Signals and Systems

Lecture # 2

Signals Classification and Transformation

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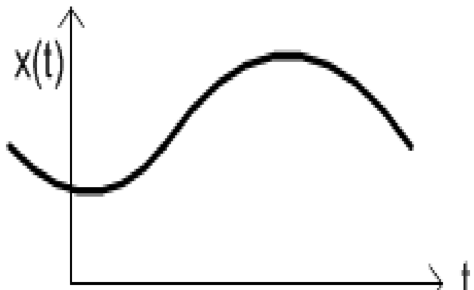
Topics of the lecture:

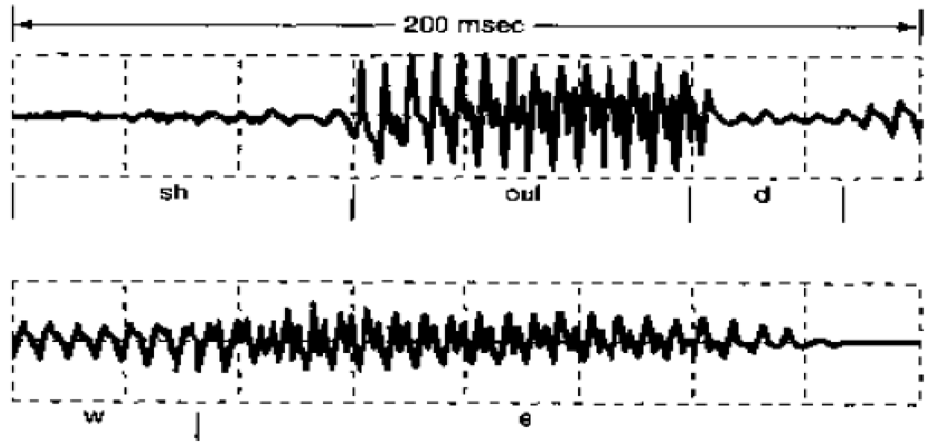
- **Signals Classification according to the Independent Variable.**
- **Signal's Energy and Power.**
- **Signals Transformations of Independent Variable.**

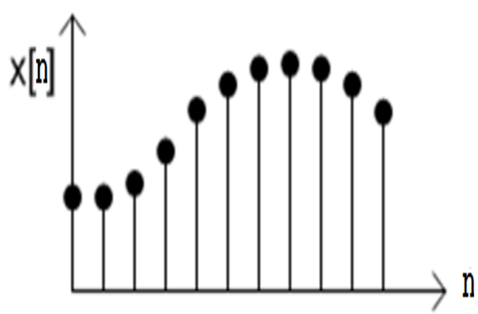
➢ Signal Classification according the independent variable

Signals are either *continuous in time* , that are defined at any time instant in its time domain (e.g. voltages and currents in electrical circuits or sound signals), OR, *discrete in time* , that are defined at integer time instants only in its time domain(e.g. closed stock market average , crime rate , or total population).

