Signals and Systems

Lecture # 3

Signal Transformations (followed), Odd and Even Signals, and Signal Periodicity

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Topics of the lecture:

- **Signals Transformations of Independent Variable.**
- **Even and Odd Signals.**
- **Signal Periodicity.**

➢ Signals Transformation (2- Of Dependent Variable)

An issue of fundamental importance in the study of signals and systems is the use of systems to process or manipulate signals. This issue usually involves a combination of some basic operations. In particular, we may identify two classes of operations, as described here.

B. *Operations performed on dependent variables.*

Amplitude scaling Let $x(t)$ denote a continuous-time signal. The signal $y(t)$ resulting from amplitude scaling applied to $x(t)$ is defined by

$$y(t) = cx(t) \qquad (1)$$

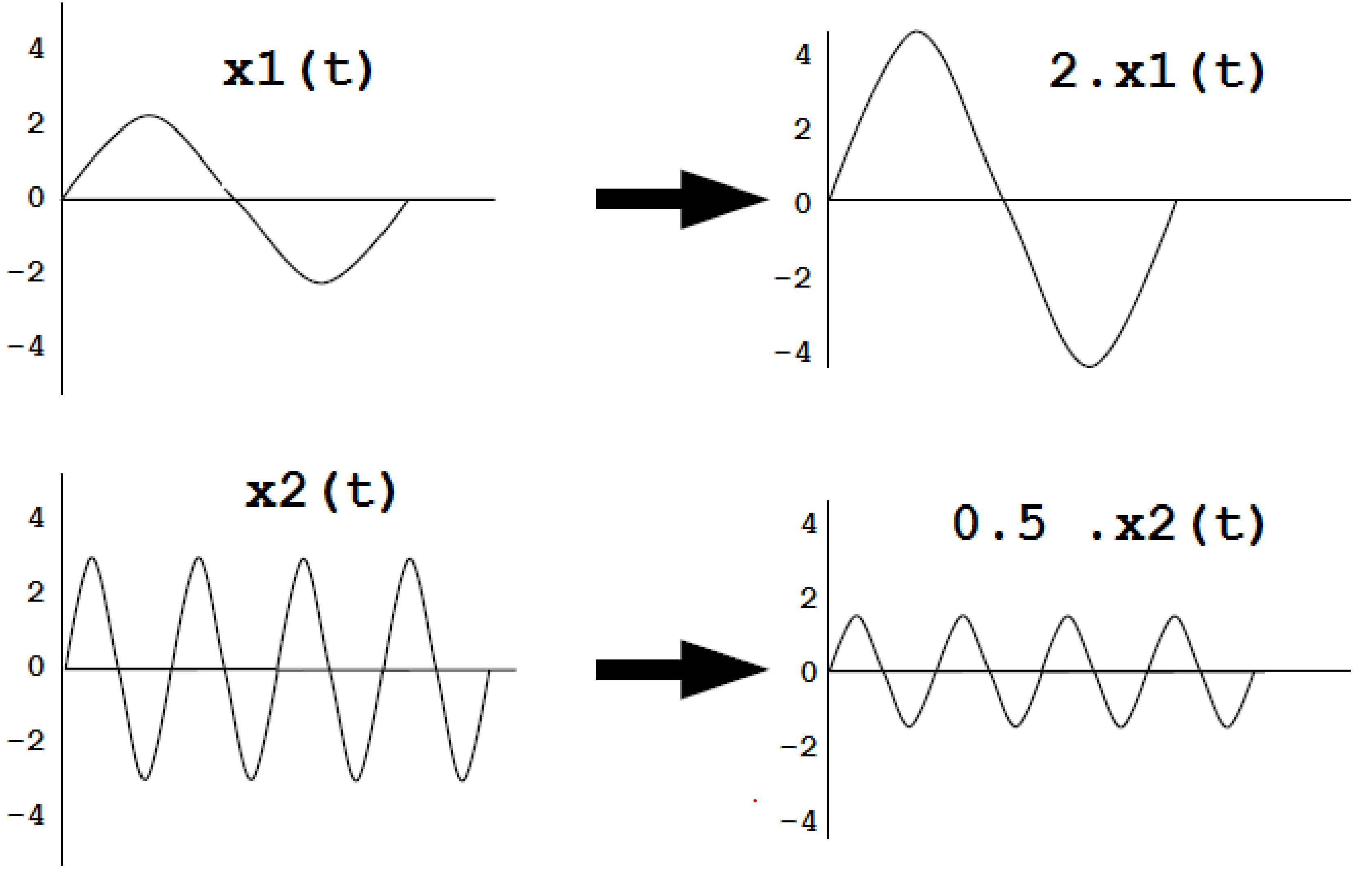
where c is the scaling factor. According to Eq. (1), the value of $y(t)$ is obtained by multiplying the corresponding value of $x(t)$ by the scalar c. A physical example of a device that performs amplitude scaling is an electronic *amplifier*. A resistor also performs amplitude scaling when $x(t)$ is a current, c is the resistance, and $y(t)$ is the output voltage.

