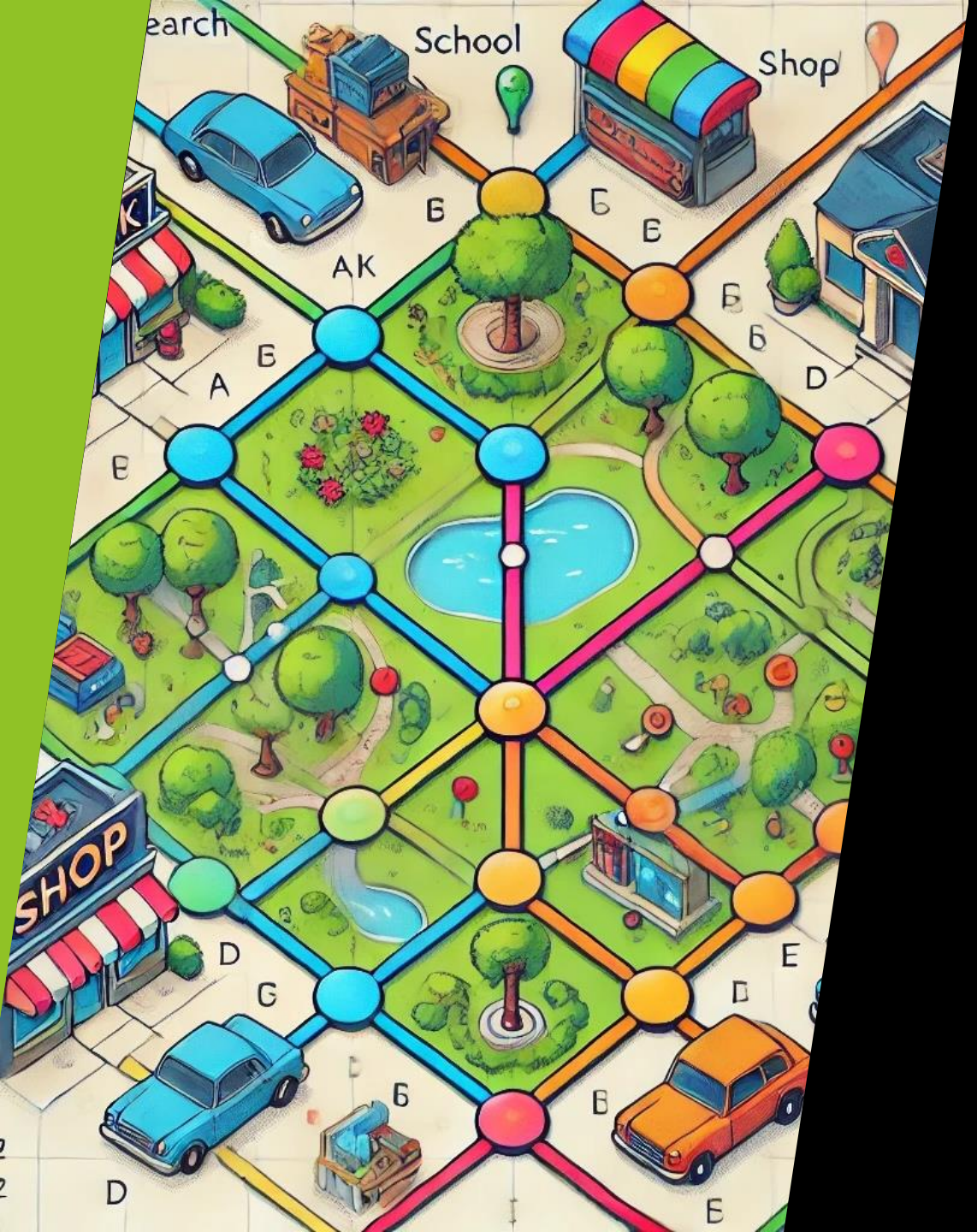


ECE2810J
Data Structures and Algorithms

Graph Search

Learning Objectives:

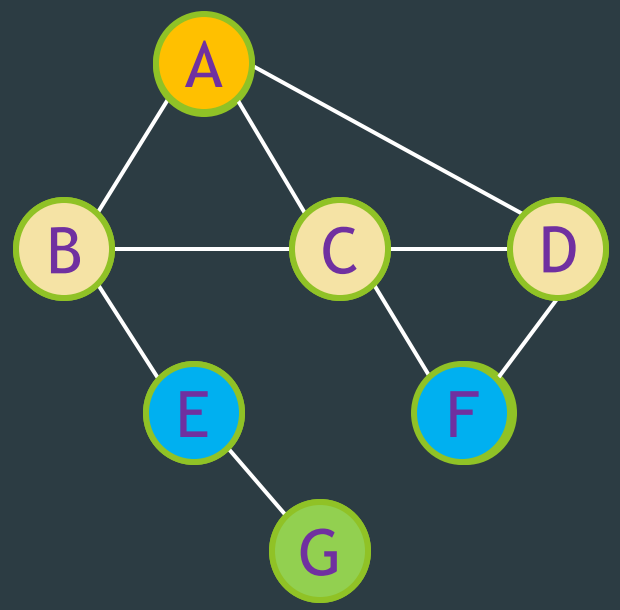
- Know two widely-used graph search algorithms, breadth-first search and depth-first search
- Know their runtime complexity

Graph Search

- A node u is reachable from a node v if and only if there is a path from v to u.
- A graph search method starts at a given node v and visits every node that is reachable from v exactly once.
- Many graph problems are solved using a search method.
  - Find a path from one node to another.
  - Find if the graph is connected.
- Commonly used search methods:
  - Breadth-first search.
  - Depth-first search.

Breadth-First Search (BFS)

- Breath = broad / wide
- Given a start node, visit all directly connected neighbors first, then nodes 2 hops away, 3 hops away, and so on.



start node

direct neighbor

nodes 2 hops away

nodes 3 hops away

A→B→C→D→E→F→G

Breadth-First Search (BFS)
Implementation

- BFS can be implemented using a queue.

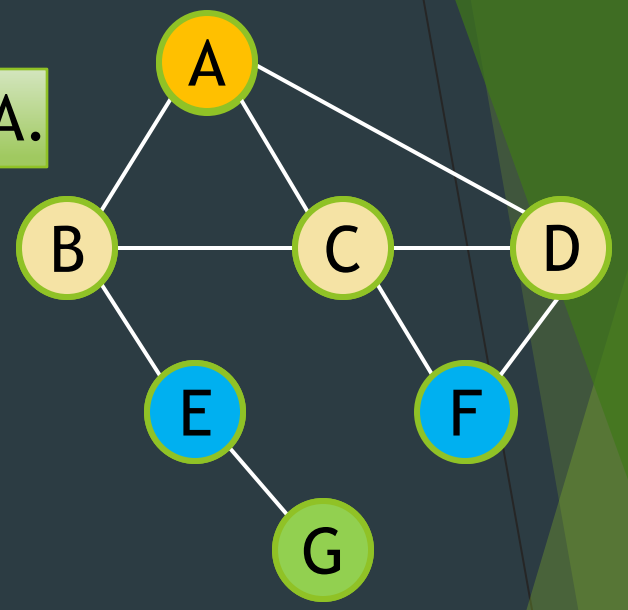
```
BFS(s) {
  queue q; // An empty queue
  visit s and mark s as visited;
  q.enqueue(s);

  while(!q.isEmpty()) {
    v = q.dequeue();
    for(each node u adjacent to v) {
      if(u is not visited) {
        visit u and mark u as visited;
        q.enqueue(u);

      }
    }
  }
}
```

Breadth-First Search (BFS)

Example

Start node is node A.

```
BFS(s) {
  queue q; // An empty queue
  visit s and mark s as visited;
  q.enqueue(s);
  while(!q.isEmpty()) {
    v = q.dequeue();
    for(each node u adjacent to v) {
      if(u is not visited) {
        visit u and mark u as visited;
        q.enqueue(u);
      }
    }
  }
}
```



Queue: A B C D E F G

Visit Order: A B C D E F G

Breadth-First Search (BFS)

Time Complexity

- If graph is implemented as adjacency matrix:
  - Visit each node exactly once: $O(V)$.
  - The row of each node in the adjacency matrix is scanned once: $O(|V|)$ for each node.
  - Total running time: $O(|V|^2)$.
- If graph is implemented as adjacency list:
  - Visit each node exactly once: $O(|V|)$.
  - Adjacency list of each node is scanned once.
  - Size of entire adjacency list is $2|E|$ for undirected graph and $|E|$ for directed graph.
  - Total running time: $O(|V| + |E|)$.

