Signals and Systems

Lecture # 1

Introduction
and
Complex Numbers Review

Prepared by:

Dr. Mohammed Refaey

Topics of the lecture:

- **Course Outlines.**
- **Introduction.**
- **Review of Complex Numbers.**
  - **Imaginary Number.**
  - **Complex Numbers Representations.**
  - **Complex Conjugate and its Properties.**
  - **Complex Number Magnitude and its Properties.**
  - **Complex Numbers Mathematical Operations**

Course Outline.

- **The Course Textbooks will be:**
  1- **"Signals and Systems", 2nd Edition , 1997**
  A.V.Oppenheim & A.S. Willsky (Prentice Hall)
  2- **"Signals and Systems": 2nd Edition, 2011**
  Edward A. Lee & Pravin Varaiya

- **The Course prerequisites (include but not limited to):**
  The Calculus (especially differentiation and Integration), The Partial Fractions, The Complex Numbers, Trigonometry, and Differential Equations ... etc

- **The Course Total Degrees is 100:**
  - **60 % Final Exam.**
  - **20 % Midterm Exam.**
  - **20 % Two Quizzes and Assignments.**

➢ Introduction

Are you interested to study Signals and Systems?

Do not answer now!

Let us first know some applications of signals and Systems:

1- Communications. (e.g. the internet carrier signal, the mobile communications…etc)

2- Aeronautics علوم الطيران. (Study, design, and manufacturing of air flight-capable machines.)

3- Astronautics الملاحة الفضائية . (The science and technology of space flight.)

4- Circuit Design. (Design and test electrical circuits.)

5- Acoustics الصوتيات. (e.g. how to make the sound clear and effective to the audience)

6- Seismology علم الزلازل. (e.g. detecting earthquakes.)

7- Biomedical Engineering الهندسة الطبية. (e.g. design of medical imaging devices.)

8- Energy Generation and Distribution Systems. (e.g. Microwaves and Heaters.)

9- Chemical Process Control. (e.g. Adjusting the mix parameters according to sensors.)

10- Speech Processing. (e.g. Speaker Identification, Text to Speech, Speech Recognition.)

11- Image Processing. (e.g. Image restoration and enhancement…etc)

⋮

➢ Introduction

Are you involved? No

OK, let us see some examples:

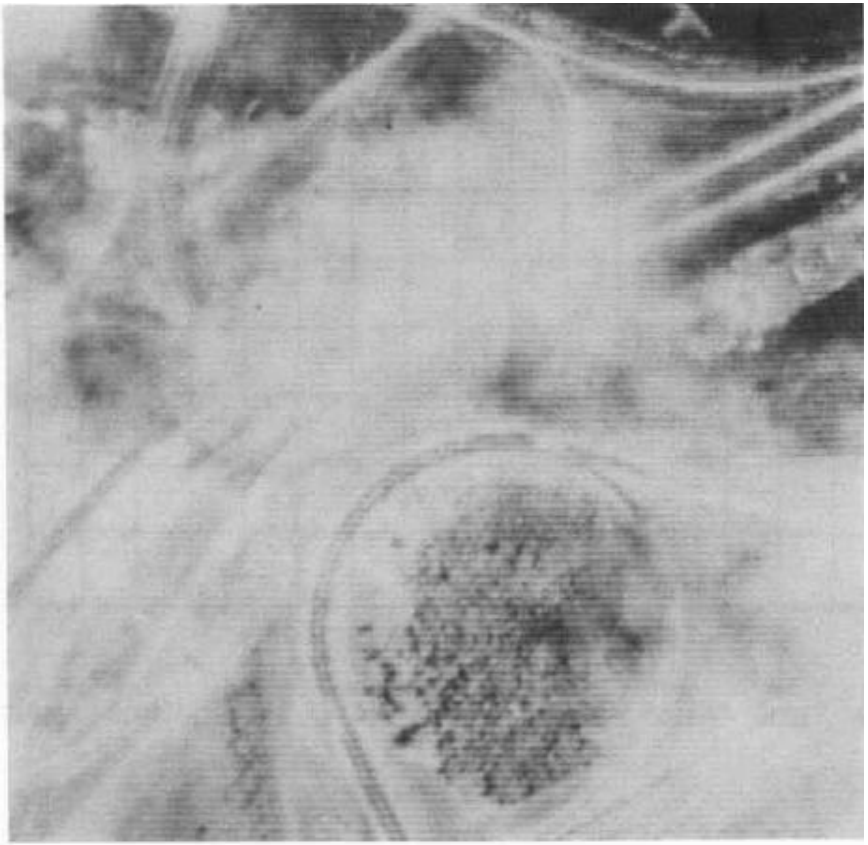
Aeronautics example
صناعة

Astronautics example
قيادة

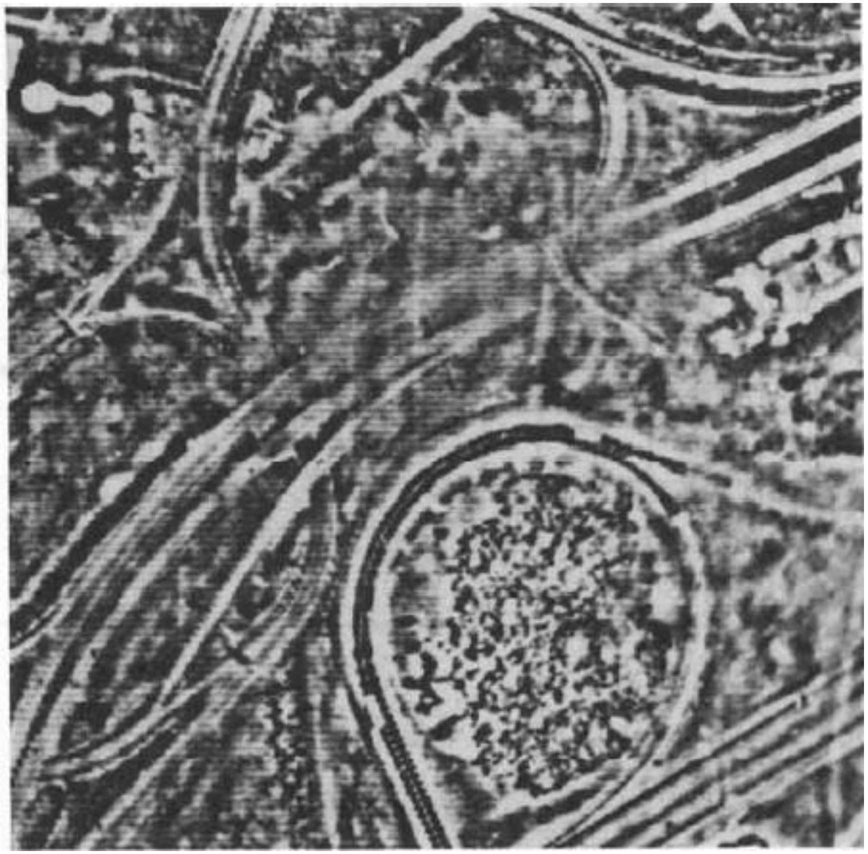
➢ Introduction

Are you involved? No

OK, let us see more examples:



Satellite image Before Enhancement

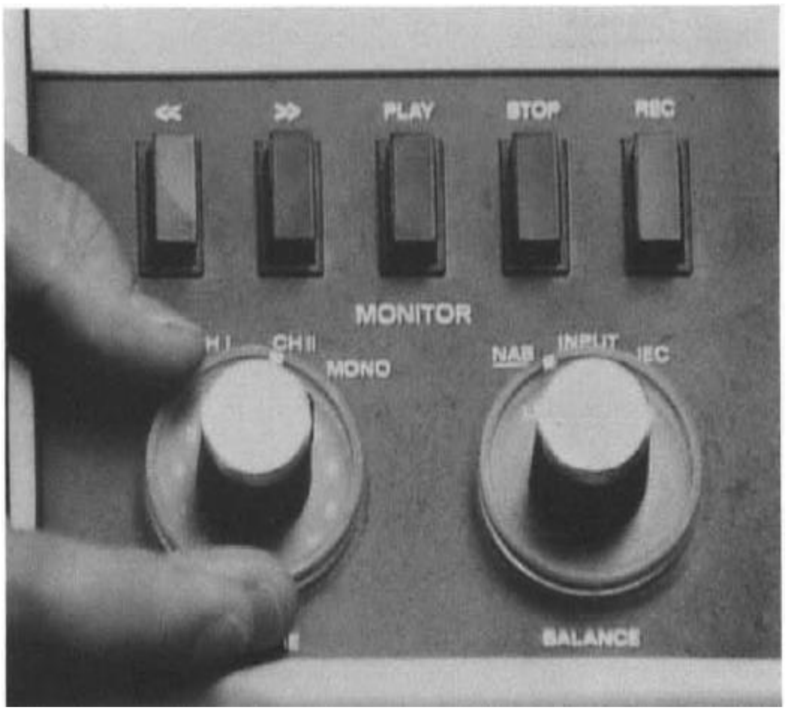


After Enhancement

➢ Introduction

Are you involved? No

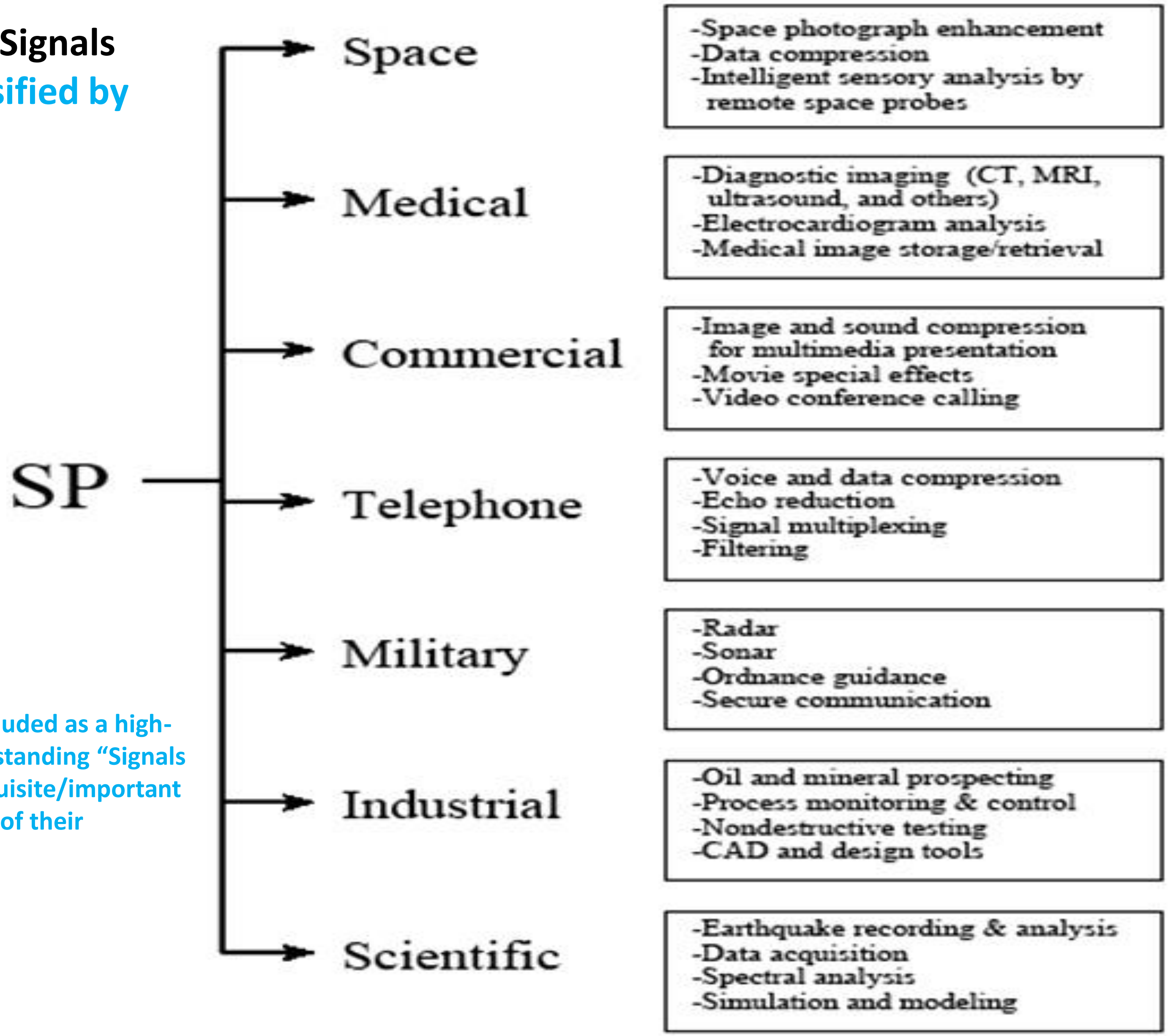
OK, let us see more examples:



Recording Enhancement Process

➢ Introduction

Applications of Signals Processing **classified by field**:



Note: it may not be included as a high-end product but understanding "Signals and Systems" is prerequisite/important to development/usage of their products/machines.

