

# LA1 - Version 2 - Exercise 6

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```
# initial setup
options(scipen = 999)
options(tinytex.verbose = TRUE)
library(matlib)
library(knitr)
library(rmarkdown)
library(quarto)
library(tinytex)
library(pandoc)
knitr::opts_chunk$set(echo=TRUE, message=FALSE, warning=FALSE, fig.width=6, fig.height=6)
```

The matrix to be solved:

```
A <- matrix(
  c( 0,-2, 2,-1,
     1, 2,-2, 1,
     2,-1, 0, 1,
    -2, 0, 1,-1), 4, 4, byrow = TRUE)
b <- c(-1, 2, 4,-4)
```

Show (in Latex)

```
showEqn(A, b, fractions = TRUE, latex = TRUE)
```

$$\begin{array}{rrrrrr} 0 \cdot x_1 & - & 2 \cdot x_2 & + & 2 \cdot x_3 & - & 1 \cdot x_4 & = & -1 \\ 1 \cdot x_1 & + & 2 \cdot x_2 & - & 2 \cdot x_3 & + & 1 \cdot x_4 & = & 2 \\ 2 \cdot x_1 & - & 1 \cdot x_2 & + & 0 \cdot x_3 & + & 1 \cdot x_4 & = & 4 \\ -2 \cdot x_1 & + & 0 \cdot x_2 & + & 1 \cdot x_3 & - & 1 \cdot x_4 & = & -4 \end{array}$$

Matrix Ab1

```
Ab1 <- matrix(c( -1,-2, 2,-1,
                 2, 2,-2, 1,
                 4,-1, 0, 1,
                -4, 0, 1,-1), 4, 4, byrow = TRUE)
```

Determinant of Ab1 - step by step solution:

```
Det(Ab1, verbose = T, fractions = TRUE)
```

```
##
## Initial matrix:
##      [,1] [,2] [,3] [,4]
## [1,] -1   -2    2   -1
## [2,]  2    2   -2    1
## [3,]  4   -1    0    1
## [4,] -4    0    1   -1
##
## row: 1
##
```

```

## exchange rows 1 and 3
##      [,1] [,2] [,3] [,4]
## [1,]  4   -1   0   1
## [2,]  2    2  -2   1
## [3,] -1   -2   2  -1
## [4,] -4    0   1  -1
##
## multiply row 1 by 1/4
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/4   0  1/4
## [2,]    2    2  -2   1
## [3,]   -1   -2   2  -1
## [4,]   -4    0   1  -1
##
## multiply row 1 by 2 and subtract from row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/4   0  1/4
## [2,]    0  5/2  -2  1/2
## [3,]   -1   -2   2  -1
## [4,]   -4    0   1  -1
##
## multiply row 1 by 1 and add to row 3
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/4   0  1/4
## [2,]    0  5/2  -2  1/2
## [3,]    0 -9/4   2 -3/4
## [4,]   -4    0   1  -1
##
## multiply row 1 by 4 and add to row 4
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/4   0  1/4
## [2,]    0  5/2  -2  1/2
## [3,]    0 -9/4   2 -3/4
## [4,]    0   -1   1    0
##
## row: 2
##
## multiply row 2 by 2/5
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/4   0  1/4
## [2,]    0    1 -4/5  1/5
## [3,]    0 -9/4   2 -3/4
## [4,]    0   -1   1    0
##
## multiply row 2 by 1/4 and add to row 1
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -1/5 3/10
## [2,]    0    1 -4/5  1/5
## [3,]    0 -9/4   2 -3/4
## [4,]    0   -1   1    0
##
## multiply row 2 by 9/4 and add to row 3
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -1/5 3/10
## [2,]    0    1 -4/5  1/5
## [3,]    0    0  1/5 -3/10
## [4,]    0   -1   1    0

```

