

# ECE2810J

## Data Structures and Algorithms

### Binary Tree Traversal

#### Learning Objectives:

- ▶ Know the effect and procedure of pre-order, post-order, and in-order depth-first traversal
- ▶ Know the effect and procedure of level-order traversal
- ▶ Application with traversal



# Binary Tree Traversal

- ▶ Many binary tree operations are done by performing a **traversal** of the binary tree.
- ▶ In a traversal, each node of the binary tree is visited **exactly once**.
- ▶ During the visit of a node, all actions (making a clone, displaying, evaluating the operator, etc.) with respect to this node are taken.





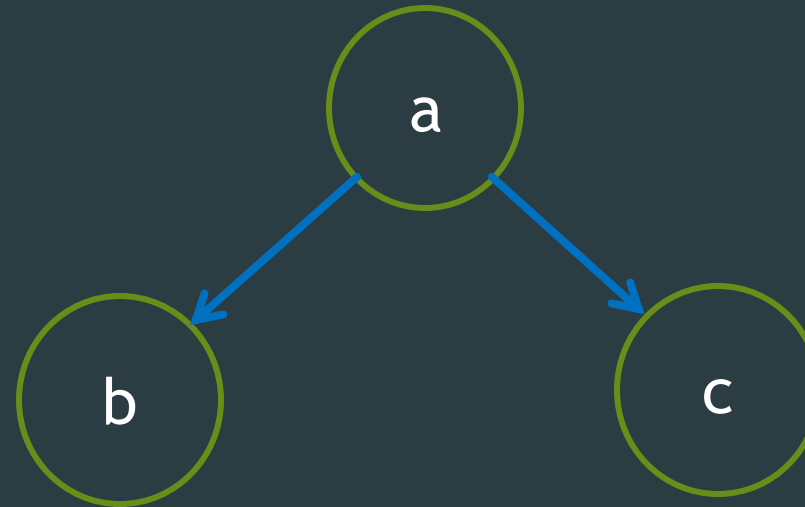
# Binary Tree Traversal Methods

- ▶ Depth-first traversal
  - ▶ Pre-order
  - ▶ Post-order
  - ▶ In-order
- ▶ Level-order traversal