➢ Signals and systems Definition

Now let us know what is the signal? And what is the system?

The Signal:

It is a mean to convey information **(or *an abstraction of any measurable quantity*) that is usually have some form of variations. The contained information point to the behavior or nature of some phenomena.**

Mathematically:

The signal is a function of one or more independent variable **that maps a domain, often time or space, into a range, often a physical measure such as air pressure or light intensity. We usually called the independent variable time even if it is not actually time.**

The System:

***It is a tool that transform a signal to get another signal or process a signal to obtain a desired behavior or for extracting a piece of information*.**

Mathematically:

***A system is a function that maps signals from its domain—its input signals—into a signal in its range—its output signals*. The domain and the range are both sets of signals; we call a set of signals a signal space. Thus, systems are functions whose domains and ranges are signal spaces.**

➢ Signals and systems Definition

Examples:

Input	System	Output
Sequence of bits stored on a memory	Audio Player	High Quality Stereo Sound
Spoken Words	Interactive Voice Response (IVR)	Sequence of commands to a computer/machine
Sound	Ultrasound Imaging System	Images
Driver Actions	Automobile (Car, Train, Airplane)	Increased/Decrease Automobile's speed, and/or direction
Images	Image Processing System	Recognized Objects/ Detecting manufacturing errors

➢ Motivations

What are the types of problems that signals and systems techniques try to answer?

1- ***System Characterization*** **in detail to understand how it will respond to different inputs. (e.g. aircraft/ electrical circuit)**

2- ***System design*** **to react to inputs in a specific way. This usually involves a signal enhancement or restoration. (e.g. air traffic control tower in the airport to avoid the loud noise around when communicating with a pilot)**

3- ***Extracting specific pieces of information*****.** **(e.g. electrocardiogram estimation of heart rates)**

4- ***Design of Signals with particular properties*****.** **(e.g. the carrier signal in long distance communications)**

5- ***Modification and control*** **the characteristics** ***of a given system*****.** **(e.g. chemical process control through sensors)**

➢Review of Complex Numbers.

Imaginary Number:

An Imaginary Number, when squared, gives a negative result

imaginary^2 → negative

- **Unit Imaginary Number:**

Each numbering system should have a unit to can be counted !!!
The "unit" Imaginary Number (the equivalent of 1 for Real Numbers) is $\sqrt{-1}$ (the square root of negative one). In mathematics we use (*i*) (for imaginary) but in electronics use (*j*) (because (*i*) already means current, and the next letter after (*i*) is (*j*)). We will use (*j*) in this course!

It is "*imaginary* " and useful ?!

- imaginary numbers give us the ability to find solutions (roots) for *quadratic equations* like : $x^2 + 1 = 0$

- using imaginary numbers and real numbers together makes it a lot easier to do the calculations in many applications as they *encode the magnitude and phase together in a compact simpler form*.

- the imaginary numbers are *not imaginary, they are exist* and *fill a gab in math*. For example: *imaginary* x *imaginary* = *real* ➔ i.e. exist !

➢Review of Complex Numbers.

- **Dealing with imaginaries gives you the ability to work with the square root of negatives. BUT, in the same time you lose something !**

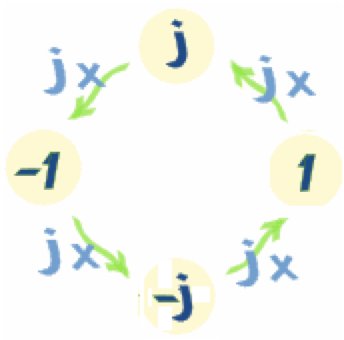
$$\because \quad j = \sqrt{-1}$$

$$and\ as\ (A^a)^b = \left(A^b\right)^a$$

$$then: j^2 = \left(\sqrt{-1}\right)^2 = \sqrt{(-1)^2} = \sqrt{1} = \pm 1$$

$$HERE\ this\ is\ not\ true, as\ you\ should\ always\ make\ j^2 = -1 \quad not\ +1$$

- **Interesting Property**:



$$\underline{\underline{Note}}: \quad j = j^5 = j^9 = \sqrt{-1} \quad ; \quad j^2 = j^6 = j^{10} = -1$$

$$j^3 = j^7 = j^{11} = -j \quad ; \quad j^4 = j^8 = j^{12} = 1$$

$$in\ general,\ j^m = j^{4\times n}.j^k = j^k, where \quad m, n, and\ k\ are\ \ integers$$

$$for\ example: \quad j^{99} = j^{96+3} = j^{4\times 24}.j^3 = j^3 = -j$$

➢Review of Complex Numbers.