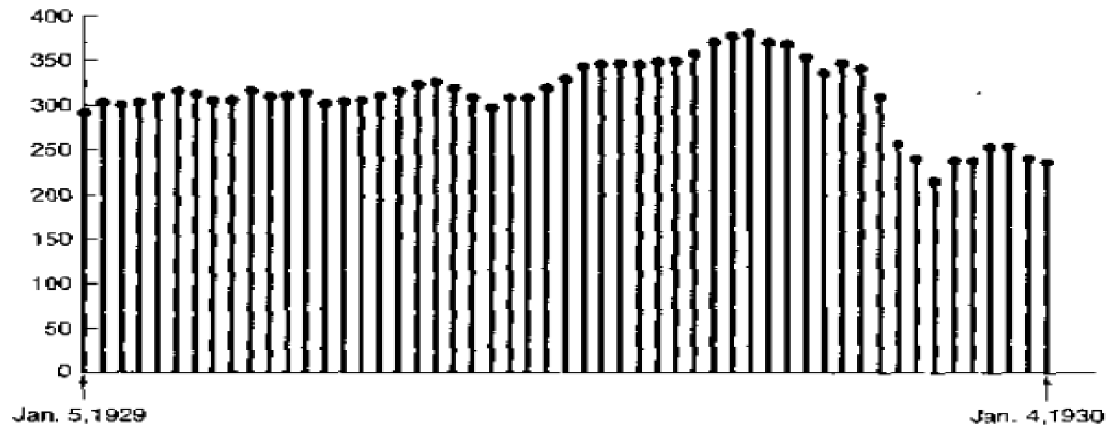


Continuous-Time





Discrete-Time



An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

➢ Signal Classification according the independent variable

	Continuous-Time Signals	Discrete-Time Signals
- **Domain:**	Its Domain is a continuous interval of Time $f(t)=x \quad , \quad t1 < t < t2$	Its Domain is a sequence of Time samples $f[n] = y \quad ,$ $n \in \{ n1 , n1+1 , n1+2 , \ldots, n2\}$
- **Range:**	Its range is a real valued e.g. $f(t)=1.43$	Its range is real valued e.g. $f[n]=9.32$
- **Symbol of IV:**	t	n
- **Function form:**	( )	[ ]
- **Examples:**	• Speech Signal • Voltage and Current	• Digital Images • Stock Market Index

- **In some books, if the domain and range are discrete the signal is called digital signal**

➢ Signal's Energy and Power

- **As too many signals are related to *physical quantities capturing energy and power*, it is useful to define and measure the signal's energy and power.**

$$in\, electrical\, circuit,\ the\ power\ consumed:$$

$$P(t) = V(t).I(t) = \frac{1}{R}V^2(t), \qquad V\ is\, the\, voltage$$

$$in\, autombile, the\ power\ consumed\, through\ \ friction:$$

$$P(t) = b.V^2(t), \qquad V\ is\, the\, automobile\, speed$$

- **In the above two examples, though they are different they have something in common, that is the *power is* a *constant* (that could be ignored for analysis purposes) *times a square of the system variable*.**

Signal's Energy and Power

Energy: **is the capacity for doing work. You must have energy to accomplish work - it is like the "currency" for performing work.**

Power: **is the rate of doing work or the rate of using energy.**

- For most of this class we will use a broad definition of power and energy that applies to any signal $x(t)$ or $x[n]$
- **Instantaneous signal power**

$$P(t) = |x(t)|^2 \qquad\qquad P[n] = |x[n]|^2$$

- **Signal energy**

$$E(t_0, t_1) = \int_{t_0}^{t_1} |x(t)|^2 \, \mathrm{d}t \qquad\qquad E(n_0, n_1) = \sum_{n=n_0}^{n_1} |x[n]|^2$$

- **Average signal power**

$$P(t_0, t_1) = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} |x(t)|^2 \, \mathrm{d}t$$

$$P(n_0, n_1) = \frac{1}{n_1 - n_0 + 1} \sum_{n=n_0}^{n_1} |x[n]|^2$$

|.| means magnitude of possibly complex valued x(t) or x[n]

➢ Signal's Energy and Power

Usually, the limits are taken over an infinite time interval to get the total Energy:

$$E_\infty \triangleq \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt = \int_{-\infty}^{+\infty}|x(t)|^2\,dt,$$

in discrete time→ $$E_\infty \triangleq \lim_{N\to\infty}\sum_{n=-N}^{+N}|x[n]|^2 = \sum_{n=-\infty}^{+\infty}|x[n]|^2$$

and total Average Power: $$P_\infty \triangleq \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt$$

The division may lead to undefined limits

in discrete time→ $$P_\infty \triangleq \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|x[n]|^2$$

➢ Signal's Energy and Power

We will encounter many types of signals :