In a manner similar to Eq. (1), for discrete-time signals we write

$$y[n] = cx[n]$$

➢ Signals Transformation (2- Of Dependent Variable)

Signal Scaling Example:



➢ Signals Transformation (2- Of Dependent Variable)

$\boxed{\textit{Addition}}$ Let $x_1(t)$ and $x_2(t)$ denote a pair of continuous-time signals. The signal $y(t)$ obtained by the addition of $x_1(t)$ and $x_2(t)$ is defined by

$$y(t) = x_1(t) + x_2(t) \qquad (2)$$

A physical example of a device that adds signals is an audio *mixer*, which combines music and voice signals.

In a manner similar to Eq. (2), for discrete-time signals we write

$$y[n] = x_1[n] + x_2[n]$$

$\boxed{\textit{Multiplication}}$ Let $x_1(t)$ and $x_2(t)$ denote a pair of continuous-time signals. The signal $y(t)$ resulting from the multiplication of $x_1(t)$ by $x_2(t)$ is defined by

$$y(t) = x_1(t)x_2(t) \qquad (3)$$

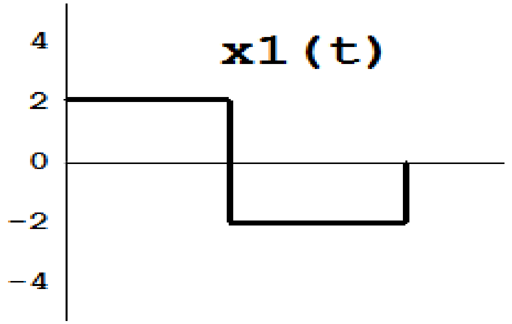
That is, for each prescribed time t the value of $y(t)$ is given by the product of the corresponding values of $x_1(t)$ and $x_2(t)$. A physical example of $y(t)$ is an *AM radio signal*, in which $x_1(t)$ consists of an audio signal plus a dc component, and $x_2(t)$ consists of a sinusoidal signal called a carrier wave.

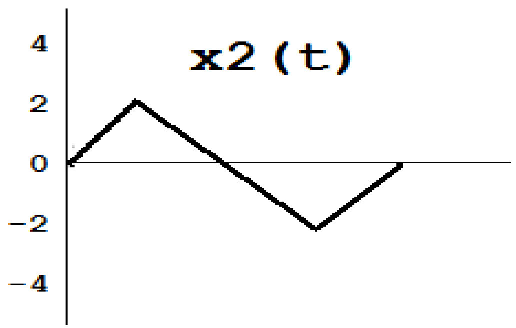
In a manner similar to Eq. (3), for discrete-time signals we write

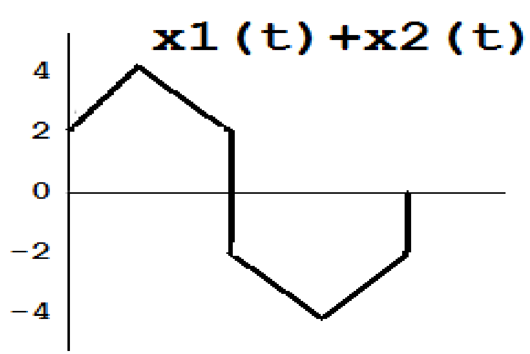
$$y[n] = x_1[n]x_2[n]$$

➢ Signals Transformation (2- Of Dependent Variable)

