Stock Price Forecasting using Machine Learning and Deep Learning Techniques

Bota Paul[1], Osagie Eboigbe[2], and Raul Alexander Velita Zorilla[3]

[1]*Worldquant University*
[2]*Worldquant University*
[3]*Worldquant University*

[1]*botapaul@gmail.com*, [2]*johanneseboigbe55@yahoo.com*, [3]*ra.velitazorrilla@gmail.com*

Abstract

Stock trading is a highly demanding task consisting of many different and connected parts. While algorithmic exchanging has acquired advantages like decreased expense and diminished dormancy, it likewise carries with it huge difficulties for individual and retail speculators who don't have the essential innovation to fabricate such frameworks. In this research project, we will build up a model that can be effectively utilized by individual and retail financial specialists and that will utilize different AI and profound learning methods to anticipate the development of stock costs and to decide whether to exchange or not. We will use the following models to implement our solution: Moving Average, Autoregressive integrated moving average (ARIMA), Artificial Neural Networks (ANN), and Long Short Term Memory (LSTM). To measure the performance of our model, we will use the Root Mean Square Error (RMSE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE), and Correlation.

Keywords: *Machine Learning, Deep Learning, Moving Average, Autoregressive Integrated Moving Average (ARIMA), Artificial Neural Networks (ANN), Long Short-Term Memory (LSTM), Root Mean Square Error (RMSE), Correlation.*

Introduction

Stock trading is a highly demanding task consisting of many different and connected parts, and that has up to recently been a domain reserved for professional stockbrokers [1]. Additionally, the application of machine learning and algorithmic trading systems has further continued to change the way stock markets operate with algorithms now generating most of the trading volumes in equity futures.

Machine learning and deep learning have the potential to ease the whole processes of analysing large chunks of data, spotting significant patterns and generating a single output that navigates traders towards a particular decision based on predicted asset prices. In this project, we will develop a model for forecasting stock prices using machine learning and deep learning techniques that can be used by individual investors or traders.

Problem statement

While algorithmic exchanging has brought in advantages like reduced cost and reduced latency, it likewise carries with it huge difficulties for individual and retail investors who don't have the essential innovation to fabricate such frameworks [2].

Breaking down value practices and securities exchange developments is incredibly testing and the ground-breaking calculations which have been utilized for exchanging in the business sectors to create high benefits are kept secret and restrictive and the examination or system behind them is for the most part never distributed [2].

While we may not be able to totally fill this void, it is our longing to build up a model that can be effectively utilized by individual and retail investors and that will utilize different AI and profound learning methods to anticipate the development of stock costs and to decide whether to exchange or not.

Goals

The point of this exploratory examination is to discover what sorts of highlights are associated with foreseeing stock costs and how various models perform in this setting. Anticipating stock costs precisely is troublesome: there are numerous components that impact stock costs and a ton of clamour. This exploratory examination doesn't intend to deliver a best in class, superior to anything benchmark-purchase and-hold (exchange costs included) exchanging procedure - that is amazingly troublesome and is a test in any event, for top exchanging firms.

Literature Review

Many widely recognized empirical studies show that financial markets are predictable to some extent [3]. In this section, we review the studies previously done in this area and also gather information about the mechanisms that are currently being used for the prediction of Stock market [4].

Myer [5] observed that analysts utilize various techniques in trying to determine where the stock price might be headed. Some of these techniques include time series forecasting, technical analysis, and machine learning modelling [4].

Shen et al., [6] write that the potential in forecasting financial markets by Machine learning algorithms have been widely studied. These algorithms try to forecast the movement of stock prices based on training given with the past value movements [7].

Moving averages are indicators that analysts can use to assess the trend of the stock by averaging the daily price over a fixed period [8]. They achieve this by generating Buy and sell signals when a moving average of a shorter duration crosses that of a longer duration. Additionally, the moving average is a useful tool for observing longer-term trends and smoothing out volatility.

Time Series analysis considers time to be a very important parameter to generate series of stock price movement [7]. Ariyo, Adewumi and Ayo [9] recommend the use of autoregressive integrated moving average model (ARIMA) for predicting stock prices.

Kimoto et al., [10] presented a prediction system of ANNs for predicting stocks listed on the Tokyo Stock Exchange and achieved excellent profits in a simulation exercise. A buying and selling alert system proposed by Tsang et al., [11] in the Hong Kong Stock Exchange using ANNs to forecast stock prices obtained an overall ratio of over 70%. Also, studies conducted by Kim and Kim [12] find that Artificial neural networks (ANNs) can detect nonlinear relationships in the characteristics of data.

Competitor Analysis

Analysing price behaviours and stock market movements is extremely challenging due to the markets' noisy, dynamic, non-stationary, nonlinear, nonparametric, and chaotic nature. Stock markets are affected by many highly interrelated factors that include company-specific variables, political, economic, and psychological [13].

There are two main approaches to analyse the financial market namely Fundamental analysis and Technical analysis [13]. Fundamental analysis forecast by attempting to measure the intrinsic value of the stock, while Technical analysts identify trends and patterns that suggest what direction a stock will take in the future [8].

Fundamental analysts investigate many things ranging from the overall industry and economic conditions to the management and financial condition of companies. Assets, earnings, liabilities, and expenses, are all crucial characteristics to fundamental analysts. Technical analysis on the other hand only uses the stock's price and volume as the only inputs with the assumption that all known fundamentals are factored into the price [8].

Sadia et al., [4] write that there are various methods and ways of implementing a prediction system. Due to the many number of options, there can also be many models that can be used to predict the price of the stock. Furthermore, they found that more and more researchers are investing their time every day in devising ways to obtain techniques that can continue to improve the accuracy of the prediction models.

Any approach a model of prediction uses should be robust, accurate, and reliable [4]. It should also consider all the variables that could affect the performance and the stock's value.