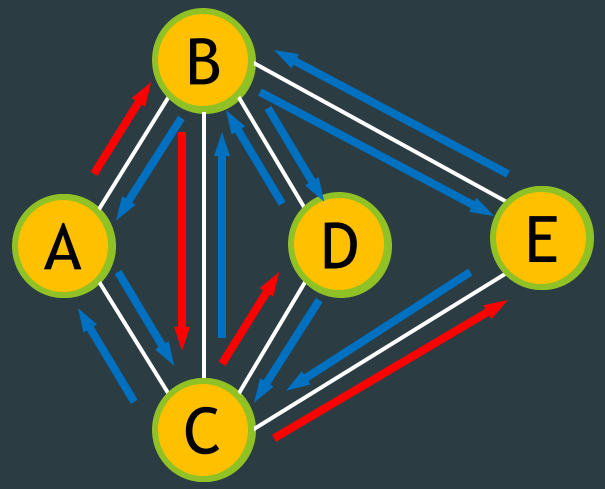
Depth-First Search (DFS)

```
DFS(v) {
  visit v;
  mark v as visited;
  for(each node u adjacent to v)
    if(u is not visited) DFS(u);
}
```

- How to mark a node "visited"?
  - Keep a "visited" field in the node

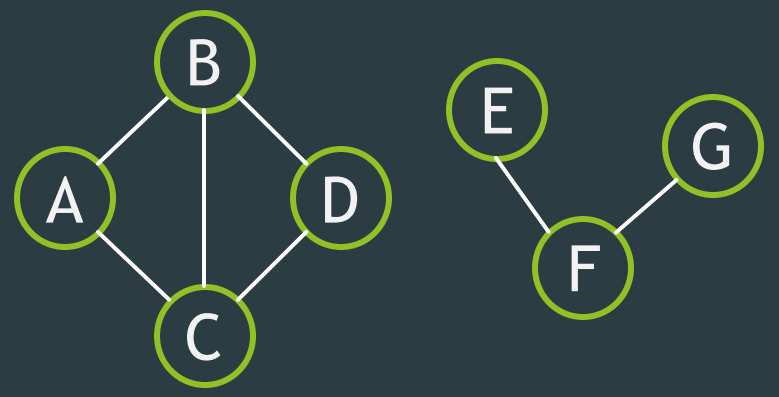
Time complexity?

Same as BFS



Traverse All the Nodes in a Graph

- The graph may not be connected. How can we traverse all the nodes in the graph?




```
for(each node v in the graph)
  if(v is not visited)
    DFS(v);
```

Graph Search Applications

Route Planning

Graph Search Applications

Maze Problem

Question:
- do we use DFS or BFS?

Exercise 1

LeetCode: Problem 1926. Nearest Exit from Entrance in Maze

Description | Editorial | Solutions | Submissions

1926. Nearest Exit from Entrance in Maze

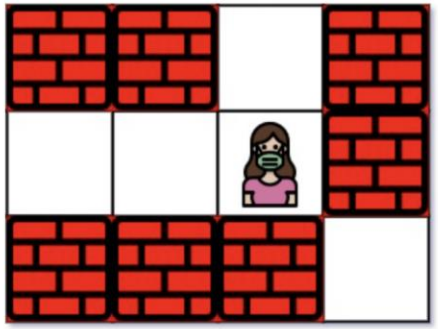
Medium | Topics | Companies | Hint

You are given an `m x n` matrix `maze` (**0-indexed**) with empty cells (represented as `'.'`) and walls (represented as `'+'`). You are also given the `entrance` of the maze, where `entrance = [entrance_row, entrance_col]` denotes the row and column of the cell you are initially standing at.

In one step, you can move one cell **up**, **down**, **left**, or **right**. You cannot step into a cell with a wall, and you cannot step outside the maze. Your goal is to find the **nearest exit** from the `entrance`. An **exit** is defined as an **empty cell** that is at the **border** of the `maze`. The `entrance` **does not count** as an exit.