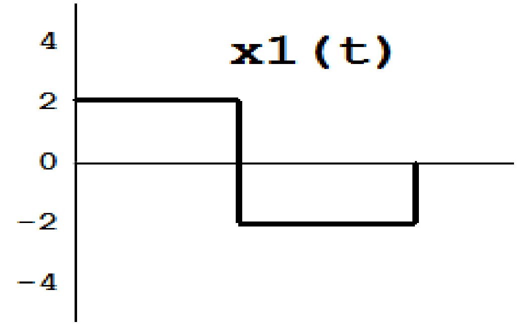
Signal Addition Example:

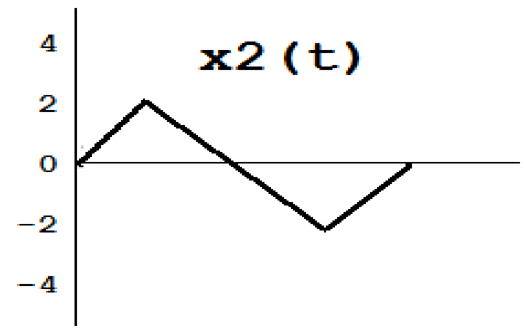


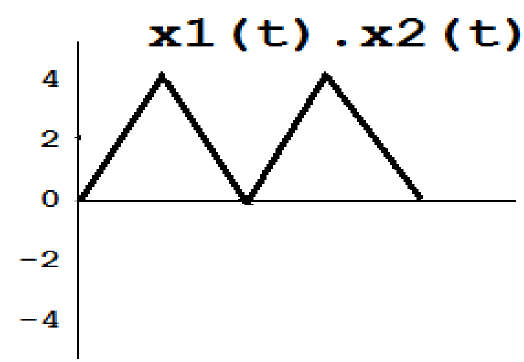




Signal Multiplication Example:







➢ Signals Transformation (2- Of Dependent Variable)

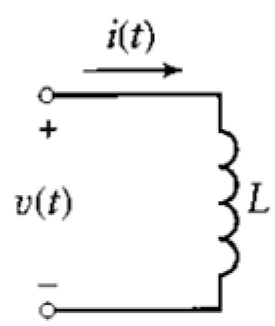
Differentiation Let $x(t)$ denote a continuous-time signal. The derivative of $x(t)$ with respect to time is defined by

$$y(t) = \frac{d}{dt} x(t) \qquad (4)$$

For example, an *inductor* performs differentiation. Let $i(t)$ denote the current flowing through an inductor of inductance L, as shown in Fig. . The voltage $v(t)$ developed across the inductor is defined by

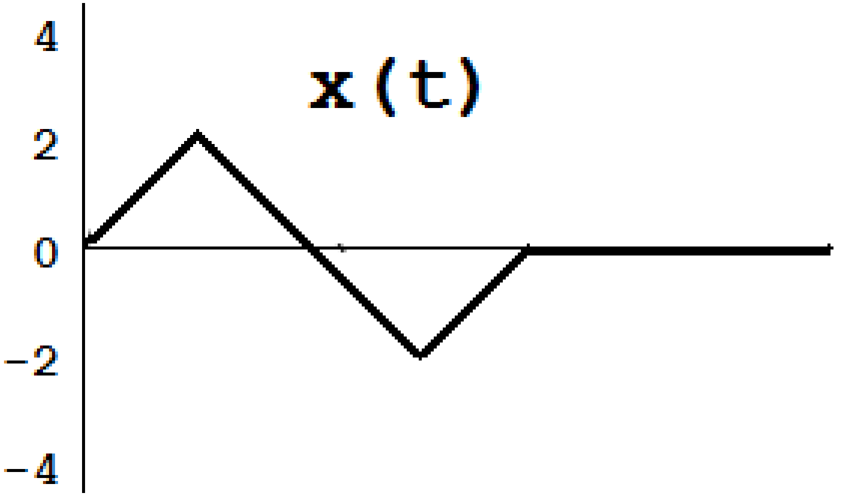
$$v(t) = L \frac{d}{dt} i(t) \qquad (5)$$

An **inductor**, also called a **coil** or **reactor**, is a passive two-terminal electrical component which resists changes in electric current passing through it. The Voltage across the coil is built according to **the rate of change in the input current**.