- Some have infinite average power, energy, or both
- A signal is called an **energy signal** if $E_\infty < \infty$
- A signal is called a **power signal** if $0 < P_\infty < \infty$
- A signal can be an energy signal, a power signal, or neither type
- A signal can not be both an energy signal and a power signal

Signal Energy & Power Tips

- There are a few rules that can help you determine whether a signal has finite energy and average power
- Signals with finite energy have zero average power:
  $E_\infty < \infty \Rightarrow P_\infty = 0$
- Signals of finite duration and amplitude have finite energy:
  $x(t) \leq K$ for $|t| > c$, $K < \infty \Rightarrow E_\infty < \infty$
- Signals with finite average power have infinite energy:
  $P_\infty > 0 \Rightarrow E_\infty = \infty$

➢ Signal's Energy and Power

Determine whether the energy and average power of each of the following signals is finite.

a- $$x(t) = \begin{cases} 8 & |t| < 5 \\ 0 & \text{otherwise} \end{cases}$$

b- $$x[n] = j$$

c- $$x[n] = A\cos(\omega n + \phi)$$

d- $$x(t) = \begin{cases} e^{at} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

e- $$x[n] = e^{j\omega n}$$

➢ Signal's Energy and Power

a-

$$E_\infty = \int_{-\infty}^{+\infty} |x(\sigma)|^2 d\sigma = \int_{-5}^{+5} |8|^2 d\sigma = \int_{-5}^{+5} 64 d\sigma = 640$$

$$P_\infty = 0 \quad as\ E_\infty\ is\ finite$$

b-

$$E_\infty = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |j|^2 = \sum_{n=-\infty}^{+\infty} 1^2 = \infty$$

$$P_\infty = \lim_{N->\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 = \lim_{N->\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |j|^2$$

$$= \lim_{N->\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} 1^2 = \lim_{N->\infty} \frac{1}{2N+1} (2N+1)$$

$$= \lim_{N->\infty} 1 = 1$$

➢ Signal's Energy and Power

a-

$$x(t) = \begin{cases} 8 & |t| < 5 \\ 0 & \text{otherwise} \end{cases}$$

➢ Signal's Energy and Power

b- $x[n] = j$

➢ Signal's Energy and Power

c− $$x[n] = A\cos(\omega n + \phi)$$

➢ Signal's Energy and Power

d-

$$x(t) = \begin{cases} e^{at} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

➢ Signal's Energy and Power

e–

$$x[n] = e^{j\omega n}$$

➢ Signals Transformation

Signal Transformation plays a central concept in signals and systems analysis. Where you need to know the shape of signal after deformation(s)/processing or how to construct a signal from another group of signals

Examples:

Pilot Actions → **Through electrical and mechanical system** → **Changes in aircraft thrust قوة الدفع, OR Positions in the control surfaces**

→ **Through Dynamics and Kinematics** → **Changes in velocity or heading توجيه**

Elements of a communication system. The transmitter changes the message signal into a form suitable for transmission over the channel. The receiver processes the channel output (i.e., the received signal) to produce an estimate of the message signal.

➢ Signals Transformation (1- Of Independent Variable (time))

