28.10.24

VAM LA1 | D.Ú. 2 LA1

SALAMON
SAL0110

PŘ 1 verze 1

1.

$$\left[\begin{array}{cccc|c} 3 & 2 & -1 & 4 & 0 \\ 1 & 1 & 1 & -2 & -1 \\ 3 & -2 & 1 & -1 & 3 \\ 6 & 2 & 3 & 1 & 5 \end{array}\right] \underset{4.\check{r}-(2)\cdot 3.\check{r}}{\overset{1.\check{r}-3.\check{r}}{\sim}} \left[\begin{array}{cccc|c} 0 & 4 & -2 & 5 & -3 \\ 1 & 1 & 1 & -2 & -1 \\ 3 & -2 & 1 & -1 & 3 \\ 0 & 6 & 1 & 3 & 2 \end{array}\right] \overset{3.\check{r}-(3)\cdot 2.\check{r}}{\sim}$$

$$\left[\begin{array}{cccc|c} 0 & 4 & -2 & 5 & -3 \\ 1 & 1 & 1 & -2 & -1 \\ 0 & -5 & -2 & 5 & 6 \\ 0 & 6 & 1 & 3 & 2 \end{array}\right] \underset{4.\check{r}\times\left(\frac{5}{6}\right)}{\overset{1.\check{r}\times\frac{3}{2}}{\sim}} \left[\begin{array}{cccc|c} 0 & 6 & -3 & \frac{15}{2} & -\frac{9}{2} \\ 1 & 1 & 1 & -2 & -1 \\ 0 & -5 & -2 & 5 & 6 \\ 0 & 5 & \frac{5}{6} & \frac{5}{2} & \frac{10}{6} \end{array}\right] \overset{1.\check{r}+3.\check{r}}{\underset{4.\check{r}+3.\check{r}}{\sim}} \left[\begin{array}{cccc|c} 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 1 & 1 & 1 & -2 & -1 \\ 0 & -5 & -2 & 5 & 6 \\ 0 & 0 & -\frac{7}{6} & \frac{15}{2} & \frac{46}{6} \end{array}\right]$$

$$\overset{3.\check{r}\cdot\left(-\frac{1}{5}\right)}{\underset{4.\check{r}\cdot(6)}{\sim}} \left[\begin{array}{cccc|c} 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 1 & 1 & 1 & -2 & -1 \\ 0 & -1 & -\frac{2}{5} & 1 & \frac{6}{5} \\ 0 & 0 & -7 & 45 & 46 \end{array}\right] \overset{3.\check{r}+1.\check{r}}{\sim} \left[\begin{array}{cccc|c} 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 1 & 1 & 1 & -2 & -1 \\ 0 & 0 & -\frac{27}{5} & \frac{27}{2} & \frac{27}{5} \\ 0 & 0 & -7 & 45 & 46 \end{array}\right] \overset{1.\check{r}\leftrightarrow 2.\check{r}}{\underset{3.\check{r}\cdot\left(-\frac{1}{27}\right)}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & -27 & \frac{135}{2} & 27 \\ 0 & 0 & -7 & 45 & 46 \end{array}\right]$$

$$\overset{4.\check{r}\cdot\left(-\frac{1}{7}\right)}{\underset{3.\check{r}\cdot\left(-\frac{1}{27}\right)}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{135}{54} & -1 \\ 0 & 0 & 1 & -\frac{45}{7} & -\frac{46}{7} \end{array}\right] \overset{4.\check{r}-3.\check{r}}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{135}{54} & -1 \\ 0 & 0 & 0 & & \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & -\frac{1}{5} & \frac{1}{2} & \frac{1}{5} \\ 0 & 0 & -7 & 45 & 46 \end{array}\right] \underset{4.\check{r}\cdot(-7)}{\overset{3.\check{r}\cdot(35)}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & -7 & \frac{35}{2} & 7 \\ 0 & 0 & +7 & -45 & -46 \end{array}\right] \overset{4.\check{r}+3.\check{r}}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & -7 & \frac{35}{2} & 7 \\ 0 & 0 & 0 & -\frac{55}{2} & -39 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & -7 & \frac{35}{2} & 7 \\ 0 & 0 & 0 & -\frac{55}{2} & -39 \end{array}\right] \underset{4.r.\cdot(-2)}{\overset{3.r.\cdot(-\frac{1}{7})}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{5}{2} & -1 \\ 0 & 0 & 0 & 55 & 78 \end{array}\right] \overset{4.r.\cdot\frac{1}{55}}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & -5 & \frac{25}{2} & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{5}{2} & -1 \\ 0 & 0 & 0 & 1 & \frac{78}{55} \end{array}\right]$$