Sadia et al., [4] used the Random Forest algorithm and found that it was easy to use and that it obtained high accuracy rates in predicting; French et al., [14] recommended using the generalized autoregressive conditional heteroscedasticity (GARCH) model to forecast stock prices using the relationship between a stock’s return and its volatility; Fama and French [15] proposed two-factor models using size and book-to-market equity, both of which utilize a company’s fundamental information, to forecast stock prices; Cao and Tay [16] utilized support vector machine (SVM) in their financial forecasting and compared it with the regularized radial basis function (RBF) neural network and the multilayer back-propagation (BP) neural network; Pai and Lin, [17] thought of utilizing the ARIMA and SVM (support vector machines) in forecasting stock prices; Kwon and Moon [18] proposed a hybrid neurogenetic system for stock trading; while Hegazy, Osmanand Abdul Salam [19] proposed a machine learning algorithm which integrates Particle swarm optimization (PSO), and Least-square support vector machine (LS-SVM).

Due to the vast number of options available [4], determining the methods to use in our model was not easy. We considered among other factors the ease of use of the method, popularity, reliability, and expected forecasting accuracy to arrive at the choices we made.

Our choice of the Moving Average was inspired by Majaski [8] who found this method to be one of the most popular forms of technical analysis and Moving Average [20] who asserted that it is a very useful for smoothing out volatility and for observing longer-term trends.

Also, our choice of the autoregressive integrated moving average (ARIMA) was guided by Tsai et al., [21] who noted that the most well-known conventional time series forecasting approach is the ARIMA, and findings by Ariyo, Adewumi and Ayo [9] who observed that ARIMA models compete well against many emerging techniques that are used today for forecasting, as well as for short term prediction.

Das, Mokashiand Culkin [22] utilized neural networks to predict the movement of the S&P 500 Index and observed that prior stock price movements could be used to predict their future direction. A prediction system of ANNs presented by Kimoto et al., [10] achieved excellent profits in a simulation exercise of predicting stocks listed on the Tokyo Stock Exchange.

Equally, Shao, Wu and Liao [23] wrote that the LSTM to be more accurate than other machine learning models, such as multilayer perceptron, random forest, and pseudo-random models. These findings were complemented by studies conducted by Kim and Kim [12] whose findings indicate that the long short-term memory (LSTM) was superior for learning temporal patterns.

Data

It is of great importance to obtain robust, accurate and reliable data for the implementation of this research project. We will use SPY (Stock symbol for SPDR S&P 500) to implement and demonstrate our research. In our data collection, we will utilize a Python data mining function to scrape data from the New York Stock exchange.

Design and Methodology

Raw data is usually incomplete, inconsistent and contains many errors. We Pre-process this data transforming it from raw form into a more coherent format. This involves checking for missing values, looking for categorical values, and splitting the data-set into training and testing sets and performing a feature scaling.

Our Machine Learning workflow:

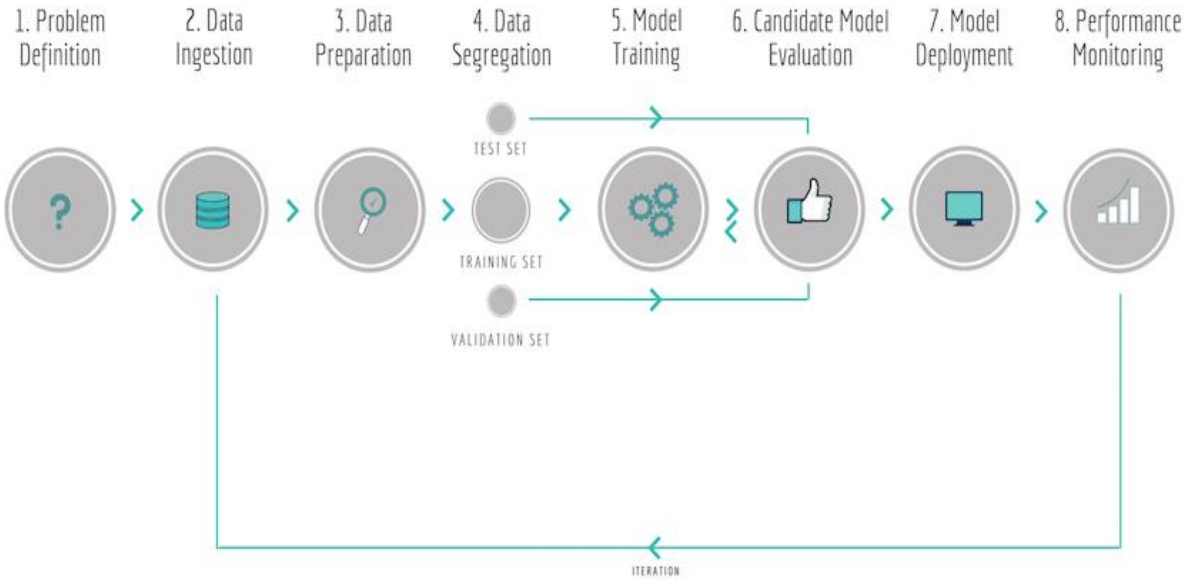

Figure 1: Machine learning workflow:

We will use the following models to implement our solution: Moving Average, Autoregressive integrated moving average (ARIMA), Artificial Neural Networks (ANN), and Long Short Term Memory (LSTM).

Moving Average

We create a data frame containing the *Date* and *Close* price columns, which will then be split into training and validation sets to verify our predictions. For each day, the predicted closing price will be the average of previously observed values. The moving average will utilize the latest set of values for every prediction. It will consider the predicted values for each subsequent step while removing the oldest observed values [24].