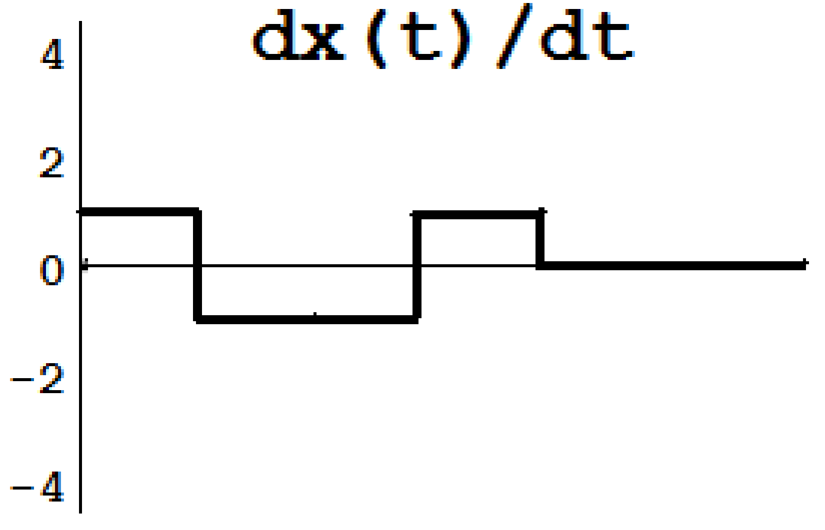


Inductor with current $i(t)$, inducing voltage $v(t)$ across its terminals.

➢ Signals Transformation (2- Of Dependent Variable)



Signal Differentiation Example:



➢ Signals Transformation (2- Of Dependent Variable)

$\boxed{\textit{Integration}}$ Let $x(t)$ denote a continuous-time signal. The integral of $x(t)$ with respect to time t is defined by

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad (6)$$

where τ is the integration variable. For example, a capacitor performs integration. Let $i(t)$ denote the current flowing through a capacitor of capacitance C, as shown in Fig. 2 The voltage $v(t)$ developed across the capacitor is defined by

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau \qquad (7)$$

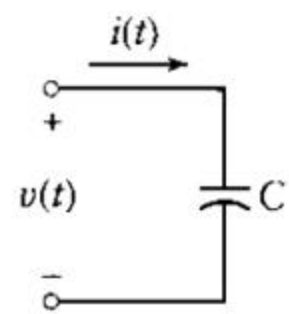
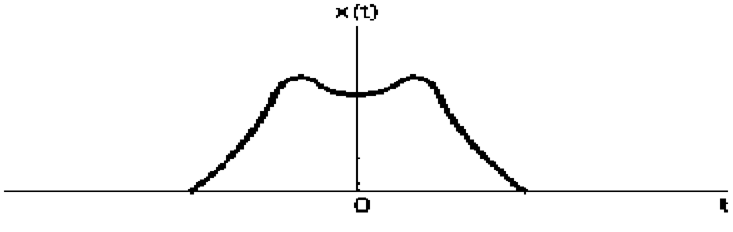


FIGURE 2 Capacitor with voltage $v(t)$ across its terminals, inducing current $i(t)$.

➢ Even and Odd Signals

The signal is called "Even" if
it is identical to its time-reverse.
i.e. The even signal has the property that:

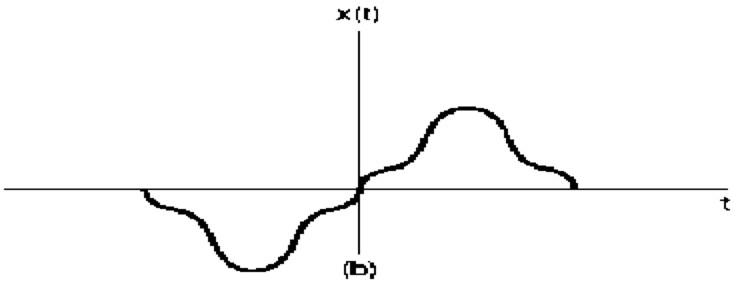
$$x(t) = x(-t)$$



(a)

