Signals and Systems

Lecture # 5

Basic Signals

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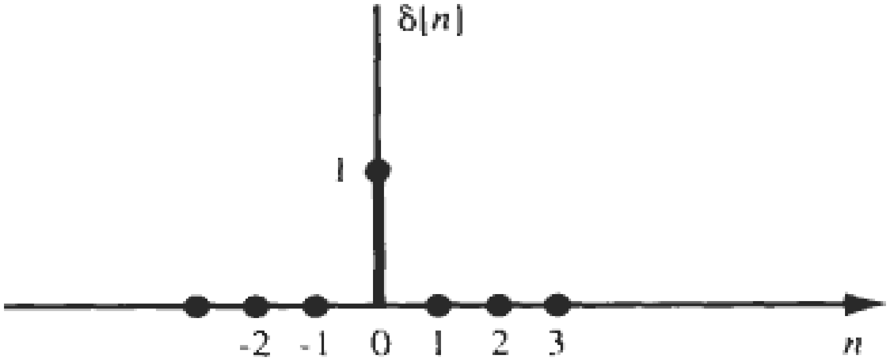
Topics of the lecture:

- **Discrete-Time Unit Impulse and Unit step Signals.**
- **Continuous-Time Unit Impulse and Unit step Signals.**

➢ Basic Signals

The discrete-time unit Impulse Signal:

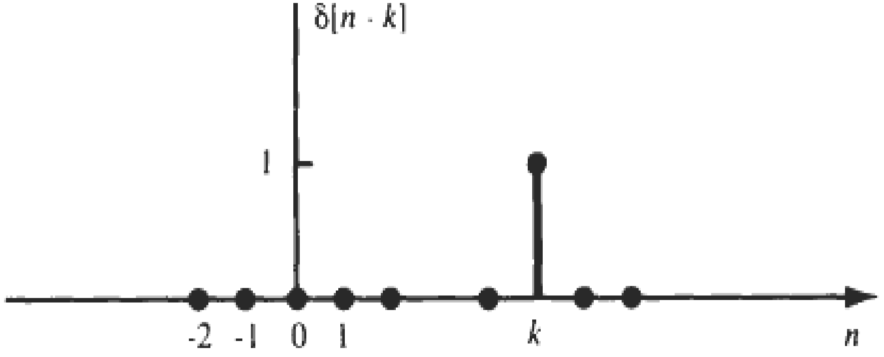
$$\delta[n] = \begin{cases} 1 & ; \quad n = 0 \\ 0 & ; \quad n \neq 0 \end{cases}$$



It is also called unit sample signal

The discrete-time shifted unit impulse Signal:

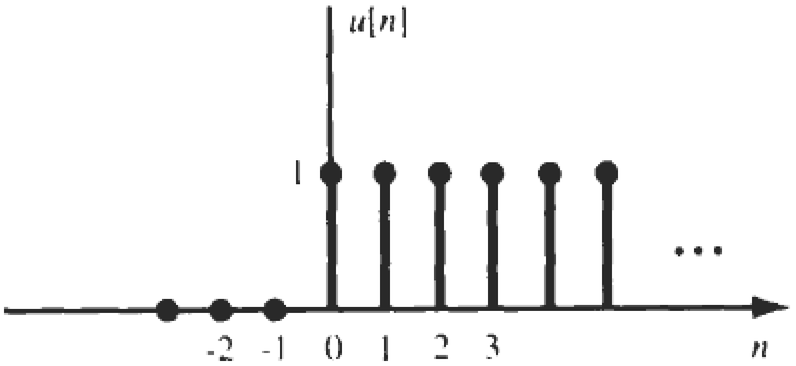
$$\delta[n-k] = \begin{cases} 1 & ; \quad n = k \\ 0 & ; \quad n \neq k \end{cases}$$



Basic Signals

The discrete-time unit step Signal:

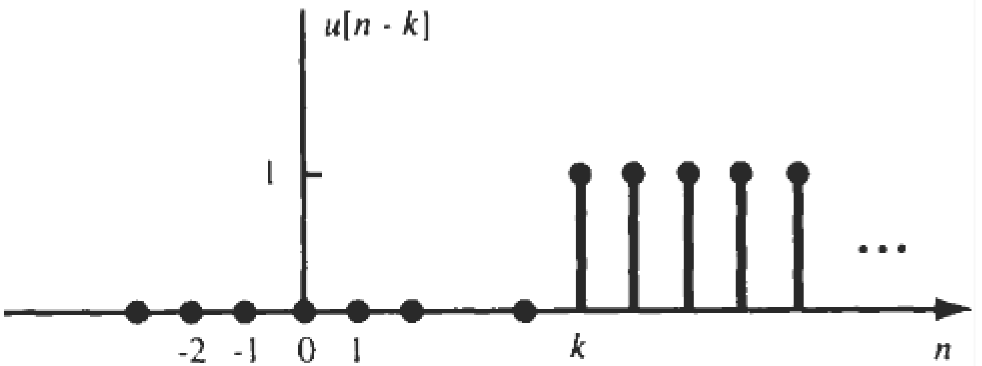
$$u[n]=\begin{cases}1 & ; \quad n\geq 0\\ 0 & ; \quad n<0\end{cases}$$



It is also called unit sequence signal

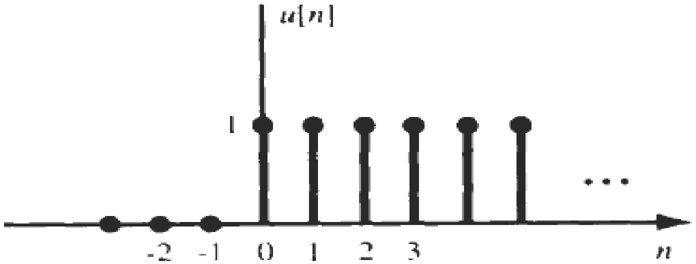
The discrete-time shifted unit step Signal:

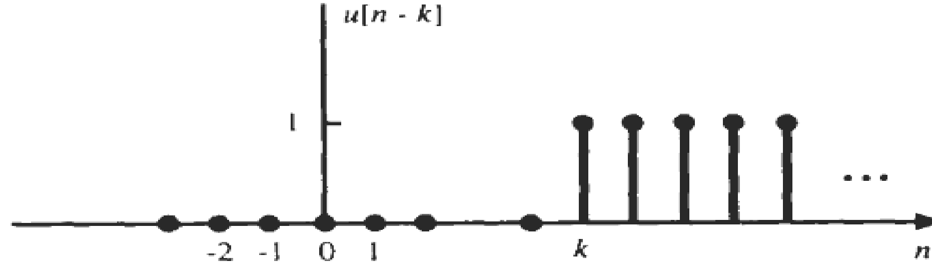
$$u[n-k]=\begin{cases}1 & ; \quad n\geq k\\ 0 & ; \quad n<k\end{cases}$$



➢ Basic Signals

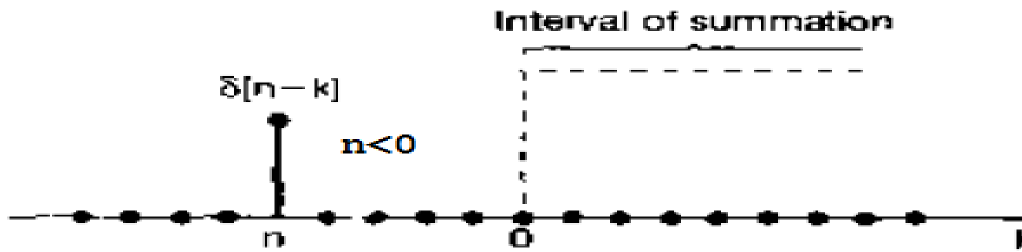
The relationship between discrete-time unit impulse and unit step signals:





$$\delta[n] = u[n] - u[n-1]$$

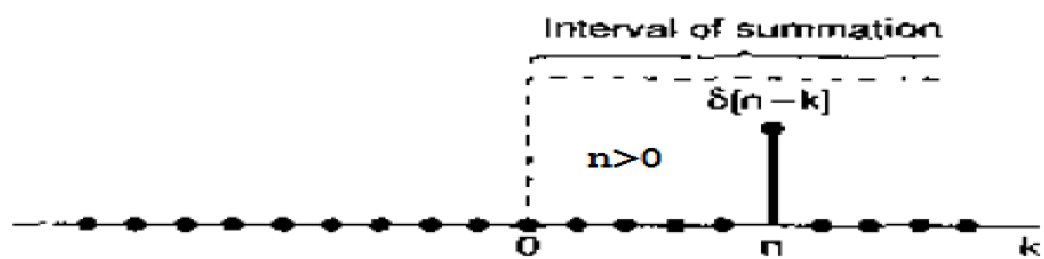
$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$



(a)

Let $m = n - k$ **OR** $k = n - m$

$$\therefore \quad u[n] = \sum_{k=+\infty}^{0} \delta[n-k]$$



(b)

$$\therefore \quad u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

➢ Basic Signals

The unit impulse sequence can be used to sample the value of a signal at $n = 0$. In particular, since $\delta[n]$ is nonzero (and equal to 1) only for $n = 0$, it follows that

$$x[n]\delta[n] = x[0]\delta[n] = x[0]$$

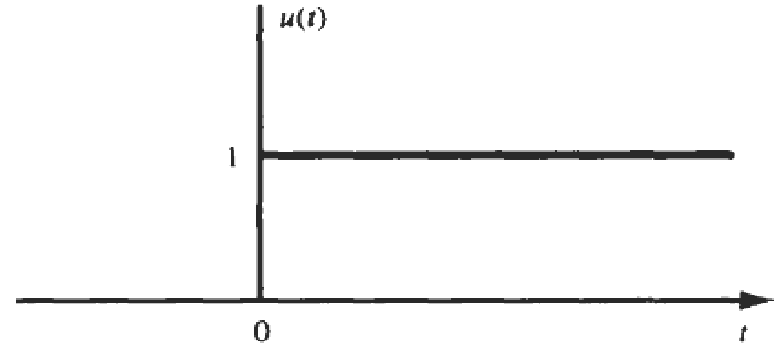
More generally, if we consider a unit impulse $\delta[n - n_0]$ at $n = n_0$, then

$$x[n]\delta[n - n_0] = x[n_0]\delta[n - n_0] = x[n_0]$$

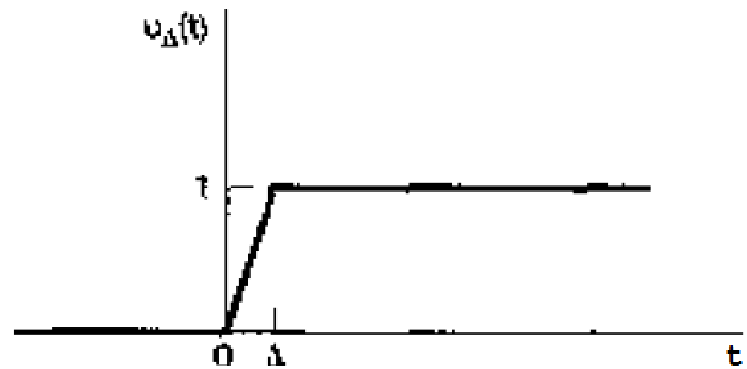
➢ Basic Signals

The continuous-time unit step Signal:

$$u(t)=\begin{cases}1 & ; \quad t>0\\ 0 & ; \quad t<0\end{cases}$$



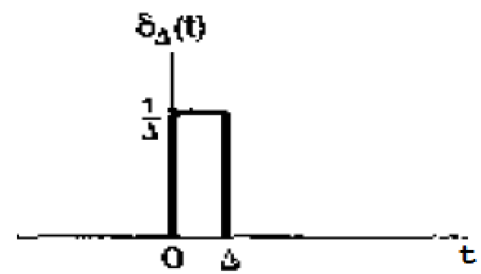
As the there is no such sudden change in real practical application. So an approximation of the ideal case is usually is what is happen.



