

## Tasks for Part II

5)

- a) What is the risk of melt-down in the power plant during a day if no observations have been made? What if there is icy weather?
- b) Suppose that both warning sensors indicate failure. What is the risk of a meltdown in that case? Compare this result with the risk of a melt-down when there is an actual pump failure and water leak. What is the difference? The answers must be expressed as conditional probabilities of the observed variables,  $P(\text{Meltdown}|\dots)$ .
- c) The conditional probabilities for the stochastic variables are often estimated by repeated experiments or observations. Why is it sometimes very difficult to get accurate numbers for these? What conditional probabilities in the model of the plant do you think are difficult or impossible to estimate?
- d) Assume that the "IcyWeather" variable is changed to a more accurate "Temperature" variable instead (don't change your model). What are the different alternatives for the domain of this variable? What will happen with the probability distribution of  $P(\text{WaterLeak} | \text{Temperature})$  in each alternative?

6)

- a) What does a probability table in a Bayesian network represent?
- b) What is a joint probability distribution? Using the chain rule on the structure of the Bayesian network to rewrite the joint distribution as a product of  $P(\text{child}|\text{parent})$  expressions, calculate manually the particular entry in the joint distribution of  $P(\text{Meltdown}=F, \text{PumpFailureWarning}=F, \text{PumpFailure}=F, \text{WaterLeakWarning}=F, \text{WaterLeak}=F, \text{IcyWeather}=F)$ . Is this a common state for the nuclear plant to be in?
- c) What is the probability of a meltdown if you know that there is both a water leak and a pump failure? Would knowing the state of any other variable matter? Explain your reasoning!
- d) Calculate manually the probability of a meltdown when you happen to know that  $\text{PumpFailureWarning}=F$ ,  $\text{WaterLeak}=F$ ,  $\text{WaterLeakWarning}=F$  and  $\text{IcyWeather}=F$  but you are not really sure about a pump failure.

## Answers for Part II

5)

- a) If no observations have been made, we can't know the value of any of the variables. So looking to the Bayesian network we can say that the probability of a melt-down is **0'03**. On the other hand, if we know that there is icy weather, we should change the value of IcyWeather to true. After making this change, we can observe that the probability for a melt-down has not changed, it's **0'03**.

- b) If both warning sensors indicate failure, we must change their variables to true. After doing this, we can observe that the probability for a melt-down is **0'15**. In the other hand, if there has been a Pump Failure and a Water Leak, we can see that the probability for a melt-down is bigger (**0'20**).

$$P(\text{Meltdown} \mid \text{PumpFailure}, \text{WaterLeak}) > P(\text{Meltdown} \mid \text{PumpFailureWarning}, \text{WaterLeakWarning})$$

- c) That is because we need to repeat the process many times to get the result but, sometimes, our model oversimplifies the real situation. This could be because we don't have enough knowledge or because the computation is too difficult. In our model all the probabilities are able to estimate with relatively ease.
- d) If we change the IcyWeather variable to Temperature, we should estimate if the given Temperature is considered to be "Icy". For example, we could say that every temperature below 5 °C is considered to be "icy" so we would measure the probability of the temperature been higher or lower than 5 °C.

$$P(\text{Meltdown} \mid \text{temperature} < 5) = P(\text{Meltdown} \mid \text{IcyWeather})$$

$$P(\text{Meltdown} \mid \text{temperature} \geq 5) = P(\text{Meltdown} \mid \sim \text{IcyWeather})$$

6)

- a) A probability table represents the probability of the given node being true based on his parent nodes and other factors.
- b) Meltdown: m, Pumpfailure: p, PumpFailureWarning: pw, WaterLeak: w, WaterLeakWarning: ww, IcyWeather: i

$$P(\neg m \wedge \neg p \wedge \neg pw \wedge \neg w \wedge \neg ww \wedge \neg i)$$

$$= P(\neg m \mid \neg p, \neg w) * P(\neg pw \mid \neg p) * P(\neg p) * P(\neg ww \mid \neg w) * P(\neg w \mid \neg i) * P(\neg i)$$

$$= (1.0) * (0.95) * (0.9) * (0.95) * (0.9) * (0.95)$$

$$= 0.69447$$

- c) If we know that there has been a pump failure and a water leak, the probability for a meltdown is **0'80**. As these nodes are the last ones connecting with the meltdown, knowing the state of any other variable doesn't matter. The other variables only influence WaterLeak and PumpFailure, and we already know their value.
- d)  $P(m \mid \neg pw \wedge \neg w \wedge \neg ww \wedge \neg i) =$

$$= \alpha \{ [P(m \mid \neg p, \neg w) * P(\neg pw \mid \neg p) * P(\neg p) * P(\neg ww \mid \neg w) * P(\neg w \mid \neg i) * P(\neg i)]$$

$$+ [P(m \mid p, \neg w) * P(\neg pw \mid p) * P(p) * P(\neg ww \mid \neg w) * P(\neg w \mid \neg i) * P(\neg i)] \}$$

