# ECE 6882

# HW# 1

Date:

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Q1. Given RV $X$ with PDF:

$$P_X(x) = \begin{cases} 1/2 & x = -1 \\ 1/4 & x = 0 \\ 1/4 & x = 1 \end{cases}$$

a) Find $E[X]$ and $E[X^2]$.

$$E[X] = \sum_a a P_X(a)$$

$$= \frac{1}{2} \times -1 + \frac{1}{4} \times 0 + \frac{1}{4} \times 1$$

$$= \frac{-1}{2} + \frac{1}{4}$$

$$\boxed{E[X] = -\frac{1}{4} = -0.25}$$

$$E[X^2] = \sum_a a^2 P_X(a)$$

$$= \frac{1}{2} \times (-1)^2 + 0 + \frac{1}{4} \times (1)^2$$

$$\boxed{E[X^2] = \frac{3}{4} = 0.75}$$

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b) Find $Var[X]$ and $\sigma$.

We know that,

$$Var[X] = E[X - E[X]]^2 = E[X^2] - (E[X])^2$$

$$\Rightarrow Var[X] = E[X^2] - (E[X])^2$$

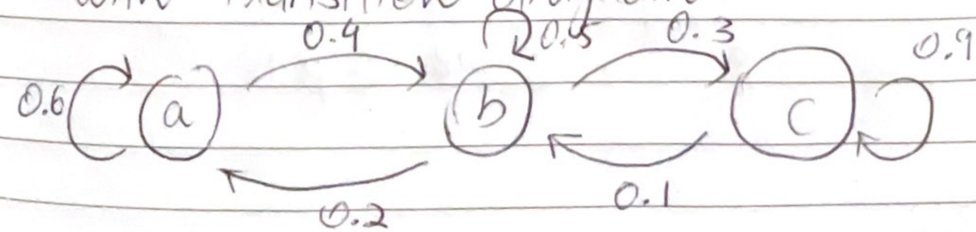
Subtituting from part (a)

$$\Rightarrow Var[X] = 0.75 - (-0.25)^2$$
$$= 0.75 - 0.0625$$
$$\boxed{Var[X] = 0.6875}$$

We know that,

$$Var[X] = \sigma^2 = 0.6875$$
$$\Rightarrow \sigma = \sqrt{0.6875}$$
$$\Rightarrow \boxed{\sigma = \pm 0.82916}$$

Q2. Consider Markov Chain $\{x_n, n = 0, 1, \ldots\}$ with transition diagram:



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a) Compute the transition matrix, given $x = \{a, b, c\}$.

$$
\text{Transition Matrix } (P) = \begin{array}{c} \\ a \\ b \\ c \end{array}
\begin{array}{c}
\begin{array}{ccc} a & b & c \end{array} \\
\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.1 & 0.9 \end{bmatrix}
\end{array}
$$

b) Compute $p(x_k = b \mid x_{k-1} = a)$ & $p(x_k = b \mid x_{k-2} = a)$.

→ $p(x_k = b \mid x_{k-1} = a) = p_{ab} = ?$

This is a 1-step transition probability, i.e. time period = 1.

$\boxed{p_{ab}^{(1)} = p_{12}^{(1)} = 0.4}$ (directly from state matrix)

→ $p(x_k = b \mid x_{k-2} = a) = p_{ab}^{(2)}$

This is a 2-step transition probability

$p_{ab}^{(2)} = p_{12}^{(2)}$, so we compute $P^2$.

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$$P^2 = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.1 & 0.9 \end{bmatrix} \times \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.1 & 0.9 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.44 & 0.44 & 0.12 \\ 0.22 & 0.36 & 0.42 \\ 0.02 & 0.14 & 0.84 \end{bmatrix}$$

$$\Rightarrow \boxed{p_{ab}^{(2)} = p_{12}^{(2)} = 0.44}$$

Thus,

$$\Rightarrow p(x_k = b \mid x_{k-1} = a) = 0.4$$

&

$$p(x_k = b \mid x_{k-2} = a) = 0.44$$

Another way to compute the two-state probability is;

$$p_{ab}^{(2)} = (a \to a) \times (a \to b) + (a \to b) \times (b \to b)$$

$$= 0.6 \times 0.4 + 0.4 \times 0.5$$

$$= 0.44 \text{ (which is the same)}$$

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