$$x_4 = \frac{78}{55}$$

$$x_3 - \frac{5}{2} x_4 = -1$$

$$x_3 - \frac{\cancel{5}}{2} \cdot \frac{78}{\cancel{55}_{11}} = -1$$

$$x_3 - \frac{78}{22} = -1$$

$$x_3 = -1 + \frac{39}{11}$$

$$x_3 = -\frac{11}{11} + \frac{39}{11}$$

$$x_3 = \frac{28}{11}$$

$$x_2 - 5 \cdot x_3 + \frac{25}{2} \cdot x_4 = \frac{3}{2}$$

$$x_2 - 5 \cdot \frac{28}{11} + \frac{\cancel{25}^{5}}{\cancel{2}_{1}} \cdot \frac{\cancel{78}^{39}}{\cancel{55}_{11}} = \frac{3}{2}$$

$$x_2 - \frac{1540}{11} + \frac{195}{11} = \frac{3}{2} \quad / \cdot 11$$

$$11x_2 - 1540 + 195 = \frac{33}{2}$$

$$11x_2 - 1345 = \frac{33}{2}$$

$$11x_2 = \frac{33}{2} + \frac{2690}{2}$$

$$11x_2 = \frac{2723}{2} \quad / \cdot \frac{1}{11}$$

$$x_2 = \frac{2723}{22}$$

$$2723 : 22 = 123{,}7$$

$$\begin{array}{r} 55 \\ 28 \\ \hline 440 \\ 110 \\ \hline 1540 \end{array}$$

$$\begin{array}{r} 71 \\ 25 \\ 3 \end{array}$$

$$x_4 = -\frac{39}{1} \cdot -\frac{2}{55} = \frac{78}{55}$$

D.Ú. LA1 - Verze 1

7.11.24

SALAMON
SALO110

Př. 2 Vyp. hodnotu výrazu $A^{-1}Bu + A^{-1}Bv$, kde

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 5 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$A^{-1}: \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 1 & 0 \\ 2 & 5 & 3 & 0 & 0 & 1 \end{array}\right] \underset{1.\check{r}\cdot(2)}{\overset{2.\check{r}-1.\check{r}}{\sim}} \left[\begin{array}{ccc|ccc} 2 & 2 & 2 & 2 & 0 & 0 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 2 & 5 & 3 & 0 & 0 & 1 \end{array}\right] \overset{3.\check{r}-1.\check{r}}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 2 & 2 & 0 & 0 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 & 1 \end{array}\right] \overset{1.\check{r}-2.\check{r}}{\sim} \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 3 & -1 & 0 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & 3 & 1 & -2 & 0 & 1 \end{array}\right] 3.\check{r} - \left(\frac{3}{2}\right)\cdot 2.\check{r}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 3 & -1 & 0 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{7}{2} & -\frac{3}{2} & 1 \end{array}\right] \underset{3.\check{r}\cdot(-2)}{\sim} \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 3 & -1 & 0 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -7 & 3 & -2 \end{array}\right] 1.\check{r} - 3.\check{r} \sim$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 10 & -4 & 2 \\ 0 & 2 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -7 & 3 & -2 \end{array}\right] \underset{2.\check{r}-3.\check{r}}{\overset{1.\check{r}\cdot\left(\frac{1}{2}\right)}{\sim}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -2 & 1 \\ 0 & 2 & 0 & 6 & -2 & 2 \\ 0 & 0 & 1 & -7 & 3 & -2 \end{array}\right] 2.\check{r}\cdot\frac{1}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -2 & 1 \\ 0 & 1 & 0 & 3 & -1 & 1 \\ 0 & 0 & 1 & -7 & 3 & -2 \end{array}\right]$$

$$A^{-1}$$

$$AA^{-1}: \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 5 & -2 & 1 \\ 3 & -1 & 1 \\ -7 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$1.ř \cdot (3) + 4.ř$