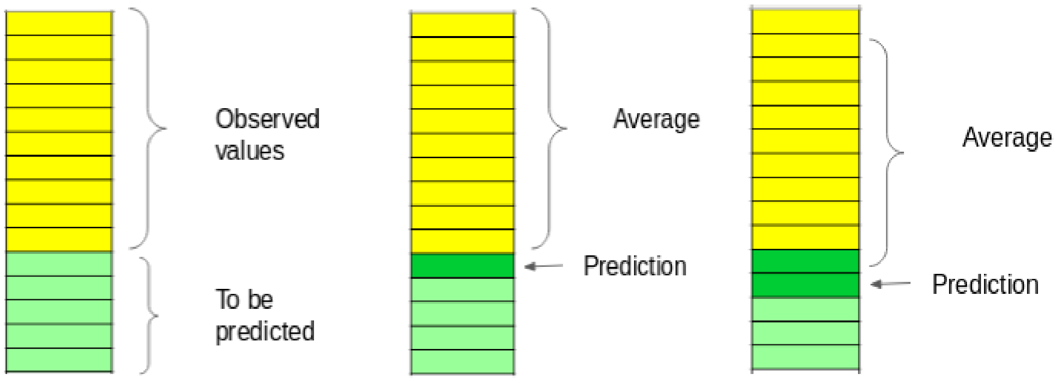

Figure 2: A demonstration of moving averages [24].

Autoregressive Integrated Moving Average model (ARIMA)

The ARIMA model is a popular tool for time series forecasting. This tool will utilize past values to forecast the future values. There are three main parameters that the ARIMA uses:

- p (past values used for forecasting the next value)
- q (past forecast errors used to predict the future values)
- d (order of differencing)

Parameter tuning for ARIMA is likely to consume a lot of time. We will therefore use auto ARIMA which will automatically select the best combination of *p*, *q*, and *d* and provide the least error.

Artificial Neural Networks

Artificial neurons are modelled from biological neurons, as described in the figures below.

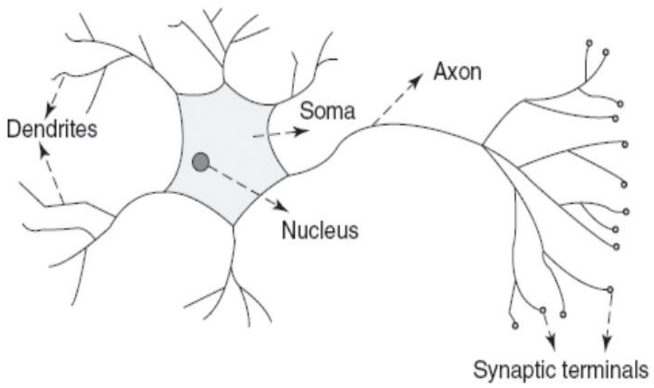

Figure 3: Biological Neuron [25].

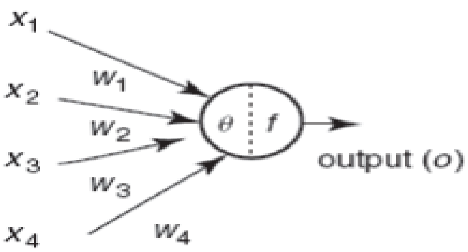

Figure 4: Artificial Neuron [25].

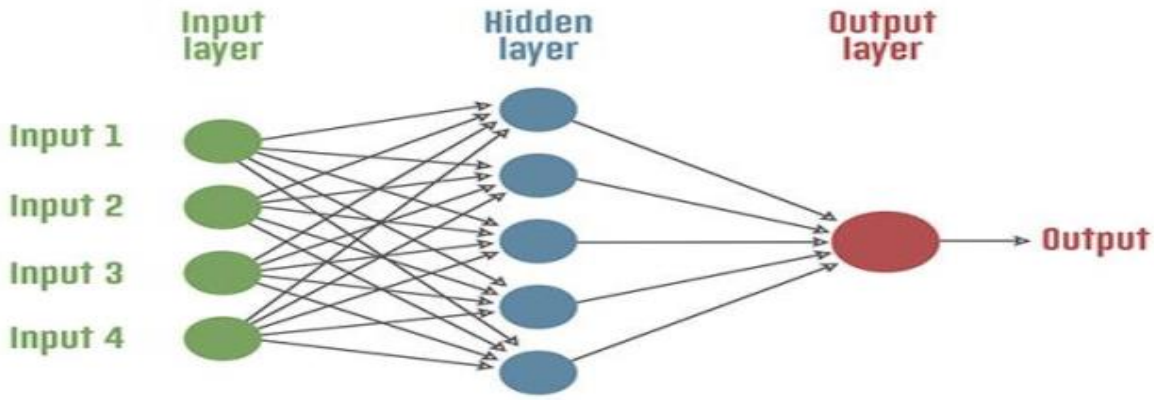

Figure 5: Architecture of artificial neural network

Artificial Neural Networks (ANN's) use three neuron layers namely: input layer, hidden layer, and the output layers to obtain the forecasted prices. Artificial nodes or neurons are the main processing elements of artificial neural networks. The synapses are related to input signals and show connection weights. The weighted sum of the input signals represents the neuron impulse and is computed then transformed using a transfer function. A transfer function defines the neurons nonlinear characteristics. The neuron is able to obtain its learning capability by setting the weights according to the chosen learning algorithm [25].

Long Short-Term Memory (LSTM)

Long Short-Term Memory (LSTM) networks are the gold standard to building RNN's in practice today and they have proven to be highly effective for sequence prediction problems. They rely on gated cells to track information throughout many time steps thereby learning long-term dependencies and overcoming the vanishing gradient problem.

LSTM are a modified version of recurrent neural networks well-suited to predict time series given time lags of unknown durations. LSTM will train the model using back-propagation [26], and will store useful past information while forgetting any information that is not.

LSTM will use the following three gates:

The input gate: Adds information to the cell state

The forget gate: Removes no longer required information

The output gate: selects the information to be displayed as output [24].