The signal is called "Odd" if
it is mirrored around the point (0,0)
odd signal has the property that:

$$x(t) = -x(-t)$$



(b)

Odd signals must be zero at t=0

Any signal can be written as the sum of an odd signal and an even signal

$$x(t) = x_e(t) + x_o(t)$$

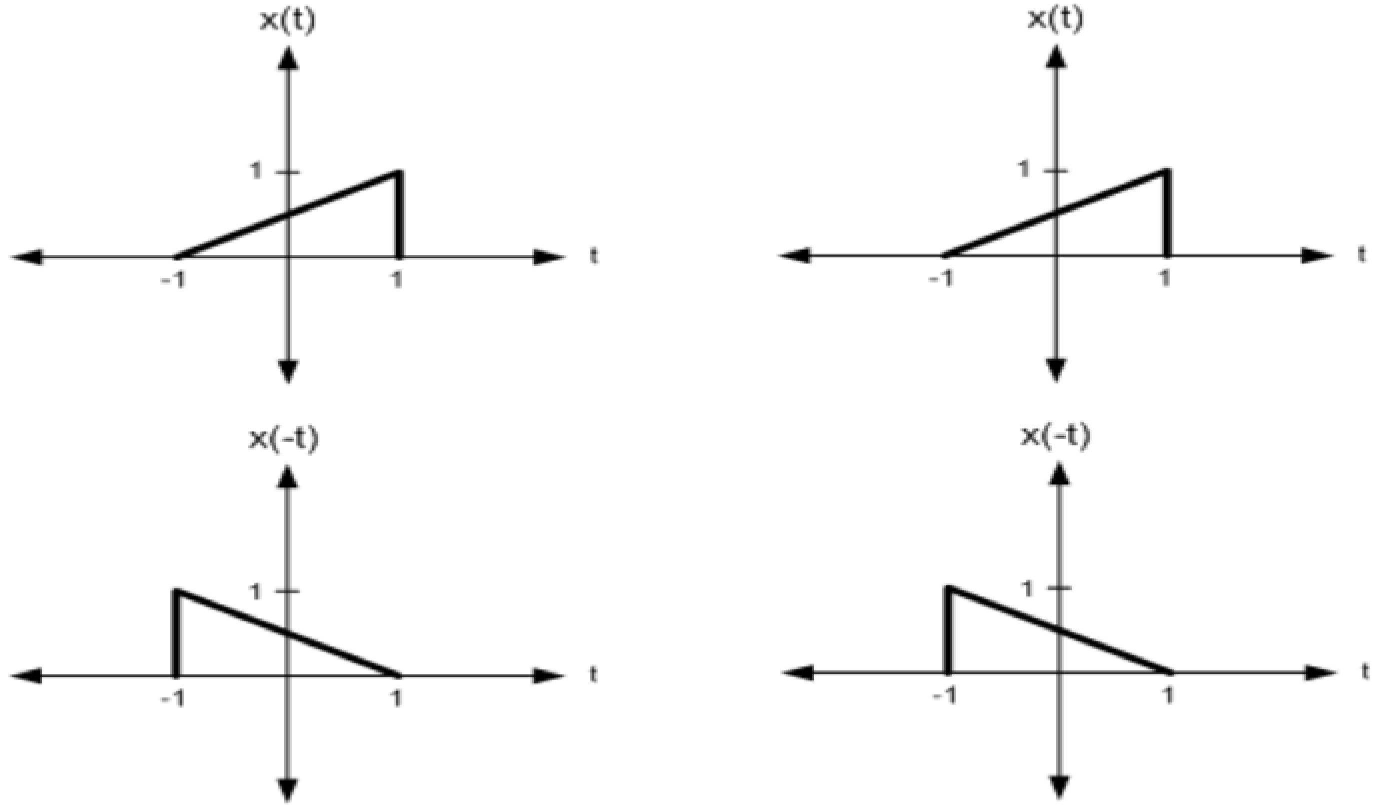
Where,

$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right) \longrightarrow \mathcal{E}v\{x(t)\}$$