```

##
## multiply row 2 by 1 and add to row 4
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -1/5 3/10
## [2,]    0    1 -4/5 1/5
## [3,]    0    0 1/5 -3/10
## [4,]    0    0 1/5 1/5
##
## row: 3
##
## multiply row 3 by 5
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -1/5 3/10
## [2,]    0    1 -4/5 1/5
## [3,]    0    0 1 -3/2
## [4,]    0    0 1/5 1/5
##
## multiply row 3 by 1/5 and add to row 1
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 0 0
## [2,]    0    1 -4/5 1/5
## [3,]    0    0 1 -3/2
## [4,]    0    0 1/5 1/5
##
## multiply row 3 by 4/5 and add to row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 0 0
## [2,]    0    1 0 -1
## [3,]    0    0 1 -3/2
## [4,]    0    0 1/5 1/5
##
## multiply row 3 by 1/5 and subtract from row 4
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 0 0
## [2,]    0    1 0 -1
## [3,]    0    0 1 -3/2
## [4,]    0    0 0 1/2
##
## row: 4
##
## multiply row 4 by 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 0 0
## [2,]    0    1 0 -1
## [3,]    0    0 1 -3/2
## [4,]    0    0 0 1
##
## multiply row 4 by 1 and add to row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 0 0
## [2,]    0    1 0 0
## [3,]    0    0 1 -3/2
## [4,]    0    0 0 1
##
## multiply row 4 by 3/2 and add to row 3
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 0 0

```

```
## [2,] 0    1    0    0
## [3,] 0    0    1    0
## [4,] 0    0    0    1
##
## det = (-1)^1 x 4 x 5/2 x 1/5 x 1/2 = -1
```

### Matrix Ab2

```
Ab2 <- matrix(
  c( 0,-1, 2,-1,
     1, 2,-2, 1,
     2, 4, 0, 1,
    -2,-4, 1,-1), 4, 4, byrow = TRUE)
```

### Determinant of Ab2 - step by step solution:

```
Det(Ab2, verbose = TRUE, fractions = TRUE)
```

```
##
## Initial matrix:
##      [,1] [,2] [,3] [,4]
## [1,]  0   -1   2   -1
## [2,]  1    2  -2    1
## [3,]  2    4   0    1
## [4,] -2   -4   1   -1
##
## row: 1
##
## exchange rows 1 and 3
##      [,1] [,2] [,3] [,4]
## [1,]  2    4   0    1
## [2,]  1    2  -2    1
## [3,]  0   -1   2   -1
## [4,] -2   -4   1   -1
##
## multiply row 1 by 1/2
##      [,1] [,2] [,3] [,4]
## [1,]  1    2   0  1/2
## [2,]  1    2  -2    1
## [3,]  0   -1   2   -1
## [4,] -2   -4   1   -1
##
## subtract row 1 from row 2
##      [,1] [,2] [,3] [,4]
## [1,]  1    2   0  1/2
## [2,]  0    0  -2  1/2
## [3,]  0   -1   2   -1
## [4,] -2   -4   1   -1
##
## multiply row 1 by 2 and add to row 4
##      [,1] [,2] [,3] [,4]
## [1,]  1    2   0  1/2
## [2,]  0    0  -2  1/2
## [3,]  0   -1   2   -1
## [4,]  0    0   1    0
##
## row: 2
##
```

```

##  exchange rows 2 and 3
##      [,1] [,2] [,3] [,4]
## [1,]  1   2   0 1/2
## [2,]  0  -1   2  -1
## [3,]  0   0  -2 1/2
## [4,]  0   0   1   0
##
##  multiply row 2 by -1
##      [,1] [,2] [,3] [,4]
## [1,]  1   2   0 1/2
## [2,]  0   1  -2   1
## [3,]  0   0  -2 1/2
## [4,]  0   0   1   0
##
##  multiply row 2 by 2 and subtract from row 1
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   4 -3/2
## [2,]  0   1  -2   1
## [3,]  0   0  -2 1/2
## [4,]  0   0   1   0
##
## row: 3
##
##  multiply row 3 by -1/2
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   4 -3/2
## [2,]  0   1  -2   1
## [3,]  0   0   1 -1/4
## [4,]  0   0   1   0
##
##  multiply row 3 by 4 and subtract from row 1
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   0 -1/2
## [2,]  0   1  -2   1
## [3,]  0   0   1 -1/4
## [4,]  0   0   1   0
##
##  multiply row 3 by 2 and add to row 2
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   0 -1/2
## [2,]  0   1   0 1/2
## [3,]  0   0   1 -1/4
## [4,]  0   0   1   0
##
##  subtract row 3 from row 4
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   0 -1/2
## [2,]  0   1   0 1/2
## [3,]  0   0   1 -1/4
## [4,]  0   0   0 1/4
##
## row: 4
##
##  multiply row 4 by 4
##      [,1] [,2] [,3] [,4]
## [1,]  1   0   0 -1/2
## [2,]  0   1   0 1/2

```