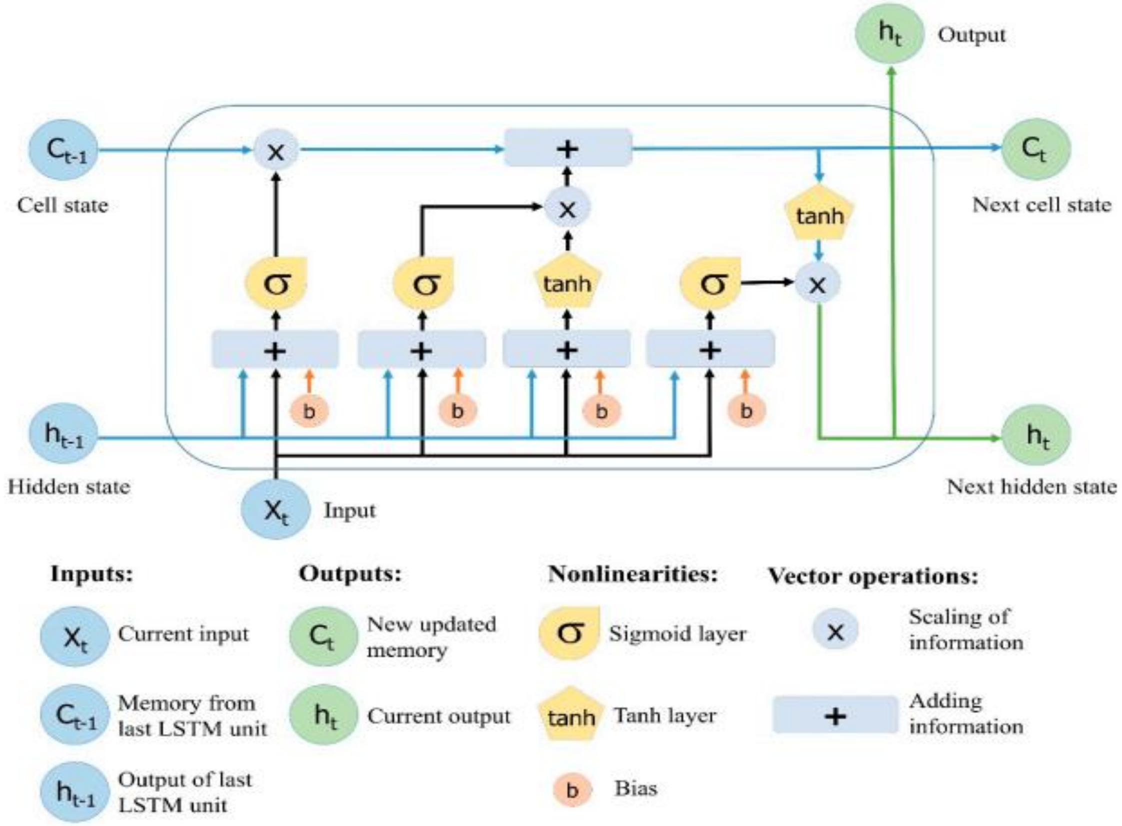

Figure 6: diagrammatic representation of LSTM

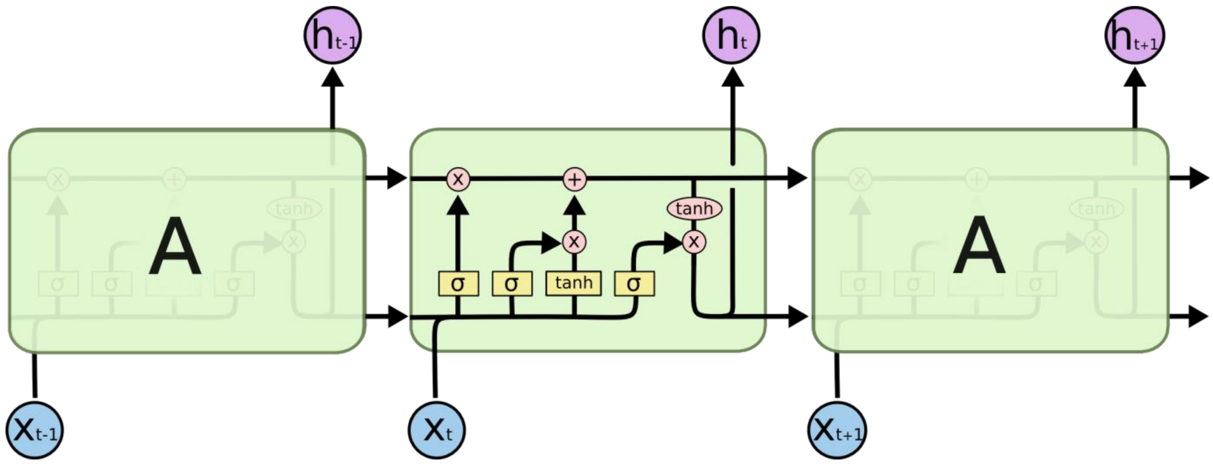

Figure 7: Architecture of LSTM network

Measuring Performance

Vandeput [29] observed that Measuring forecast error (or accuracy) is very challenging "as there is no one-size-fits-all indicator". Only by experimentation can one be able to obtain the Key Performance Indicator (KPI) suiting a specific situation. An indicator may be prone to certain pitfalls in given situations, but may overcome them in others.

The use of RMSE (Root Mean Square Error) is very common and that it makes an excellent general purpose error metric for numerical predictions [27]. Vandeput [29] notes that the Mean Percentage

Error (MPE) and Mean Absolute Percentage Error (MAPE) are some of the most commonly used to measure forecast accuracy. To measure the performance of our modelS, we will use the MPE, MAPE, and RMSE.