Continuous approximation to the unit step, $u_{\Delta}(t)$.

Similarly to the discrete-time case the unit continuous-time impulse signal is the differentiation of the unit step signal

$$\delta_{\Delta}(t)=\frac{d}{dt}u_{\Delta}(t)$$

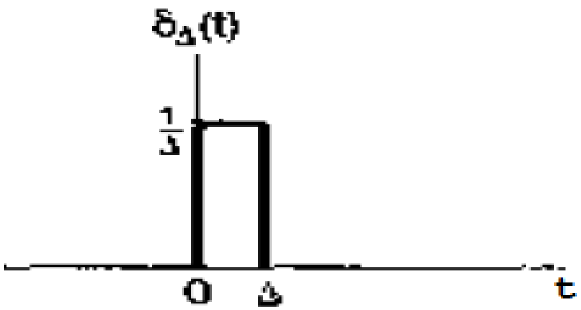


Derivative of $u_{\Delta}(t)$.

Basic Signals

The continuous-time unit impulse Signal:

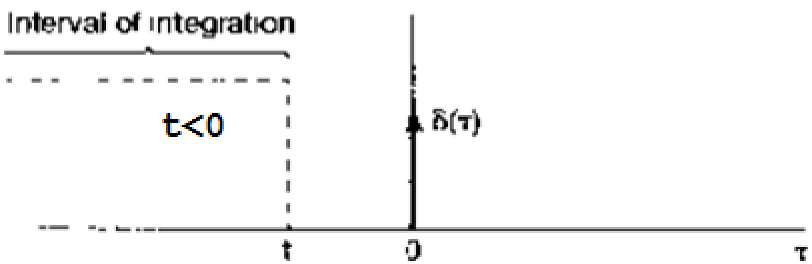
$$\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)$$



Derivative of $u_\Delta(t)$.

Note that $\delta_\Delta(t)$ is a short pulse of duration Δ and unit area. As the Δ becomes smaller the $\delta_\Delta(t)$ becomes narrower and higher maintaining the unit area. As the Δ goes to zero the $\delta_\Delta(t)$ goes to ∞

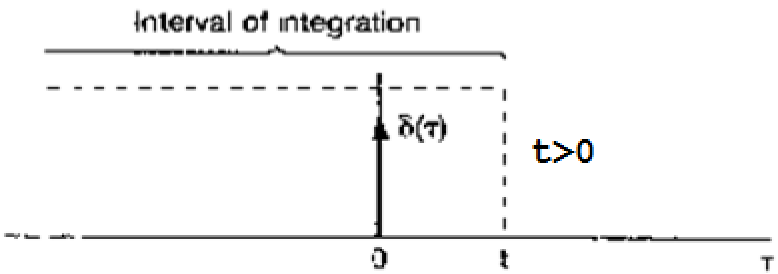
$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$



(a)