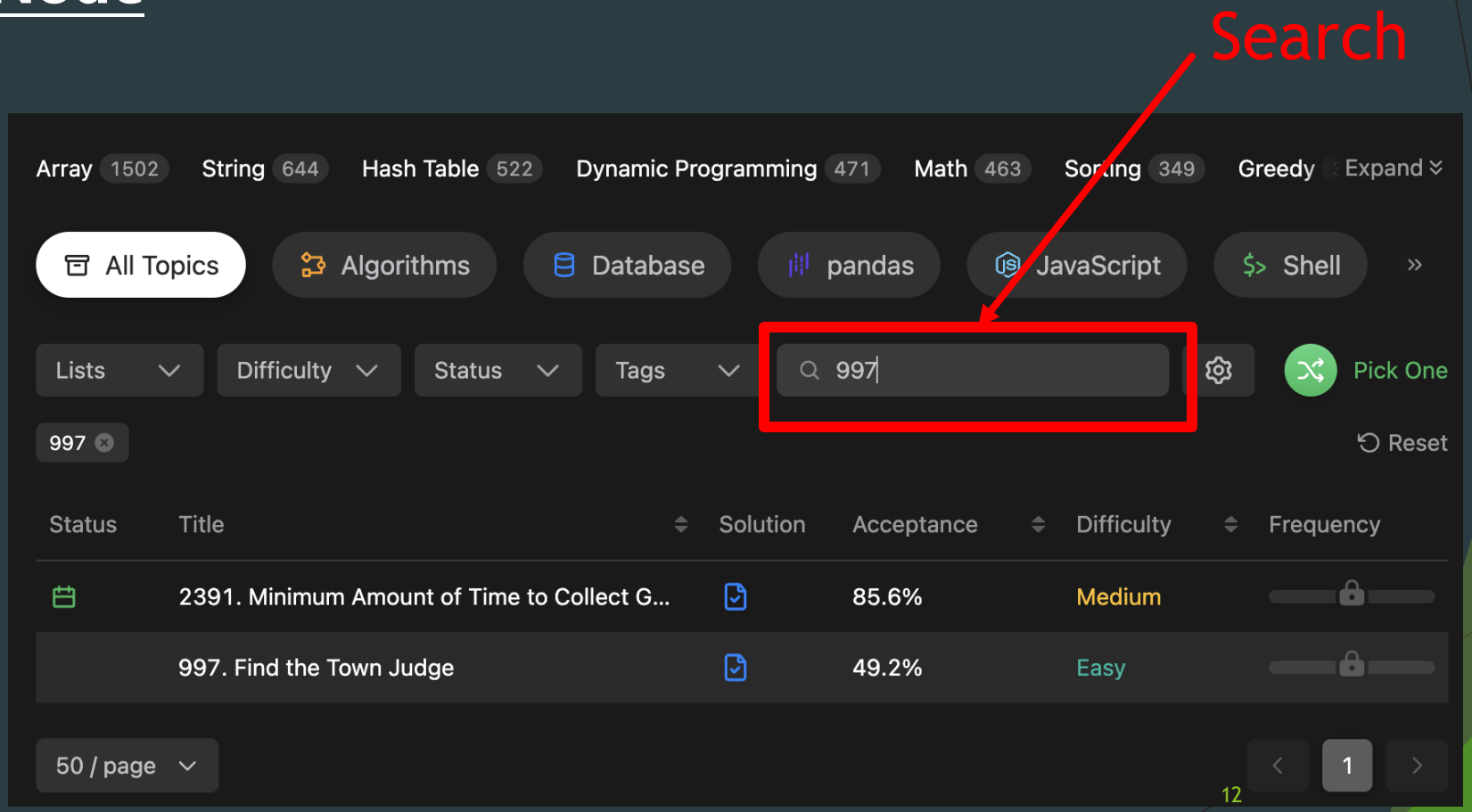
Return *the **number of steps** in the shortest path from the* `entrance` *to the nearest exit, or* `-1` *if no such path exists.*

Example 1:



Exercise 2

LeetCode: Problem 116. Populating Next Right Pointers in Each Node



Exercise 3

LeetCode: Problem 200. Number of Islands