```
## [3,]    0    0    1 -1/4
## [4,]    0    0    0    1
##
## multiply row 4 by 1/2 and add to row 1
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0  1/2
## [3,]    0    0    1 -1/4
## [4,]    0    0    0    1
##
## multiply row 4 by 1/2 and subtract from row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1 -1/4
## [4,]    0    0    0    1
##
## multiply row 4 by 1/4 and add to row 3
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
##
## det = (-1)^2 x 2 x -1 x -2 x 1/4 = 1
```

### Matrix Ab3

```
Ab3 <- matrix(
  c( 0,-2,-1,-1,
     1, 2, 2, 1,
     2,-1, 4, 1,
    -2, 0,-4,-1), 4, 4, byrow = TRUE)
```

### Determinant of Ab3 - step by step solution:

```
Det(Ab3, verbose = TRUE, fractions = TRUE)
```

```
##
## Initial matrix:
##      [,1] [,2] [,3] [,4]
## [1,]    0   -2   -1   -1
## [2,]    1    2    2    1
## [3,]    2   -1    4    1
## [4,]   -2    0   -4   -1
##
## row: 1
##
## exchange rows 1 and 3
##      [,1] [,2] [,3] [,4]
## [1,]    2   -1    4    1
## [2,]    1    2    2    1
## [3,]    0   -2   -1   -1
## [4,]   -2    0   -4   -1
##
## multiply row 1 by 1/2
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2    2  1/2
```

```

## [2,]    1    2    2    1
## [3,]    0   -2   -1   -1
## [4,]   -2    0   -4   -1
##
## subtract row 1 from row 2
##
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2    2  1/2
## [2,]    0  5/2    0  1/2
## [3,]    0   -2   -1   -1
## [4,]   -2    0   -4   -1
##
## multiply row 1 by 2 and add to row 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2    2  1/2
## [2,]    0  5/2    0  1/2
## [3,]    0   -2   -1   -1
## [4,]    0   -1    0    0
##
## row: 2
##
## multiply row 2 by 2/5
##
##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2    2  1/2
## [2,]    0    1    0  1/5
## [3,]    0   -2   -1   -1
## [4,]    0   -1    0    0
##
## multiply row 2 by 1/2 and add to row 1
##
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    2  3/5
## [2,]    0    1    0  1/5
## [3,]    0   -2   -1   -1
## [4,]    0   -1    0    0
##
## multiply row 2 by 2 and add to row 3
##
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    2  3/5
## [2,]    0    1    0  1/5
## [3,]    0    0   -1 -3/5
## [4,]    0   -1    0    0
##
## multiply row 2 by 1 and add to row 4
##
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    2  3/5
## [2,]    0    1    0  1/5
## [3,]    0    0   -1 -3/5
## [4,]    0    0    0  1/5
##
## row: 3
##
## multiply row 3 by -1
##
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    2  3/5
## [2,]    0    1    0  1/5
## [3,]    0    0    1  3/5
## [4,]    0    0    0  1/5
##
## multiply row 3 by 2 and subtract from row 1

```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0 -3/5
## [2,]    0    1    0  1/5
## [3,]    0    0    1  3/5
## [4,]    0    0    0  1/5
##
## row: 4
##
## multiply row 4 by 5
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0 -3/5
## [2,]    0    1    0  1/5
## [3,]    0    0    1  3/5
## [4,]    0    0    0    1
##
## multiply row 4 by 3/5 and add to row 1
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0  1/5
## [3,]    0    0    1  3/5
## [4,]    0    0    0    1
##
## multiply row 4 by 1/5 and subtract from row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1  3/5
## [4,]    0    0    0    1
##
## multiply row 4 by 3/5 and subtract from row 3
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
##
## det = (-1)^1 x 2 x 5/2 x -1 x 1/5 = 1
```