# Pre-Order Depth-First Traversal Procedure

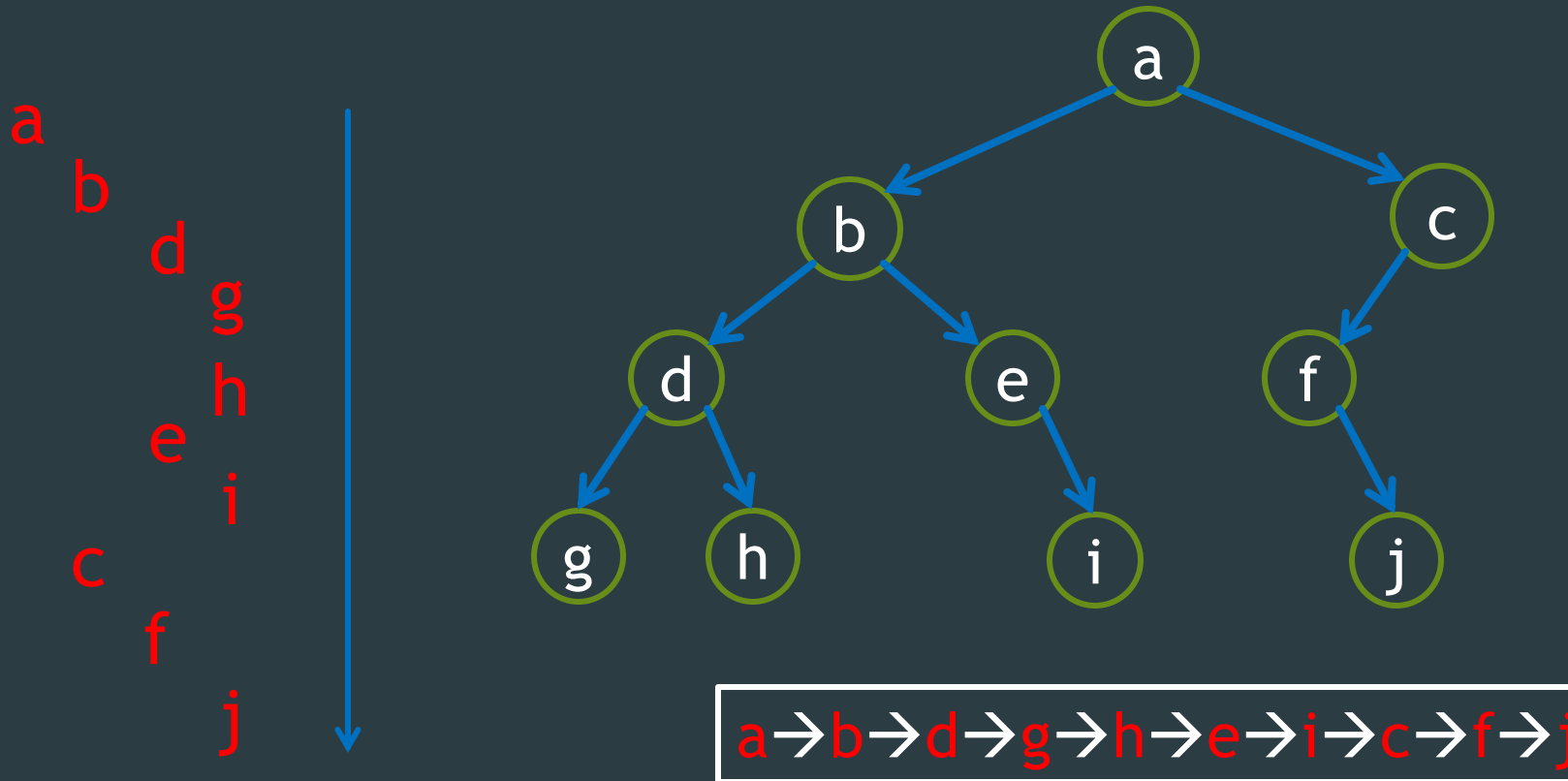
- ▶ Visit the node
- ▶ Visit its left subtree
- ▶ Visit its right subtree

```
void preOrder(node *n) {  
    if(!n) return;  
    visit(n);  
    preOrder(n->left);  
    preOrder(n->right);  
}
```



$a \rightarrow b \rightarrow c$

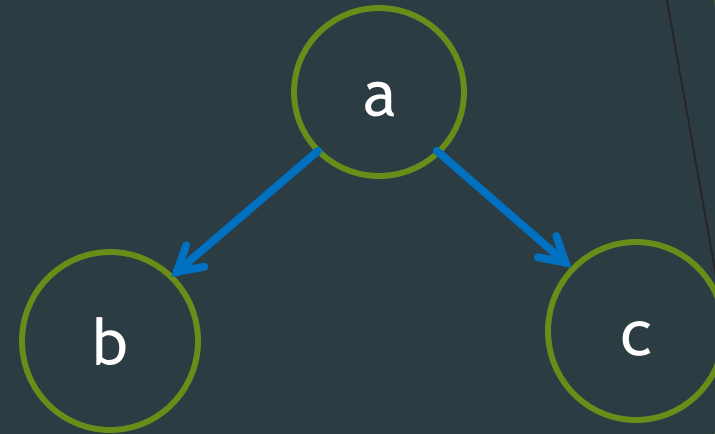
# Pre-Order Depth-First Traversal Example