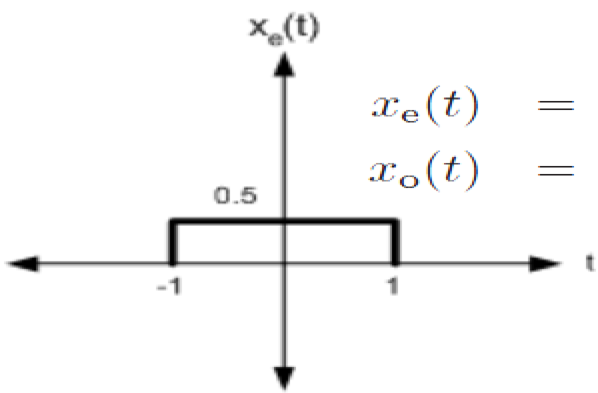
$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right) \longrightarrow \mathcal{O}d\{x(t)\}$$

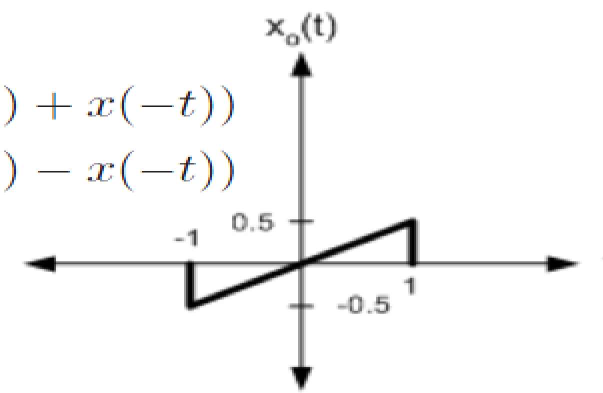
➢ Even and Odd Signals

Example:



$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$
$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$





➢ Even and Odd Signals

Note that

the product of two even signals or of two odd signals is an even signal

and that

the product of an even signal and an odd signal is an odd signal

➢ Even and Odd Signals

EXAMPLE

Determine whether or not the following functions are even or odd:

(a) $x(t) = t \cos t$
(b) $x(t) = \cos t \sin^2 t$
(c) $x(t) = t \sin t$

SOLUTION

(a) We have an odd function t multiplied by an even function $\cos t$. The product of an odd function with an even function is odd, so in this case $x(t)$ is odd.

(b) Let's look at this one in two steps. First, we note that $\cos t$ is even. Now $\sin t$ is odd, and we have $\sin^2 t = \sin t \sin t$, which is an odd function times an odd function. An odd function times an odd function is even, and so $\sin^2 t$ is even. In the end we have $x(t) = \cos t \sin^2 t$, which is an even function times an even function, so this signal is even.

(c) In this case we have an odd function times an odd function. We have $x(-t) = -t \sin(-t) = t \sin t$, so this is an even function.

➢ Periodic Signals

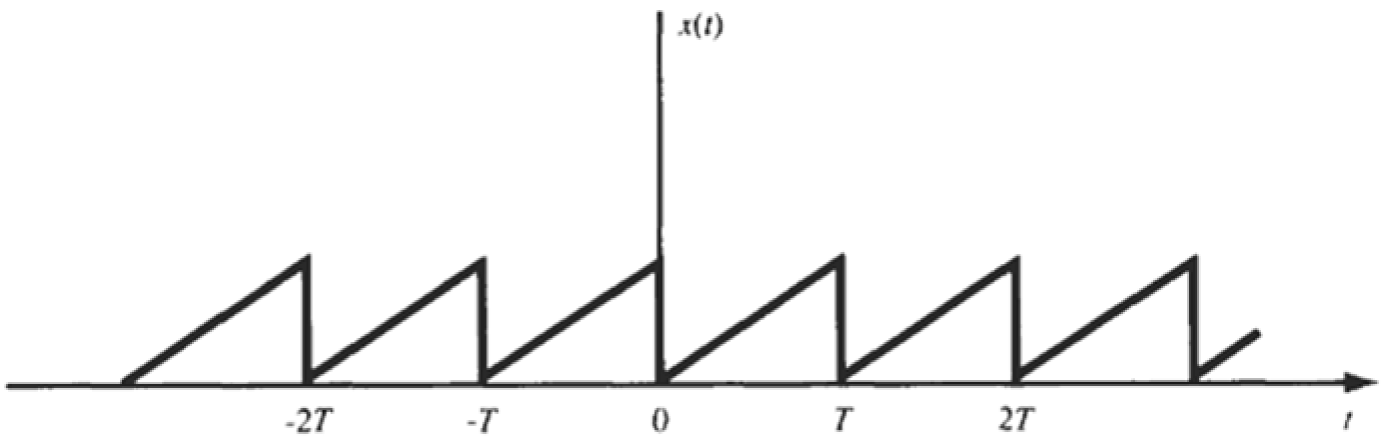
A continuous-time signal $x(t)$ is said to be *periodic with period* T if there is a positive nonzero value of T for which