#### Ab4

```
Ab4 <- matrix(
  c( 0,-2, 2,-1,
    1, 2,-2, 2,
    2,-1, 0, 4,
    -2, 0, 1,-4), 4, 4, byrow = TRUE)
```

#### Determinant of Ab4 - step by step solution:

```
Det(Ab4, verbose = TRUE, fractions = TRUE)
```

```
##
## Initial matrix:
##      [,1] [,2] [,3] [,4]
## [1,]    0   -2    2   -1
## [2,]    1    2   -2    2
## [3,]    2   -1    0    4
## [4,]   -2    0    1   -4
##
```



```

## row: 1
##
##  exchange rows 1 and 3

##      [,1] [,2] [,3] [,4]
## [1,]  2   -1   0   4
## [2,]  1    2  -2   2
## [3,]  0   -2   2  -1
## [4,] -2    0   1  -4
##
##  multiply row 1 by 1/2

##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2   0   2
## [2,]    1    2  -2   2
## [3,]    0   -2   2  -1
## [4,]   -2    0   1  -4
##
##  subtract row 1 from row 2

##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2   0   2
## [2,]    0  5/2  -2   0
## [3,]    0   -2   2  -1
## [4,]   -2    0   1  -4
##
##  multiply row 1 by 2 and add to row 4

##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2   0   2
## [2,]    0  5/2  -2   0
## [3,]    0   -2   2  -1
## [4,]    0   -1   1   0
##
## row: 2
##
##  multiply row 2 by 2/5

##      [,1] [,2] [,3] [,4]
## [1,]    1 -1/2   0   2
## [2,]    0    1 -4/5   0
## [3,]    0   -2   2  -1
## [4,]    0   -1   1   0
##
##  multiply row 2 by 1/2 and add to row 1

##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -2/5   2
## [2,]    0    1 -4/5   0
## [3,]    0   -2   2  -1
## [4,]    0   -1   1   0
##
##  multiply row 2 by 2 and add to row 3

##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -2/5   2
## [2,]    0    1 -4/5   0
## [3,]    0    0  2/5  -1
## [4,]    0   -1   1   0
##
##  multiply row 2 by 1 and add to row 4

##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -2/5   2
## [2,]    0    1 -4/5   0

```

```

## [3,]    0    0 2/5  -1
## [4,]    0    0 1/5   0
##
## row: 3
##
## multiply row 3 by 5/2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0 -2/5   2
## [2,]    0    1 -4/5   0
## [3,]    0    0   1 -5/2
## [4,]    0    0 1/5   0
##
## multiply row 3 by 2/5 and add to row 1
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   0   1
## [2,]    0    1 -4/5   0
## [3,]    0    0   1 -5/2
## [4,]    0    0 1/5   0
##
## multiply row 3 by 4/5 and add to row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   0   1
## [2,]    0    1   0  -2
## [3,]    0    0   1 -5/2
## [4,]    0    0 1/5   0
##
## multiply row 3 by 1/5 and subtract from row 4
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   0   1
## [2,]    0    1   0  -2
## [3,]    0    0   1 -5/2
## [4,]    0    0   0  1/2
##
## row: 4
##
## multiply row 4 by 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   0   1
## [2,]    0    1   0  -2
## [3,]    0    0   1 -5/2
## [4,]    0    0   0   1
##
## subtract row 4 from row 1
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   0   0
## [2,]    0    1   0  -2
## [3,]    0    0   1 -5/2
## [4,]    0    0   0   1
##
## multiply row 4 by 2 and add to row 2
##      [,1] [,2] [,3] [,4]
## [1,]    1    0   0   0
## [2,]    0    1   0   0
## [3,]    0    0   1 -5/2
## [4,]    0    0   0   1
##
## multiply row 4 by 5/2 and add to row 3

```

```
##      [,1] [,2] [,3] [,4]
## [1,] 1    0    0    0
## [2,] 0    1    0    0
## [3,] 0    0    1    0
## [4,] 0    0    0    1
##
## det = (-1)^1 x 2 x 5/2 x 2/5 x 1/2 = -1
```

**Final solution of equations**

```
x1 = (det Ab1) / (det A) = -1 / -1 = 1
x2 = (det Ab2) / (det A) = 1 / -1 = -1
x3 = 1 / -1 = -1
x4 = -1 / -1 = 1
```