Signals and Systems

Lecture # 7

System Properties (continued)

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Topics of the lecture:

- **Systems Properties.(continued)**
  5. **Time-Invariance**
  6. **Linearity**

➢ Systems Properties

5- Time-Invariance:

The system is said to be time-invariant if the behavior and characteristics of the system are (not change) fixed over time.

Example: in RC circuit if the values of resistance (R) and capacitance (C) are not changed over time then the RC circuit will be time-invariant. Then you expect to get the same results if you repeat the same experiment at two different times. But, if the values of (R) and (c) changed/fluctuate over time, then the results of repeated experiment will not be the same as the system becomes time-variant.

In signals and systems language, the system is said to be time-invariant if a time-shift in the input signal results in identical time-shift in the output signal.

So, any time-varying gain system is not time-invariant nor stable.

➢ Systems Properties

5- Time-Invariance:

1- let : $$x_1(t) \xrightarrow{S} y_1(t)$$

And let $$x_2(t) = x_1(t - t_o) \xrightarrow{S} y_2(t)$$

2- Get $y_2(t)$ in terms of $x_1(t)$ using the system equation. → I

3- Get the expression $y_1(t\text{-}t_o)$ by replacing each (t) by $(t\text{-}t_o)$ → II

4- check if the result of I and II are equal?

- **If Yes → time-invariant system.**
- **If No → not time-invariant system**

➢ Systems Properties

5- Time-Invariance:

Examples to be solved on the board :

$$y(t) = \sin(x(t))$$

$$y[n] = n\,x[n]$$

$$y(t) = x(2t)$$

➢ Systems Properties

5- Time-Invariance: $y(t) = \sin(x(t))$

➢ Systems Properties

<u>5- Time-Invariance:</u> $y[n] = n\,x[n]$

➢ Systems Properties

5- Time-Invariance: $y(t) = x(2t)$

➢ Systems Properties

6- Linearity:

The system is said to be linear if :

$$x_1(t) \xrightarrow{S} y_1(t)$$

$$x_2(t) \xrightarrow{S} y_2(t)$$

And system has the following two properties:

1- Additive property: $x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$

2- Scaling/Homogeneity property:

$$ax_1(t) \xrightarrow{S} ay_1(t)$$

The two conditions can be meet together through satisfying the superposition property that:

$$ax_1(t) + bx_2(t) \xrightarrow{S} ay_1(t) + by_2(t)$$

i.e. a linear combination of inputs result in the same linear combination of outputs.

➢ Systems Properties

Linearity check algorithm:

1- let :

$$x_1(t) \xrightarrow{S} y_1(t)$$
$$x_2(t) \xrightarrow{S} y_2(t)$$

And let

$$x_3(t) = ax_1(t) + bx_2(t) \xrightarrow{S} y_3(t)$$

2- Get y_3(t) in terms of x_1(t) and x_2(t) using the system equation. → I

3- Get the expression ay_1(t) + b y_2(t) in terms of x_1(t) and x_2(t). → II

4- check if the result of I and II are equal?

- **If yes → linear system.**
- **If No → not linear system.**

➢ Systems Properties

6- Linearity:

Examples to be solved on the board:

1- $y(t) = t\,x(t)$

2- $y(t) = x^2(t)$

3- $y[n] = Re\{x[n]\}$

4- $y[n] = 2x[n] + 3$

➢ Systems Properties

6- Linearity:

1- $$y(t) = t\,x(t)$$

➢ Systems Properties

<u>6- Linearity:</u>

2- $$y(t) = x^2(t)$$

➢ Systems Properties

6- Linearity:

$$y[n] = Re\{x[n]\}$$

➢ Systems Properties

6- Linearity:

$$y[n] = 2x[n] + 3$$

➢ Review on topics till now

Topics covered:

1- Complex Numbers.

2- Signals and Systems definitions and classifications.

3- Energy and Power.

4- Signals Transformations both for dependent and independent variables.

5- Even and Odd signals.

6- Periodic Signals.

7- Continuous-time Exponential Signals.

8- Discrete-time Exponential Signals.

9- The differences between Continuous-time Exponential and Discrete-time Exponential Signals.

10- Unit impulse and unit step signals.

11- System Properties.