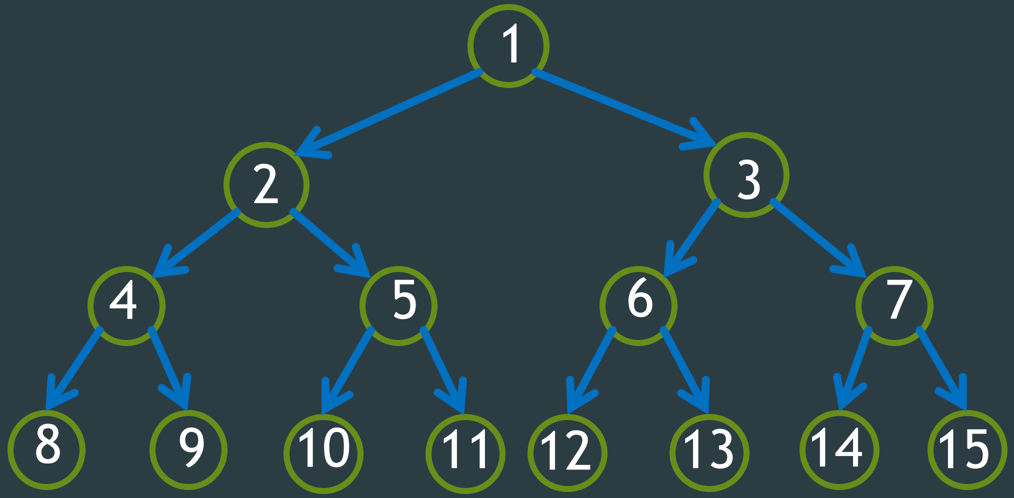


Numbering Nodes In a Perfect Binary Tree

- Numbering nodes from 1 to $2^{h+1} - 1$.
- Numbering from top to bottom level.
- Within a level, numbering from left to right.



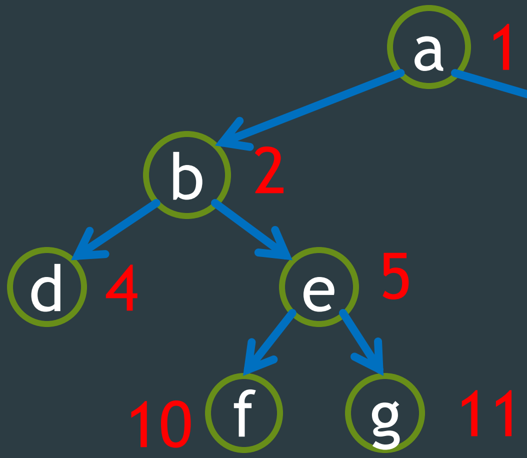
Numbering Nodes In a Perfect Binary Tree



- What is the parent of node i?
  - For $i \neq 1$, it is $\lfloor i/2 \rfloor$. For node 1, it has no parent.
- What is the left child of node i? Let n be the number of nodes.
  - If $2i \leq n$, it is $2i$; If $2i > n$, no left child.
- What is the right child of node i?
  - If $2i + 1 \leq n$, it is $2i + 1$; If $2i + 1 > n$, no right child.

Representing Binary Tree Using Array

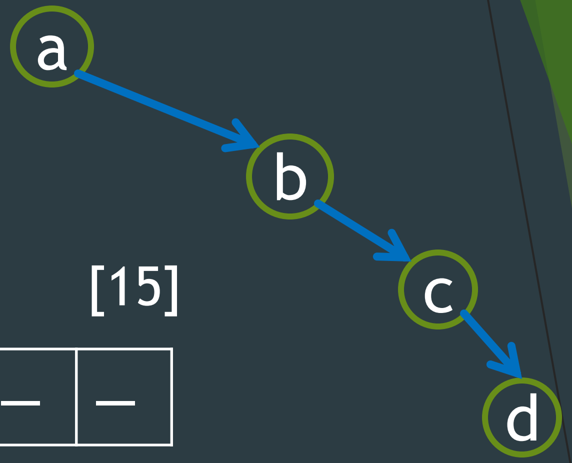
- Based on the numbering scheme for a perfect binary tree.
- If the number of the node in a perfect binary tree is i, then the node is put at index i of the array.



a	b	c	d	e	–	–	–	–	f	g
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]

How Would You Represent a **Right-skewed** Binary Tree?

- Assume array index starts from 1.