The complex number:

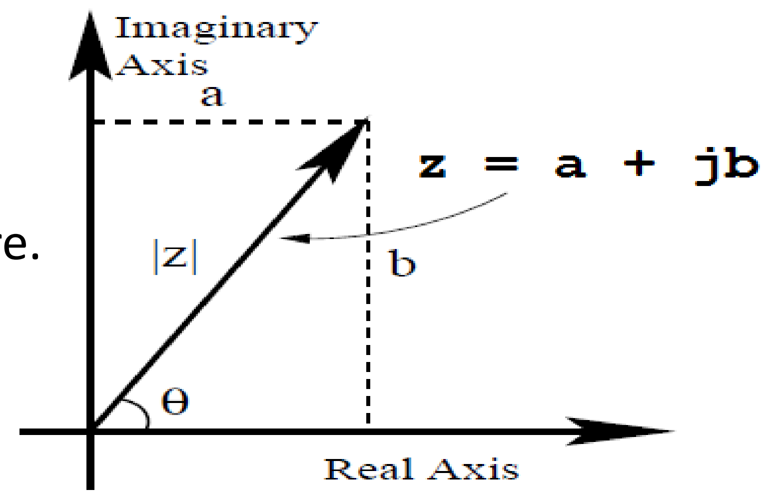
is made up of both real and imaginary components and its usually represented as: $Z = a + j\,b$

Where a is called the real part of the complex number Z, or $a = Re\{ Z \}$,

b is called the imaginary part of the complex number Z, or $b = Im\{ Z \}$,

and (j) is the square root of (-1), i.e. $j = \sqrt{-1}$

The Complex Plane:



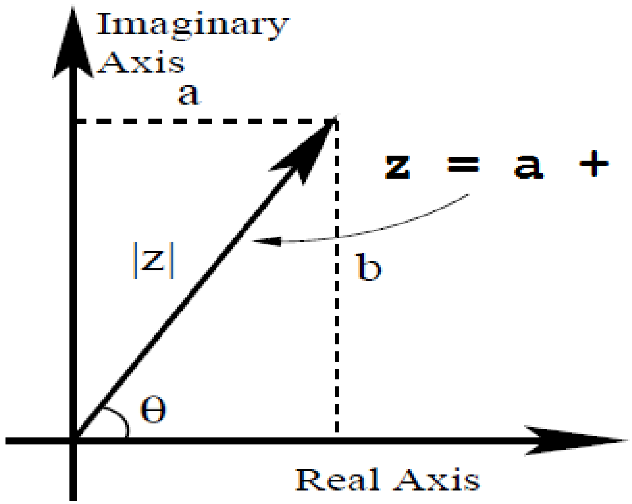
- A complex number can be visualized in a two-dimensional number plane, known as an **Argand diagram**, or the complex plane as shown in Figure.

- It is conventional to represent a complex number as a **vector** in the complex plane, usually called a **phasor**.

If $Z_1 = a + j\,b$ and $Z_2 = c + j\,d$, then $Z_1 = Z_2$ iff $a = c$ and $b = d$

➢Review of Complex Numbers.