We use the formulas to calculate the various measures of performance:

Residual Error, $e_t = f_t - d_t$ (1)

Absolute Error, $|e_t| = |f_t - d_t|$ (2)

Bias = $\frac{1}{n}\sum_n e_t$ (3)

M.P.E = $\left(\frac{1}{n}\sum\frac{e_t}{d_t}\right) * 100\%$ (4)

M.A.E. = = $\frac{1}{n}\sum|e_t|$ (5)

M.A.P.E =$\left(\frac{1}{n}\sum\frac{|e_t|}{d_t}\right) * 100\%$ (6)

M.S.E. = $\frac{1}{n}\sum e_t^2$ (7)

R.M.S.E. = $\sqrt{\frac{1}{n}\sum e_t^2}$ (8)

Where:
f_t= Forecasted value
d_t= Actual value
n = Number of observations

Results

In this section, we present the results we obtained by our models. We use graphs, figures, and tables, to represent the information in a logical order.

a.) Moving Average

Our moving average model obtained the following results:

M.P.E. = -0.38697506147037447
M.A.P.E. = 0.38697506147037447
R.M.S.E. = 78.90635347461239

The graph below displays the stock price movement from the years 2000 - 2016.

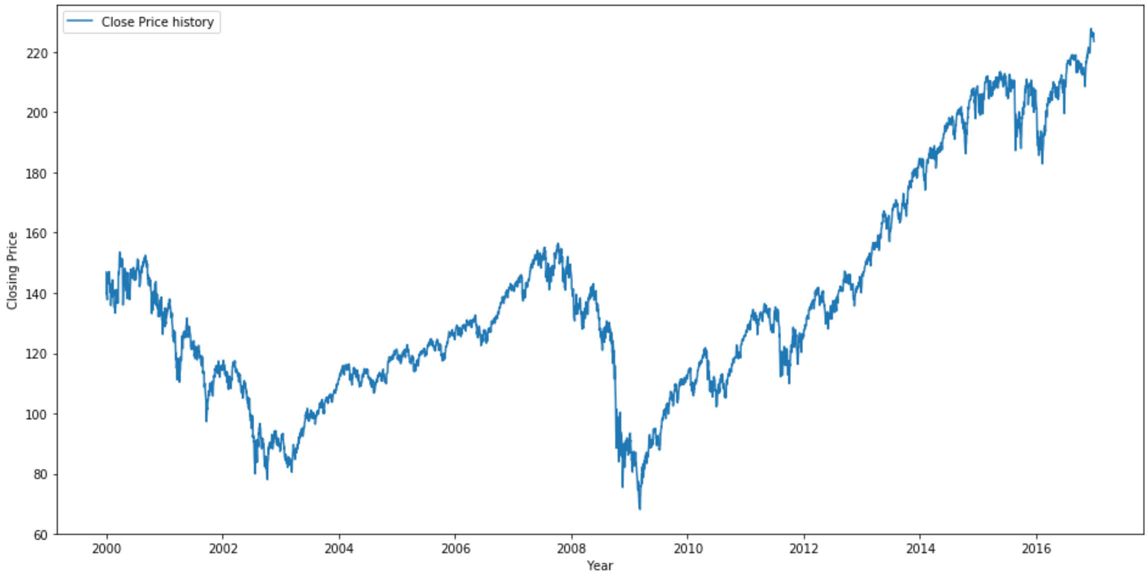

Figure 8: Stock price movement from the years 2000 - 2016.

The graph below displays the predicted values and the actual values as obtained from our moving average model:

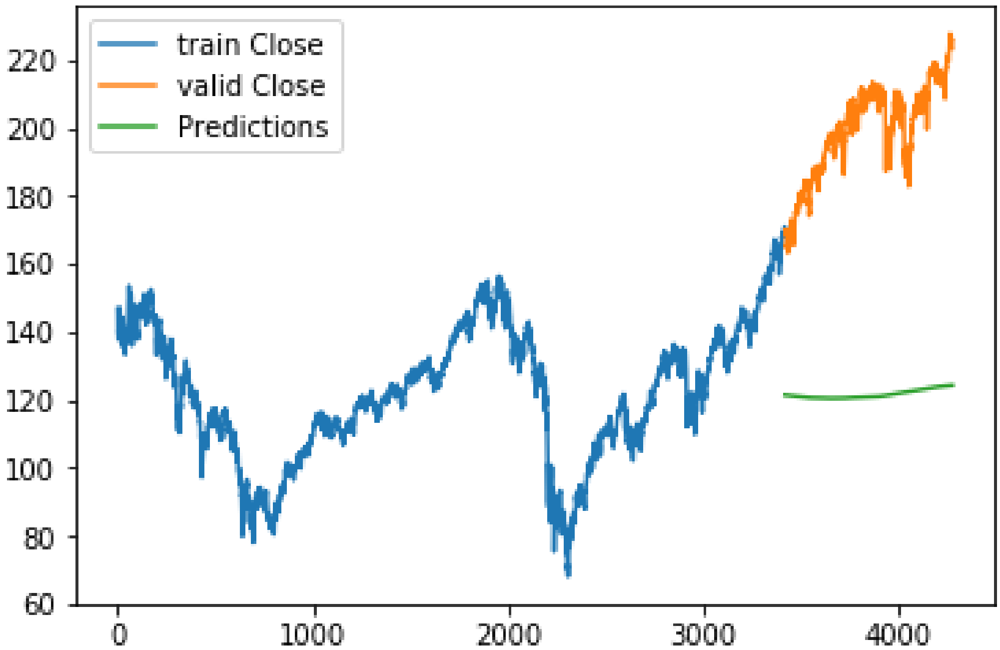

Figure 9: Graph of our moving average model

b.) ARIMA

Our Auto ARIMA model achieved the following results:

M.P.E. = 0.037798547684222236
M.A.P.E. = 0.05856440705241665
R.M.S.E. = 15.785715855636036
Correlation = 0.8129733087369445