# Post-Order Depth-First Traversal Procedure

- ▶ Visit the left subtree
- ▶ Visit the right subtree
- ▶ Visit the node

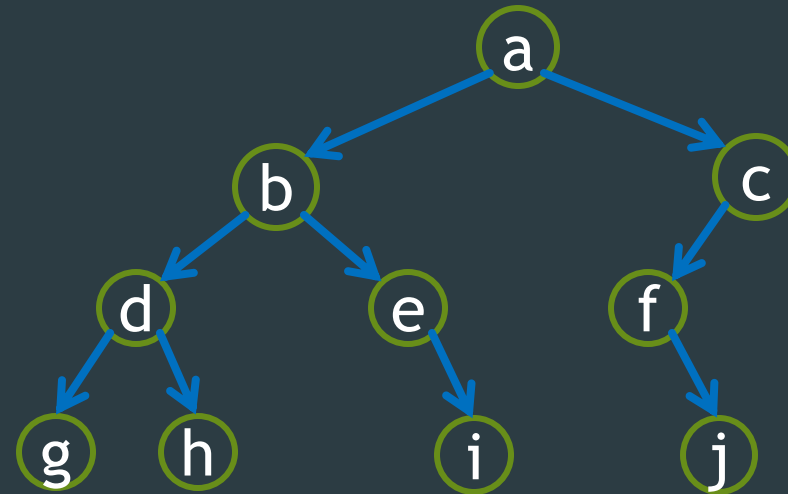
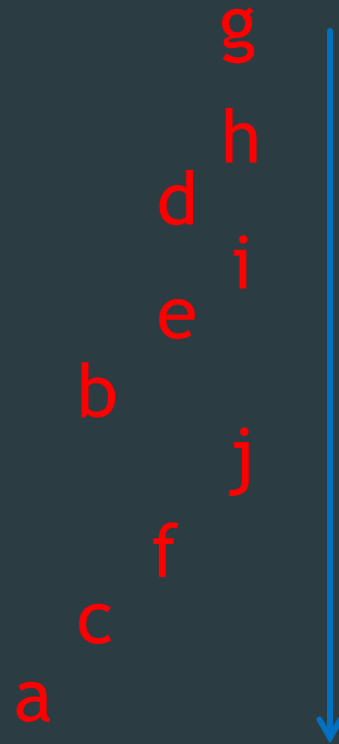
```
void postOrder(node *n) {  
    if(!n) return;  
    postOrder(n->left);  
    postOrder(n->right);  
    visit(n);  
}
```



$b \rightarrow c \rightarrow a$

# Post-Order Depth-First Traversal

## Example

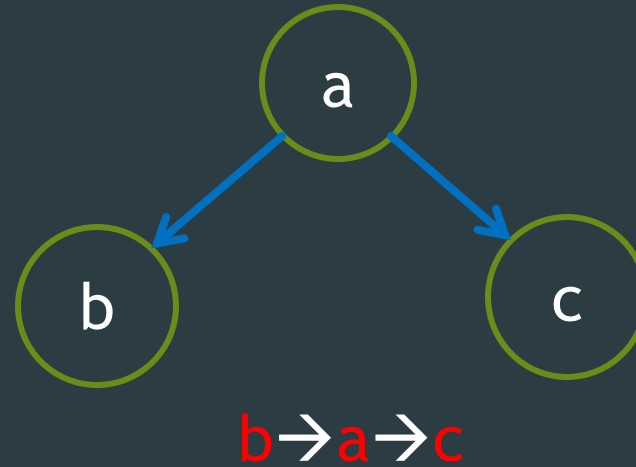


g→h→d→i→e→b→j→f→c→a

# In-Order Depth-First Traversal Procedure

- ▶ Visit the left subtree
- ▶ Visit the node
- ▶ Visit the right subtree

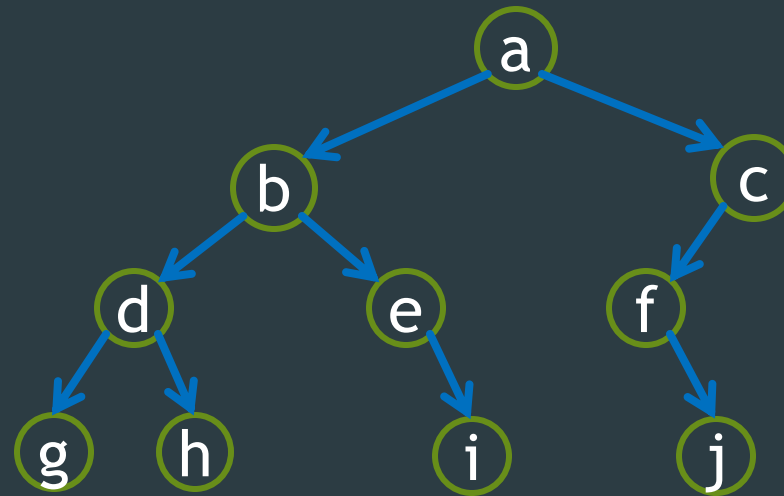
```
void inOrder(node *n) {  
    if(!n) return;  
    inOrder(n->left);  
    visit(n);  
    inOrder(n->right);  
}
```