Then u(t) can be thought as a running integral of $\delta(t)$

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$



➢ Basic Signals

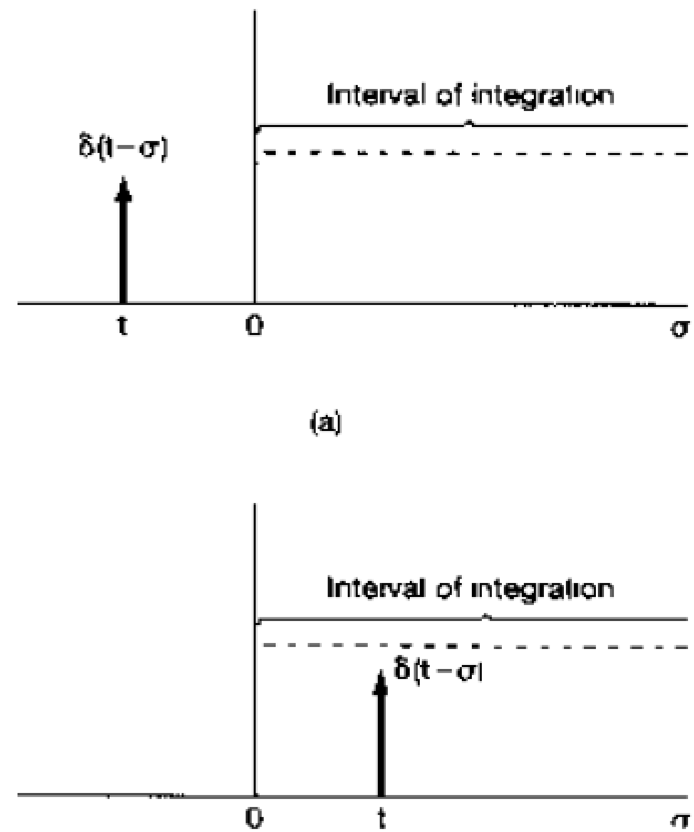
<u>The relationship between continuous-time unit impulse and unit step signals:</u>

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

Let ***σ = t – τ*** **OR** ***τ = t - σ***

$$u(t) = \int_{\infty}^{0} \delta(t-\sigma)(-d\sigma)$$

$$u(t) = \int_{0}^{\infty} \delta(t-\sigma)\,d\sigma$$



➢ Basic Signals

$$x_1(t) = x(t)\delta_\Delta(t).$$

In Figure (a) we have depicted the two time functions $x(t)$ and $\delta_\Delta(t)$, and in Figure (b) we see an enlarged view of the nonzero portion of their product. By construction, $x_1(t)$ is zero outside the interval $0 \le t \le \Delta$. For Δ sufficiently small so that $x(t)$ is approximately constant over this interval,

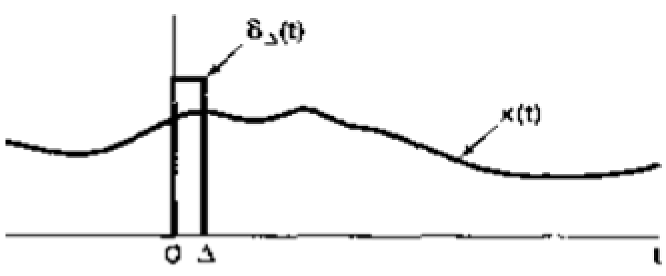
$$x(t)\delta_\Delta(t) \approx x(0)\delta_\Delta(t).$$

Since $\delta(t)$ is the limit as $\Delta \to 0$ of $\delta_\Delta(t)$, it follows that

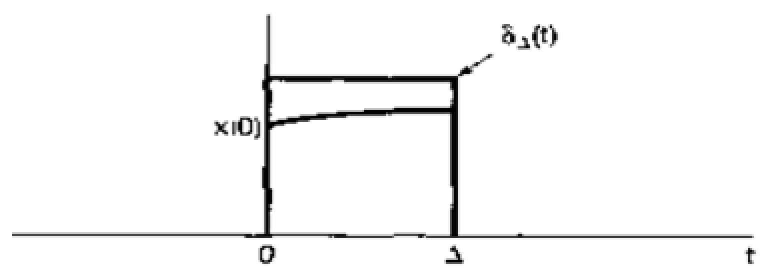
$$x(t)\delta(t) = x(0)\delta(t).$$

By the same argument, we have an analogous expression for an impulse concentrated at an arbitrary point, say, t_0. That is,

$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0).$$



(a)



(b)