ECE2810J
Data Structures and Algorithms

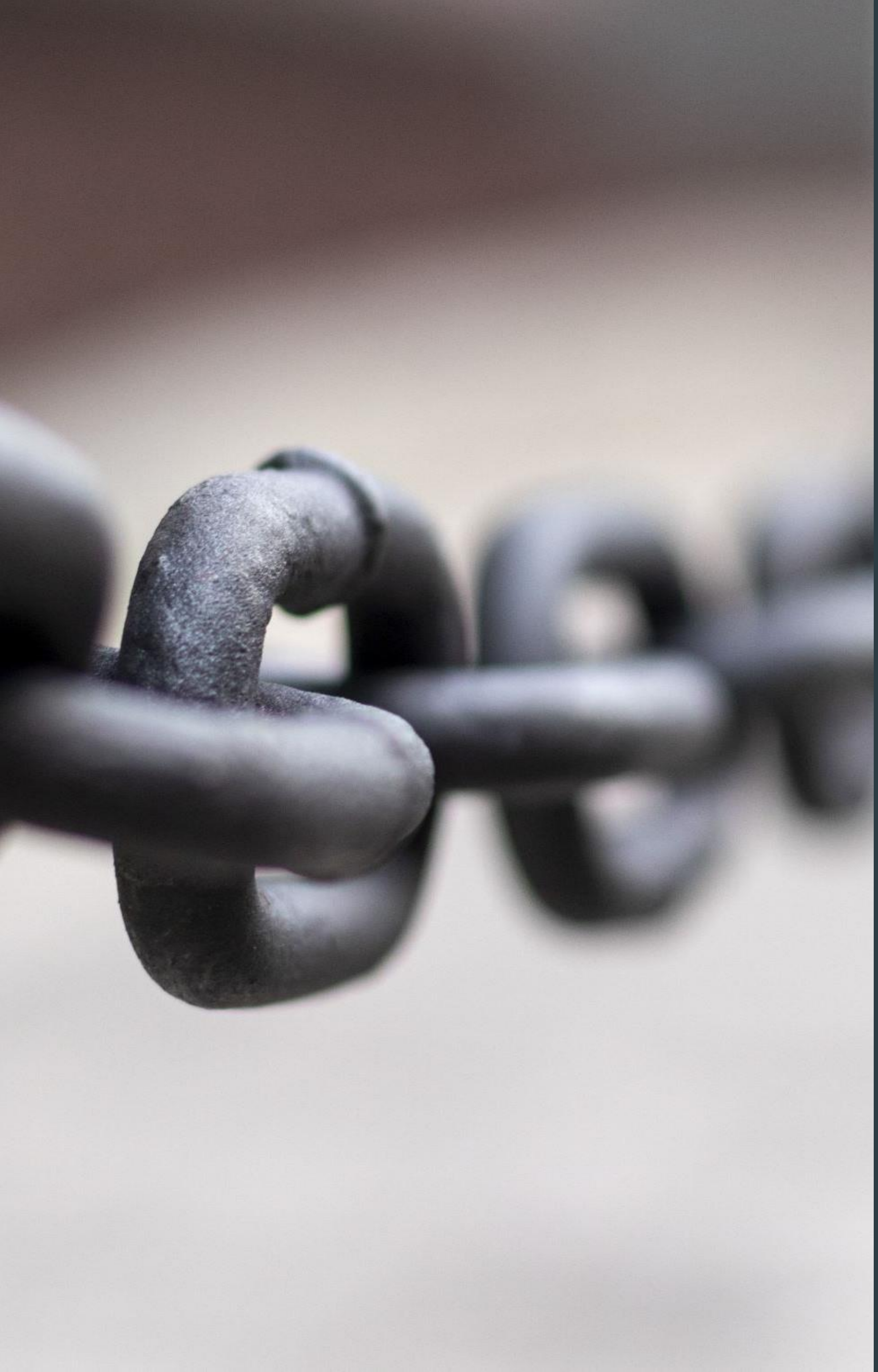
Hashing: Collision Resolution

Learning Objectives:

- Understand separate chaining
- Understand the general idea of open addressing
- Know three basic ways of open addressing and their advantages and disadvantages

Code Exercise: Hashing by Modulo (~10 mins)