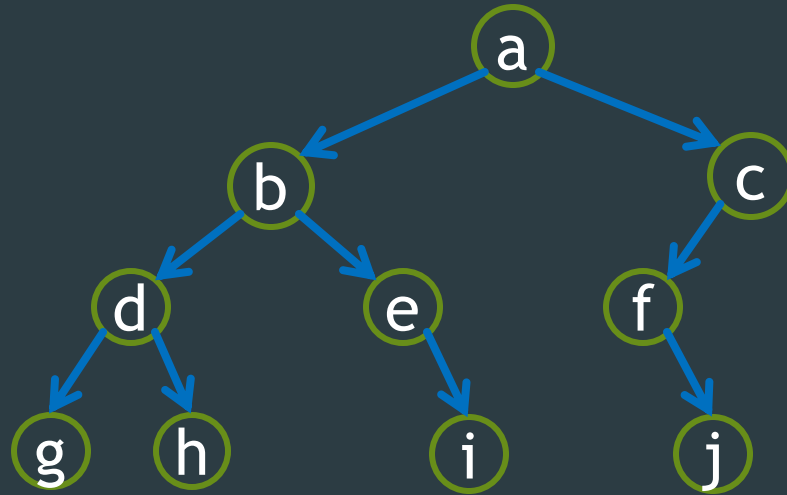
# What Is the Result of In-Order Depth-First Traversal?

- A. g, d, h, b, e, i, a, c, f, j
- B. g, d, h, b, e, i, a, f, j, c
- C. g, d, h, b, i, e, a, j, f, c
- D. g, d, h, b, i, e, a, f, j, c



# Level-Order Traversal

- ▶ We want to traverse the tree level by level **from top to bottom**.
- ▶ Within each level, traverse **from left to right**.



How can we implement this traversal?

**a→b→c→d→e→f→g→h→i→j**

# Level-Order Traversal

## Procedure

► Use a queue!

Loop

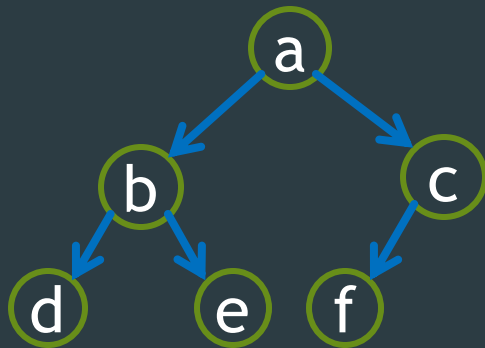


1. Enqueue the root node into an empty queue.
2. While the queue is not empty, dequeue a node from the front of the queue.
  1. Visit the node.
  2. Enqueue its left child (if exists) and right child (if exists) into the queue.

# Level-Order Traversal

## Code and Example

```
void levelOrder(node *root) {  
    queue q; // Empty queue  
    q.enqueue(root);  
    while(!q.isEmpty()) {  
        node *n = q.dequeue();  
        visit(n);  
        if(n->left) q.enqueue(n->left);  
        if(n->right) q.enqueue(n->right);  
    }  
}
```



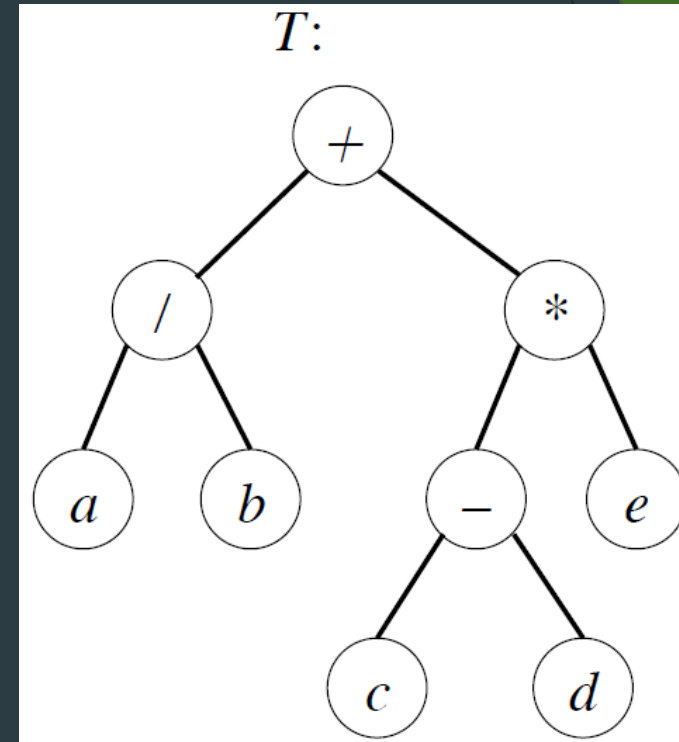
Queue: 