Here is a summary of the output obtained from our ARIMA model:

```
print(model.summary())
```

```
                                 Statespace Model Results
==========================================================================================
Dep. Variable:                                  y   No. Observations:                 3419
Model:             SARIMAX(1, 1, 1)x(1, 1, 2, 12)   Log Likelihood               -6221.857
Date:                            Sun, 15 Dec 2019   AIC                          12457.714
Time:                                    20:44:13   BIC                          12500.647
Sample:                                         0   HQIC                         12473.058
                                           - 3419
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
intercept      0.0003      0.000      1.170      0.242      -0.000       0.001
ar.L1          0.5964      0.078      7.658      0.000       0.444       0.749
ma.L1         -0.6730      0.069     -9.708      0.000      -0.809      -0.537
ar.S.L12      -0.8284      0.077    -10.798      0.000      -0.979      -0.678
ma.S.L12      -0.1303      0.072     -1.819      0.069      -0.271       0.010
ma.S.L24      -0.8546      0.070    -12.126      0.000      -0.993      -0.716
sigma2         2.2278      0.032     70.410      0.000       2.166       2.290
===================================================================================
Ljung-Box (Q):                       81.74   Jarque-Bera (JB):              2907.71
Prob(Q):                              0.00   Prob(JB):                         0.00
Heteroskedasticity (H):               0.79   Skew:                            -0.26
Prob(H) (two-sided):                  0.00   Kurtosis:                         7.50
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
```

Figure 10: summary of the output as obtained from our ARIMA model

The graph below displays the predicted values and the actual values by our ARIMA model

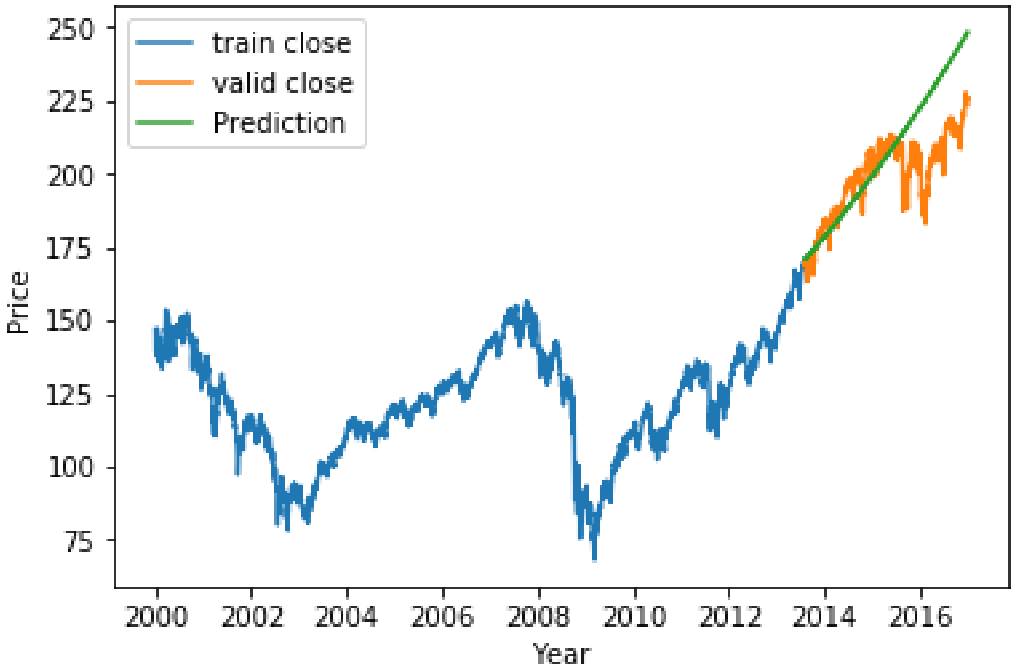

Figure 11: Actual and predicted values as obtained by our ARIMA model

c.) LSTM

Our LSTM model obtained the following results:

M.P.E. = 0.07690187666147934
M.A.P.E. = 0.07690187666147934
R.M.S.E. = 18.9628767142347

Here is a summary obtained from our LSTM model

```
print(model.summary())
```

```
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
lstm_1 (LSTM)                (None, 60, 12)            672
_________________________________________________________________
lstm_2 (LSTM)                (None, 8)                 672
_________________________________________________________________
dense_1 (Dense)              (None, 1)                 9
=================================================================
Total params: 1,353
Trainable params: 1,353
Non-trainable params: 0
_________________________________________________________________
None
```

Figure 12: Summary from our LSTM model

The graph below displays the actual and forecasted prices as obtained from our LSTM model

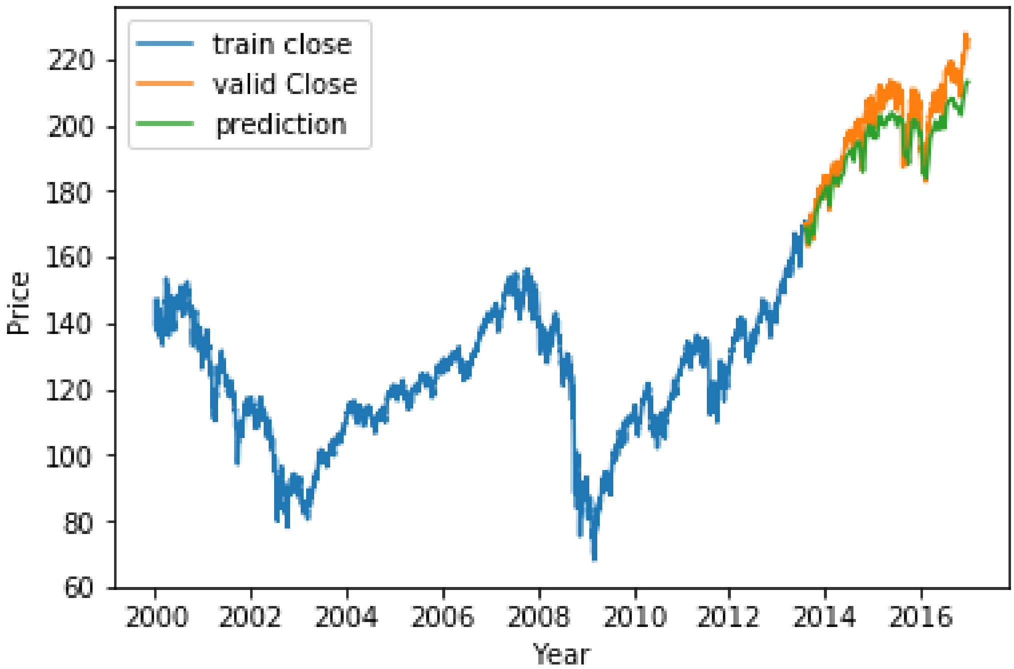

Figure 13: Actual and forecasted prices as obtained from our LSTM model

d.) ANN

Our Artificial Neural Network model obtained the following results:

MPE = 0.18390504429003657
MAPE = 0.18390504429003657
RSME = 40.056562611549495

Below is a summary of the ANN output

```
print(model1.summary())
```

```
Model: "sequential_2"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_2 (Dense)              (None, 128)               7808
_________________________________________________________________
dense_3 (Dense)              (None, 128)               16512
_________________________________________________________________
dense_4 (Dense)              (None, 1)                 129
=================================================================
Total params: 24,449
Trainable params: 24,449
Non-trainable params: 0
_________________________________________________________________
None
```

Figure 14: Summary from our ANN model

The graph below displays the actual and forecasted prices as obtained from our ANN model

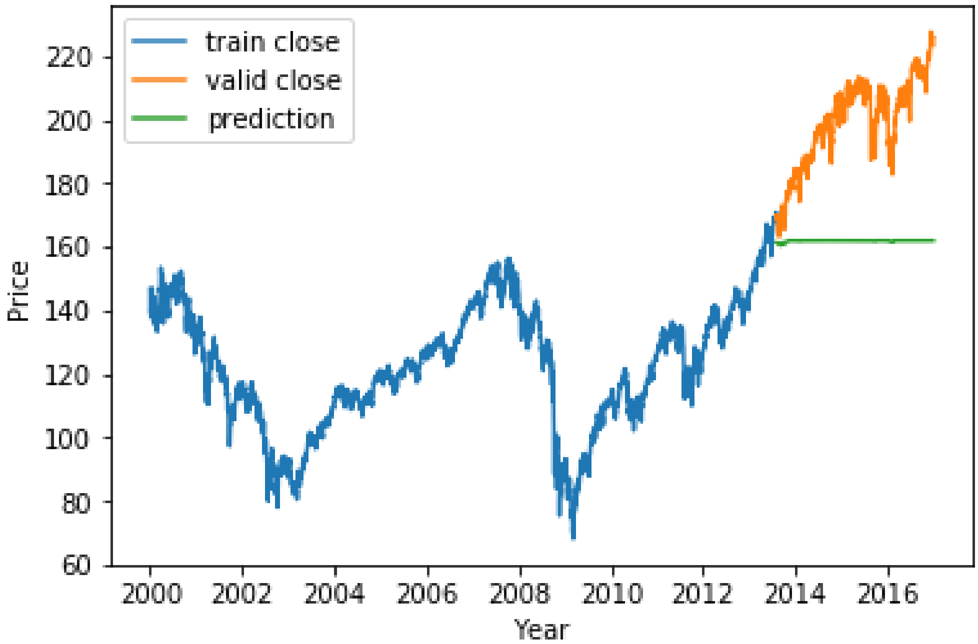

Figure 15: Actual and forecasted prices as obtained from our ANN model

e.) Summary of the Results

Here is a summary of the measures of performance obtained above

Model	M.P.E	M.A.P.E	R.M.S.E
Moving Average	-0.38697506147037447	0.38697506147037447	78.90635347461239
ARIMA	0.037798547684222236	0.05856440705241665	15.785715855636036
LSTM	0.07690187666147934	0.07690187666147934	18.9628767142347
ANN	0.18390504429003657	0.18390504429003657	40.056562611549495

Figure 16: A Summary of the metrics obtained

Discussion

Our findings agree with Vandeput [29] who writes that Measuring forecast error (or accuracy) is very challenging "as there is no one-size-fits-all indicator". They also agree with many widely recognized empirical studies that financial markets are predictable to some extent [3]. Our models were able to capture a trend in the movement of stock prices with varying degrees of accuracy. Predicted values obtained by the moving average model were not very promising as can be observed in Figure 9 above. The moving average obtained an R.M.S.E. of 78.90635347461239 and M.A.P.E. of 0.38697506147037447, which to say the least, was not a good performance.

With an M.A.P.E. of 0.18390504429003657 and an R.S.M.E. of 40.056562611549495, the ANN performed better as compared with the moving average, but was also not good enough as can be observed in Figure 15 above.

Predicted values obtained by both the ARIMA and LSTM models were in the same range as the actual values. Both the ARIMA and LSTM models obtained very good great results, with the ARIMA model achieving M.A.P.E. of 0.05856440705241665 and an R.M.S.E. of 15.785715855636036, while the LSTM model achieved a M.A.P.E. of 0.07690187666147934 and an R.M.S.E. of 18.9628767142347. This was a very good performance as can be observed in Figure 11 (for the ARIMA) and Figure 12 (for the LSTM).

We also observed that the LSTM model can be tuned for various parameters, for instance, changing the amount of LSTM layers, including dropout regard or growing the amount of ages.

Conclusion

Machine learning and deep learning techniques have the potential to ease the whole processes of analysing large chunks of data, spotting significant patterns and generating a single output that navigates traders towards a particular decision based on predicted asset prices.

While we were able to achieve significant results using our models, we recognize that many financial engineers and other researchers are spending sleepless nights trying to improve the performance of machine learning and deep learning models. We propose to open source our architecture to give an opportunity to likeminded engineers and researchers to contribute to the improvement of this code and to encourage collaboration and continual improvement.