- Code can be found in :
  - Canvas -> Code Exercise -> Hashing



Outline

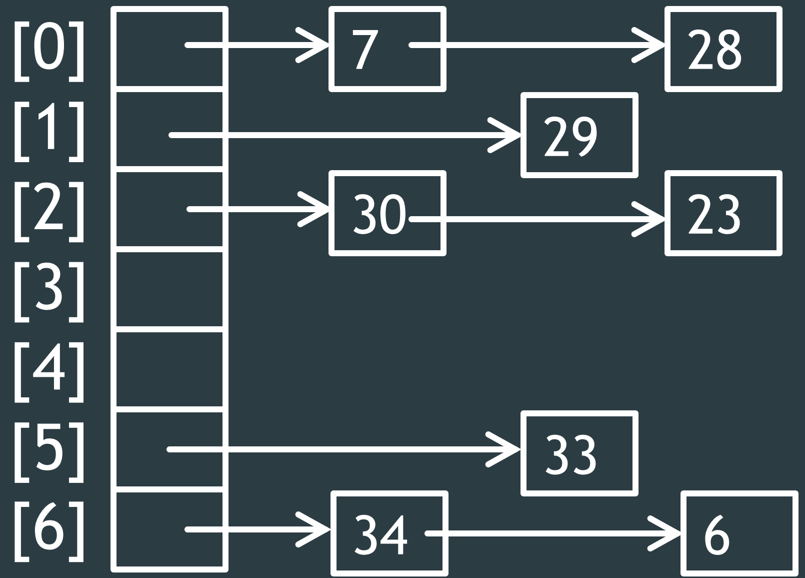
- Collision Resolution: Separate Chaining
- Collision Resolution: Open Addressing
  - Linear Probing
  - Quadratic Probing and Double Hashing
  - Performance of Open Addressing

Collision Resolution Scheme

- **Collision-resolution scheme**: assigns distinct locations in the hash table to items involved in a collision.
- Two major scheme:
  - Separate chaining
  - Open addressing

Separate Chaining

- Each bucket keeps a linked list of all items whose home buckets are that bucket.
- Example: Put pairs whose keys are 6, 23, 34, 28, 29, 7, 33, 30 into a hash table with $n = 7$ buckets.
  - `homeBucket = key % 7`
  - **Note**: we insert object at the beginning of a linked list.



Separate Chaining

- `Value find(Key key)`
  - Compute `k = h(key)`
  - Search in the linked list located at the k-th bucket (e.g., check every entry) with the key.

- `void insert(Key key, Value value)`
  - Compute `k = h(key)`
  - Search in the linked list located at the k-th bucket. If found, update its value; otherwise, insert the pair at the beginning of the linked list in O(1) time.

Separate Chaining

- `Value remove(Key key)`
  - Compute `k = h(key)`
  - Search in the linked list located at the k-th bucket. If found, remove that pair.

Separate Chaining Exercise

- Canvas -> Exercise -> Collision Resolution -> Separate Chaining

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Open Addressing

- Reuse empty space in the hash table to hold colliding items.

- To do so, search the hash table in some systematic way for a bucket that is empty.
  - Idea: we use a sequence of hash functions h_0, h_1, h_2, . . . to probe the hash table until we find an empty slot.
    - I.e., we **probe** the hash table buckets mapped by h_0(key), h_1(key), ..., in sequence, until we find an empty slot.
    - Generally, we could define $h_i(x) = h(x) + f(i)$

Open Addressing

- Three methods:
  - Linear probing:
    $h_i(x) = (h(x) + i) \% n$
  - Quadratic probing:
    $h_i(x) = (h(x) + i^2) \% n$
  - Double hashing:
    $h_i(x) = (h(x) + i*g(x)) \% n$

n is the hash table size

Linear Probing

$\mathbf{h_i(key)\ =\ (h(key)+i)\ \%\ n}$

- Apply hash function h_0, h_1, …, in sequence until we find an empty slot.
  - This is equivalent to doing a linear search from `h(key)` until we find an empty slot.

- Example: Hash table size `n = 9`, `h(key) = key%9`
  - Thus `h_i(key) = (key%9+i)%9`
  - Suppose we insert 1, 5, 11, 2, 17, 21, 31 in sequence

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	1	11			5			

How about 2?