$$x(t+T)=x(t) \qquad \text{all } t \tag{1}$$

An example of such a signal is given in Fig. (*a*). From Eq. (1) or Fig. (*a*) it follows that

$$x(t+mT)=x(t) \tag{2}$$

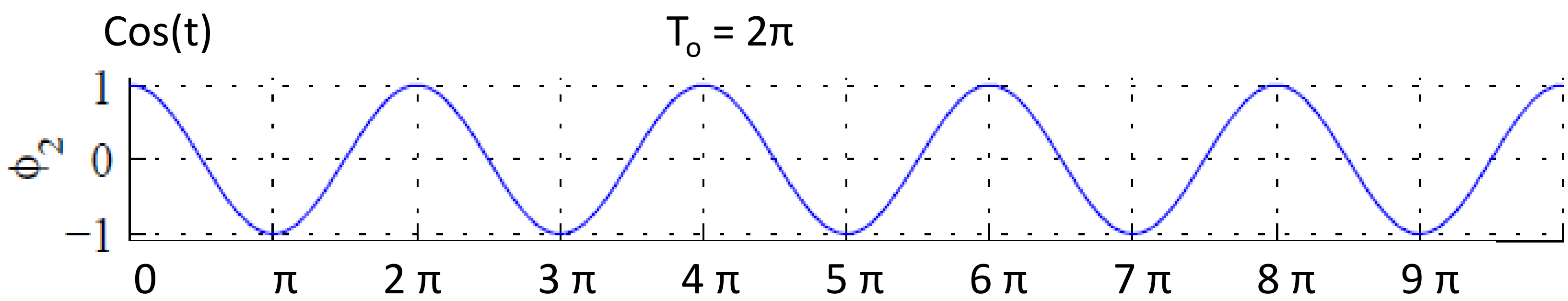
for all t and any integer m. The *fundamental period* T_0 of $x(t)$ is the smallest positive value of T for which Eq. (1) holds.

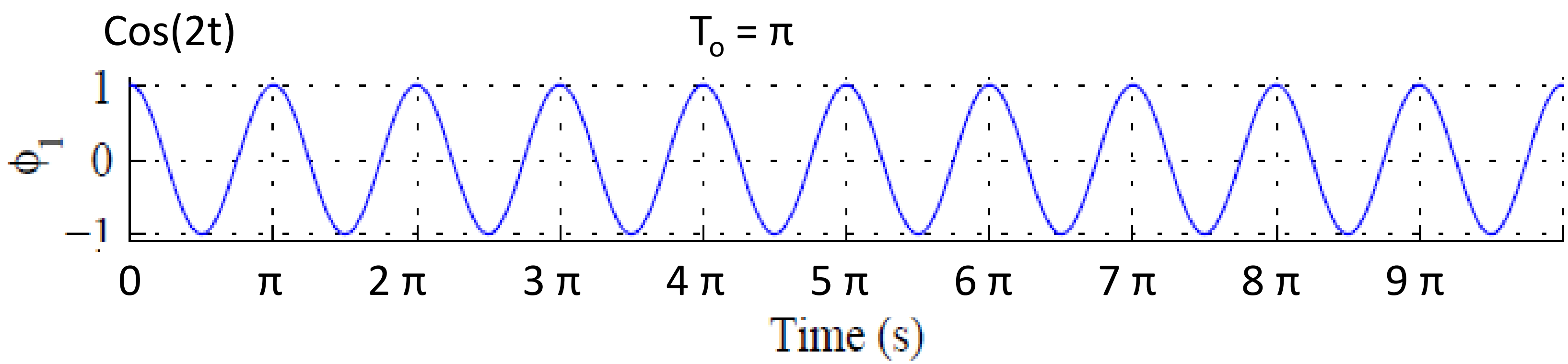


(*a*)