$2.ř - 3.ř$

$$\left[\begin{array}{cccc|c} 3 & 1 & 7 & 9 & 38 \\ 0 & \frac{1}{6} & \frac{5}{3} & -\frac{1}{6} & \frac{11}{3} \\ 1 & \frac{4}{3} & \frac{1}{3} & \frac{2}{3} & \frac{28}{3} \\ 0 & -5 & -2 & -3 & -22 \end{array}\right] \begin{array}{c} \\ 2.ř \cdot (6) \\ 3.ř \cdot (3) \\ \\ \end{array} \left[\begin{array}{cccc|c} 3 & 1 & 7 & 9 & 38 \\ 0 & 1 & 10 & -1 & 22 \\ 3 & 4 & 1 & 2 & 28 \\ 0 & -5 & -2 & -3 & -22 \end{array}\right] \begin{array}{c} \\ 3.ř - 1.ř \\ 4.ř + 2.ř \cdot (5) \\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c} 3 & 1 & 7 & 9 & 38 \\ 0 & 1 & 10 & -1 & 22 \\ 0 & 3 & -6 & -7 & -10 \\ 0 & 0 & 48 & -8 & 88 \end{array}\right] \begin{array}{c} 1.ř/(3) \\ \\ 3.ř/(3) \\ 4.ř/(8) \end{array} \left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 1 & 10 & -1 & 22 \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 6 & -1 & 11 \end{array}\right] \begin{array}{c} \\ 2.ř - 3.ř \\ \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 0 & 12 & \frac{4}{3} & \frac{76}{3} \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 6 & -1 & 11 \end{array}\right]$$

$4.ř - 2.ř \cdot (\frac{1}{2})$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 0 & 12 & \frac{4}{3} & \frac{76}{3} \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 0 & -\frac{5}{3} & -\frac{5}{3} \end{array}\right] \begin{array}{c} \\ 2.ř \leftrightarrow 3.ř \\ 3.ř/12 \\ 4.ř/-\frac{5}{3} \end{array} \left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 1 & \frac{1}{9} & \left(\frac{228}{12} \cdot \frac{1}{12}\right) \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 1 & \frac{1}{9} & \frac{228}{144} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 1 & \frac{1}{9} & \frac{114}{72} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & \frac{1}{3} & \frac{7}{3} & 3 & \frac{38}{3} \\ 0 & 1 & -2 & -\frac{7}{3} & -\frac{10}{3} \\ 0 & 0 & 1 & \frac{1}{9} & \frac{57}{36} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

d) Najdu vš. řešení soustavy rovnic:

$$\underline{\underline{x_4 = 1}}$$

$$x_3 + \frac{1}{9} \cdot 1 = \frac{57}{36}$$

$$x_3 + \frac{4}{36} = \frac{57}{36}$$

$$x_3 = \frac{57}{36} - \frac{4}{36} = \frac{53}{36}$$

$$\underline{\underline{x_3 = \frac{53}{36}}}$$

$$x_2 - 2 \cdot \frac{53}{36} - \frac{7}{3} \cdot 1 = -\frac{10}{3}$$

$$x_2 - \frac{106}{36} - \frac{7}{3} \cdot 1 = -\frac{10}{3} \quad / \cdot 3$$

$$3x_2 - \frac{106}{12} - 7 = -10 \quad / \cdot 12$$

$$36x_2 - 106 - 84 = -120$$

$$36x_2 - 190 = -120$$

$$36x_2 = 70$$

$$\underline{\underline{x_2 = \frac{35}{18}}}$$

$$x_1 + \frac{1}{3} \cdot \frac{35}{18} + \frac{7}{3} \cdot \frac{53}{36} + 3 \cdot 1 = \frac{38}{3} \quad / \cdot 3$$