Linear Probing Example

- Hash table size $n = 9$, $h(key) = key\%9$
  - Thus $h_i(key) = (key\%9+i)\%9$
  - Suppose we insert 1, 5, 11, 2, 17, 21, 31 in sequence.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	1	11	2	21	5	31		17

  - $h_0(2) = 2$. Not empty!
  - So we try $h_1(2) = 3$. It is empty, so we insert there!
  - $h_0(21) = 3$. Not empty!
  - $h_1(21) = 4$. It is empty, so we insert there!
  - $h_0(31) = 4$. Not empty!
  - $h_1(31) = 5$. Not empty!
  - $h_2(31) = 6$. It is empty, so we insert there!

Linear Probing

find()

	1	11	2	21	5	31		17
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

- With linear probing `hi(key) = (key%9+i)%9`
  - How will you **search** an item with key = 31?
  - How will you **search** an item with key = 10?

- Procedure: probe in the buckets given by h_0(key), h_1(key), …, in sequence **until**
  - we find the key,
  - or we find an empty slot, which means the key is not found.

Linear Probing

remove()

	1	11	2	21	5	31		17
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

- With linear probing `h`$_i$ `(key) = (key%9+i)%9`
  - How will you **remove** an item with key = 11?
  - If we just find 11 and delete it, will this work?

	1		2	21	5	31		17
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

What is the result for searching key = 2 with the above hash table?

Linear Probing

remove()

cluster

	1		2	21	5	31		17
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

- After deleting 11, we need to **rehash** the following “cluster” to fill the vacated bucket.
- However, we cannot move an item **beyond** its actual hash position. In this example, 5 cannot be moved ahead.

	1		2	21	5	31		17
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]

Linear Probing

Alternative implementation of remove()

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	1	del	2	21	5	31		17

- Lazy deletion: we mark deleted entry as "**deleted**".
  - "deleted" is not the same as "empty".
  - Now each bucket has three states: "occupied", "empty", and "deleted".
- We can overwrite the "deleted" entry when inserting.
- When we search, we will keep looking if we encounter a "deleted" entry.

Linear Probing: Clustering Problem

cluster

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
	1	11	2	21	5	31		17

- Clustering: when contiguous buckets are all occupied.
- **Claim**: Any hash value inside the cluster adds to **the end** of that cluster.
- Problems with a large cluster:
  - It becomes more likely that the next hash value will collide with the cluster.
  - Collisions in the cluster get more expensive to resolve.

Linear Probing: Clustering Problem

- **Best**: least number of probes to find an empty slot
- Assuming input size N, table size 2N:
  - What is the best-case cluster distribution?

X		X		X		X	

  - What is the worst-case cluster distribution?

		X	X	X	X		

Which Statements Are Correct?

- Assuming input size N, table size 2N. Analyze the average number of probes to <u>find an empty slot</u> for best-case and worst-case clusters. Select all the correct answers.

A. The average number for best-case cluster is 1.5.

B. The average number for best-case cluster is 1.

C. The average number for worst-case cluster is roughly $\frac{1}{4}N$.

D. The average number for worst-case cluster is roughly $\frac{1}{2}N$.

Best Case	X		X		X		X	
Worst Case			X	X	X	X		

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Quadratic Probing

$$h_i(key) = (h(key)+i^2) \% n$$

- It is less likely to form large clusters.
- Example: Hash table size $n = 7$, $h(key) = key\%7$
  - Thus $h_i(key) = (key\%7+i^2)\%7$
  - Suppose we insert 9, 16, 11, 2 in sequence.

[0]	[1]	[2]	[3]	[4]	[5]	[6]
		9	16	11		2

  - $h_0(16) = 2$. Not empty!
  - $h_1(16) = 3$. It is empty, so we insert there.
  - $h_0(2) = 2$. Not empty!
  - $h_1(2) = 3$. Not empty!
  - $h_2(2) = 6$. It is empty, so we insert there.

Conflicts in Linear Probing

- Inserting X
- X conflicts with Y if h(X) = b(Y)
- X conflicts with Z if X conflicts Y and Z conflicts with Y
- X conflicts with W if X conflicts with Y and b(W) = b(Y)+1

0	7	14	3	10	17	
[0]	[1]	[2]	[3]	[4]	[5]	[6]

Conflicts in Linear Probing

- Inserting X
- X conflicts with Y if h(X) = b(Y)
- X conflicts with Z if X conflicts Y and Z conflicts with Y
- X conflicts with W if X conflicts with Y and b(W) = b(Y)+1

