# ECE2810J

## Data Structures and Algorithms

## Non-Comparison Sort

**Learning Objective:**

- Understand three non-comparison sorts, counting sort, bucket sort, and radix sort

# Outline

- Non-comparison Sort
  - Counting Sort
  - Bucket Sort
  - Radix Sort

# Counting Sort: A Simple Version

- Sort an array **A** of integers in the range $[0, k]$, where $k$ is known.

1. Allocate an array **`count[k+1]`**.
2. Scan array **A**. For i=**1** to **N**, increment **`count[A[i]]`**.
3. Scan array **`count`**. For i=**0** to **k**, print **`i`** for **`count[i]`** times.

- Time complexity: $O(N + k)$.

- The algorithm can be converted to sort integers in some other known range $[a, b]$.
  - Minus each number by $a$, converting the range to $[0, b - a]$.

# Example

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |

## Pseudo code:

Sort an array **A** of integers in the range $[0, k]$, where $k$ is known.

1. Allocate an array **count[k+1]**.
2. Scan array **A**. For i=**1** to **N**, increment **count[A[i]]**.
3. Scan array **count**. For i=0 to **k**, print **i** for **count[i]** times.

# Counting Sort: A General Version

- In the previous version, we print **`i`** for **`count[i]`** times
  - Simple but only works when sorting integer keys alone
  - How to sort items when there is "***additional***" information with each key? Furthermore, how to guarantee the stability?
- A general version:

1. Allocate an array **`C[k+1]`**
2. Scan array **`A`**. For i=**`1`** to **`N`**, increment **`C[A[i]]`**
3. For **`i=1`** to **`k`**, **`C[i]=C[i-1]+C[i]`**
   - **`C[i]`** now contains number of items less than or equal to **`i`**
4. For **`i=N`** downto **`1`**, put **`A[i]`** in new position **`C[A[i]]`** and decrement **`C[A[i]]`**

# Counting Sort: Example

1. Allocate an array `C[k+1]`.
2. Scan array **A**. For `i=1` to `N`, increment `C[A[i]]`.
3. For `i=1` to `k`, `C[i]=C[i-1]+C[i]`
4. For `i=N` downto `1`, put `A[i]` in new position `C[A[i]]` and decrement `C[A[i]]`.

k=5

A

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |

C

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | 0 | 2 | 3 | 0 | 1 |

C

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | 2 | 4 | 7 | 7 | 8 |

# Counting Sort Example

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| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 2 | 2 | 4 | 7 | 7 | 8 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | | | | | | 3 | |

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 2 | 2 | 4 | 6 | 7 | 8 |

Why putting 3 at location 7 is correct?

# Counting Sort Example

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C

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | 2 | 4 | 6 | 7 | 8 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
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| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | 0 | | | | 3 | 3 | |

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|---|---|---|---|---|---|---|---|
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| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 1 | 2 | 3 | 5 | 7 | 8 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | | 2 | | 3 | 3 | |

| | 0 | 1 | 2 | 3 | 4 | 5 |
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|---|---|---|---|---|---|---|---|---|
| A | 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 0 | 2 | 3 | 5 | 7 | 8 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | | 2 | 3 | 3 | 3 | |

| | 0 | 1 | 2 | 3 | 4 | 5 |
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# Counting Sort Example

1. Allocate an array `C[k+1]`.

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3. For `i=1` to `k`, `C[i]=C[i-1]+C[i]`

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| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
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| A | 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |

| | 0 | 1 | 2 | 3 | 4 | 5 |
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| C | 0 | 2 | 3 | 4 | 7 | 8 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | | 2 | 3 | 3 | 3 | 5 |

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 0 | 2 | 3 | 4 | 7 | 7 |

# Counting Sort Example

1. Allocate an array `C[k+1]`.

2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.

3. For `i=1` to `k`, `C[i]=C[i-1]+C[i]`

4. For `i=N` downto `1`, put `A[i]` in new position `C[A[i]]` and decrement `C[A[i]]`.

k=5

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| A | 2 | 5 | 3 | 0 | 2 | 3 | 0 | 3 |

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 0 | 2 | 3 | 4 | 7 | 7 |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 2 | 2 | 3 | 3 | 3 | 5 |

Done!

