

LA1 - exercise 1

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January, 2025

Example 1 (ver2)

```
library(matlib)
library(knitr)
library(rmarkdown)
library(quarto)
library(tinytex)
library(pandoc)
```

Find all solution of systems of linear equations of the form $Ax=b$, where:

```
A <- matrix(
  c(-2, 1, 0, 2,
    3, -2, 0, -5,
    -2, 3, 2, 12,
    2, -2, -1, -7), 4, 4, byrow=TRUE)
```

```
b <- c(-3, 5, -3, 3)
```

Show equations (in Latex):

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 0 & 2 \\ 3 & -2 & 0 & -5 \\ -2 & 3 & 2 & 12 \\ 2 & -2 & -1 & -7 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -3 \\ 3 \end{pmatrix}$$

Solve equation:

```
Solve(A, b, vars = paste0('x', 1:4))
```

```
## x1      + x4 = 1
##  x2  + 4*x4 = -1
##  x3  + x4 = 1
##           0 = 0
```

```
Solve(A, b, verbose = TRUE, fractions = TRUE) ### arrives to solution by going to
```

```
##
```

```
## Initial matrix:
```

```
## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn()
```

```
## approaches
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] -2   1   0   2  -3
## [2,]  3  -2   0  -5   5
## [3,] -2   3   2  12  -3
## [4,]  2  -2  -1  -7   3
##
```

```

## row: 1
##
## exchange rows 1 and 2

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]  3   -2   0   -5   5
## [2,] -2    1   0    2  -3
## [3,] -2    3   2   12  -3
## [4,]  2   -2  -1   -7   3
##
## multiply row 1 by 1/3

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1 -2/3   0 -5/3  5/3
## [2,]  -2    1   0    2  -3
## [3,]  -2    3   2   12  -3
## [4,]    2   -2  -1   -7   3
##
## multiply row 1 by 2 and add to row 2

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1 -2/3   0 -5/3  5/3

```

```

## [2,]    0 -1/3    0 -4/3  1/3
## [3,]   -2    3    2  12   -3
## [4,]    2   -2   -1   -7    3
##
## multiply row 1 by 2 and add to row 3

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1 -2/3    0 -5/3  5/3
## [2,]    0 -1/3    0 -4/3  1/3
## [3,]    0  5/3    2 26/3  1/3
## [4,]    2   -2   -1   -7    3
##
## multiply row 1 by 2 and subtract from row 4

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1 -2/3    0 -5/3  5/3
## [2,]    0 -1/3    0 -4/3  1/3
## [3,]    0  5/3    2 26/3  1/3
## [4,]    0 -2/3   -1 -11/3 -1/3
##
## row: 2
##
## exchange rows 2 and 3

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(

```

```
## approaches
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1 -2/3    0 -5/3  5/3
## [2,]    0  5/3    2 26/3  1/3
## [3,]    0 -1/3    0 -4/3  1/3
## [4,]    0 -2/3   -1 -11/3 -1/3
```

```
##
```

```
## multiply row 2 by 3/5
```

```
## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn()
```

```
## approaches
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1 -2/3    0 -5/3  5/3
## [2,]    0    1  6/5 26/5  1/5
## [3,]    0 -1/3    0 -4/3  1/3
## [4,]    0 -2/3   -1 -11/3 -1/3
```

```
##
```

```
## multiply row 2 by 2/3 and add to row 1
```

```
## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn()
```

```
## approaches
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0  4/5  9/5  9/5
## [2,]    0    1  6/5 26/5  1/5
## [3,]    0 -1/3    0 -4/3  1/3
## [4,]    0 -2/3   -1 -11/3 -1/3
```

```
##
```

```
## multiply row 2 by 1/3 and add to row 3
```

```

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0 4/5  9/5  9/5
## [2,]    0    1 6/5 26/5  1/5
## [3,]    0    0 2/5  2/5  2/5
## [4,]    0 -2/3  -1 -11/3 -1/3
##
## multiply row 2 by 2/3 and add to row 4

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0 4/5  9/5  9/5
## [2,]    0    1 6/5 26/5  1/5
## [3,]    0    0 2/5  2/5  2/5
## [4,]    0    0 -1/5 -1/5 -1/5
##
## row: 3
##
## multiply row 3 by 5/2

## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0 4/5  9/5  9/5
## [2,]    0    1 6/5 26/5  1/5
## [3,]    0    0    1    1    1

```

```

## [4,]    0    0 -1/5 -1/5 -1/5
##
## multiply row 3 by 4/5 and subtract from row 1
## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    1    1
## [2,]    0    1 6/5 26/5 1/5
## [3,]    0    0    1    1    1
## [4,]    0    0 -1/5 -1/5 -1/5
##
## multiply row 3 by 6/5 and subtract from row 2
## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    1    1
## [2,]    0    1    0    4   -1
## [3,]    0    0    1    1    1
## [4,]    0    0 -1/5 -1/5 -1/5
##
## multiply row 3 by 1/5 and add to row 4
## Warning in printMatrix(A): Function is deprecated. See latexMatrix() and Eqn(
## approaches
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    1    1