The Complex Magnitude :



From the Figure, it can be easily seen (using the Pythagorean theorem) that the magnitude, or length, of the vector representing the complex number is:

$$(|z|)^2 = a^2 + b^2$$

$$|z| = \sqrt{a^2 + b^2}$$

It is *also* called **absolute value** or **modulus.**

The Phase Angle of a complex number:

It is the angle of the phasor of the complex number with the positive Real-Axis:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

It is usually called the Argument of the complex number. $\theta = Arg\{Z\} = \angle Z$

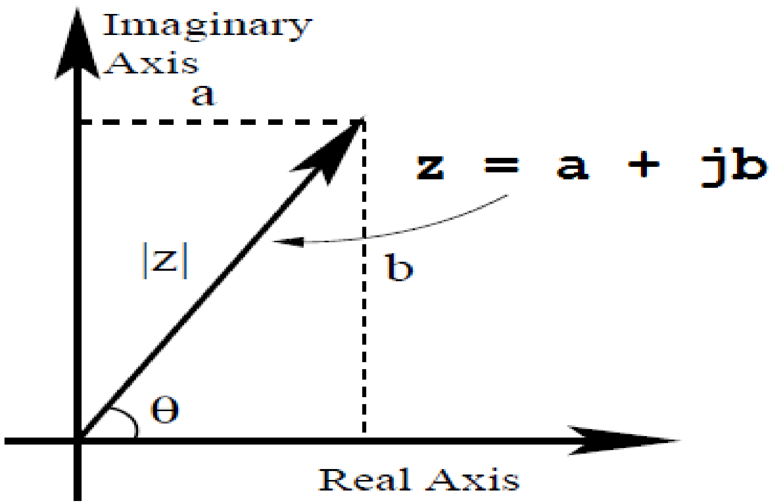
$$0 \leq \theta < 2\pi$$

➢Review of Complex Numbers.

Polar Form:

If you have Z = a + j b (Rectangular Form)

$$from\ the\ graph:$$



$$\sin(\theta) = \frac{b}{|Z|} \quad \Rightarrow \quad b = |Z|\sin(\theta)$$

$$and \quad \cos(\theta) = \frac{a}{|Z|} \quad \Rightarrow \quad a = |Z|\cos(\theta)$$

Then Z can be rewritten as :

$$\begin{aligned} Z &= |Z|\cos(\theta) + j\ |Z|\sin(\theta) \\ &= |Z|\left(\cos(\theta) + j\ \sin(\theta)\right) \end{aligned}$$

$$From\ Euler's\ Formula: e^{j\theta} = \cos(\theta) + j\ \sin(\theta)$$

$$Then \quad Z = |Z|e^{j\theta} = |Z|e^{j\angle Z}, \quad if \quad r = |Z| \quad \Rightarrow \quad \underline{\underline{Z = re^{j\theta}}}$$

$$which\ is\ \underline{\underline{called\ the\ polar\ form}}\ of\ the\ complex\ number\ (Z)$$

➢Review of Complex Numbers.

Addition and Subtraction:

If you have $Z_1 = a + j\,b$ and $Z_2 = c + j\,d$

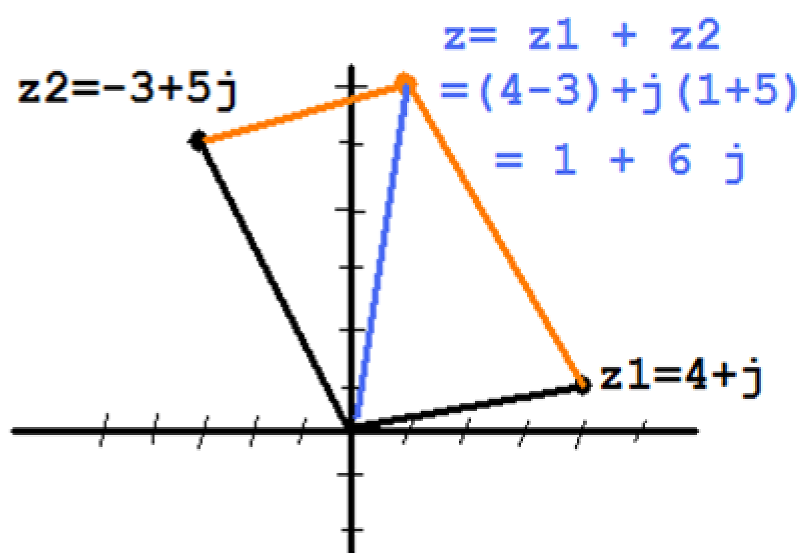
Then: $Z_1 + Z_2 = a + j\,b + c + j\,d = (a + c) + j\,(b + d)$

i.e. add the real part to the real part and the imaginary part to the imaginary part.

And : $Z_1 - Z_2 = (a - c) + j\,(b - d)$

i.e. subtract the real part from the real part and the imaginary part from the imaginary part

Complex numbers addition is similar to vectors sum.