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| C | 0 | 2 | 2 | 4 | 7 | 7 |

Is counting sort stable? Yes!

# Code Exercise: Counting Sort (~10 mins)

- Goal: Implement your own Counting sort

- Code can be found in:
  - Canvas -> Non Comparison Sort -> Code Exercise -> Counting Sort

# Outline

- Non-comparison Sort
  - Counting Sort
  - Bucket Sort
  - Radix Sort

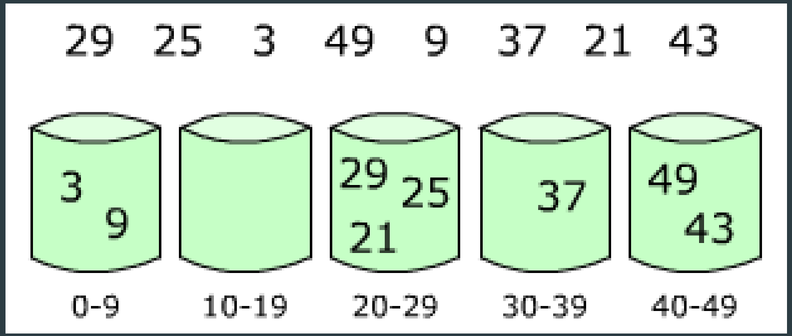
# Bucket Sort

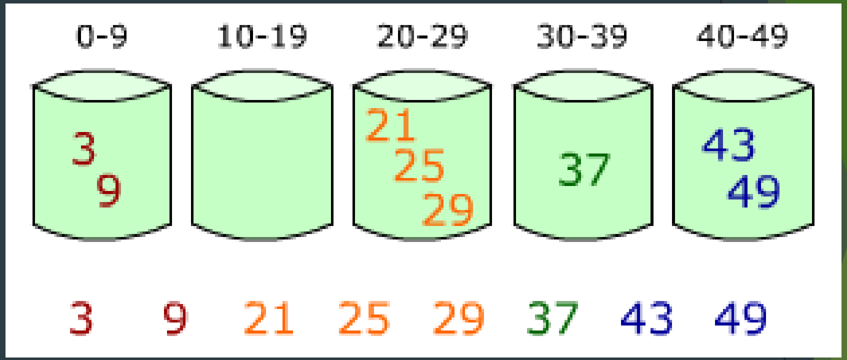
- Instead of simple integer, each key can be a complicated record, such as a real value.
- Then instead of incrementing the count of each bucket, distribute the records by their keys into appropriate buckets.
- Algorithm:

1. Set up an array of initially empty “buckets”.
2. Scatter: Go over the original array, putting each object in its bucket.
3. Sort each non-empty bucket <u>by a comparison sort</u>.
4. Gather: Visit the buckets in order and put all elements back into the original array.

# Bucket Sort

- Example




- Time complexity
  - Suppose we are sorting $cN$ items and we divide the entire range into $N$ buckets.
  - Assume that the items are **uniformly distributed** in the entire range.
  - The average case time complexity is $O(N)$.

# Code Exercise: Bucket Sort (~10 mins)

- Goal: Implement your own Bucket sort
- Code can be found in:
  - Canvas -> Non Comparison Sort -> Code Exercise -> Bucket Sort

# Outline

- Non-comparison Sort
  - Counting Sort
  - Bucket Sort
  - Radix Sort

# Radix Sort

- Radix sort sorts integers by looking at one digit at a time.
- Procedure: Given an array of integers, from the least significant bit (LSB) to the most significant bit (MSB), repeatedly do stable bucket sort according to the current bit.

- For sorting base-$b$ numbers, bucket sort needs $b$ buckets.
  - For example, for sorting decimal numbers, bucket sort needs 10 buckets.

# Radix Sort
## Example

- Sort 815, 906, 127, 913, 098, 632, 278.
- Bucket sort 815, 906, 127, 913, 098, 632, 278 according to the least significant bit:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| | | 632 | 913 | | 815 | 906 | 127 | 098<br>278 | |

- Bucket sort 632, 913, 815, 906, 127, 098, 278 according to the second bit.