```

```
## [2,] 0 1 0 4 -1
## [3,] 0 0 1 1 1
## [4,] 0 0 0 0 0
```

```
##
```

```
## row: 4
```

```
## x1 + x4 = 1
```

```
## x2 + 4*x4 = -1
```

```
## x3 + x4 = 1
```

```
## 0 = 0
```

```
# Showing solution verbose step by step, ie. the row operations, created by `Sol
```

```
Initial matrix:
```

```
      x1 x2 x3 x4
```

```
[1,] -2 1 0 2 -3
```

```
[2,] 3 -2 0 -5 5
```

```
[3,] -2 3 2 12 -3
```

```
[4,] 2 -2 -1 -7 3
```

```
row: 1
```

```
exchange rows 1 and 2
```

```
      x1 x2 x3 x4
```

```
[1,] 3 -2 0 -5 5
```

```
[2,] -2 1 0 2 -3
```

```
[3,] -2 3 2 12 -3
```

```
[4,] 2 -2 -1 -7 3
```

```
multiply row 1 by 1/3
```


	x1	x2	x3	x4	
[1,]	1	-2/3	0	-5/3	5/3
[2,]	-2	1	0	2	-3
[3,]	-2	3	2	12	-3
[4,]	2	-2	-1	-7	3

multiply row 1 by 2 and add to row 2

	x1	x2	x3	x4	
[1,]	1	-2/3	0	-5/3	5/3
[2,]	0	-1/3	0	-4/3	1/3
[3,]	-2	3	2	12	-3
[4,]	2	-2	-1	-7	3

multiply row 1 by 2 and add to row 3

	x1	x2	x3	x4	
[1,]	1	-2/3	0	-5/3	5/3
[2,]	0	-1/3	0	-4/3	1/3
[3,]	0	5/3	2	26/3	1/3
[4,]	2	-2	-1	-7	3

multiply row 1 by 2 and subtract from row 4

	x1	x2	x3	x4	
[1,]	1	-2/3	0	-5/3	5/3
[2,]	0	-1/3	0	-4/3	1/3
[3,]	0	5/3	2	26/3	1/3
[4,]	0	-2/3	-1	-11/3	-1/3

row: 2

exchange rows 2 and 3

	x1	x2	x3	x4	
[1,]	1	-2/3	0	-5/3	5/3
[2,]	0	5/3	2	26/3	1/3
[3,]	0	-1/3	0	-4/3	1/3
[4,]	0	-2/3	-1	-11/3	-1/3

multiply row 2 by 3/5

	x1	x2	x3	x4	
[1,]	1	-2/3	0	-5/3	5/3
[2,]	0	1	6/5	26/5	1/5
[3,]	0	-1/3	0	-4/3	1/3
[4,]	0	-2/3	-1	-11/3	-1/3

multiply row 2 by 2/3 and add to row 1

	x1	x2	x3	x4	
[1,]	1	0	4/5	9/5	9/5
[2,]	0	1	6/5	26/5	1/5
[3,]	0	-1/3	0	-4/3	1/3
[4,]	0	-2/3	-1	-11/3	-1/3

multiply row 2 by 1/3 and add to row 3

	x1	x2	x3	x4	
[1,]	1	0	4/5	9/5	9/5
[2,]	0	1	6/5	26/5	1/5

[3,]	0	0	2/5	2/5	2/5
[4,]	0	-2/3	-1	-11/3	-1/3

multiply row 2 by 2/3 and add to row 4

	x1	x2	x3	x4	
[1,]	1	0	4/5	9/5	9/5
[2,]	0	1	6/5	26/5	1/5
[3,]	0	0	2/5	2/5	2/5
[4,]	0	0	-1/5	-1/5	-1/5

row: 3

multiply row 3 by 5/2

	x1	x2	x3	x4	
[1,]	1	0	4/5	9/5	9/5
[2,]	0	1	6/5	26/5	1/5
[3,]	0	0	1	1	1
[4,]	0	0	-1/5	-1/5	-1/5

multiply row 3 by 4/5 and subtract from row 1

	x1	x2	x3	x4	
[1,]	1	0	0	1	1
[2,]	0	1	6/5	26/5	1/5
[3,]	0	0	1	1	1
[4,]	0	0	-1/5	-1/5	-1/5

multiply row 3 by 6/5 and subtract from row 2

	x1	x2	x3	x4	
[1,]	1	0	0	1	1
[2,]	0	1	0	4	-1
[3,]	0	0	1	1	1
[4,]	0	0	-1/5	-1/5	-1/5

multiply row 3 by 1/5 and add to row 4

	x1	x2	x3	x4	
[1,]	1	0	0	1	1
[2,]	0	1	0	4	-1
[3,]	0	0	1	1	1
[4,]	0	0	0	0	0

row: 4

x1		+ x4	=	1
x2		+ 4*x4	=	-1
x3		+ x4	=	1