a	b	c	d	e	f
---	---	---	---	---	---

Output: a b c d e f

# Binary Tree Traversal Application

- ▶ The expression  $a/b + (c - d)e$  has been encoded as a tree  $T$ .
  - ▶ The leaves are **operands**.
  - ▶ The internal nodes are **operators**.
- ▶ How would you traverse the tree  $T$  to print out the expression (ignoring parentheses)?
  - ▶ In-order depth-first traversal.
- ▶ What is the expression printed out by post-order depth-first traversal?
  - ▶  $ab/cd - e * +$
  - ▶ **Reverse Polish Notation**





# In Class Exercise 1 (15 mins)

## 94. Binary Tree Inorder Traversal

Easy

Topics

Companies

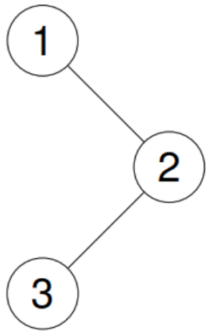
Given the `root` of a binary tree, return the *inorder traversal* of its nodes' values.

### Example 1:

Input: `root = [1,null,2,3]`

Output: `[1,3,2]`

Explanation:



### Example 2:

Input: `root = [1,2,3,4,5,null,8,null,null,6,7,9]`

Output: `[4,2,6,5,7,1,3,9,8]`

Explanation:

# In Class Exercise 2 (15 mins)

## 107. Binary Tree Level Order Traversal II

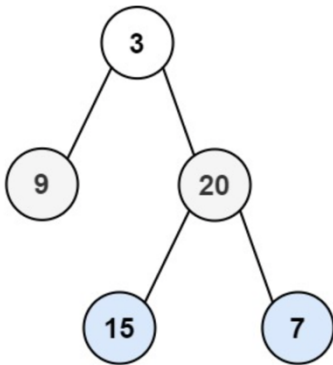
Medium

Topics

Companies

Given the `root` of a binary tree, return the *bottom-up level order traversal* of its nodes' values. (i.e., from left to right, level by level from leaf to root).

Example 1:



Input: `root = [3,9,20,null,null,15,7]`

Output: `[[15,7],[9,20],[3]]`

Example 2:

Input: `root = [1]`

Output: `[[1]]`

Example 3:

Input: `root = []`

Output: `[]`

Constraints:

- The number of nodes in the tree is in the range `[0, 2000]`.
- `-1000 <= Node.val <= 1000`

# In Class Exercise 3 (15 mins)

## 1993. Operations on Tree

Medium

Topics

Companies

Hint

You are given a tree with  $n$  nodes numbered from  $0$  to  $n - 1$  in the form of a parent array `parent` where `parent[i]` is the parent of the  $i^{\text{th}}$  node. The root of the tree is node  $0$ , so `parent[0] = -1` since it has no parent. You want to design a data structure that allows users to lock, unlock, and upgrade nodes in the tree.

The data structure should support the following functions:

- **Lock:** **Locks** the given node for the given user and prevents other users from locking the same node. You may only lock a node using this function if the node is unlocked.
- **Unlock:** **Unlocks** the given node for the given user. You may only unlock a node using this function if it is currently locked by the same user.
- **Upgrade:** **Locks** the given node for the given user and **unlocks** all of its descendants **regardless** of who locked it. You may only upgrade a node if **all 3** conditions are true:
  - The node is unlocked,
  - It has at least one locked descendant (by **any** user), and
  - It does not have any locked ancestors.

Implement the `LockingTree` class:

`rent()` initializes the data structure with the parent array.

`er)` returns `true` if it is possible for the user with id `user` to lock the node `num`, or `false` otherwise. If it is will become **locked** by the user with id `user`.

`user)` returns `true` if it is possible for the user with id `user` to unlock the node `num`, or `false` otherwise. If it is will become **unlocked**.

`user)` returns `true` if it is possible for the user with id `user` to upgrade the node `num`, or `false` otherwise. If `num` will be **upgraded**.