# Radix Sort
## Example

- Bucket sort 632, 913, 815, 906, 127, 098, 278 according to the second bit.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 906 | 913<br>815 | 127 | 632 | | | | 278 | | 098 |

- Bucket sort 906, 913, 815, 127, 632, 278, 098 according to the most significant bit.

# Radix Sort
## Example

- Bucket sort 906, 913, 815, 127, 632, 278, 098 according to the most significant bit.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 098 | 127 | 278 | | | | 632 | | 815 | 906<br>913 |

- The final sorted order is: 098, 127, 278, 632, 815, 906, 913.

# Radix Sort: Correctness

- Claim: after bucket sorting the $i$-th LSB, the numbers are sorted according to their last $i$ digits

- Proof by mathematical induction
- Base case is obviously true
- Inductive step
  - Assume that according to the last $i$ digits, order is $a_1 < \cdots < a_n$
  - For two adjacent numbers $a_k$ and $a_{k+1}$ if they are not in the same bucket, they are sorted according to their last $i^{\text{th}}$ digits
  - If they are in the same bucket, then $a_k$ < $a_{k+1}$ for the last $(i-1)$ bits. They are also sorted due to stability of bucket sort

# Radix Sort
## Time Complexity

- Let $k$ be the maximum number of digits in the keys and $N$ be the number of keys.
- We need to repeat bucket sort $k$ times.
  - Time complexity for the bucket sort is $O(N)$.

- The total time complexity is $O(kN)$.

# Radix Sort

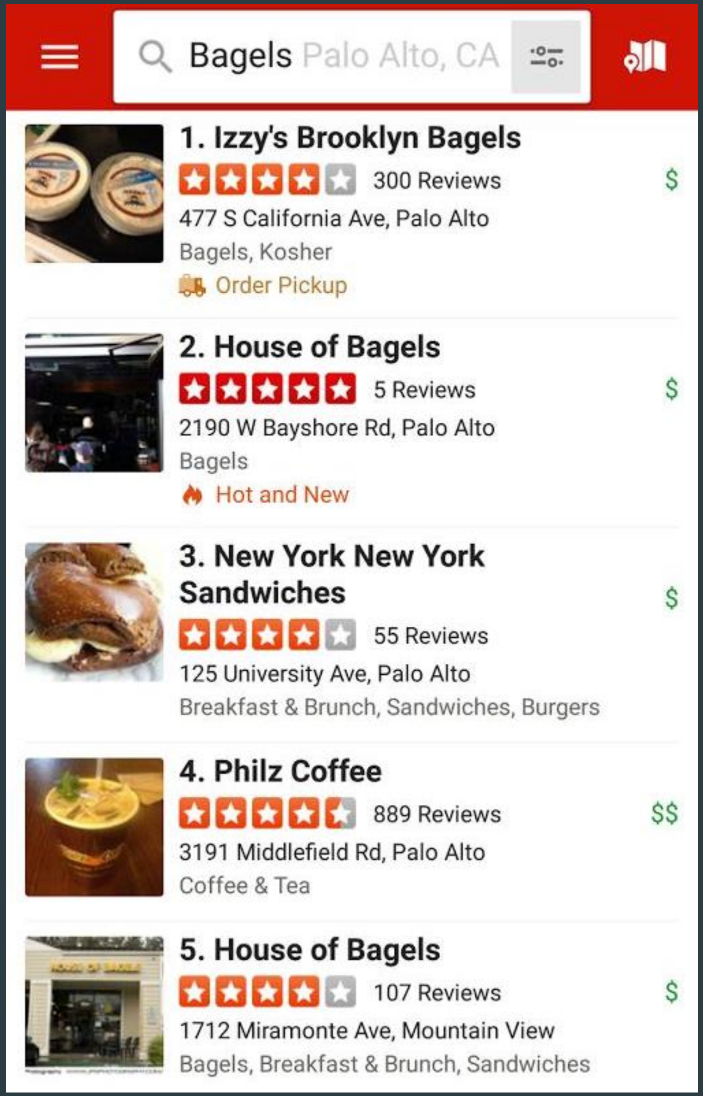
- Radix sort can be applied to sort keys that are built on positional notation.
  - Positional notation: all positions uses the same set of symbols, but different positions have different weight.
  - Decimal representation and binary representation are examples of positional notation.
  - Strings can also be viewed as a type of positional notation. Thus, radix sort can be used to sort strings.
- We can also apply radix sort to sort records that contain multiple keys.
  - For example, sort records (year, month, day).

