HW3 (Due: 04/03/2023)

Problem 1.

Consider the following system with two states $s_k \in \{s^1 = 0, s^2 = 1\}$.
There are two possible actions: a^1 and a^2. The transition probabilities can be expressed as:

$$p(s'|s,a^1)\begin{cases} 1 & s = 0, s' = 0 \\ 0 & s = 0, s' = 1 \\ 0 & s = 1, s' = 0 \\ 1 & s = 1, s' = 1 \end{cases} \qquad p(s'|s,a^2)\begin{cases} 0 & s = 0, s' = 0 \\ 1 & s = 0, s' = 1 \\ 1 & s = 1, s' = 0 \\ 0 & s = 1, s' = 1 \end{cases}$$

Reward function is as follows: $\begin{cases} moving\ to\ state\ s^2: +1 \\ moving\ to\ state\ s^1: 0 \\ action\ a^1\ and\ a^2: 0 \end{cases}$

Start with a random policy $\pi^0(s^1) = a^1, \pi^0(s^2) = a^1, \gamma = 0.9, \theta = 0.85.$ Use Policy Iteration to compute $\pi^1(s^1),\ \pi^1(s^2)$. Use $V_0(s^1) = V_0(s^2) = 0$, for initialization of Policy Evaluation.

Problem 2.

Consider the problem defined in Problem 1.

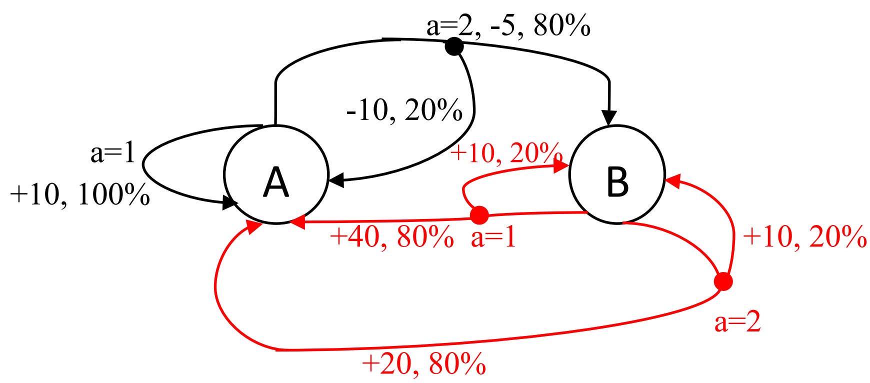
a) Given $\begin{bmatrix} V_0(s^1) \\ V_0(s^2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\gamma = 0.9$, perform Value Iteration method to compute V_1, V_2, V_3 .

b) Compute $\pi(s = 0)\ and\ \pi(s = 1)$ associated with V_3.

Problem 3.

Consider the following MDP having two states: A, B. In each state, there are two possible actions: 1 and 2. The transition model and reward are shown in the diagram below. Apply Policy Iteration to determine the optimal policy and state values of A and B. Assume the initial policy is action 2 for both staters, $\gamma = 0.9$.
For evaluation of policy, you need to solve two set of linear equations for the following form, instead of iterative steps of policy evaluation:

$$V^{\pi}(s) = \sum_{s',r} P\big(s' \big| s, \pi(s)\big)[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$



*Here is an example of transition and reward from the diagram:
In state A, action 2 moves the agent to state B with probability 0.8 with the corresponding reward -5, and make the agent stay at state A with probability 0.2 and corresponding reward -10.