Finally, it is important to note that stock prices can be affected by many other highly interrelated factors that include company-specific variables, political, economic, and psychological [13]. Natural factors like climate change, drought, hurricanes, floods, earthquakes and other factors like demonetization or merger/demerger of the organizations could also affect stock prices. Many of these factors can be difficult to anticipate and may need additional tools to properly forecast their effects.

References

[1] M. Abdolmohammadi, & J. Sultan, “Ethical reasoning and the use of insider information in stock trading”: JBE JBE. *Journal of Business Ethics, 37*(2), (2002), 165-173.

[2] D. Shah, H. Isah, & F. Zulkernine “Stock Market Analysis: A Review and Taxonomy of Prediction Techniques”. *International Journal of Financial Studies, 7*(2), (2019), 26. MDPI AG.

[3] E. Chong, H. Chilwoo, and F. C. Park, “Deep learning networks for stock markets analysis and prediction; Methodology, data representations, and case studies”. *Expert Systems with Applications* 83: (2017), 187-205.

[4] K. Sadia, A. Sharma, A. Paul, S. Padhi, and S. Sanyal *Volume-8 Issue-4 | Stock Market Prediction Using Machine Learning Algorithms*. [online] International Journal of Engineering and Advanced Technology(TM) (2019)

[5] R. Myer, “Moving averages are a main signal on the track”: TECHNICAL ANALYSIS. *The Age* (2011, Nov 22).

[6] S. Shen, J. Haomiao, and T. Zhang. “*Stock Market Forecasting Using Machine Learning Algorithms”*. Stanford: Department of Electrical Engineering, Stanford University, (2012), pp. 1–5.

[7] S. Prasanna, and D. Ezhilmaran, “*A Survey of Stock Price Prediction & Estimation Using Data Mining Techniques”*. [online] Google.com.(2016).

[8] C. Majaski, “*The Difference Between Fundamental vs. Technical Analysis?”*. [online] Investopedia.(2019).

[9] A. Ariyo, A. Adewumi, and C. Ayo, “*Stock Price Prediction Using the ARIMA Model - IEEE Conference Publication”*. [online] Ieeexplore.ieee.org. (2014).

[10] T. Kimoto, K. Asakawa, M. Yoda, and M. Takeoka, “*Stock market prediction system with modular neural networks - IEEE Conference Publication”*. [online] Ieeexplore.ieee.org. (1990).

[11] P. Tsang, P. Kwok, S. Choy, R. Kwan, S. Ng, J. Mak, J. Tsang, K. Koong, and T. Wong, “*Design and implementation of NN5 for Hong Kong stock price forecasting”*. [online] Engineering Applications of Artificial Intelligence, vol. 20. (2019).

[12] T. Kim, and H. Kim, “*Forecasting stock prices with a feature fusion LSTM-CNN model using different representations of the same data”*. [online].(2019).

[13] T. H. Nguyen, K. Shirai, and J. Velcin, “Sentiment analysis on social media for stock movement prediction”. *Expert Syst. Appl.* 42, 24 (December 2015), 9603-9611.

[14] K. French, G. Schwert, and R. Stambaugh, “Expected stock returns and volatility”, *Journal of Financial Economics,* 19, issue 1, (1987), p. 3-29.

[15] E. Fama, and K. French, “The cross-section of expected stock returns”. J Finance [Internet]. 1992;47(2):427–65.

[16] L. J. Cao and F. E. H. Tay, "Support vector machine with adaptive parameters in financial time series forecasting," in *IEEE Transactions on Neural Networks*, vol. 14, no. 6, pp. 1506-1518, Nov. 2003.

[17] P. Pai, and C. Lin, “*A hybrid ARIMA and support vector machines model in stock price forecasting”*. [online] Elsevier. (2005).

[18] Y. Kwon, and B. Moon, “*A Hybrid Neurogenetic Approach for Stock Forecasting” - IEEE Journals & Magazine*. [online] Ieeexplore.ieee.org. (2007).

[19] H. Osman, O. S. Soliman, and A. S. Mustafa. “A Machine Learning Model for Stock Market Prediction”. International Journal of Computer Science and Telecommunications. 4. (2013), 17-23.

[20] Moving average. “*Moving average”* [online] FT.Com. (2009).

[21] M. Tsai, C. Cheng, M. Tsai, and H. Shiu, “*Forecasting leading industry stock prices based on a hybrid time-series forecast model”*. [online] PLoS One. 2018; 13(12): e0209922.

[22] S. R. Das, K. Mokashi, and R. Culkin, “Are Markets Truly Efficient? Experiments Using Deep Learning Algorithms for Market Movement Prediction”. Algorithms 2018, 11, 138.

[23] X. SHAO, K. WU, and B. LIAO, “Norm Multikernel Learning Approach for Stock Market Price Forecasting”. *Computational Intelligence and Neuroscience : CIN,* **2012**, pp. 601296.

[24] A. Singh, “*Predicting the Stock Market Using Machine Learning and Deep Learning”*. [online] Analytics Vidhya. (2018).

[25] K. Turhan, B. Kurt, and Y. Z. Engin, “Estimation Of Student Success With Artificial Neural Networks”. *Egitim ve Bilim,* (2013), **38**(170).

[26] A. Mittal, “*Understanding RNN and LSTM”*. [online] Medium. (2019).

[27] S. Katariya, and S. Jain, “*Stock Price Trend Forecasting using Supervised Learning”*. [online] Slideshare.net. (2019).

[28] P. Y. Chen, and P. M. Popovich, "*Correlation: Parametric and nonparametric measures*". Thousand Oaks, CA: Sage Publications. (2002).

[29] Vandeput, N. (2019). *Forecast KPI: RMSE, MAE, MAPE & Bias*. [online] Medium. Available at: https://medium.com/analytics-vidhya/forecast-kpi-rmse-mae-mape-bias-cdc5703d242d [Accessed 10 Dec. 2019].