$$x_1 + \frac{35}{18} + \frac{371}{36} + 9 = 38$$

$$x_1 + \frac{441}{36} + \frac{324}{36} = \frac{1368}{36}$$

$$x_1 = \frac{603}{36} =$$

$$= \underline{\underline{16{,}75}}$$

5.11.24

SARAMON SALO110

D.Ú. z LA1

VAM LA1

Př. 1 verze 3

(1)

$$\left[\begin{array}{cccc|c} 0 & -1 & -3 & 1 & 1 \\ 3 & 1 & 9 & 1 & -6 \\ -1 & 0 & -2 & -1 & 2 \\ 1 & 1 & 5 & 0 & -4 \end{array}\right] \underset{3.ř + 4.ř}{\overset{2.ř \cdot (\frac{1}{3})}{\sim}} \left[\begin{array}{cccc|c} 0 & -1 & -3 & 1 & 1 \\ 1 & \frac{1}{3} & 3 & \frac{1}{3} & -2 \\ 0 & 1 & 3 & -1 & -2 \\ 1 & 1 & 5 & 0 & -4 \end{array}\right] \underset{4.ř - 2.ř}{\overset{3.ř + 1.ř}{\sim}}$$

$$\left[\begin{array}{cccc|c} 0 & -1 & -3 & 1 & 1 \\ 1 & \frac{1}{3} & 3 & \frac{1}{3} & -2 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & \frac{2}{3} & 2 & -\frac{1}{3} & -2 \end{array}\right] \underset{4.ř \cdot (3)}{\overset{1.ř \cdot (2)}{\sim}} \left[\begin{array}{cccc|c} 0 & -2 & -6 & 2 & 2 \\ 1 & \frac{1}{3} & 3 & \frac{1}{3} & -2 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 2 & 6 & -1 & -6 \end{array}\right] \underset{}{\overset{4.ř + 1.ř}{\sim}}$$

$$\left[\begin{array}{cccc|c} 0 & -2 & -6 & 2 & 2 \\ 1 & \frac{1}{3} & 3 & \frac{1}{3} & -2 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -4 \end{array}\right]$$

7.12.24

VAM LA1 Kombi

D.Ú. z LA1

VERZE 4

SALAMON O.
SAL0110

**P.Ř. 1** Najděte vš. řeš. soustavy lin. rovnic:

$$x_1+2x_2+3x_3+4x_4=20$$
$$2x_1+3x_2+4x_3+x_4=26$$
$$3x_1+4x_2+x_3+2x_4=28$$
$$4x_1+x_2+2x_3+3x_4=26$$

a) Daný systém lin. rovnic lze určit maticí soustavy $A=[a_{ij}]$ spolu s vektorem pravé strany $b=[b_1 \dots b_4]$. Tento zápis označujeme jako „maticová rovnice":

$$A^{4,4}\bar{x}=\bar{b}$$

(kde $\bar{x}$ je vektor neznámých.)

b) V dalším kroku přidám vektor $\bar{b}$ (jako (pravý) poslední sloupec) k matici soustavy:

$$[A|b]=\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 20 \\ 2 & 3 & 4 & 1 & 26 \\ 3 & 4 & 1 & 2 & 28 \\ 4 & 1 & 2 & 3 & 26 \end{array}\right]$$

c) Soustavu pak řeším tzv. Gaussovou eliminační metodou, tj. pomocí tzv. elementárních řádkových operací převedu rozšířenou matici $[A|b]$ na tzv. schodovou matici. (Najít pak řešení soustavy ze schodového tvaru je obvykle pak snadné.)

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 20 \\ 2 & 3 & 4 & 1 & 26 \\ 3 & 4 & 1 & 2 & 28 \\ 4 & 1 & 2 & 3 & 26 \end{array}\right] \underset{4.ř-3.ř}{\sim} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 20 \\ 2 & 3 & 4 & 1 & 26 \\ 3 & 4 & 1 & 2 & 28 \\ 0 & -5 & -2 & -3 & -22 \end{array}\right] \underset{3ř/(3)}{\sim} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 20 \\ 1 & \frac{3}{2} & 2 & \frac{1}{2} & 13 \\ 1 & \frac{4}{3} & \frac{1}{3} & \frac{2}{3} & \frac{28}{3} \\ 0 & -5 & -2 & -3 & -22 \end{array}\right]$$