$$= \alpha \{ [(0'001) * (0'95) * (0'9) * (0'95) * (0'9) * (0'95)]$$

$$+ [(0'15) * (0'1) * (0'1) * (0'95) * (0'9) * (0'95)] \}$$

$$= \alpha \{ [0'000694474] + [0'001218375] \} = \alpha(0'001912849)$$

$$\begin{aligned}
P(\neg m \mid \neg pw \wedge \neg w \wedge \neg ww \wedge \neg i) &= \\
&= \alpha \{ [P(\neg m \mid \neg p, \neg w) * P(\neg pw \mid \neg p) * P(\neg p) * P(\neg ww \mid \neg w) * P(\neg w \mid \neg i) * P(\neg i)] \\
&\quad + [P(\neg m \mid p, \neg w) * P(\neg pw \mid p) * P(p) * P(\neg ww \mid \neg w) * P(\neg w \mid \neg i) * P(\neg i)] \} \\
&= \alpha \{ [(0'999) * (0'95) * (0'9) * (0'95) * (0'9) * (0'95)] \\
&\quad + [(0'85) * (0'1) * (0'1) * (0'95) * (0'9) * (0'95)] \} \\
&= \alpha \{ [0'693779276] + [0'006904125] \} = \alpha(0'700683401)
\end{aligned}$$

$$\alpha = 1 / (0'001912849 + 0'700683401) = 1'423292538$$

$$\text{RESULT} = 1'423292538 * 0'001912849 = P(m \mid \neg pw \wedge \neg w \wedge \neg ww \wedge \neg i) = 0'002722544$$

### Tasks for Part III

- a) During the lunch break, the owner tries to show off for his employees by demonstrating the many features of his car stereo. To everyone's disappointment, it doesn't work. How did the owner's chances of surviving the day change after this observation?
- b) The owner buys a new bicycle that he brings to work every day. The bicycle has the following properties:
  - $P(\text{bicycle\_works}) = 0.9$
  - $P(\text{survives} \mid \neg \text{moves} \wedge \text{melt-down} \wedge \text{bicycle\_works}) = 0.6$
  - $P(\text{survives} \mid \text{moves} \wedge \text{melt-down} \wedge \text{bicycle\_works}) = 0.9$

How does the bicycle change the owner's chances of survival?

- c) It is possible to model any function in propositional logic with Bayesian Networks. What does this fact say about the complexity of exact inference in Bayesian Networks? What alternatives are there to exact inference?

### Answers for Part III

- a) As the radio doesn't work, there is a chance that the cause is that the battery is not working and the car won't move. Because this, there is a small decrement on the probability of the owner to survive.

$$P(\text{OwnerSurvives}) = 0.99$$

$$P(\text{OwnerSurvives} \mid \sim \text{RadioWorks}) = 0.98$$

- b) With the bicycle added, the chances of the owner to survive increase to **1.0**.

$$P(\text{OwnerSurvives}) = 1$$

- c) Yes, because Bayesian Networks are based in propositional logic, the limitation comes when you try to model a function from, for example, first order logic because this would require a very a very large implementation. When you are calculating the exact inference, you have to loop several times over the variable's different values, which results in a very expensive computational cost. Another method for calculating this is the variable elimination algorithm, that avoid repeated calculations.

### Tasks for Part IV

- a) The owner had an idea that instead of employing a safety person, to replace the pump with a better one. Is it possible, in your model, to compensate for the lack of Mr H.S.'s expertise with a better pump?
- b) Mr H.S. fell asleep on one of the plant's couches. When he wakes up he hears someone scream: "There is one or more warning signals beeping in your control room!". Mr H.S. realizes that he does not have time to fix the error before it is too late (we can assume that he wasn't in the control room at all). What is the chance of survival for Mr H.S. if he has a car with the same properties as the owner? **Hint:** This question involves a **disjunction** (A or B)

which can not be answered by querying the network as is. How could you answer such questions? Maybe something could be added or modified in the network.

- c) What unrealistic assumptions do you make when creating a Bayesian Network model of a person?
- d) Describe how you would model a more dynamic world where for example the "IcyWeather" is more likely to be true the next day if it was true the day before. You only have to consider a limited sequence of days.

### Answers for Part IV

- a) Of course! As role of Mr. H.S. depends directly from the alarms, and develops itself as another factor to modify the owner's chances to survive (with not very good results), a better pump would decrease the chances of meltdown directly and, by extension, the chance of the owner dying.
- b) You can add an extra node called "OneOrMoreAlarm", this node would be influenced by the alarms and will act as an logic "or" gate (0.0 for FF, 1.0 for the others) then you just have to make an observation in this node to see how the assumption propagates through the network and modify the outcome.
- c) When creating a Bayesian Network model of a person, we must assume that the person will act within the established parameters. As a human is not a machine, the probabilities for failure (for example) can change if the person has a "bad day" or by various external agents.

As measuring and specifying all the factors that make a human act on different ways is impossible, we must assume that the human will act depending on our limited factors. This can be a unrealistic assumption.

- d) As you cannot give "feedback" to you BN to determine if the day before was "IcyWeather", you have to add as many nodes as you are interested in order to simulate this:

Once you have added this, the exact inference calculation would require the same method, as Dynamic Bayesian Networks are still Bayesian Networks.