➢ Periodic Signals

Examples:





➢ Periodic Signals

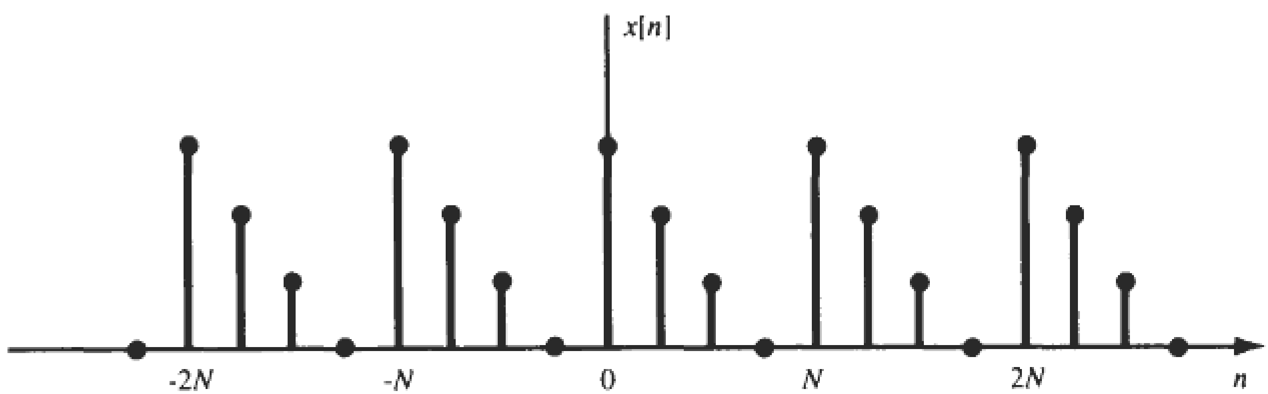
Periodic discrete-time signals are defined analogously. A sequence (discrete-time signal) $x[n]$ is *periodic with period N* if there is a positive integer N for which

$$x[n+N]=x[n] \qquad \text{all } n \tag{3}$$

An example of such a sequence is given in Fig. (*b*). From Eq. (3) and Fig. (*b*) it follows that

$$x[n+mN]=x[n] \tag{4}$$

for all n and any integer m. The fundamental period N_0 of $x[n]$ is the smallest positive integer N for which Eq. (3) holds.



(*b*)

➢ Periodic Signals

Note 1: the fundamental period of a constant continuous-time signal is undefined! As there is no smallest positive value of T. if you take any choice you find another smaller one e.g. 0.1, 0.01, 0.001, 0.0001…etc

Note 2: the fundamental period of a constant discrete-time signal is 1.

Note 3: the uniform sampling of a periodic continuous-time signal may not be periodic!

Note 4: the sum of two continuous-time periodic signals may not be periodic BUT, the sum of two discrete-time periodic signals is always periodic

Note 5: the sum of two periodic signals is periodic only if :

K T1 = L T2 ➔ $$\frac{T1}{T2} = \frac{L}{K} = \frac{Integer}{Integer} = Rational\ \ Number$$

➢ Periodic Signals

LCM: the Least Common Multiplier

To compute the LCM of a list of numbers:

1- begins by listing all of the numbers vertically in a table

2- The process begins by dividing all of the numbers by 2. If a number does not divide evenly, just rewrite the number again.

3- Now, check if 2 divides again.

4- Once 2 no longer divides, divide by 3.

5- If 3 no longer divides, try 5 then 7 ... etc (i.e. Prime numbers). Keep going until all of the numbers have been reduced to 1.

	2	2	3	7
4	2	1	1	1
7	7	7	7	1
12	6	3	1	1
21	21	21	7	1
42	21	21	7	1

Note: if any prime can not divide any row it is neglected. Like 5 in the above mentioned example.

6- Now, LCM is the multiplications of the numbers in the top row: $2 \times 2 \times 3 \times 7 = 84$