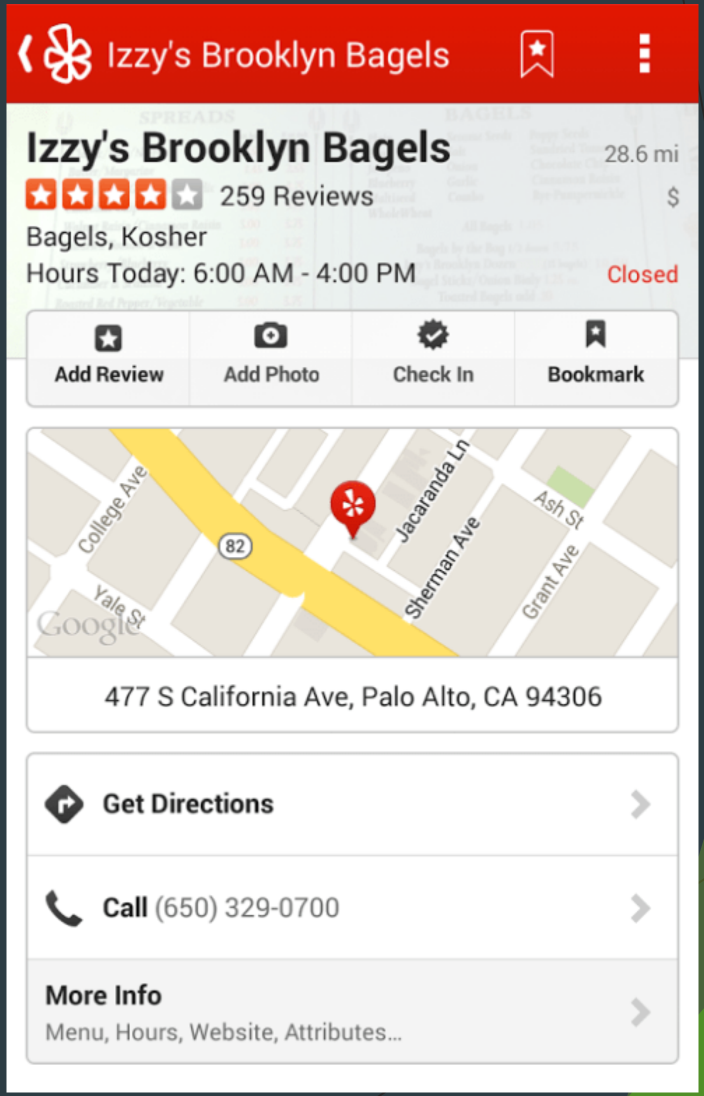
# Code Exercise: Radix Sort (~10 mins)

- Goal: Implement your own Radix sort

- Code can be found in:
  - Canvas -> Non Comparison Sort -> Code Exercise -> Radix Sort

# Code Exercise: Sorting in A Yelp-like Android APP



# Code Exercise: LeetCode Problem 164

Problem List

Description | Editorial | Solutions | Submissions

## 164. Maximum Gap

Medium | Topics | Companies

Given an integer array `nums`, return *the maximum difference between two successive elements in its sorted form*. If the array contains less than two elements, return `0`.

You must write an algorithm that runs in linear time and uses linear extra space.