➢Review of Complex Numbers.

Multiplication:

If you have $Z_1 = a + j\,b$ and $Z_2 = c + j\,d$

Then: $Z_1 \,.\, Z_2 = (a + j\,b)\,.\,(c + j\,d)$

$$= a\,(c + j\,d) + j\,b\,(c + j\,d) = a\,c + j\,a\,d + j\,b\,c - b\,d$$

$$= a\,c - b\,d + j\,(a\,d + b\,c)$$

The commutative and distributive properties hold for the product of complex numbers.

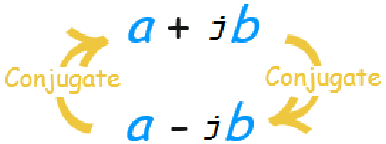
Complex Conjugation:

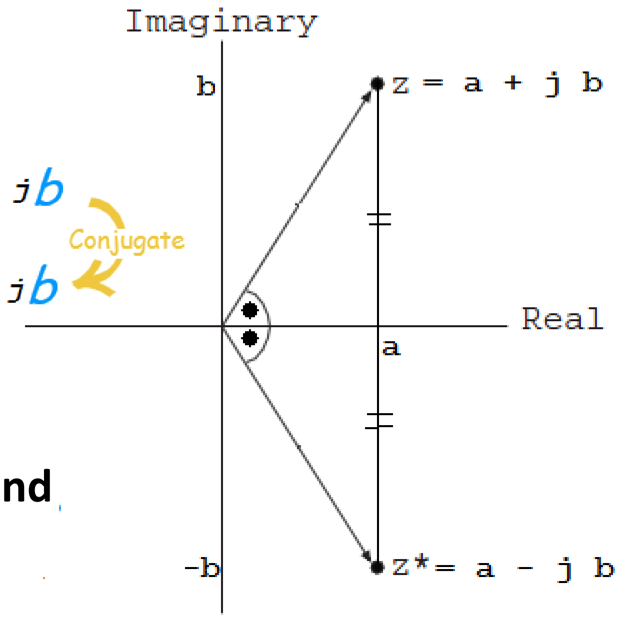
The complex conjugate of a complex number

$$Z = a + j\,b,$$

is denoted Z^*, and is defined by:

$$Z^* = a - j\,b$$





The complex conjugate Z^* has:

- the same real part but opposite imaginary part, and
- the same magnitude but opposite phase.

➢Review of Complex Numbers.

Complex Conjugation Properties:

- $(Z^*)^* = Z$
- $(Z_1 + Z_2)^* = Z_1^* + Z_2^*$
- $(Z_1 . Z_2)^* = Z_1^* . Z_2^*$
- if $Z_2 \neq 0$, $\left(\dfrac{z_1}{z_2}\right)^* = \dfrac{z_1^{\,*}}{z_2^{\,*}}$
- $\left(z^n\right)^* = \left(z^*\right)^n$
- if Z is real, then $Z = Z^*$

➢Review of Complex Numbers.

Complex Conjugation Properties:

$$prove\ that: \quad \left(Z^{n}\right)^{*} = \left(Z^{*}\right)^{n}$$

$$let \quad Z = r\left(\cos(\theta) + j\sin(\theta)\right)$$

$$then\ by\ DeMoivre's\ Theroem:$$

$$Z^{n} = \left[r\left(\cos(\theta) + j\sin(\theta)\right)\right]^{n} = r^{n}\left(\cos(n\theta) + j\sin(n\theta)\right)$$

$$\therefore \left(Z^{n}\right)^{*} = \left(r^{n}\left(\cos(n\theta) + j\sin(n\theta)\right)\right)^{*} = r^{n}\left(\cos(n\theta) - j\sin(n\theta)\right)$$

$$= \left[r\left(\cos(\theta) - j\sin(\theta)\right)\right]^{n}$$

$$= \left(Z^{*}\right)^{n}$$

Review of Complex Numbers.

The Complex Magnitude Properties:

- $|Z| = 0$ iff $Z = 0$
- $|Z| = |Z^*|$
- $|Z_1 . Z_2| = |Z_1| . |Z_2|$
- if $Z \neq 0$, then $\left|\frac{1}{z}\right| = \frac{1}{|z|}$
- $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$
- $Z . Z^* = |Z|^2$

Review of Complex Numbers.

