Quantifying Uncertainty

Belief States

Defintion

Belief State

A representation of all possible world states with likelihoods for all of them

- Expensive to compute
- Only likely to be used for small systems

Decision Making

- Sometimes there is no "correct" answer or path
- Sometimes there is a **qualification problem** where there are factors hard to explain
- Decisions must still be made by the system based on rationality

Summarizing Uncertainty

Def

Probability Theory

A way to represent likelihoods of current states from a limited amount of information about the environment

- Explaining an effect from a list of causes can also be impossible
    - Must be represented with rules of uncertainty
    - Failure to correctly identify an effect could stem from
        - Laziness
        - Theoretical Ignorance
        - Practical Ignorance
- **Degrees of belief** must be used to represent likely effects from causes using probability theory

Rational Decisions

Def

Utility Theory

A way to represent preferences based on their **relative** usefulness to each other.

Decision Theory
Probability Theory + Utility Theory

- Rational Decisions are ones use the **Maximum expected utility** of a state.

Probability Lingo

Def

Marginal Probability
Probability of a particular trait across all possible other trait combinations

Marginalization
Summing individual probabilities to get a marginal probability

Conditioning
Representing $P(X \cap Y)$ as $P(X|Y)P(Y)$

Independence
When $P(X|Y) = P(X)$

Bayes' Rule
$P(Y|X \cap Z) = \frac{P(X|Y \cap Z)P(Y|Z)}{P(X|Z)}$

Conditional Independence
In A and B where
$P(A \cap B|C) = P(A|C)P(B|C)$
but not necessarily
$P(A \cap B) = P(A)P(B)$
Said to be A and B are independent, only given C

Naive Bayes' Rule

Assuming conditional independence across all other variables given one of them.

$$P(X_1, \ldots, X_n) = P(X_1) \prod_{i=2}^{n} P(X_i|X_1)$$

Posterior, Likelihood, Prior, and Marginal Probabilities

$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

Posterior: $P(A|B)$
Likelihood: $P(B|A)$
Prior: $P(A)$
Marginal: $P(B)$