21

[0]	[1]	[2]	[3]	[4]	[5]	[6]
0	7	14	3	10		

Conflicts in Linear Probing

- Inserting X
- X conflicts with Y if h(X) = b(Y)
- X conflicts with Z if X conflicts Y and Z conflicts with Y
- X conflicts with W if X conflicts with Y and b(W) = b(Y)+1

21

0	7	14	3	10		
[0]	[1]	[2]	[3]	[4]	[5]	[6]

Conflicts in Linear Probing

- Inserting X
- X conflicts with Y if h(X) = b(Y)
- X conflicts with Z if X conflicts Y and Z conflicts with Y
- X conflicts with W if X conflicts with Y and b(W) = b(Y)+1

21

0	7	14	3	10		
[0]	[1]	[2]	[3]	[4]	[5]	[6]

Conflicts in Quadratic Probing

1

0	7			14		
[0]	[1]	[2]	[3]	[4]	[5]	[6]

Conflicts in Quadratic Probing

1

0	7			4	11	
[0]	[1]	[2]	[3]	[4]	[5]	[6]

Problem of Quadratic Probing

- However, sometimes we will never find an empty slot even if the table isn't full!

- Luckily, if the load factor $L \leq 0.5$, we are guaranteed to find an empty slot.
  - Table size must be a **prime** number!
  - Definition: given a hash table with n buckets that stores m objects, its load factor is

$$L = \frac{m}{n} = \frac{\#\text{objects in hash table}}{\#\text{buckets in hash table}}$$

More on Load Factor of Hash Table

- Question: which collision resolution strategy is feasible for load factor larger than 1?
  - Answer: separate chaining.
  - Note: for open addressing, we require $L \leq 1$.
- Claim: $L = O(1)$ is a necessary condition for operations to run in constant time.

Double Hashing

$$h_i(x) = (h(x) + i*g(x)) \% n$$

- Uses 2 distinct hash functions.
- Increment differently depending on the key.
  - If h(x) = 13, g(x) = 17, the probe sequence is 13, 30, 47, 64, …
  - If h(x) = 19, g(x) = 7, the probe sequence is 19, 26, 33, 40, …
  - For linear and quadratic probing, the incremental probing patterns are the same for all the keys.

Double Hashing Example

- Hash table size $\mathbf{n = 7}$, $\mathbf{h(key) = key\%7}$, $\mathbf{g(key) = (5-key)\%5}$
  - Thus $\mathbf{h_i(key) = (key\%7+(5-key)\%5*i)\%7}$
  - Suppose we insert 9, 16, 11, 2 in sequence.

[0]	[1]	[2]	[3]	[4]	[5]	[6]
		9		11	2	16

- $\mathbf{h_0(16) = 2}$. Not empty!
- $\mathbf{h_1(16) = 6}$. It is empty, so we insert there.
- $\mathbf{h_0(2) = 2}$. Not empty!
- $\mathbf{h_1(2) = 5}$. It is empty, so we insert there.

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Performance of Open Addressing

- Hard to analyze rigorously.
- The runtime is dominated by the **number of comparisons.**
- The number of comparisons depends on the **load factor** L.
- Define the expected number of comparisons in an unsuccessful search as $U(L)$.
- Define the expected number of comparisons in a successful search as $S(L)$.

Expected Number of Comparisons

- Linear probing

$$U(L) = \frac{1}{2}\left[1 + \left(\frac{1}{1-L}\right)^2\right]$$

$$S(L) = \frac{1}{2}\left[1 + \frac{1}{1-L}\right]$$

L	$U(L)$	$S(L)$
0.5	2.5	1.5
0.75	8.5	2.5
0.9	50.5	5.5

$L \leq 0.75$ is recommended

Probabilistic analysis of linear probing [Knuth (1962)]

Expected Number of Comparisons

- Quadratic probing and double hashing

$$U(L) = \frac{1}{1-L}$$

$$S(L) = \frac{1}{L}\ln\frac{1}{1-L}$$

L	$U(L)$	$S(L)$
0.5	2	1.4
0.75	4	1.8
0.9	10	2.6

Which Strategy to Use?

- Both separate chaining and open addressing are used in real applications

- Some basic guidelines:
  - If resizing is frequent, better to use open addressing
  - If need removing items, better to use separate chaining
    - `remove()` is tricky in open addressing
  - In mission critical application, prototype both and compare

Exercise Linear Probing

- Canvas Exercise -> linear Probing

281 One More Thing