Division:

If you have $Z_1 = a + j\,b$ and $Z_2 = c + j\,d$

Then:

$$\frac{Z_1}{Z_2} = \frac{a + jb}{c + jd}$$

Is usually rewritten by rationalizing the denominator to make it simpler. i.e. can be represented as a ratio.

$$\frac{Z_1}{Z_2} = \frac{a + jb}{c + jd} \times \frac{c - jd}{c - jd}$$

$$= \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2}$$

$\underline{Example:}$

$$Express \quad \frac{5 + 5j}{1 + 3j} \quad in\, the\;\; form\, of\;\; (a + jb)$$

We multiply the numerator and denominator by the conjugate of the denominator (1 - 3j).

$$\left(\frac{5 + 5j}{1 + 3j}\right) \times \left(\frac{1 - 3j}{1 - 3j}\right) = \frac{5 - 15j + 5j + 15}{1 - 3j + 3j + 9}$$

$$= \frac{20 - 10j}{10}$$

$$= \frac{20}{10} - \frac{10}{10}j = 2 - j$$

➢Review of Complex Numbers.

Examples to be solved on the board:

$$1-show\ that\ the\quad Arg\{Z^{*}\}=-Arg\{Z\}?$$

$$2-Get\ the\ value\ of\quad e^{-1+j\frac{\pi}{6}}\quad ?$$

$$3-Simplify\quad \frac{-20+\sqrt{-75}}{5}\ ?$$

$$4-Show\ that\ when\ multiplying\ two\ complex\ numbers,$$
$$actually\ we\ multiply\ their\ magnitudes\ and$$
$$add\ their\ phase\ angles?$$

$$5-Proof\ that:\qquad \cos^{2}(\theta)+\sin^{2}(\theta)=1$$
$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$

$1-show\ that\ the\quad Arg\{Z^*\}=-Arg\{Z\}?\qquad 2-Get\ the\ value\ of\quad e^{-1+j\frac{\pi}{6}}\quad ?$

$3 - \mathit{Simplify}\ \dfrac{-20 + \sqrt{-75}}{5}\ ?$

$4 - \mathit{Show\ that\ when\ multiplying\ two\ complex\ numbers,\ actually}$
$\mathit{we\ multiply\ their\ magnitudes\ and\ add\ their\ phase\ angles}\ ?$

$5-Proof\ \ that:$ $\cos^2(\theta)+\sin^2(\theta)=1$

$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$

➢Review of Complex Numbers.

<u>Assignment:</u>

1- Proof the complex conjugate properties?

2- Proof the complex magnitude properties?

$$3 - Proof\ \ that: \qquad \sin(\alpha+\beta)\ = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

➢Review of Complex Numbers.

4- Consider a series AC electrical circuit with two resistors and a capacitor. The output complex voltage

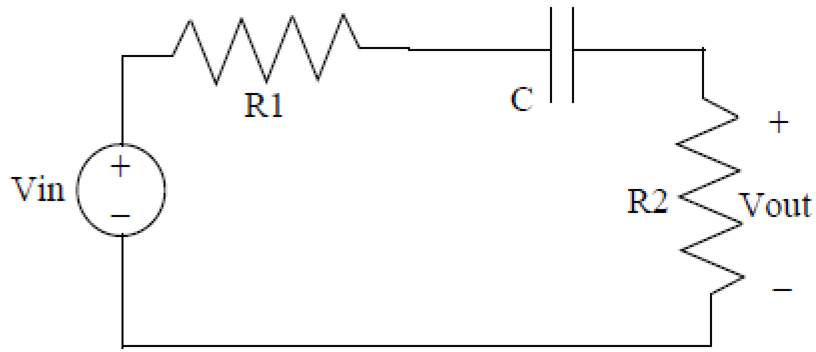


Figure : A simple AC circuit.

is related to the input complex voltage by the voltage divider law

$$\hat{V}_{out} = \frac{R_2}{R_1 + R_2 - i/(\omega C)} \hat{V}_{in}$$

If $R_1 = 100\Omega$, $R_2 = 200\Omega$, $C = 50\mu F$, and $\omega = 2\pi(60)$ cycles/s, and $\hat{V}_{in} = 100V$, then what is the

a) magnitude of the output voltage,
b) phase of the output voltage.
c) plot $|\hat{V}_{out}/\hat{V}_{in}|$ as a function of differents ω's.