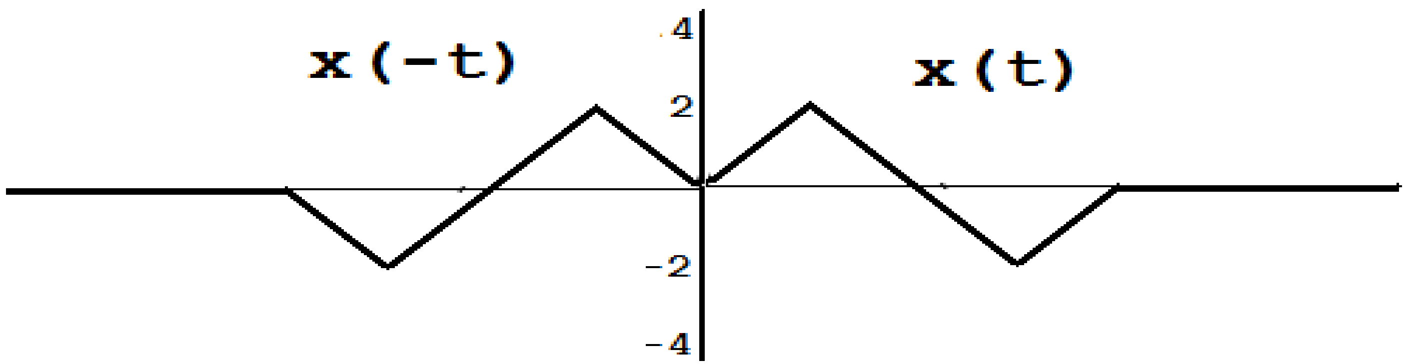
A- Basic Operations on the independent Variable:

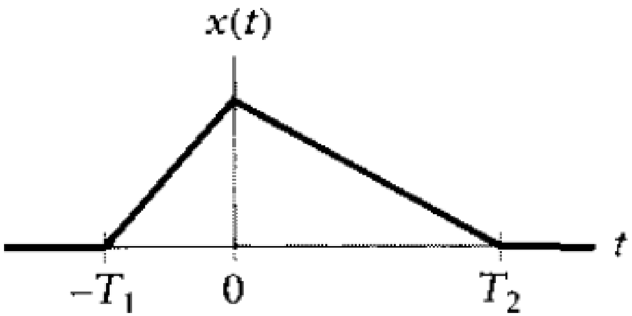
1- <u>Time Reversal</u>:

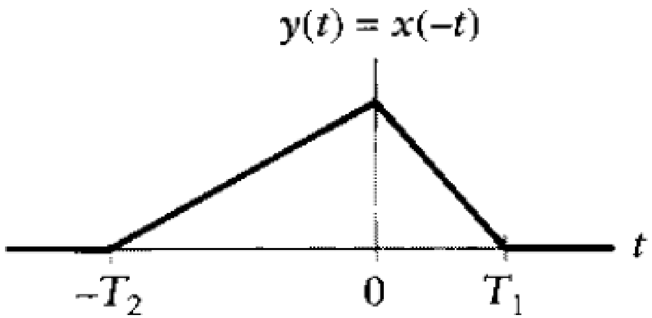
It is a reflection of the signal around the vertical axis (i.e. reversing it) at t=0 for the continuous-time signals, OR at n=0 for discrete-time signals.

e.g. if x(t) is an audio file, x(-t) is the same file but played backward.

Mirror







➢ Signals Transformation (1- Of Independent Variable (time))

2- Time Shift: $y(t) = x(t - t_o)$

It is the same signal in shape but moved either to right ($t_o > 0$, and called Time-Delay) or to the left ($t_o < 0$, and called Advance) of the original signal. e.g. Any application has a transmitter and multiple receivers.

Example:

The original signal f(t) at (t = -5) will occur at a new location of the signal f(t-2) at:

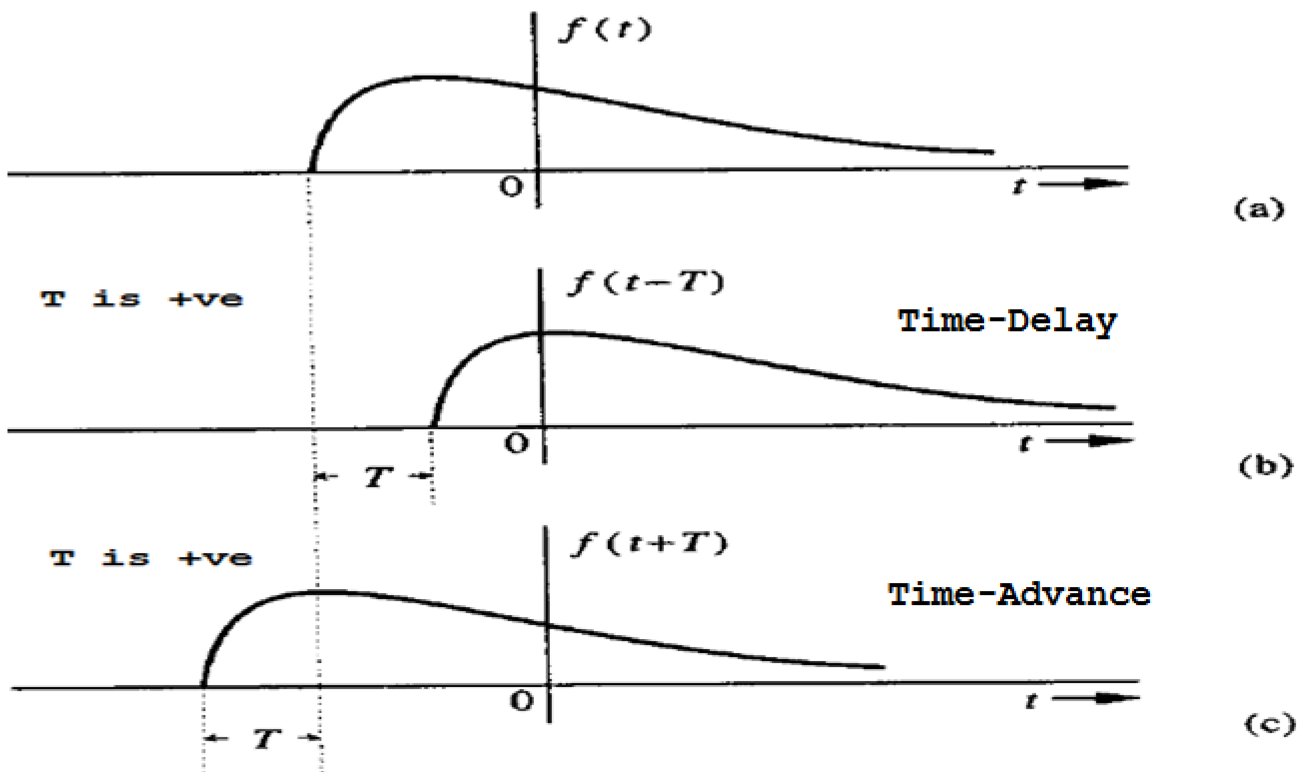
t-2 = -5 → t= -3

i.e. the new location at (t=-3)

Similarly, its new location of the signal f(t+2) at:

t+2=-5 → t= -7

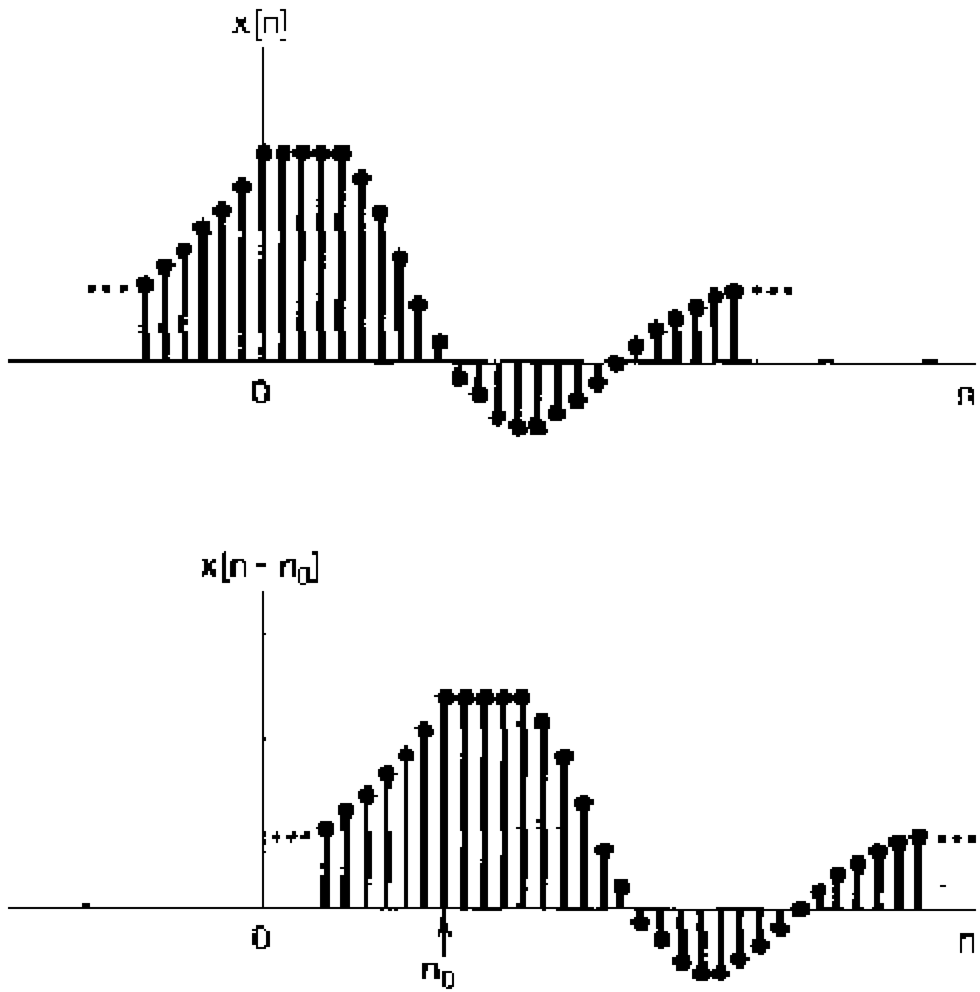
i.e. the new location will be at (t= -7)



Time shifting a signal.

➢ Signals Transformation (1- Of Independent Variable (time))

2- Time Shift:



Discrete-time signals related by a time shift. In this figure $n_0 > 0$, so that $x[n - n_0]$ is a delayed version of $x[n]$ (i.e., each point in $x[n]$ occurs later in $x[n - n_0]$).

3- <u>Time Scaling</u>:

$$y(t) = x(at)$$

It is a signal similar to the original signal in shape but it is either compressed **(a > 1, and called Time-Shrinking or Time-Compression)** or stretched **(a < 1, and called Time-Expansion or Time-Stretch)** version of the original signal.

e.g. An audio file either played at double speed or half speed.