**Example 1:**

```
Input: nums = [3,6,9,1]
Output: 3
Explanation: The sorted form of the array is [1,3,6,9], either (3,6) or (6,9) has the maximum difference 3.
```

**Example 2:**

```
Input: nums = [10]
Output: 0
Explanation: The array contains less than 2 elements, therefore return 0.
```

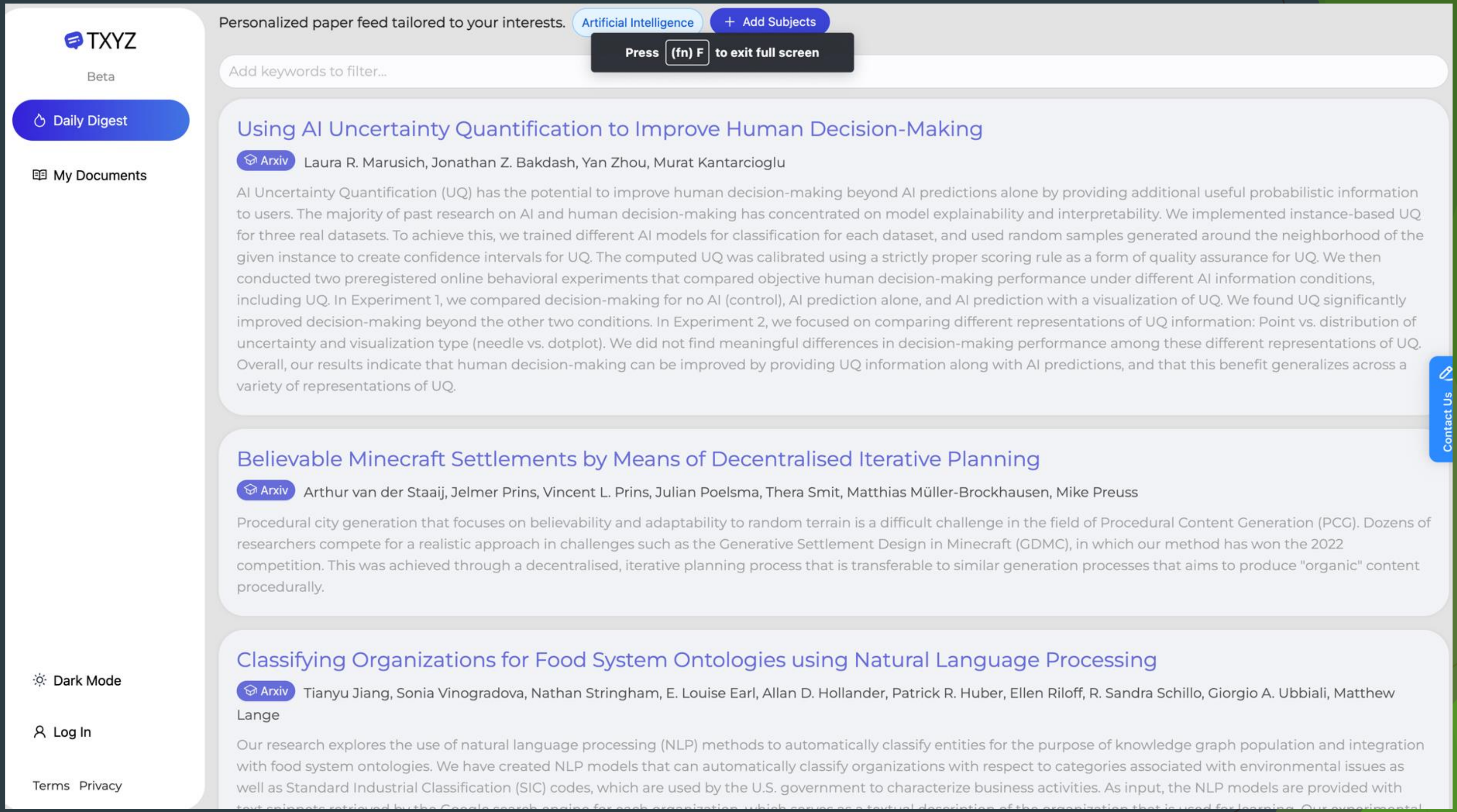
**Constraints:**

- $1 <= \texttt{nums.length} <= 10^5$
- $0 <= \texttt{nums[i]} <= 10^9$

# That is All for today!

# Any questions?

# Today’s 281 One More Thing




Txyz.ai