	[1]		[3]		[5]		[7]		[9]		[11]		[13]		[15]
A.	a	b	–	c	–	–	–	d	–	–	–	–	–	–	–
B.	a	b	c	d	–	–	–	–	–	–	–	–	–	–	–
C.	a	–	b	–	–	–	c	–	–	–	–	–	–	–	d
D.	a	–	b	–	c	–	d	–	–	–	–	–	–	–	–

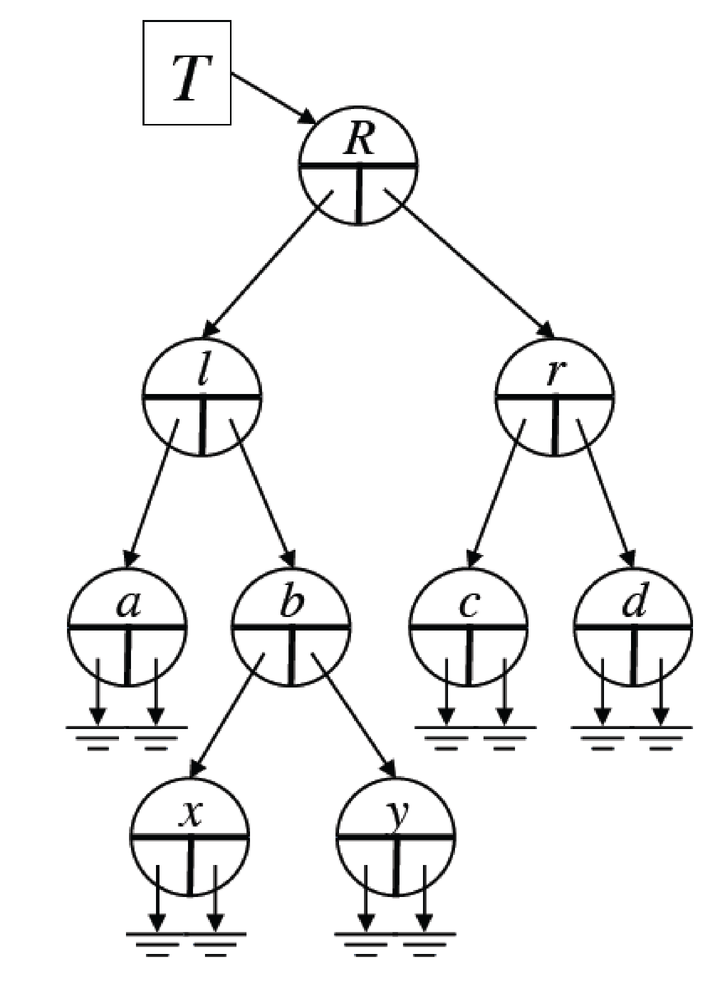
[1] [3] [5] [7] [9] [11] [13] [15]

An n node binary tree needs an array whose length is between n and $2^n - 1$.

Representing Binary Tree Using Linked Structure

```
struct node {
  Item item;
  node *left;
  node *right;
};
```

- `left/right` points to a left/right subtree.
  - If the subtree is an empty one, the pointer points to NULL.
- For a leaf node, both its `left` and `right` pointers are NULL.



LeetCode Exercise: Path Sum II (~15 mins)

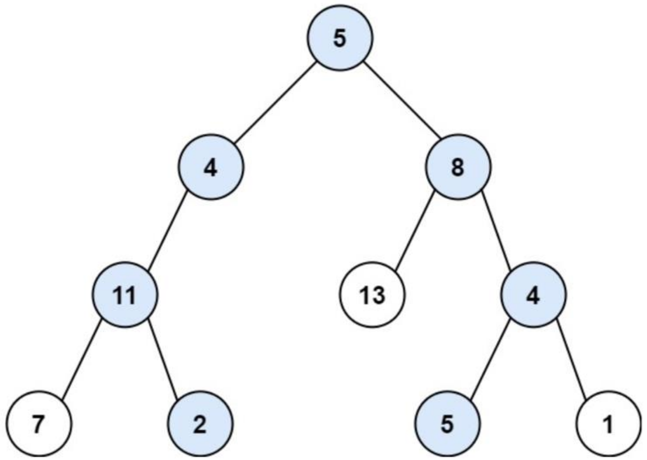
113. Path Sum II

Medium Topics Companies

Given the `root` of a binary tree and an integer `targetSum`, return *all* ***root-to-leaf*** *paths where the sum of the node values in the path equals* `targetSum`*. Each path should be returned as a list of the node* ***values****, not node references.*

A **root-to-leaf** path is a path starting from the root and ending at any leaf node. A **leaf** is a node with no children.

Example 1:




```
Input: root = [5,4,8,11,null,13,4,7,2,null,null,5,1], targetSum = 22
Output: [[5,4,11,2],[5,8,4,5]]
Explanation: There are two paths whose sum equals targetSum:
5 + 4 + 11 + 2 = 22
5 + 8 + 4 + 5 = 22
```