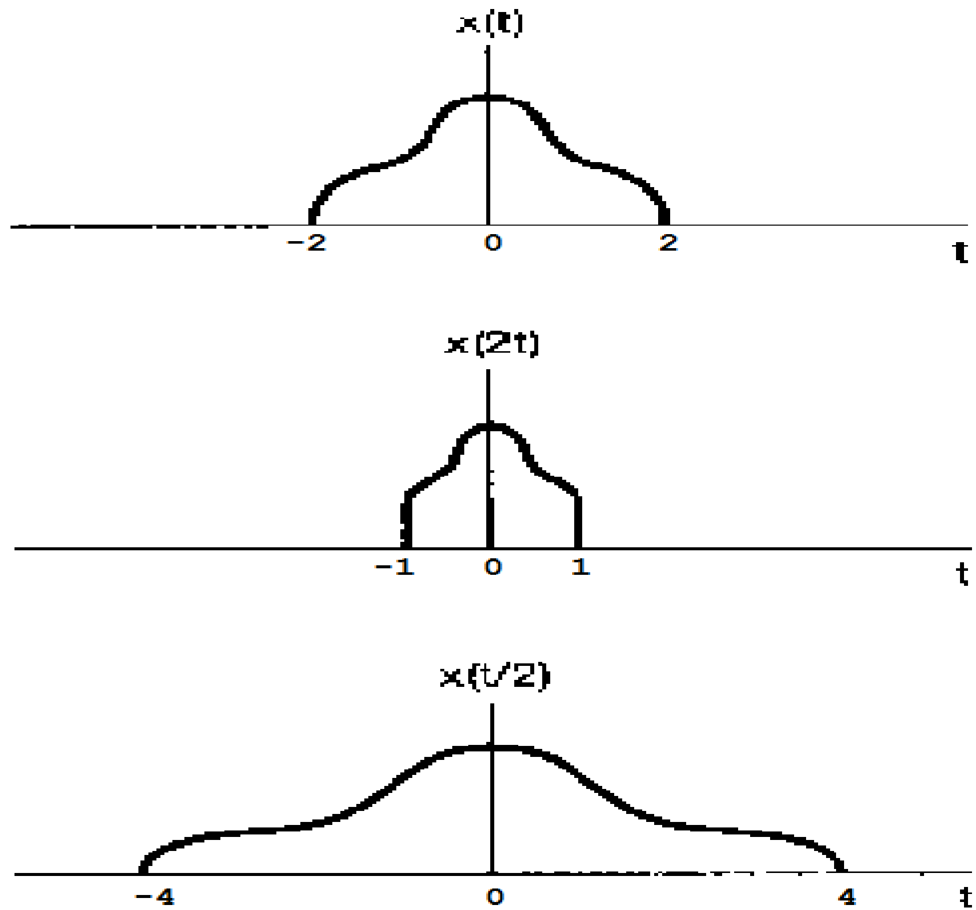


Figure 1.12 Continuous-time signals related by time scaling.

➢ Signals Transformation (1- Of Independent Variable (time))

In general:

$y(t) = x(a.t - b)$

If $b \neq 0$

There is a Time-Shift:

- -Time-delay (move to right) if (t)and (b) have different signs
- - Time advance(move to left) if (t)and (b) have same signs

If $a = -ve$

There is a Time-Reversal:

The result is similar in shape to the original signal but mirrored/reversed version of it

If $|a| \neq 1$

There is a Time-Scaling:

- -Time-Shrinking/Time-Compression if $|a| > 1$
- - Time-Expansion or Time-Stretch if $|a| < 1$

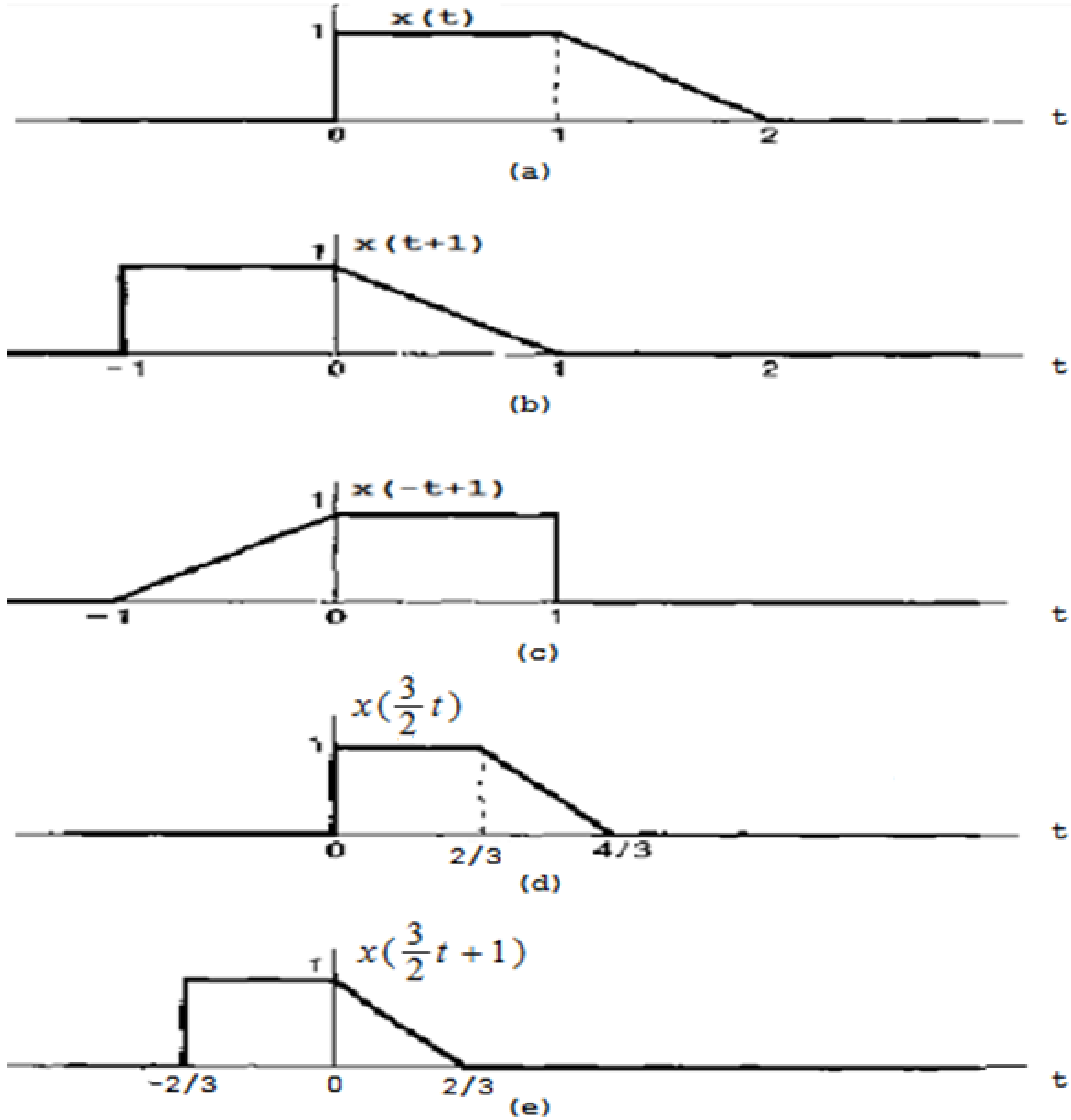
A systematic approach to get $y(t) = x(a.t - b)$ from $x(t)$:

1- Do the time-shift i.e. get → $y1(t) = x(t - b)$

2- Do the time-reverse/scaling on the resulting signal, i.e get → $y(t) = y1(a.t)$

➢ Signals Transformation (1- Of Independent Variable (time))

Example:



➢ Signals Transformation (1- Of Independent Variable (time))

Key-points approach to compute the independent transformations result:

1- Determine the Key-points in time.

Where the key-point is the point at which there is a change in the signal behavior.

In the last example in the previous slide for example three key-points:

At t=0, t=1, and t=2

2- let the required transformation as a Left-Hand-Side of an equation.

In the last example the second required transformation was (–t+1).

And sequentially find the new key-points by let each old key-point in the right-Hand-Side of the equation and solve for t to get the new key-point's location.

For the last example:

For t=0 → -t+1=0 → t=1 , then what was happened at t=0 will happen at t=1
For t=1 → -t+1=1 → t=0 , then what was happened at t=1 will happen at t=0
For t=2 → -t+1=2 → t=-1, then what was happened at t=2 will happen at t=-1

3- draw the result through connecting between key-points.

➢ Signals Transformation (1- Of Independent Variable (time))

If you have to perform composite transformation x(a.t + b) on the independent variable for a given signal x(t), what is the correct order:

1- perform time-shift then time-scaling/reverse ?

2- perform time-scaling/reverse then time-shift ?

And How to perform either choice? Justify your answer ...

➢ Signals Transformation (1- Of Independent Variable (time))

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And How to perform either choice? Justify your answer ...